Modeling and μ- synthesis Robust Control of Two-link Flexible Manipulators

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Abstract

Two-link robot manipulators are commonly used in industrial sectors such as manufacturing. Some manipulators are often bulky and their power consumption is relatively high. Others, such as the arm on the space shuttle, are driven slow to prevent the onset of flexible oscillations. The efficiency of these manipulators can be improved by reducing the weight of some of these arms and/or increasing the speed of others. These modifications complicate the dynamic behavior of the system due to the possible onset of low frequency oscillations.

This paper addresses the issue of modeling and end-point robust control of two-link flexible manipulators using the μ-synthesis technique. The Timoshenko beam theory along with the assumed modes method are used to derive reference equations of motion for the flexible manipulator. Discrepancies between the control design model and the actual dynamics of the manipulator are attributed to neglected non-linearities such as cross-coupling which should be included in the controller design. A linear estimation of these errors will be identified and used in the control design to compensate for the unmodelled dynamics of the flexible arm and parameter uncertainties. The μ-synthesis control design techniques are then employed to synthesize controllers for the two-link flexible robot manipulator.

1 Introduction

It was shown by Karkoub et al. [8] that the p-synthesis technique along with a good model can lead to the design of robust controllers for the single-link flexible robot manipulator. This technique is extended to two-link flexible manipulators and the results are discussed in this paper. First, let us shed the light on what has been done previously in the modeling and control of multi-degree of freedom flexible manipulators. Due to the long list of contributors to this area, only a representative few will be discussed here.

For the past decade or so, researchers have tried to come up with dynamic models that describe the motion of two-link flexible manipulators accurately ([7], [11], and [13]). The complexity of the distributed system requires the making of simplifying assumptions. Few of the models such as the ones derived by Oakley and Feliu ([5] and [11]) assume that the first link and all the joints are rigid bodies. A model for a two-link flexible manipulator was derived by Yang and Donath [13] in which the joint flexibility and all non linear terms are neglected.

A Timoshenko model for the two-link flexible robot manipulator which includes the flexibility of the joints and the links is derived [9]. The time response of the manipulator is solved for using the assumed modes method. Only the first two modes of each link will be used in the solution scheme and all other modes and deflections in the directions other than the transverse ones will be neglected.

In the literature, there are few control techniques that have been used to maneuver two degree-of-freedom flexible manipulators. These control models are based on a linear version of the Euler-Bernoulli dynamic model ([3], [5], and [11]) . The most common of these is the linear quadratic gaussian (LQG) [3]. In this paper, the p-synthesis technique is used to design a robust controller for the two-link flexible manipulator. The designed controller will be robust to high frequency dynamics, noise corruption, and uncertainty in the frequency variations. Different combinations of feedback signals will be used to derive the control laws. All designed controllers will use hub angle information and one of the following measurement sensor information: relative tip deflections, relative tip accelerations, or hub angular rates.

2 Dynamic Model

The two-link flexible manipulator consists of a flexible joint, a flexible arm, another flexible joint, a flexible forearm, and two tip masses (see Figure 1). The masses at the tip of the arm and the forearm can be used to account for the presence of the joint motor and the payload respectively. The joints of the two-link flexible robot manipulator are represented by spring-damper systems whose stiffness and damping ratio can be determined experimentally. Both the arm and forearm can vibrate in the horizontal direction only and motion in all other directions is considered negligible.

The total energy of the two-link flexible manipulator is the sum of the energy of the first link and the second link. The derivation of the dynamics of the first link of the two-link flexible manipulator is done in [8]. Therefore, one needs to derive the dynamics of the second link only. The angle of rotation of the arm θ_1 and the forearm θ_2 are the sum of an elastic deflection angle and a rigid one:

$$\theta_1 = \theta_{1r} + \theta_{1e} \tag{1}$$

and

$$\theta_2 = \theta_{2r} + \theta_{2e} \tag{2}$$

The potential energy of the manipulator is given by:

$$U = \frac{1}{2}\sum_{i=1}^{2} E_i I_i \int_0^{L_i} u_i'^2 dx_i + \frac{1}{2}\sum_{i=1}^{2} k_i G_i A_i \int_0^{L_i} v_i^2 dx_i + \frac{1}{2}\sum_{i=1}^{2} K_{s_i}\left(\theta_{ie} + \frac{\partial y_i}{\partial x_i}\,|_{x_i=0}\right)^2 \tag{3}$$

The kinetic energy of the manipulator is given by

$$\begin{aligned} T = {} & \frac{1}{2}\sum_{i=1}^{2} I_{h_i}\ddot{\theta}_{ir} + \frac{1}{2}\sum_{i=1}^{2}\rho_i A_i \int_0^{L_i} \dot{X}_i^T \dot{X}_i dx_i \\ & + \frac{1}{2}\sum_{i=1}^{2} M_{pi}\dot{X}_i^T(L_i)\dot{X}_i(L_i) \\ & + \frac{1}{2}\sum_{i=1}^{2}\rho_i I_i \int_0^{L_i}\left\{\dot{u}_i(x_i) + \dot{\theta}_{ir} + \dot{\theta}_{ie}\right\}^2 dx_i \\ & + \frac{1}{2}\sum_{i=1}^{2} J_i\left\{\dot{u}_i(L_i) + \dot{\theta}_{ir} + \dot{\theta}_{ie}\right\}^2 \end{aligned} \tag{4}$$

Using the Hamiltonian principle and calculus of variation, a model for the two-link flexible jointed, flexible manipulator is obtained [9].

3 μ-Synthesis Control Design

The equations of motion of the two-link flexible manipulator derived in the previous section are linearized to obtain a control system model. The dynamic system has two inputs and at least two outputs. The two inputs are two possible torques at the arm and the forearm joints. The outputs can be the arm and forearm tip deflections, tip accelerations, hub angles, or hub angular rates,

3.1 Control Design

Vibration suppression and tip tracking with the two-link flexible manipulator is the goal of this paper. The designed control laws have to be robust to modeling errors and noise corruption. To achieve that, performance and uncertainty weights are selected and included in the control system (see Figure 3). Two p-synthesis controllers are designed to maneuver the tip of a two-link flexible manipulator using several feedback signals. The first controller uses hub angles and tip displacement feedback. The second controller is designed using hub angles and arm and forearm tip accelerations feedback.

Figure 3 shows a diagram of the control system used to derive the p-synthesis control law for the two-link flexible manipulator. The unmodelled high frequency dynamics is represented by the " w_additive" block. Two first order systems are stacked in the "w-actuator" block to account for the actuation errors and at the same time prevent pole zero cancellation at the origin, The arm and forearm hub and tip performance weights are included in the "w_hub_perf" and "w_tip_perf' blocks respectively. The performance weights contain information about the magnitudes and durations of the oscillations, Since all measured signals are corrupted with noise, a constant noise source " w_sens_nois" is included in the control system to simulate that effect. Finally, the "flexible manipulator dynamics" block is a set of transfer functions that provide hub angles and angular rates, or hub angles and arm and forearm tip deflection, or hub angles and arm and forearm tip accelerations for a given set of input torques.

3.2 μ-synthesis

The p-synthesis technique ([1] and [2]) will be used to design robust controllers for the two-link flexible manipulator. The control design diagram shown in Figure 3 can be rearranged to look like Figure 4. The uncertainty matrix, Δ, is block diagonal with each block Δ_j having an infinity norm less than 1 ($\|\Delta_j\|_\infty \leq 1$). The μ-synthesis technique computes upper and lower bounds for the infinity norm of the closed-

[1] The linearization was done using a packaged software called ACSL (Advanced Continuous Simulation Language)

loop system. The μ-synthesis function is given by:

$$\mu(P(j\omega)) = \begin{cases} \left[\min\limits_{\Delta \in B\mathbf{\Delta}} \bar{\sigma}(\Delta(j\omega)) : \quad det(I - P\Delta) = 0\right]^{-1} \\ Otherwise, \\ 0 \quad if \quad det[I - P\Delta] \neq 0 \quad \forall \quad \Delta \in B\mathbf{\Delta} \end{cases} \tag{5}$$

where P is the closed-loop system -the nominal plant closed with an H_∞ controller-, A is the matrix containing all uncertainties and the performance requirements, and

$$\begin{aligned} &\mathbf{\Delta} = diag[\delta_1 I_{r1}, ..., \delta_s I_{rs}, \Delta_1, ..., \Delta_F] \\ &\delta_i \in C, \Delta_j \in C^{m_j \times m_j} \\ &B\mathbf{\Delta} = \{\Delta \in \mathbf{\Delta} : \bar{\sigma}(\Delta) \leq 1\} \end{aligned} \tag{6}$$

The symbol $\bar{\sigma}$ denotes the maximum singular value.

It can be shown that for the feedback system to remain stable for all stable and bounded Δ's, μ has to be less than 1 over all frequencies [1]:

$$\sup_{\omega} \quad \mu(P(j\omega)) \leq 1 \tag{7}$$

4 Discussion

Two controllers are designed for a two-link, flexible jointed, flexible manipulator using the μ-synthesis technique ([1] and [2]). The first controller was designed using hub angles and relative tip displacement feedback:

$$\{Y\} = [\theta_1, \theta_2, y_1, y_2] \tag{8}$$

The designed controller is robust and lead to good performance. The arm and forearm reach steady-state after 3.5 seconds and the corresponding hubs settle after 2 seconds respectively. The control law was used with the non-linear model and the results are superimposed on the linear simulations for comparison. The non-linear simulations show that the control law is robust to the approximations made while deriving the control model (see Figures 5 through 10).

Another controller was derived using the hub angles and relative tip acceleration feedback:

$$\{Y\} = [\theta_1, \theta_2, \ddot{y}_1, \ddot{y}_2] \tag{9}$$

The resultant controller is also robust and lead to similar results as in the previous design. The hub angles settle after 2 seconds and the tips after approximately 3.5 second. The results of the linear and non-linear simulations are shown in Figures 11 through 16. The non-linear simulations proves that the designed controller is robust to the discrepancies between the linear control design model and the actual dynamics of the manipulator system.

5 Concluding Remarks

There are slight discrepancies between the linear and non-linear simulations of the controllers designed with tip deflections and tip accelerations feedback. This result was expected since as the links slew too fast, several unmodelled phenomena take place, such as large deflections. When the tip deflections are relatively small, the linear and non-linear simulations are in good agreement. It is worth mentioning that when hub angles and hub angular rates feedback were used, we were not able to design a robust controller for the two-link flexible robot manipulator.

From the above results, it is clear that additional sensors at the tips of the links of the manipulator can improve the performance of the manipulator tremendously, When accelerometers are used to control the two-link flexible manipulator, it lead to as good results as the ones achieved with tip deflection feedback. As expected, the torque output of the controller is higher for tip acceleration feedback than for tip deflection feedback (see Figures 9, 10, 15, and 16). However, from an economic point of view, it is recommended that accelerometers be used to improve the performance of flexible manipulators.

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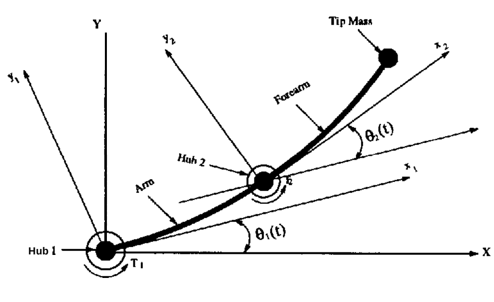

Figure 1: Schematic Diagram of the Flexible Manipulator

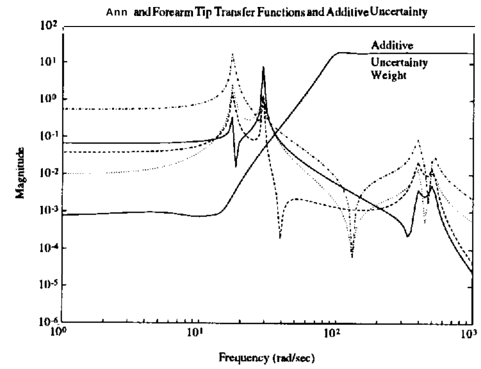

Figure 2: Additive Uncertainty and torque inputs to arm and forearm tip deflections transfer functions: arm joint torque input to arm tip deflection transfer function (solid), forearm joint torque input to arm tip deflection transfer function (dashed), arm joint torque input to forearm tip deflection transfer function (dotted), forearm joint torque input to forearm tip deflection transfer function (dash-dotted).

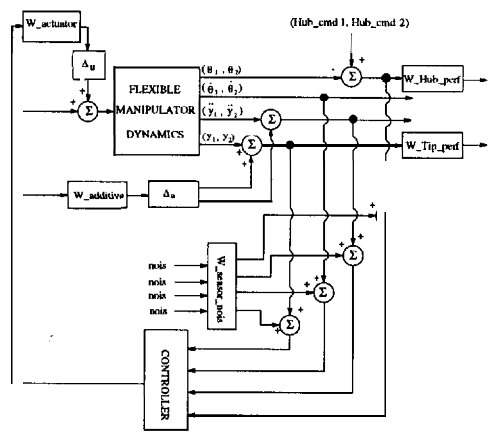

Figure 3: Block Diagram for the Control System for the two-link flexible manipulator

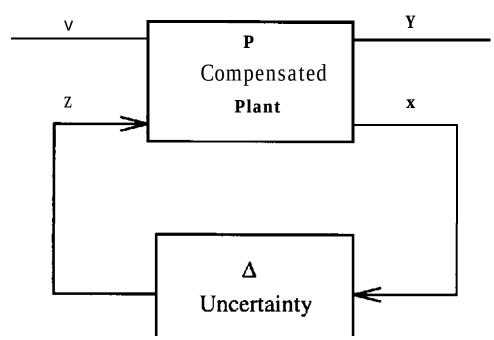

Figure 4: Simplified Block Diagram for u-synthesis

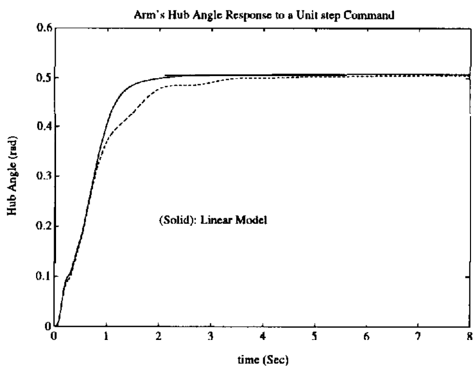

Figure 5: Time Response of the Hub of the Arm of the Flexible Manipulator Using the µ-synthesis Controller with Hub Angle and Tip Displacement Feedback: Linear and Non-linear Models

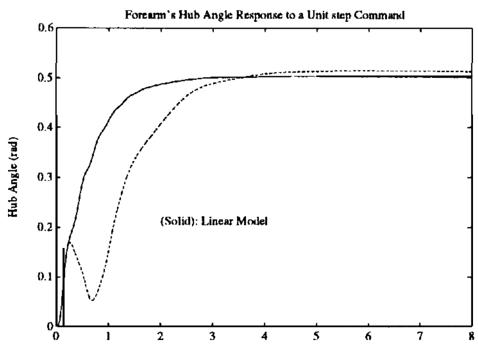

Figure 6: Time Response of the Hub of the Forearm of the Flexible Manipulator Using the µ-synthesis Controller with Hub Angle and Tip Displacement Feedback: Linear and Non-linear Models

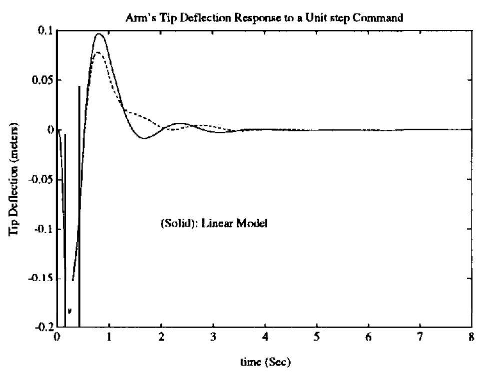

Figure 7: Time Response of the Tip of the Arm of the Flexible Manipulator Using the µ-synthesis Controller with Hub Angle and Tip Displacement Feedback: Linear and Non-linear Models

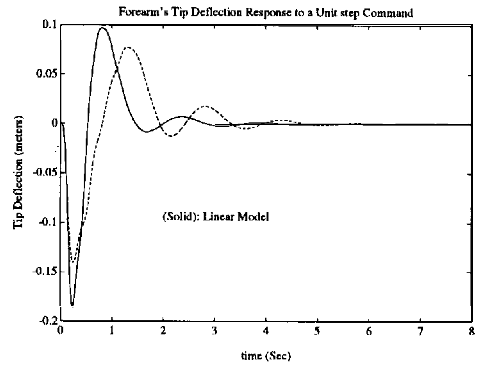

Figure 8: Time Response of the Tip of the Forearm of the Flexible Manipulator Using the µ-synthesis Controller with Hub Angle and Tip Displacement Feedback: Linear and Non-linear Models

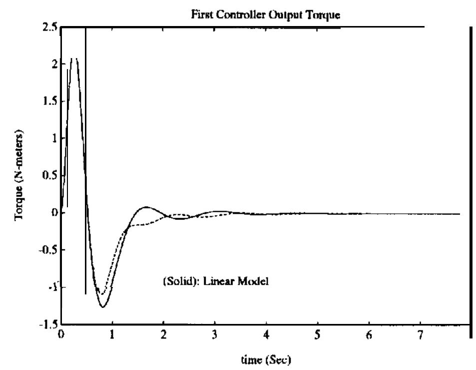

Figure 9: Controller Torque Output (Hub Angle and Tip Displacement Feedback: Linear and Non-linear Models)

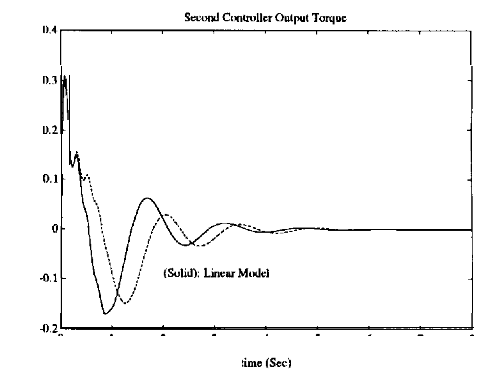

Figure 10: Controller Torque Output (Hub Angle and Tip Displacement Feedback: Linear and Non-linear Models)

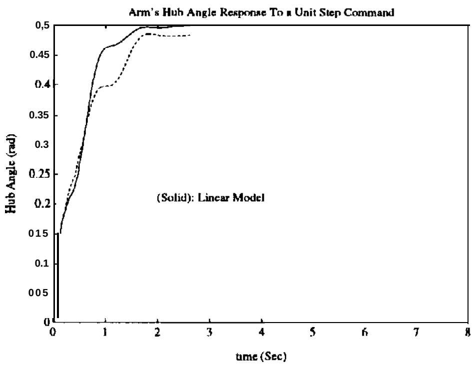

Figure 11: Time Response of the Hub of the Arm of the Flexible Manipulator Using the μ-synthesis Controller with Hub Angle and Tip Acceleration Feedback: Linear and non-linear Models

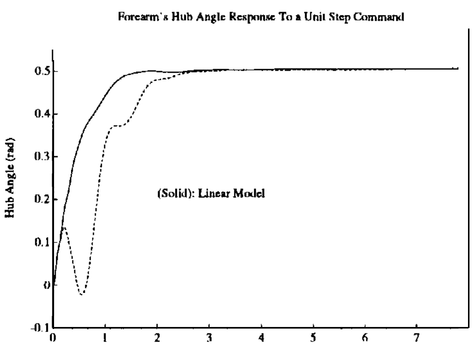

Figure 12: Time Response of the Hub of the Forearm of the Flexible Manipulator Using the μ-synthesis Controller with Hub Angle and Tip Acceleration Feedback: Linear and Non-linear Models

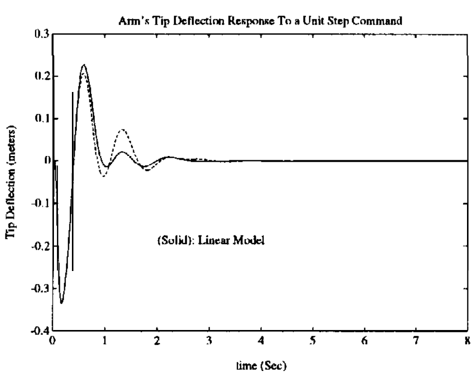

Figure 13: Time Response of the Tip of the Arm of the Flexible Manipulator Using the μ-synthesis Controller with Hub Angle and Tip Acceleration Feedback: Linear and Non-linear Models

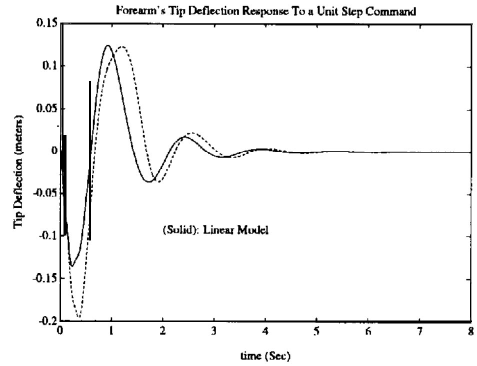

Figure 14: Time Response of the Tip of the Forearm of the Flexible Manipulator's Forearm's Tip Using the μ-synthesis Controller with Hub Angle and Tip Acceleration Feedback: Linear and Non-linear Models

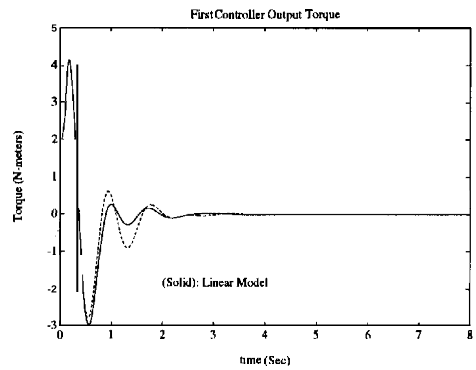

Figure 15: Controller Torque Output (Hub Angle and Tip Acceleration Feedback: Linear and Non-linear Models)

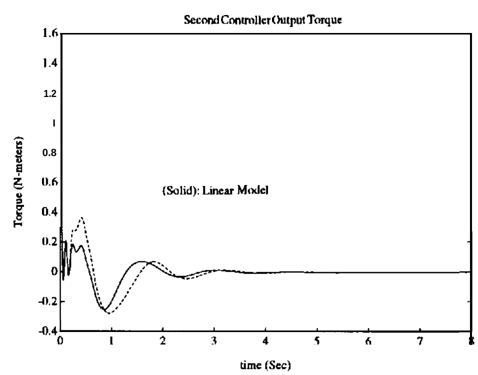

Figure 16: Controller Torque Output (Hub Angle and Tip Acceleration Feedback: Linear and Non-linear Models)