ROBUST STABILITY ANALYSIS OF GPC: AN APPLICATION TO DEAD-BEAT AND MEAN-LEVEL PREDICTIVE CONTROLLERS

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ABSTRACT

Most stability results in model predictive control are based on the assumption that the model describes the real plant perfectly. In real systems the model is always different to the process and a robust stability analysis is needed. In this paper it is shown how the Extreme Point Results theory can be used to analyse the robust stability of predictive controllers in a natural way with low computational cost.

1. INTRODUCTION

Stability analysis of Model Predictive Controllers (MPC) in closed loop systems is an open and interesting research area. A source of criticism for them has been that most stability guarantees apply only for limiting (with theoretically infinite horizons) or particular (dead-beat, mean-level, ...) cases, and assuming that the model describes the process perfectly. In reality the model is always different to the process and no mathematical model capable of exactly describing a physical process exists. Moreover, the behaviour of the plant itself changes in time and these changes are rarely captured in the models. Whatever the synthesis technique used, controllers are always designed based on information about the dynamic behaviour of the process and, as this is necessarily incomplete, modelling errors may adversely affect the behaviour of the control system.

Because of this model-process mismatch the existing theorems, developed for the nominal system, do not guarantee the stability of the real system and a robust stability analysis is needed.

This topic has been generally studied assuming that the nominal system is under unstructured disturbances and therefore applying H_2 and H_∞ results. For instance, Yoon and Clarke [1], using the small gain theorem, propose a relationship between the real plant and the model polynomials used to design a Generalized Predictive Controller (GPC) that guarantees stability.

Camacho and Bordóns [2] assume a process with structured uncertainties and find the limits of the stability region by numerically solving the closed loop characteristic equation in an iterative way.

Finally, the theory of Extreme Point Results has been used by Mañoso *et al.* [3] to study the robust stability of a family of closed loop characteristic polynomials with structured uncertainties.

This paper focuses on GPC as one of the most representative predictive controllers. It emphasises on the necessity of a robust analysis of the closed loop characteristic equation and points out the application of the Extreme Point Results in a simple way with low computational effort. To do so, section 2 introduces MPC fundamentals, paying special attention to GPC and some of its stability results. It is shown how slight variations in the process coefficients lead to instability. For this reason in section 3 the robust stability analysis is performed applying Extreme Point Results. Finally, section 4 draws the main conclusions of this paper.

2. BACKGROUND AND MOTIVATION

Model Predictive Control refers to a class of algorithms that compute a sequence of manipulated variable adjustments in order to optimise the future behaviour of a plant.

The methodology of all the controllers belonging to the MPC family is characterised by the following strategy [4]. Firstly, the future outputs for a given prediction horizon, N_2, are predicted at each instant t using a process model. Secondly, the set of future control signals for the prediction horizon is calculated by optimising a given criterion in order to keep the process as close as possible to the reference trajectory. Some assumptions about the structure of the future control law are also made in some cases (for instance, it is assumed that it will be constant from a given instant $t+ N_u$, the so-called control horizon). Finally, the control signal for the current instant t is sent to the process, while the next control signals are rejected. This strategy is repeated at the next sampling time.

The GPC method was proposed by Clarke *et al.* [5], [6] and has become one of the most popular MPC methods in both industry and academia. It is based on a controlled auto-regressive and integrated moving average (CARIMA) model:

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + \frac{T(z^{-1})}{\Delta}\xi(t) \qquad (1)$$

where $\Delta = 1 - z^{-1}$, $A_0(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na}$, $B_0(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{nb} z^{-nb}$, $y(t)$ is the plant output, $u(t)$ is the plant input, $\xi(t)$ is an uncorrelated random sequence and $T(z^{-1})$ is a coloring filter that does not appear in the closed loop characteristic equation. For simplicity and without loss of generality, in the following the T polynomial is chosen to be 1.

The control is obtained by minimising the cost function

$$J = E\left\{ \sum_{j=N_1}^{N_2} \mu(j)\left[y(t+j|t) - r(t+j|t)\right]^2 + \sum_{j=1}^{N_u} \lambda(j)\left[\Delta u(t+j-1|t)\right]^2 \right\} \tag{2}$$

where $E\{\cdot\}$ is the expectation operator and $y(t+j|t)$ is the predicted output. N_1 and N_2 are the minimum and maximum prediction horizons, N_u is the control horizon, $\mu(j)$ and $\lambda(j)$ are weighting sequences and $r(t+j|t)$ is the future reference trajectory. In the following and without loss of generality $\mu(j) = 1$ and $\lambda(j) = \lambda$.

In order to obtain the closed loop characteristic equation, the control structure can be always described as in fig. 1, where R_0 and S_0 are polynomials in z^{-1} that only depend on the controller parameters (N_1, N_u, N_2 and λ) and the model transfer function B_0/A_0, but not on the real transfer function B/A [7].

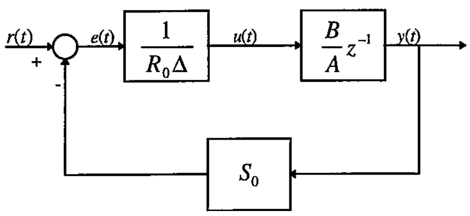

Figure 1. MPC structure

The characteristic equation of this general closed loop system is given by:

$$\delta(z^{-1}) = R_0 A \Delta + B S_0 z^{-1} \tag{3}$$

As it was said in the introduction, there exist some stability theorems for limiting (with theoretically infinite horizons) or particular (dead-beat, mean-level,...) cases. The stability of a GPC controller with arbitrary values of its design parameters N_1, N_u, N_2 and λ cannot be guaranteed in advance. (Clarke *et al.* [5] have proposed some thumb rules that work for *most* situations.)

Some of these stability results for unconstrained GPC can be found in [5], [6]. These theorems are derived using a state-space representation of the model augmented by an integrator

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}\Delta u(t)$$
$$y(t) = \mathbf{c}^{\mathrm{T}}\mathbf{x}(t) \tag{4}$$

where one of the eigenvalues of $\mathbf{A}$ is at 1 due to the integral state and n, the number of states, is $\max(n_a+1, n_b)$.

Theorem 2.1 For a stabilizable and detectable process which is also open loop stable, the closed loop under GPC control is stable and tends to a mean-level law for $N_u = N_1 = 1$, $\lambda = 0$ and $N_2 \to \infty$. ♦

Theorem 2.2 GPC results in a stable state dead-beat controller if:

(1) the above system ($\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$) is completely controllable and observable with state dimension n, and

(2) $N_1 = n$, $N_2 \geq 2n-1$, $N_u = n$ and $\lambda = 0$ ♦

Theorem 2.3 The closed loop under GPC control is stable if the system model ($\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$) is stabilizable and detectable and if

(1) N_u, $N_2 \to \infty$ with $N_u = N_2$ and $\lambda > 0$, or:

(2) N_u, $N_2 \to \infty$ with $N_u = N_2-n+1$ and $\lambda = 0$, provide there is no plant zero on the stability boundary. ♦

Part (1) of theorem 2.3 is a standard LQ controller whilst part (2) is essentially Peterka's control [8].

These theorems are based on the assumption that a single linear model can describe the system behaviour. Because this assumption is only an approximation, a large uncertainty in the value of the process parameters might result. The sources of this uncertainty are parameters that change in time, unmodelled dynamics, etc. Hence it is very important to consider the effect of these uncertain parameters on the stability.

The following examples will show how a slight difference between the process and the model coefficients can lead to instability, i.e., the application of the above theorems is not enough to guarantee the stability of a real system with uncertainties.

Example 1: Mean-level controller

In this example the process and the model are described by the following polynomials:

$$A_0 = 1 - 1.7z^{-1} + 0.9z^{-2}$$
$$B_0 = 0.9 - 0.6z^{-1}$$

After studying its settling time we can conclude that N_2 equal to 150 is an acceptable approximation for infinite. The following controller parameters $N_1 = 1$, $N_2 = 150$, $N_u = 1$, $\lambda = 0$ and $T = 1$ are selected. When the model is equal to the process the closed loop characteristic equation is

$$\delta_0 = 1 - 1.7042z^{-1} + 0.8907z^{-2}$$

which corresponds to a stable system (fig. 2) with all its poles at the open loop locations due to the mean-level control law, according to theorem 2.1.

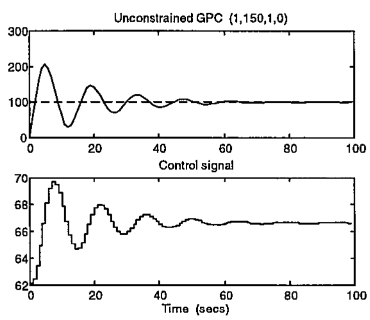

Figure 2. GPC with model equal to process and $N_1 = 1$, $N_2 = 150$, $N_u = 1$, $\lambda = 0$

Now, let us assume that some of the coefficients of the process transfer function have slightly changed:

$$A = (1 - 1.3z^{-1} + 0.9z^{-2})$$
$$B = 0.9 - 0.7z^{-1}$$

The closed loop characteristic equation is:

$$\delta = 1 - 1.3042z^{-1} - 0.6030z^{-2} + 1.3200z^{-3} - 0.2885z^{-4}$$

which corresponds to an unstable closed loop system (fig. 3).

Even if the control horizon is also infinite, as in part (2) of theorem 2.3, that is a LQ control problem, the system is still closed loop unstable (fig. 4).

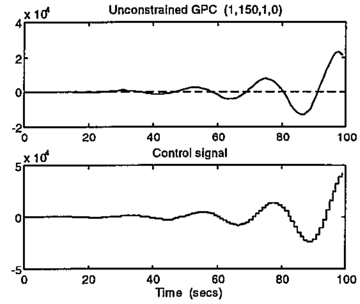

Figure 3. GPC with model different to process and $N_1 = 1, N_2 = 150, N_u = 1, \lambda = 0$

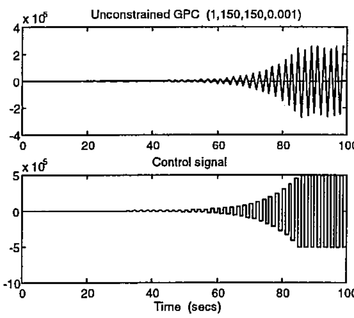

Figure 4. GPC with model different to process and $N_1 = 1, N_2 = 150, N_u = 150, \lambda = 0.001$

Example 2: Dead-beat controller

In this example the process and the model are described by the following polynomials:

$$A_0 = 1 - 2.475z^{-1} + 0.8950z^{-2} - 0.825z^{-3}$$

$$B_0 = 1$$

When the model is equal to the process and with the following controller parameters $N_1 = 4$, $N_2 = 7$, $N_u = 4$, $\lambda = 0$ and $T = 1$ the closed loop characteristic equation is:

$$\delta_0 = 1$$

This is the closed loop characteristic equation of a stable dead-beat control law (all poles are placed at the origin), which corresponds to theorem 2.2.

On the other hand, for a process with a slight variation in its coefficients:

$$A = 1 - 2.5z^{-1} + 1.3z^{-2} - 1.2z^{-3}$$

$$B = 1$$

The closed loop characteristic equation is unstable:

$$\delta = 1.0000 - 0.025z^{-1} + 0.4300z^{-2} - 0.7800z^{-3} + 0.3750z^{-4}$$

♦♦♦

These examples make clear that the stability theorems based on the assumption that the model represents perfectly the process cannot guarantee the stability of a real system with uncertainties. In practice, a robust stability analysis is needed.

3. ROBUST ANALYSIS

Let us assume a real plant under structured uncertainties, i.e., the uncertainties are in the coefficients. Then the numerator and the denominator of the real plant are given by uncertain polynomials. Affine linear uncertainty structures will be considered. Thus, given a set of real parameters $q_i, i = 0,\ldots,l$, which can vary between a maximum and a minimum value, $q_i^- \le q_i \le q_i^+, i = 0,\ldots,l$, the coefficients of the numerator and denominator polynomials are affine linear functions of the uncertain parameter vector

$$\overline{q} = (q_0,\ldots q_l) \in \Re^l$$

that is

$$a_i(\overline{q}) = \alpha_i^T \overline{q} + \beta_i, i = 0,\ldots,m,$$
$$b_i(\overline{q}) = \gamma_i^T \overline{q} + \rho_i, i = 0,\ldots,n \qquad (5)$$

where α_i and γ_i are $1 \times l$ vectors and β_i and ρ_i are scalars. Then the real plant is defined by the family of plants:

$$P(\overline{q}, z^{-1}) = \frac{B(\overline{q}, z^{-1})}{A(\overline{q}, z^{-1})} = \frac{\sum_{i=0}^{n} b_i(\overline{q}) z^{-i}}{\sum_{i=0}^{m} a_i(\overline{q}) z^{-i}}, \quad m \ge n \ge 0 \qquad (6)$$

With this structure of uncertainties, $A(\overline{q}, z^{-1})$ and $B(\overline{q}, z^{-1})$ are polytopes of polynomials in z^{-1}, and $P(\overline{q}, z^{-1})$ is a polytope of plants in z^{-1}.

The following theorem by Mañoso *et al.* [3] permit us to applied the Extreme Point Results theory to the analysis of the closed loop characteristic equation under a GPC control law:

Theorem 3.1: The family of characteristic polynomials $\delta(z^{-1})$ of the closed loop system constituted by a GPC predictive controller and the family of plants (6) is a polytope of polynomials. ♦

Lemma 3.1 The family of characteristic polynomials $\delta(z^{-1})$ of the closed loop system constituted by a GPC predictive controller with the parameters $N_u = N_1 = 1$, $\lambda = 0$ and $N_2 \to \infty$ (as in theorem 2.1) and the family of plants (6) is a polytope of polynomials given by:

$$\delta = R_0 A \Delta + \frac{B}{B_0} A_0 (1 - R_0 \Delta) \qquad (7)$$

Proof:

From theorem 3.1 the closed loop characteristic equation of a GPC controller with parameters $N_u = N_1 = 1$, $\lambda = 0$ and $N_2 \to \infty$ and a process with structured uncertainties as (6) is a polytope.

Under the assumption that the model, B_0/A_0, is equal to the stable process, B/A, and the controller tuned with the parameters $N_u = N_1 = 1$, $\lambda = 0$ and $N_2 \to \infty$, the closed loop characteristic equation is equal to the open loop characteristic equation (due to the mean-level control law), i.e., it is equal to the denominator of the transfer function of the process. So (3) can be written in the following way:

$$\delta_0 = R_0 A_0 \Delta + B_0 S_0 z^{-1} = A_0 \tag{8}$$

From here S_0 is equal to:

$$S_0 = \frac{A_0(1 - R_0\Delta)}{B_0 z^{-1}} \tag{9}$$

If the process is different to the model, (6), the closed loop characteristic equation is given by (3), and taking into account (9) the closed loop characteristic equation can be written as:

$$\delta = R_0 A\Delta + \frac{B}{B_0} A_0 (1 - R_0 \Delta)$$ ♦

Remark 3.1. If B and B_0, and A and A_0 are different, respectively, the closed loop characteristic equation (7) is different to the nominal one (8) and thus, even in the presence of slight variations in the parameters, the stability of (7) cannot be guaranteed by theorem 2.1. ♦

Lemma 3.2 The family of characteristic polynomials $\delta(z^{-1})$ of the closed loop system constituted by a GPC predictive controller with the parameters $N_1 = n$, $N_2 \geq 2n\text{-}1$, $N_u = n$ and $\lambda = 0$ (as in theorem 2.2) and the family of plants (6) is a polytope of polynomials given by:

$$\delta = R_0 A\Delta + \frac{B}{B_0}(1 - R_0 A_0 \Delta) \tag{10}$$

Proof:

From theorem 3.1 the closed loop characteristic equation of a GPC controller with parameters $N_u = N_1 = n$, $\lambda = 0$ and $N_2 \geq 2n\text{-}1$ and a process with structured uncertainties as (6) is a polytope.

Under the assumption that the model, B_0/A_0, is equal to the process, B/A, and the controller tuned with the parameters $N_u = N_1 = n$, $\lambda = 0$ and $N_2 \geq 2n\text{-}1$ the closed loop characteristic equation has all its poles at the origin. So (3) can be written in the following way:

$$\delta_0 = R_0 A_0 \Delta + B_0 S_0 z^{-1} = 1 \tag{11}$$

From here S_0 is:

$$S_0 = \frac{1 - R_0 A_0 \Delta}{B_0 z^{-1}} \tag{12}$$

If the process is different to the model, (6), the closed loop characteristic equation is given by (3), and taking into account (12) the closed loop characteristic equation can be written as:

$$\delta = R_0 A\Delta + \frac{B}{B_0}(1 - R_0 A_0 \Delta)$$ ♦

Remark 3.2. If B and B_0, and A and A_0 are different, respectively, the closed loop characteristic equation (10) does not place all the poles at the origin, as equation (11) did and thus, even in the presence of slight variations in the parameters, the stability of (10) cannot be guaranteed by theorem 2.2. ♦

Note: The study of the stability of $\delta(z^{-1})$ is equivalent to the study of the Schur stability of $\delta(z) = \delta(z^{-1})z^r$, where r is the maximum degree of $\delta(z^{-1})$. As the product of z^r by $\delta(z^{-1})$ is a polytope of polynomials in z, it is possible to study the Schur stability of a system with a predictive controller (GPC) applying all the existing results for discrete polytopes. ♦

Robust control theory against structured perturbations based on Extreme Point Results has generated a huge interest in the last years. Since Kharitonov Theorem [9] was introduced in western literature by Barmish [10], a great number of papers have appeared in relation to the analysis problem. There are powerful results to analyse the stability and the robust performance of families of polynomials formed by interval polynomials or by polytopes of polynomials. The main Extreme Point Results for the analysis of polynomial families are Kharitonov Theorem for interval polynomials and Edge Theorem [11] and Rantzer Theorem [12] for polytopes of polynomials.

One of the most significant Extreme Point Results is Edge Theorem [11]: given a simply connected domain $D \subset \mathbf{C}$, if all the roots of the exposed edges are inside the domain D, then it can be assured that all the roots of the polytope of polynomials are inside D.

Moreover, this theorem is applicable to analyse D-stability, i.e., both robust stability and robust performance can be studied. Edge Theorem gives necessary and sufficient conditions to analyse the stability of the complete family, and only the exposed edges have to be analysed.

The analysis method is simple. If all the vertex polynomials are stable (otherwise the family is obviously unstable) then the exposed edges are tested; if the exposed edges are also stable then the family is stable.

There exist several criteria to study the stability of an edge, for both Hurwitz- and Schur-stability [13], [14], [3]. The following result from Ackerman and Barmish [15] will be used to show the analysis in the z-domain:

Given a polynomial

$$P(z) = \sum_{k=0}^{n} a_k z^k = a_n \prod_{i=1}^{n} (z - z_i) \tag{13}$$

the following $(n\text{-}1) \times (n\text{-}1)$ matrix is built:

$$S(P) = \begin{bmatrix} a_n & a_{n-1} & a_{n-2} & \cdots & a_3 & a_2 - a_0 \\ 0 & a_n & a_{n-1} & \cdots & a_4 - a_0 & a_3 - a_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -a_0 & -a_1 & \cdots & a_n - a_{n-4} & a_{n-1} - a_{n-3} \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-3} & a_n - a_{n-2} \end{bmatrix} \tag{14}$$

If the coefficients $\{a_k\}$ vary continuously then the zeros $\{z_i\}$ of the polynomial $P(z)$ vary continuously [16] and if a complex pair of roots crosses the unit circle then $\det S(P) = 0$. There are two other ways for crossing the stability boundary: $P(1) = 0$ and $P(-1) = 0$. The above three cases are the critical stability constraints. Based on this fact, Ackerman and Barmish [15] propose the study of the vertex polynomials and, if they are stable, the study of the eigenvalues of the matrices

$$S(P_i)S^{-1}(P_j) \quad (15)$$

where i and j stand for the different vertex polynomials. If there are no real eigenvalues in $(-\infty, 0)$ then the family is stable.

In the following the robust stability of the above examples will be studied. Lemmas 3.1 and 3.2 guarantee the applicability of Edge Theorem to systems controlled by means of these predictive controllers, as the family of closed loop characteristic polynomials is a polytope of polynomials.

Example 1 (revisited): Mean-level controller

Under the same conditions as in example 1, let us assume that the process is different to the model. This fact is represented by uncertainties in the coefficients of the process transfer function:

$$A = 1 + (-1.7 + \alpha_1)z^{-1} + (0.9 + \alpha_2)z^{-2}$$

$$B = 0.9 + (-0.6 + \beta)z^{-1}$$

with $-1 \leq \alpha_1 \leq 1$, $\alpha_2 = 0$ and $-1 \leq \beta \leq 1$.

The polynomials R_0 and S_0 are:

$$R_0 = 1.691 - 3.0948z^{-1}$$

$$S_0 = 5.219 - 8.8612z^{-1} + 4.6422z^{-2}$$

Expression (3) is used to obtain the following family of polynomials:

$$\begin{aligned}\delta(z,\alpha_1,\alpha_2,\beta) = {} & 1.0000 + (-1.7043 + 1.000\alpha_1)z^{-1} \\ & + (-2.9233\alpha_1 + 0.89071 + 3.2434\beta + 1.000\alpha_2)z^{-2} \\ & + (-2.933\alpha_2 - 5.5069\beta + 1.9233\alpha_1)z^{-3} \\ & + (1.9233\alpha_2 + 2.8850\beta)z^{-4}\end{aligned}$$

which is the same result as in example 1 when the uncertain parameters are zero.

In order to apply the Edge Theorem to the family of characteristic polynomials the stability of the vertices is considered in the first place:

$$\begin{aligned}\delta_1(z,\alpha_1^-,\beta^-) &= 1.0000 - 2.7042z^{-1} + 0.5706z^{-2} \\ &\quad + 3.5835z^{-3} - 2.8850z^{-4} \\ \delta_2(z,\alpha_1^-,\beta^+) &= 1.0000 - 2.704z^{-1} + 7.0573z^{-2} \\ &\quad - 7.4301z^{-3} + 2.8850z^{-4} \\ \delta_3(z,\alpha_1^+,\beta^-) &= 1.0000 - 0.7042z^{-1} - 5.2759z^{-2} \\ &\quad + 7.4301z^{-3} - 2.8850z^{-4} \\ \delta_4(z,\alpha_1^+,\beta^+) &= 1.0000 - 0.7042z^{-1} + 1.2108z^{-2} \\ &\quad - 3.5835z^{-3} + 2.8850z^{-4}\end{aligned}$$

It can be concluded that the family is unstable as some of the vertices are unstable.

Example 2 (revisited): Dead-beat controller

Under the same conditions as in example 2, let us assume that the process is different to the model. This fact is represented by uncertainties in the coefficients of the process transfer function:

$$\begin{aligned}A = 1 & + (-2.475 + 0.675\alpha + 0.3\beta)z^{-1} \\ & + (0.895 + 0.025\alpha + 0.09\beta)z^{-2} \\ & + (-0.825 + 0.225\alpha + 0.1\beta)\end{aligned}$$

$$B = 1$$

with $-3.5 \leq \alpha \leq -3$ and $5 \leq \beta \leq 10$.

The polynomials R_0 and S_0 are:

$$R_0 = 1$$

$$S_0 = 3.4750 + 3.3700z^{-1} + 1.7200z^{-2} - 0.82500z^{-3}$$

and the family of closed loop characteristic polynomials is:

$$\begin{aligned}\delta(z,\alpha_1,\alpha_2,\beta) = {} & 1.0000 + (0.3000\beta + 0.67500\alpha)z^{-1} \\ & + (-0.6500\alpha - 0.2100\beta)z^{-2} \\ & + (0.2000\alpha + 0.010\beta)z^{-3} \\ & + (-0.2250\alpha_2 - 0.100\beta)z^{-4}\end{aligned}$$

which is the same result as in example 2 when the uncertain parameters are zero.

Let us consider two of the four vertices of the family:

$$\begin{aligned}\delta_1(z,\alpha^-,\beta^-) &= 1.0000 - 0.8625z^{-1} + 1.225z^{-2} \\ &\quad - 0.6500z^{-3} + 0.2875z^{-4} \\ \delta_2(z,\alpha^-,\beta^+) &= 1.0000 + 0.6375z^{-1} + 0.175z^{-2} \\ &\quad - 0.600z^{-3} - 0.2125z^{-4}\end{aligned}$$

These vertices are stable, but not all the polynomials of the edge that they define

$$E(\lambda) = \lambda\delta_1(z,\alpha^-,\beta^-) + (1-\lambda)\delta_2(z,\alpha^-,\beta^+), \lambda \in [0,1]$$

are stable. For example, when $\lambda = 0.5$ the polynomial

$$\delta^* = 1.0000 - 0.1125z^{-1} + 0.7z^{-2} - 0.625z^{-3} + 0.0375z^{-4}$$

is unstable.

In order to study the stability of the edge formally, let us apply the criterion explained previously. In the first place, the matrices associated to the vertices 1 and 2 are:

$$S(\delta_1) = \begin{pmatrix} 1.0000 & -0.8625 & 0.9375 \\ 0 & 0.7125 & -0.2125 \\ -0.2875 & 0.65 & -0.2250 \end{pmatrix}$$

$$S(\delta_2) = \begin{pmatrix} 1.0000 & 0.6375 & 0.3875 \\ 0 & 1.2125 & 1.2375 \\ 0.2125 & 0.6 & 0.8250 \end{pmatrix}$$

The eigenvalues of $S(\delta_1)S^{-1}(\delta_2)$ are 0.2484, -0.8477 and -1.7090. The edge is unstable, as some of them are negative, therefore the family is unstable. Figure 5 shows an enlargement of the stability region, which is concave, the vertices δ_1 y δ_2 and the unstable edge.

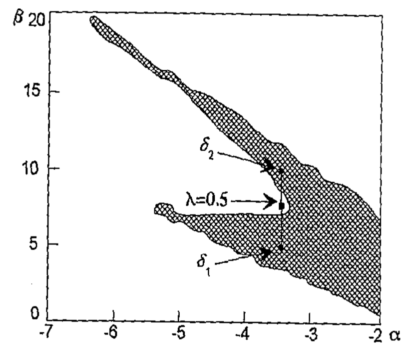

Figure 5. Enlargement of the stability region

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These examples show how the robust stability analysis of systems with predictive controllers is needed to guarantee the stability of the closed loop system in the presence of structured uncertainties. The Extreme Point Results theory is a good choice to deal with this problem in a natural way and with low computational cost. Moreover, it has been shown that the stability analysis of the edges is not superfluous for systems that are controlled by predictive controllers.

To sum up, after designing the predictive control law, the control engineer should assume a feasible family of real plants —as is given by (6)— in order to study and guarantee the robust stability of the closed loop real system using the Extreme Point Results techniques described in this paper.

4. CONCLUSIONS

In this paper it has been shown how the existing stability results for predictive controllers, derived under the assumption that the model is equal to the process, cannot guarantee the stability of real systems with uncertainties. Extreme Point Results is a mature and non-conservative theory that can be used to analyse the closed loop stability of predictive controllers in presence of uncertainties with low computational cost.

5. ACKNOWLEDGMENTS

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6. REFERENCES

[1] Yoon, T. W. and D. W. Clarke (1994). Advances in Model-Based Predictive Control. *Oxford University Press*, pp. 402-414

[2] Camacho, E. F. and C. Bordóns (1995). Model Predictive Control in the Process Industry. *Springer-Verlag*.

[3] Mañoso, C., R. Hernández, A. P. de Madrid and S. Dormido (1996). Robust stability analysis of predictive controllers using Extreme Point Results. *Proc. CESA'96 IMACS Multiconference*, vol. 1, pp. 483-488. Lille, France.

[4] Richalet, J. (1993). Pratique de la Commande Prédictive. *Hermès.*

[5] Clarke, D. W., C. Mohtadi and P. S. Tuffs (1987). Generalized Predictive Control - Part I. The Basic Algorithm, Part II. Extensions and Interpretations. *Automatica*, **23**, 137-160.

[6] Clarke, D. W. and C. Mohtadi (1989). Properties of Generalized Predictive Control. *Automatica*, **25**, 859-875.

[7] Yoon, T. W. and D. W. Clarke (1995). Observer design in Receding-Horizon Predictive Control *Int. J. Control*, **61**, 171-191.

[8] Peterka, V. (1984). Predictor-based self-tuning control. *Automatica* **20**, 39-50.

[9] Kharitonov, V. L. (1978). Asymptotic stability of an equilibrium position of a family of systems of linear differential equations. *Differential'nye Uraveniya*, **14**, 1483-1485.

[10] Barmish, B. R. (1983). Invariance of the strict Hurwitz property for polynomials with perturbed coefficients. *IEEE Trans. on Autom. Control* **29**, 935-936.

[11] Bartlett, A. C., C. V. Hollot and H. Lin (1988). Root locations of an entire polytope of polynomials: it suffices to check the edges. *Mathematics of Control, Signals and Systems*. vol. 1, pp. 61-71.

[12] Rantzer, A. (1992). Stability conditions for polytopes of polynomials. *IEEE Trans. on Autom. Control*, **37**, 79-89.

[13] Zeheb, E. (1989). Necessary and sufficient conditions for root clustering of a polytope of polynomials in a simply connected domain. *IEEE Trans. on Autom. Control*, **34**. 995-998.

[14] Chapellat, H. and S. P. Bhattacharyya (1989). An alternative proof of Kharitonov's theorem. *IEEE Trans. on Autom. Control*, **34**, 448-450.

[15] Ackermann, J. E. and B. R. Barmish (1988). Robust Schur Stability of a Polytope of Polynomials. *IEEE Trans. on Autom. Control* **33**, 984-986.

[16] Jury, E. I. and T. Pavlidis (1963). Stability and Aperiodicity Constraints for Control System Design. *IEEE Trans. Circuit*, 128-141.