# Robot Control using a sliding mode

M. BELHOCINE, M. HAMERLAIN and K. BOUYOUCEF

Laboratoire de Robotique et d'Intelligence Artificielle, CDTA.
128, Chemin Mohamed Gacem, El-Madania,
BP. N° 245.16075 ALGER. ALGERIE
Tel : 213 2 67 73 25, Fax :213 2 66 26 89, e-mail : lria@ist.cerist.dz

**Abstract :**

The dynamical model of manipulator robot is represented by equations system which are nonlinear and strongly coupled. Furthermore, the inertial parameters of the manipulator depend on the payload which is often unknown and variable. So, to avoid these problems we studied variable structure system which is well suited for robotics arms.
To this end, an application of the sliding mode control based on variable structure system for a four degrees of freedom robot is described in this paper. This technique suppresses the uncertainties due to parametric variations, external disturbances and variable payloads.
To prove these advantages, this technique is applied to the regulation (point to point) control of the SCARA robot. So the aim of this work is to show the practical realization and to demonstrate the robustness and the validity of this control law on the robot manipulator via experimental results obtained and discussed in the end.

**Keywords :** Variable structure systems, sliding mode, robot control, robust control, non linear system.

## 1. Introduction

The dynamic of the robots is described by coupled second nonlinear differential equations and the inertial parameters depends on the payload which is often unknown and changes during the task. Usually, in a classical control we must have an accurate model, so to avoid this constraint we decide to implement a robust controller based on variable structure systems (VSS).

The theory of VSS has been developed firstly in Soviet Union by Emelyanov [2], introduced after by Utkin [9] and more recently studied by several authors [5], [8], [4]. In the last years, many applications of VSS were proposed : in motion control, DC-servo-motor and robotics manipulator [6], [7], [12]. From these applications, the conclusion is that the robust nature of VSS is proved by the sliding mode. When the sliding mode occurs, the system will be forced to slide along or near the vicinity of the switching surface. The system became then robust and insensitive to the interactions, disturbances and variations. In addition, this does not require an accurate model of the robot (plant) : it is only necessary to know the boundaries of the parameter variations and load disturbances. The dynamics of the system is submerged in the dynamics of the reduced linear and free system [10]. The sliding mode control technique has already been used for robot control [4], either without decoupling of the robot equations or combined with a model of the robot used to estimate the torques necessary at the joints.

The proposed sliding mode controller is realized by linearizing the equations system by the MATLAB software; and the objective of the controller is to avoid using velocity signal. So, only measured position signal of the joints is used to control the robot arm. Like this, shaft encoder is used to sense the output position a 12 bit A/D converter provides the required signal.
To illustrate the application of the sliding mode on robotics manipulator, we implement the controller on a SCARA robot which has four degrees of freedom. But, this work is concerned with the three degrees of freedom because the fourth degree is the translation in only two positions of the end effector.
So, in this paper, the practical realization of a sliding mode controller is described. After introducing the VSS theory, we show the experimental robot with its model identification and the linear system obtained; so after we develop the calculation of the control method. In the last, we discuss the experimental results obtained and confirm the validity of the approach.

## 2. Control methodology

Variable structure systems are referred to systems which their structure changes. This kind of systems has an attractive feature for control applications, which consists in a sliding mode [11]. This mode occurs on switching surface, and the system remains insensitive to parameter variations and disturbance. So, this mode allows also elimination of interactions among the joints of the manipulator. The aim of this paper is the implementation of the sliding mode control on the linkage manipulator.

A general type of the motion equation is represented in the space state by :

$$x = f(x,t) + g(x,t).u \qquad (1)$$

Where $u$ is the control input, $x$ is the output, and the functions $f(x,t)$ and $g(x,t)$ are nonlinear and not known exactly.

The control input is :

$$u_i(x,t) = \begin{cases} u_i^+(x,t) & if \;\; S_i(x,t) > 0 \\ u_i^-(x,t) & if \;\; S_i(x,t) < 0 \end{cases} \qquad (2)$$

Where $u_i$ is the *i*th component of $u$

$S_i(x,t) = 0$ is the *i*th component of the *m* switching hypersurfaces $S(x,t) = 0 \;\; S \in R^m$ .

This system with discontinuous control is called variable structure system, since the control structure switches alternatively according to the state of the system. The sliding mode occurs on a switching surface $S(x) = 0$, which forces the original system to behave as linear time invariant system, which can be considered to be stable.

In our study the surfaces are taken to be linear and written as : $S_i(x) = x_n + \sum_{i=1}^{n} \lambda_i . x_i$

The condition for the sliding mode to exist on the *i*th surface is given by the equation :

$$\lim_{S_i \to 0^+} \dot{S}_i < 0 \quad \text{and} \quad \lim_{S_i \to 0^-} \dot{S} > 0$$

Which is equivalent to $S_i \, \dot{S}_i < 0$ in the neighborhood of $S_i(x) = 0$, when all the trajectories move towards the switching surface.

In the ideal sliding mode on $S_i$, the corresponding control is the equivalent control issued from the equation (1) and given by the equation for $\dot{S} = 0$:

$$u_{eq} = g^{-1}(x,t)\left[ x(t) - f(x,t) \right] \qquad (3)$$

So, the discontinuous control input given in (2) is written :

$$u_i = \begin{cases} u_{ieq} + \Delta u_i^+ & s_i > 0 \\ u_{ieq} + \Delta u_i^- & s_i < 0 \end{cases} \qquad (4)$$

Where $u_{eq}$ is the equivalent control (low frequency) and $\Delta u_i$ the discontinuous term (high frequency).

As we are in the practical case, the equivalent control is known by estimated value due to error modelisation and variation of the parameters. So, this yields to :

$$u^*_{eq} = u_{eq} + \Delta u_{eq} \qquad (5)$$

As we explain the formula of discontinuous control input in (4), the term of high frequency can be expressed in different manners [1] [4]; and for our experiments we choose the equation proposed by Harashima [4] :

$$\Delta u_i = [\alpha_i |e_i| + \beta_i |\dot{e}_i| + \gamma_i ] \operatorname{sgn}(S_i)$$

## 3. Design procedure

As in our experiments, we use the manipulator model issued from testing the system to identify its parameters, we present in this section the identification of the manipulator model and the different steps of the controller calculation.

### 3.1 Manipulator model

Usually the manipulator dynamics are obtained from the Lagrangian equation, they have the following form :

$$I(q).\ddot{q} + C(q,\dot{q}) + g(q) = \Gamma \qquad (6)$$

Where :

I is an inertial matrix,
C represents coriolis and centrifugal forces,
g is the torque due to the gravity,
$\Gamma$ is the torque input vector,

$(q,\dot{q},\ddot{q})$ represent the generalized position, velocity and acceleration vector.

These equations are coupled and nonlinear; since the manipulator is controlled via a computer as PC, so we can approximate the dynamic model by an identified model which is linear. This identification is done using test input signals which excite the system and the least square method is used with the measured input and output signals to estimate system parameters.
The equation (6) can be written in the following form :

$$\ddot{q}+ I^{-1}(q).H(q,\dot{q})\dot{q}+ I^{-1}(q).G(q).q = I^{-1}(q).\Gamma \quad (7)$$

Where $H(q,\dot{q}).\dot{q} = C(q,\dot{q})$ and $G(q).q = g(q)$

This work proposes an experimental study of the three degrees of freedom of the SCARA robot and its identification is achieved using the MATLAB software. By the end, we obtain a linear approximated and decoupled model dynamics which is represented by the following system :

$$\ddot{q}+ A_1.\dot{q}+ A_2.q = B.U \qquad (8)$$

Where $A_1 = I^{-1}(q).H(q,\dot{q}) = Diag[a_{i1}]$

$A_2 = I^{-1}(q).G(q) = Diag[a_{i2}]$

$B = Diag[b_i]$

$q=[q_1,q_2,q_3]^T$ the angular rotation vector,

The results obtained for the parameters to be identified : $a_{i1}, a_{i2}, b_i$; in the prediction error method from ARX method on the MATLAB software are shown in the table 1.

| i | ai1 | ai2 | bi |
|---|---|---|---|
| 1 | 201.0 | -2.4 | 0.64 |
| 2 | 560.0 | -5.4 | 0.50 |
| 3 | 413.5 | -117.5 | 20.00 |

Table 1 : Identified parameters

### 3.2 Control calculation

As showed in the previous section, the equation of the identified model yields to :

$$\ddot{q}_i+a_{i1}\,\dot{q}_i+a_{i2}q_i=b_iu_i \qquad 1\le i\le 3 \qquad (9)$$

For each joint $i$, the input control is noted as : $u_i = u^*_{ieq} + \Delta u_i$; where $u^*_{ieq} = u_{ieq} + \Delta u_{ieq}$

and $\Delta u_i = [\alpha_i|e_i| + \beta_i|\dot{e}_i| + \gamma_i].\mathrm{sgn}(S_i)$

The (*) is related to the estimated term because of the modelisation error and variation of the parameters.
We must control the system by holding it in the sliding surface. The surface has the equation :

$S_i = \lambda_i.e_i + \dot{e}_i$

where : $\lambda_i$ is a positive parameter

$e_i, \dot{e}_i$ and $\ddot{e}_i$ are the position, velocity and acceleration errors

The sliding mode is for $S_i = 0$ and $\dot{S}_i = 0$, from where we extract the equivalent control input equation :

$$u_{ieq} = \frac{1}{b_i}[\ddot{q}_{id}+a_{i1}.\dot{q}_i+a_{i2}.q_i-\lambda_i.\dot{e}_i] \quad (10)$$

The estimated equivalent control is :

$$u^*_{ieq} = \frac{1}{b^*_i}[\ddot{q}_{id}+a^*_{i1}.\dot{q}_i+a^*_{i2}.q_i-\lambda_i.\dot{e}_i] \quad (11)$$

The calculation yields to :

$$\Delta u_{ieq} = d_{i1}.\dot{e}_i + d_{i2}.e_i + d_{i3}.\dot{q}_{id} + d_{i4}.\ddot{q}_{id} + d_{i5}.q_{id}$$

With : $d_{i1} = d_{i3} - \lambda_i.d_{i4}$

The control gains are obtained from the sliding condition $S_i\dot{S}_i < 0$. So, after calculation , we obtain :

$$\dot{S}_i S_i = b_i(S_i\Delta u_{ieq} + S_i\Delta u_i)$$

Which can be written :

$$\dot{S}_i S_i = b_i(S_i\Delta u_{ieq} + (\alpha_i|e_i| + \beta_i|\dot{e}_i| + \gamma_i)Sgn(S_i)S_i)$$

$$\dot{S}_i S_i = b_i(S_i\Delta u_{ieq} + (\alpha_i|e_i| + \beta_i|\dot{e}_i| + \gamma_i)|S_i|)$$

Then :

$$\dot{S}_i S_i \le |b_i||S_i|(|\Delta u_{eq}| + (\alpha_i|e_i| + \beta_i|\dot{e}_i| + \gamma_i)$$

And we substitute $\Delta u_{eq}$ by its expression, we obtain :

$$S_i\dot{S}_i \le |b_i||S_i|((|d_{i1}|+\beta_i)|\dot{e}_i| + (|d_{i2}|+\alpha_i)|e_i| + |d_{i3}|\dot{q}_{id} + |d_{i4}|\ddot{q}_{id} + |d_{i5}|q_{id} + \gamma_i)$$

To satisfy the sliding condition $S\dot{S}<0$, it is sufficient to take :

$$\begin{cases} |d_{i3}|\dot{q}_{id} + |d_{i4}|\ddot{q}_{id} + |d_{i5}|q_{id} + \gamma_i < 0 \\ \alpha_i + |d_{i2}| < 0 \\ \beta_i + |d_{i1}| < 0 \end{cases}$$

From where we deduce :

$$\begin{cases} \alpha_i = -Sup(|d_{i2}|); \\ \beta_i = -Sup(|d_{i1}|); \\ \gamma_i = -Sup(\left|d_{i3}\cdot\dot{q}_{id} + d_{i4}\cdot\ddot{q}_{id} + d_{i5}\cdot q_{id}\right|) \end{cases}$$

## 4. Experimental results

This section describes experimental results of applying sliding mode on the robot in point to point motion (regulation). The three degrees of freedom move respectively from the initial position $q_{init}$(2.78,1.05,0.51)rd to the desired final position $q_{fin}$(1.05,2.78,5.23)rd with the following parameters of the surfaces 70, 140 and 110 for each joint.

In the figures 1, 2 and 3 the controller is tested without payload in the figures 4, 5 and 6 show the case where the robot is disturbed which consist to fight the arm of the robot with a force and to see if it doesn't make the robot disable to go the desired position.

The first figures (1, 2 and 3) give the error position for each joint, the velocity and the corresponding control input evolution. These results show the good behavior of the control algorithm. The steady states are reached after 2.5 seconds for the first joint, 2 seconds for the second and 1 second for the third joint. The position errors are around 0.02 rd.
In the figures 4, 5 and 6, we see when the robot is disturbed by an external force, that it will come back to the desired position with an error equal to 0.02 rd. This demonstrate the robustness of sliding mode controller against the disturbance. The steady states are reached after 6 seconds for the first joint, 6 seconds for the second and 4 seconds for the third joint.

## 5. Conclusion

A practical realization of robust controller using sliding mode is proposed in this paper. It was applied to the three degrees of freedom of the SCARA robot which has three degrees of freedom, and shows that nonlinear dynamic interactions of the manipulator joints are suppressed and the system is insensitive to the parameters variations. The experimental results shows also its performances against the payloads variations.

In our future study, it is interested to test our algorithm in the case of trajectory tracking. Further, it will be more interested to develop an algorithm based on generalized variable structure system to avoid chattering, which appears in the sliding mode, by the switching on the highest derivative of the input [3].

## 6. References

[1] : H.Asada et J.J.E Slotine : Robot Analysis and Control , A Wiley-Interscience Publication John Wiley and Sons, 1986.
[2] : T. Emelyanov : « Sur une classe de systèmes de régulation automatique à structure variable », Journal de l'académie des sciences d'URSS, Energétique et automatique, N°3, 1962.
[3] : M. Hamerlain : « The new robust control using the theory of generalized variable structure », IEEE International Symposium on Industrial Electronics, 10-14 July 1995, Athens, Greece.
[4] : F. Harashima, H. Hashimoto, K. Maruyama : "Practical robust control of robot arm using variable structure system", Proc. of IEEE, Int. Conf. On Robotics and Automation, San Francisco, 532-538, 1986.
[5] : H. Hashimoto : "A variable structure system with an invariant trajectory", Power electronics, Tokyo, Vol.2, 1983.
[6] : S. Nouri, M. Hamerlain, C. Mira, P. Lopez : "Variable structure model reference adaptive control using only input and output measurements for two one-link manipulators" IEEE-SMC, Le Touquet 1993.
[7] : H. Sira Ramirez, S. Ahmad, M. Zribi : "Dynamical feedback control of robotics manipulators with joint flexibility", TR.EE 90-70, December 1990, School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907.
[8] : J.J.E Slotine & J.A. Coetsee : "Adaptive sliding controller synthesis for non linear systems", Int. Jou. Control, Vol 43, N°6, 1986.
[9] : V.I Utkin : « Sliding mode and their application in variable structure systems », Moscow, 1978.
[10] : V.I. Utkin : "Sliding modes in control and optimization", Edition Springer Verlag, 1992.
[11] : K.S. Yeung & Y.P. Chen : « Sliding mode controller design of a single-link flexible manipulator under gravity », International Journal on Control, Vol.52, N°1, pp.101-117, 1990.
[12] : K.K. Young : "Variable structure control for robotics and aerospace applications", Elsevier, London, New York, Tokyo, 1993.

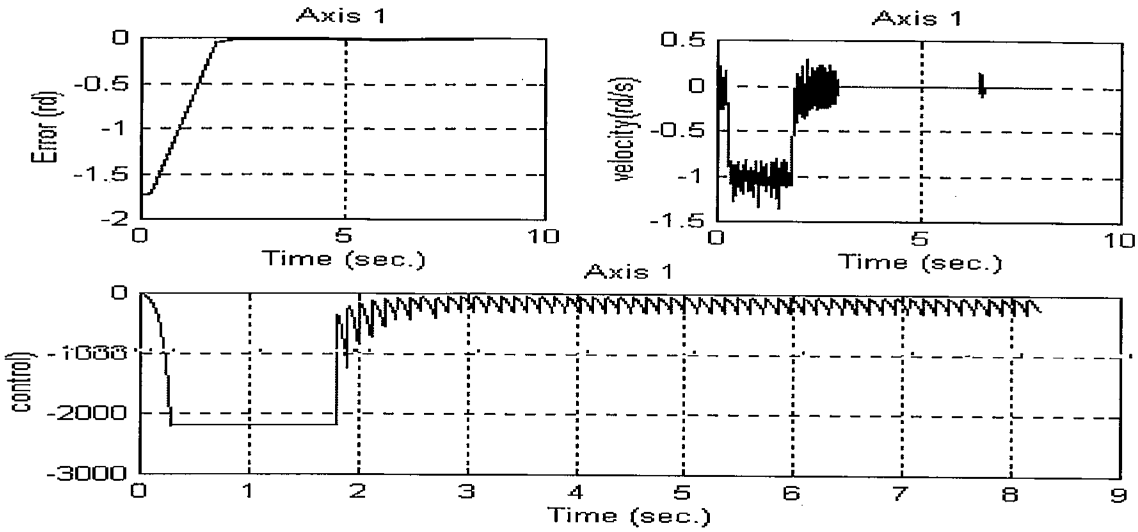


Figure 1 : VSS without disturbance

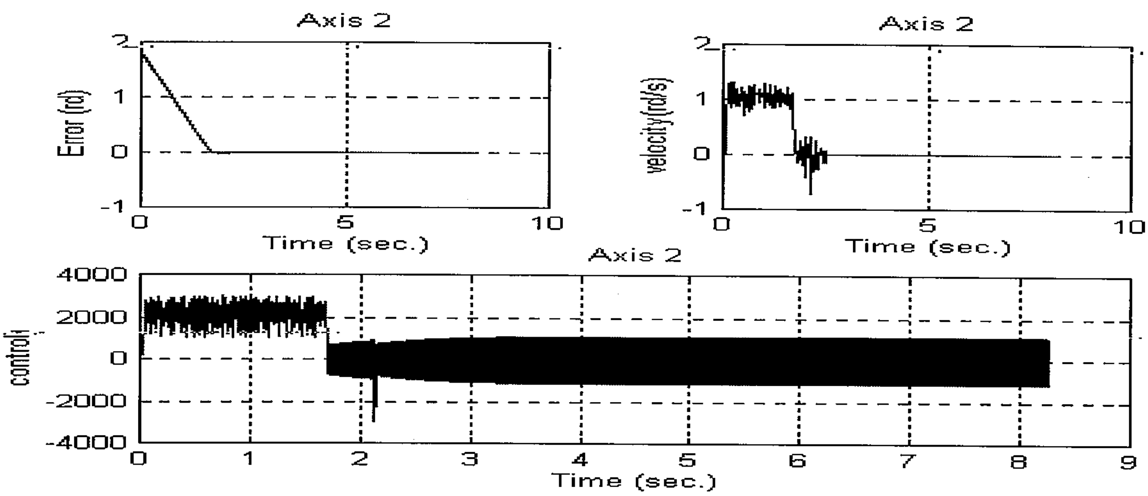


Figure 2 : VSS Without disturbance

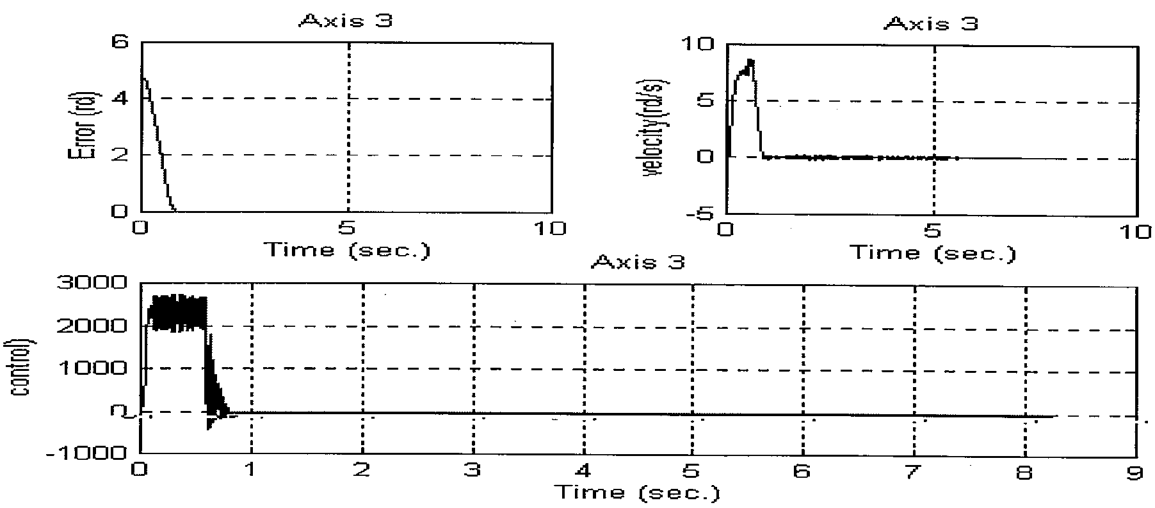


Figure 3 : VSS without disturbance

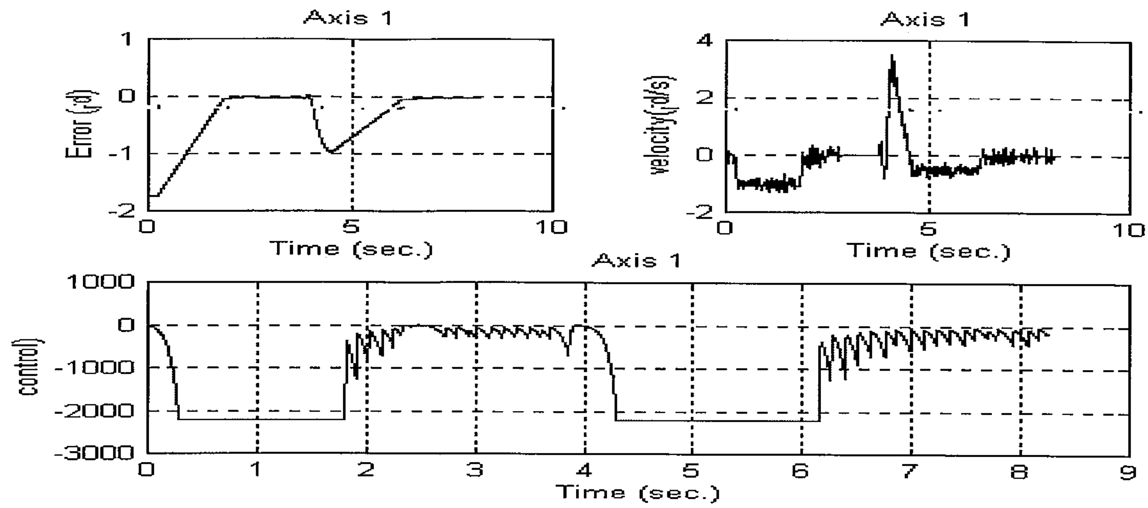


Figure 1 : VSS with disturbance

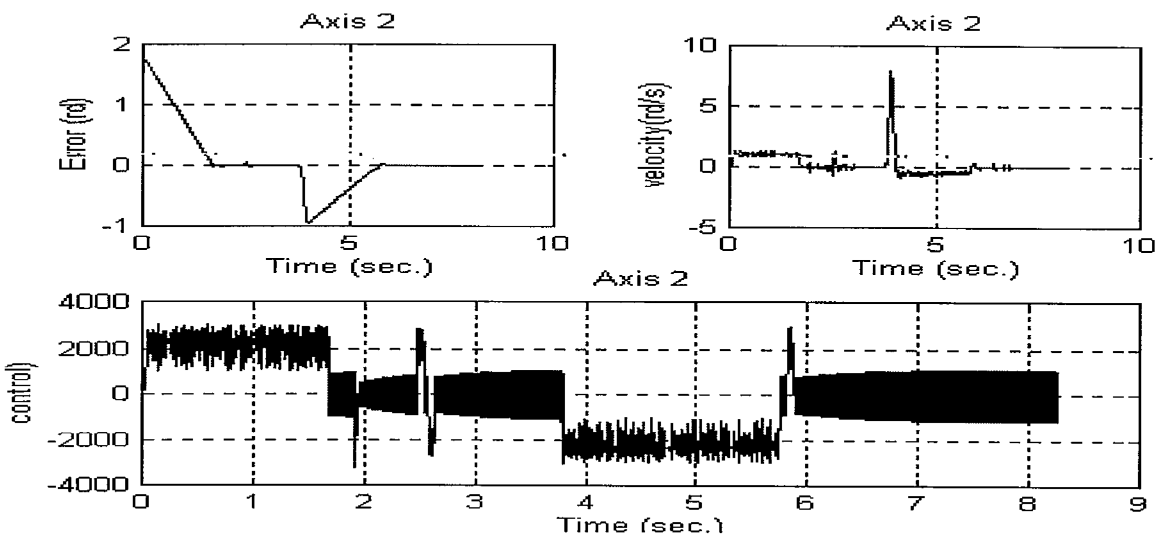


Figure 2 : VSS with disturbance

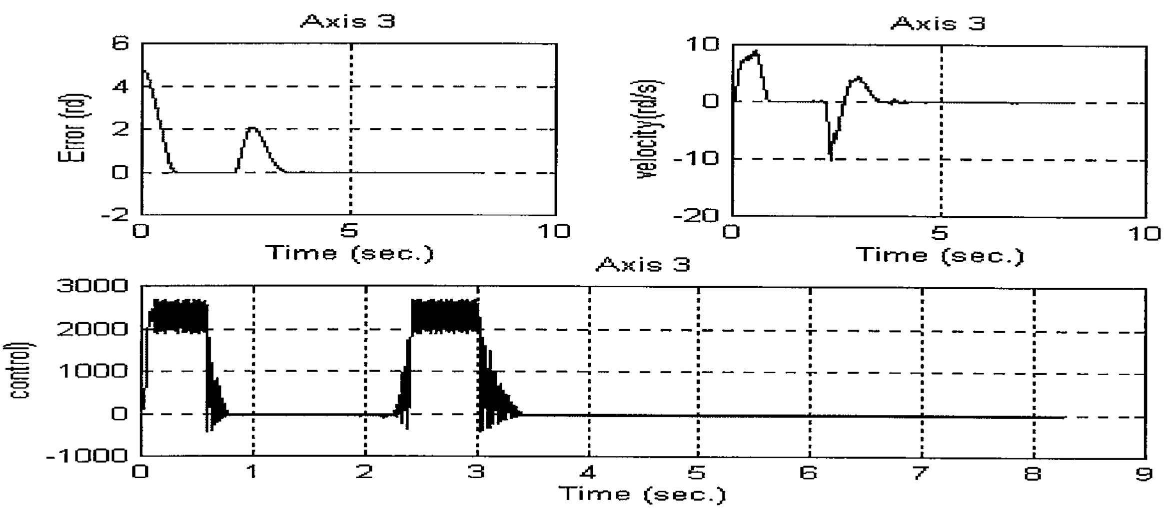


Figure 3 : VSS with disturbance