Optimal Location of Actuators for an Active Noise Control Problem

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Abstract

In this paper, we investigate the problem of finding the optimal location of controllers to achieve reduction of the noise field in an acoustic cavity. We first formulate a linear quadratic tracking problem in a Hilbert space, and then consider the problem of optimization of an appropriate performance criterion with respect to the location of the controls. Numerical examples will be presented to illustrate our theoretical results.

1 Introduction

The problem of optimal location of controls is important in active noise suppression applications where an optimum amount of reduction of noise is required without adding much to the control cost. In these problems, an arbitrarily placed controller can actually increase the sound field locally, or a controller placed at the node of the acoustic field will not be that effective at attenuating the acoustic field in other locations.

In this work, we consider optimal reduction of the noise field inside a cavity through introduction of loudspeakers that would generate a secondary acoustic field that interacts destructively with the primary noise field. This active noise control technique is useful in situations where the traditional passive damping methods which involve addition of mass are not practical. In recent years, serious research effort in this field has resulted in a vast body of literature in the acoustic field [6, 7, 8, 10]. Our effort is to formulate the problem within the theory of optimal control of infinite-dimensional system. This state-space formulation will allow us to address the important issue of optimal location of controls in a setting where optimization with respect to location can be naturally formulated.

In [1], an acoustic model based on *damped elastic boundary conditions* which preserved the frequency-dependent properties of the boundaries was proposed, and a control problem was formulated as an abstract linear quadratic tracking problem (LQT) in an infinite dimensional setting. In [3] numerical approximations based on the Legendre-tau method were performed to solve the finite-dimensional control system. The goal of this work is to first present our model based on the *impedance boundary conditions* which are suitable for frequency-independent boundaries. This model is similar to the lightly-damped enclosures that have been used and discussed extensively in acoustic literature such as [7, 11], as well as in mathematical papers [4, 5]. We present an active control strategy for reducing the acoustic fields and focus on the issue of optimizing the performance of the controls with respect to their placement.

2 Formulation of the Physical Model

For simplicity, we consider a one-dimensional domain of the interval $\Omega = (0,1)$. The governing acoustic equation is given by the linear wave equation, and the boundary conditions at the end-points are the *impedance boundary conditions,* which relate the pressure field $p(t,x)$ to the normal velocity of the fluid $v(t,x)$ at the boundary via a complex-valued impedance quantity, ζ. For this reason, we express the equations in terms of the velocity potential $\phi(t,x)$ which is related to both the acoustic pressure and the fluid velocity by the relations,

$$p(t,x) = \rho\phi_t(t,x), \qquad \text{and } v(t,x) = -\nabla\phi(t,x),$$

where ρ is the fluid density.

We model the primary noise source as a single harmonic wave: $p_1(x,t) = \hat{p}_1(x)e^{i\omega t}$. After activation of the controls (speakers) at $t = 0$, it is anticipated that the acoustic pressure field will approach a steady periodic state p_2 in a stable manner. With the period $\tau = 2\pi/\omega$, and $f_c(x,t) = \Sigma_{i=1}^{m}\chi(\hat{\Omega}_i)F_i(t)$, where $\hat{\Omega}_i$ is the support of the *ith* control, the steady state p_2 is expected to be governed by a non-homogeneous periodic wave equation. Because of the impedance boundary conditions, the state equations are expressed in terms of the velocity potential vector $\phi_2(t,x)$ related to p_2.

Now the governing equations are given as

$$\frac{\partial^2\phi_2}{\partial t^2} = c^2\triangle\phi_2 + f_c \quad \text{in } \Omega\times[0,\tau], \tag{1}$$

where c is the speed of sound in the fluid. The impedance boundary condition at the boundary is

$$-c\zeta\frac{\partial\phi_2(t,x)}{\partial\nu}=\frac{\partial\phi_2(t,x)}{\partial t},\qquad x\in\partial\Omega, \qquad (2)$$

where $\frac{\partial}{\partial\nu}$ denotes the outward normal derivative to the boundary. This equation states that the acoustic impedance $z=\frac{p}{v_{incident}}$ at the two boundary points is given by $\rho c\zeta$, where ζ is the complex-valued specific acoustic impedance of the surface [11].

We choose the state vector to be $\mathbf{x}_2(t,x)=[u_2,p_2]^T=[c\phi_2,(\phi_2)_t]^T$, where the first component of the vector refers to the velocity potential (multiplied by c) and the second component refers to the acoustic pressure (divided by ρ). Now, the state equation in the first order form is

$$\frac{\partial}{\partial t}\begin{bmatrix}u_2\\ p_2\end{bmatrix}=A\begin{bmatrix}u_2\\ p_2\end{bmatrix} \qquad (3)$$

where A is defined as

$$A=c\begin{bmatrix}0 & I\\ \partial_{xx} & 0\end{bmatrix}.$$

The problem is defined on a Hilbert space whose norm defines the energy form $||c\phi_x||^2+||\phi_t||^2=||u_x||^2+||p||^2$. Since this energy form is only a seminorm on the space $H^1(0,1)\times L^2(0,1)$, the appropriate space should exclude constant functions which have a zero derivative without being zero. Therefore the proper space is $\mathcal{H}=\bar{H}^1(0,1)\times L^2(0,1)$ where $\bar{H}^1(0,1)$ with the inner-product $\langle u,u\rangle_1=\langle u_x,u_x\rangle_{L^2}$ is the quotient space of $H^1(0,1)$ over the constant functions. With the domain of operator A defined to be a dense subset of $\mathcal{H}$ where

$$Dom(A)=\left\{\begin{array}{c}(u,p)|(u,p)\in H^2\times H^1,\\ p(0)-\zeta u_x(0)=p(1)+\zeta u_x(1)=0\end{array}\right\},$$

A is the generator of a C_0 semigroup $T(\cdot)$ in $\mathcal{H}$, which means that for an initial state $(u_{2,0},p_{2,0})\in Dom(A)$, the solution to (3) is given by $(u_2(t,x),p_2(t,x))=T(t)(u_{2,0},p_{2,0})$. From this formulation and choice of the state space we see that the velocity potential is determined only up to a constant, [5].

It can be shown that for $Re(\zeta)>0$ the solutions decay to zero exponentially and the system is uniformly exponentially stable [5]. This fact is of utmost importance in establishing a well-posed control strategy for the problem, and we shall use it in the next section.

In addition to the formulation above, we need to cast the problem in the weak form which is the natural setting for the numerical approximation of the problem. We can show that for the initial state $(u_{2,0},p_{2,0})\in Dom(A)$, the solution $(u_2(t,x),p_2(t,x))=T(t)(u_{2,0},p_{2,0})$ satisfies the following variational equations

$$\frac{\partial}{\partial t}\langle u_2(t),g\rangle_1=\langle cp_2(t),g\rangle_1 \qquad (4)$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle p_2(t),h\rangle_{L^2} &= -\langle cu_2(t),h\rangle_1\\ &\quad +c\int_\Gamma\frac{\partial u_2}{\partial\nu}h(x)dx \qquad (5)\\ &\quad +\langle f_c(t),h\rangle_{L^2}\end{aligned}$$

for all $(g,h)\in\bar{H}_1(\Omega)\times\bar{H}_1(\Omega)$. Note that for the one dimensional domain $(0,1)$ in our problem and the impedance boundary conditions at the points $x=0$, and $x=1$, the integral term in (5) reduces to

$$c\int_\Gamma\frac{\partial u}{\partial\nu}h(x)dx=-\frac{c}{\zeta}p_2(0)h(0)-\frac{c}{\zeta}p_2(1)h(1).$$

The equations (4) and (5) are called the weak formulation of the equations (1) and (2). We will use this formulation for the Galerkin approximation of the equations and the control system.

3 A Periodic Linear Quadratic Tracking Problem

The control problem we wish to solve is to find an optimal control which minimizes the total acoustic field consisting of the primary and the secondary sound fields. We formulate the problem as an (LQT) problem where the cost function consists of a term due to the total field, along with the cost associated with the controls. Because of the periodic nature of the problem, the integration in time is over the period τ. The problem now is to find an optimal control F^* which minimizes the following

$$J(F)=\int_0^\tau\{(Q[\mathbf{x}_1+\mathbf{x}_2],[\mathbf{x}_1+\mathbf{x}_2])_{\mathcal{H}}+\theta(F,F)_{L^2}\}dt$$

subject to

$$\left\{\begin{array}{rcll}\dot{\mathbf{x}}_2 &=& A\mathbf{x}_2+BF & \text{for } 0\le t\le\tau\\ \mathbf{x}_2(0) &=& \mathbf{x}_2(\tau).\end{array}\right.$$

In the above, the state and primary noise variables are

$$\begin{aligned}\mathbf{x}_2(x,t)&=\begin{bmatrix}u_2\\ p_2\end{bmatrix},\\ \mathbf{x}_1(x,t)&=\hat{\mathbf{x}}_1(x)e^{i\omega t}=\begin{bmatrix}\hat{u}_1\\ \hat{p}_1\end{bmatrix}e^{i\omega t}.\end{aligned} \qquad (6)$$

The operator Q is a self-adjoint, nonnegative operator and θ is a control design parameter, and

$$BF=\begin{bmatrix}0\\ \frac{1}{length(\hat{\Omega})}\chi(\hat{\Omega})F(t)\end{bmatrix},$$

where for simplicity we consider a single control which is normalized by its length. In [1], it is shown that the *optimal control* is given by

$$F^*(t) = -\theta^{-1}B^*G\mathbf{x}_2(t) - \theta^{-1}B^*r(t),$$

where G satisfies the *Algebraic Riccati Equation*

$$GA + A^*G + Q - \theta^{-1}GBB^*G = 0,$$

and the *tracking variable* $r(x,t)$ satisfies

$$\begin{cases} \dot{r}(x,t) = -[(A^* - \theta^{-1}GBB^*)]r - Q\mathbf{x}_1, & 0 \le t \le \tau \\ \dot{r}(0) = r(\tau). \end{cases}$$

For the primary noise source modeled as a simple harmonic wave, we have the following expression for the associated velocity potential function, $\phi_1(t,x)$, [11]:

$$\phi_1(t,x) = C(e^{i\omega(t-\frac{x}{c})} + Re^{i\omega(t+\frac{x}{c})}) \qquad (7)$$

where C is the amplitude of wave, and R is the reflection coefficient which for the impedance boundary conditions is given by

$$R = \frac{\zeta - 1}{\zeta + 1}.$$

Now by using (7) and the fact that

$$\mathbf{x}_1(x,t) = [\hat{u}_1, \hat{p}_1]^T e^{i\omega t} = [c\phi_1(x,t), (\phi_1(x,t))_t]^T$$

we can find appropriate expressions for $\hat{u}_1$ and $\hat{p}_1$.

One can show, (see [2, 3]) that the tracking variable for a harmonic primary noise source can be written as $r(x,t) = \hat{r}(x)e^{i\omega t}$ where $\hat{r}(x)$ satisfies

$$\hat{r}(x) = -[i\omega + (A^* - \theta^{-1}GBB^*)]^{-1}Q\hat{\mathbf{x}}_1.$$

To show well-posedness of this control problem we need to show that $A - \theta^{-1}BB^*G$ generates a C_0-semigroup, $S(t)$, on $\mathcal{H}$ such that $\|S(t)\|_{\mathcal{H}} \le M_1 e^{-\mu_1 t}$ $\forall t \ge 0$, for some $M_1, \mu_1 > 0$. This statement follows from uniform exponential stability of the solutions to the open-loop system where $B = 0$, (see [5]).

From the observations above, we can conclude that the *optimal state* satisfies:

$$\begin{cases} \dot{\mathbf{x}}_2 = (A - \theta^{-1}BB^*G)\mathbf{x}_2 - \theta^{-1}BB^*r & 0 \le t \le \tau \\ \mathbf{x}_2(0) = \mathbf{x}_2(\tau). \end{cases}$$

As in the case for the tracking variable, it can be shown the periodic optimal state $\mathbf{x}_2(x,t) = \hat{\mathbf{x}}_2(x)e^{i\omega t}$ satisfies the equation

$$\hat{\mathbf{x}}_2(x) = -[i\omega - (A - \theta^{-1}BB^*G)]^{-1}\theta^{-1}BB^*\hat{r}.$$

Moreover, there exists an $\hat{F}^*$ such that the optimal control is given by

$$F^* = \hat{F}^* e^{i\omega t},$$

thus the optimal control in our case is sinusoidal, [2].

4 Optimization Problem

Our goal is to optimize the following minimum LQT cost function with respect to location of the controllers, x_c:

$$J_{min}(F^*) = \int_0^\tau \{(Q\mathbf{x}_1, \mathbf{x}_1)_{\mathcal{H}} - \theta^{-1}(B^*r, B^*r)_{L^2}\}dt.$$

In this expression, only the control operator, B, and the tracking variable, r are dependent on x_c. Since the possible values for the location belong to a compact set, we can prove existence of an optimizing location by proving the cost function is weakly lower semi continuous with respect to x_c. This result can be obtained by showing that B and therefore the Riccati operator, G, are continuously dependent on x_c. In order to find the equation that the gradient of J_{min} with respect to x_c satisfies, we need to find $\dfrac{\partial G}{\partial x_c}$ and $\dfrac{\partial B}{\partial x_c}$. One can show that $\Sigma = \dfrac{\partial G}{\partial x_c}$ satisfies the following Lyapunov equation:

$$\begin{aligned} \Sigma(A - \theta^{-1}BB^*G) \quad &+ \quad (A^* - \theta^{-1}BB^*G)\Sigma \\ &= \quad \theta^{-1}G\left(\frac{\partial B}{\partial x_c}B^* + B\frac{\partial B}{\partial x_c}\right)G. \end{aligned}$$

This equation has a unique solution as long as $(A - \theta^{-1}BB^*G)$ generates a contraction semigroup that decays exponentially in time. This fact has already been established for our problem, (see [5]). Now that we have the well-posedness of the sensitivity equation, we can proceed with the numerical approximation of the optimization problem.

5 Numerical Approximations

To carry out the numerical approximation, we employ the Legendre Galerkin method to cast the infinite dimensional control system in a sequence of finite dimensional spaces of polynomials. In this method, the finite dimensional solution is expanded in terms of the Legendre polynomials, $L_n(x)$, [9]. These polynomials are orthogonal over the interval $(-1,1)$, i.e., they satisfy the following orthogonality relation

$$\int_{-1}^{1} L_n(x)L_m(x)dx = \frac{2}{2n+1}\delta_{nm}.$$

These polynomials can be generated by the following recursive relation

$$(n+1)L_{n+1}(x) = (2n+1)xL_n(x) - nL_{n-1}(x),$$

with $L_0(x) = 1$, and $L_1(x) = x$.

In addition to these properties, Legendre polynomials also satisfy

$$L_n\left(\bar{+}1\right) = \left(\bar{+}1\right)^n, \; L_n'\left(\bar{+}\right) = \left(\bar{+}1\right)^n \frac{1}{2}n(n+1).$$

The finite dimensional space, $\mathcal{H}^N$ in which the state equations as well as the control problems are posed is chosen to be the product space $\mathcal{H}^N = H_1^N \times H_1^N$, where H_1^N is the space spanned by the shifted Legendre polynomials $\left\{\hat{L}_i\right\}_{i=1}^N$ over the interval $(0,1)$. These polynomials are obtained from the Legendre polynomials over $(-1,1)$ by the transformation $t = 2x-1$. In other words

$$\hat{L}_i(x) = L_i(2x-1).$$

The new polynomials preserve the orthogonality relation over the interval $(0,1)$

$$\int_0^1 \hat{L}_i(x)\hat{L}_j(x)dx = \frac{\delta_{ij}}{2n+1}.$$

With $H_1^N = span\{\hat{L}_i\}_{i=1}^N$, we expand the approximate solutions to the equations (4)-(5) in terms of the Legendre polynomials

$$\begin{aligned} u_2^N(t,x) &= \sum_{i=1}^N u_{2,i}^N(t)\hat{L}_i(x), \\ p_2^N(t,x) &= \sum_{i=1}^N p_{2,i}^N(t)\hat{L}_i(x). \end{aligned} \tag{8}$$

The approximate solution satisfies the following equations that are analogous to the equations (4-5), and the functions g, and h are chosen to be the test functions $\hat{L}_i \in H_1^N$

$$\begin{aligned} \frac{\partial}{\partial t}\langle u_2^N(t), \hat{L}_i\rangle_1 &= \langle cp_2^N(t), \hat{L}_i\rangle_1 \\ \frac{\partial}{\partial t}\langle p_2^N(t), \hat{L}_i\rangle_{L^2} &= -\langle cu_2^N(t), \hat{L}_i\rangle_1 \\ &\quad +c\int_\Gamma \frac{\partial u_2^N}{\partial \nu}\hat{L}_i(x)dx \\ &\quad +\langle f_c, \hat{L}_i\rangle_{L^2}. \end{aligned} \tag{9}$$

Let us denote the column vector of the coefficients of the state vector as $\bar{\mathbf{x}}_2^N(t) = \left[\bar{u}_2^N, \bar{p}_2^N\right]^T$, where

$$\bar{u}_2^N = \begin{bmatrix} u_{2,1}^N(t) \\ \vdots \\ u_{2,N}^N(t) \end{bmatrix}, \quad \bar{p}_2^N = \begin{bmatrix} p_{2,1}^N(t) \\ \vdots \\ p_{2,N}^N(t) \end{bmatrix}.$$

From equations (9) and (8), we have the following first order matrix equations for the state vector coefficients:

$$\begin{aligned} &\begin{bmatrix} K^N & 0 \\ 0 & M^N \end{bmatrix}\begin{bmatrix} \dot{\bar{u}}_2^N(t) \\ \dot{\bar{p}}_2^N(t) \end{bmatrix} = \\ &c\begin{bmatrix} 0 & K^N \\ -K^N & -D^N \end{bmatrix}\begin{bmatrix} \bar{u}_2^N(t) \\ \bar{p}_2^N(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{B}^N \end{bmatrix}. \end{aligned} \tag{10}$$

The stiffness and mass matrices K^N and M^N are symmetric and positive definite and the matrix D^N is obtained from the impedance boundary conditions and is symmetric nonnegative definite. These matrices are given by

$$\begin{aligned} K_{ij}^N &= \int_0^1 (\hat{L}_i)_x(\hat{L}_j)_x dx, \\ M_{ij}^N &= \int_0^1 (\hat{L}_i)(\hat{L}_j)dx, \\ D_{ij}^N &= \frac{1}{\zeta}\hat{L}_i(1)\hat{L}_j(1) + \frac{1}{\zeta}\hat{L}_i(0)\hat{L}_j(0) \\ \tilde{B}_i^N &= \frac{1}{2a}\int_{\hat{\Omega}} \hat{L}_i(x)dx \end{aligned}$$

where for $\tilde{B}^N$, $\hat{\Omega} = [x_c - a, x_c + a]$ denotes the domain of influence of a control located at x_c with the width $2a$. From

$$W^N = \begin{bmatrix} K^N & 0 \\ 0 & M^N \end{bmatrix}, \quad A^N = c\begin{bmatrix} 0 & K^N \\ -K^N & -D^N \end{bmatrix},$$
$$B^N = \begin{bmatrix} 0 \\ \tilde{B}^N \end{bmatrix}$$

we can write the following first order equation

$$\begin{aligned} \dot{\bar{\mathbf{x}}}_2^N(t) &= \mathcal{A}^N\bar{\mathbf{x}}_2^N + \mathcal{B}^N F(t) \\ \bar{\mathbf{x}}_2^N(0) &= \bar{\mathbf{x}}_2^N(\tau). \end{aligned} \tag{11}$$

Here $\mathcal{A}^N = (W^N)^{-1}A^N$, $\mathcal{B}^N = (W^N)^{-1}B^N$. The finite-dimensional optimal control in R^{2N} is given by

$$\bar{F}_{opt}^N(t) = -\theta^{-1}\mathcal{B}^{N*}(G^N\bar{\mathbf{x}}_2^N(t) - \bar{r}^N(t)), \tag{12}$$

where G^N satisfies the matrix *Algebraic Riccati Equation*

$$G^N\mathcal{A}^N + (\mathcal{A}^N)^*G^N + Q^N - \theta^{-1}G^N\mathcal{B}^N(\mathcal{B}^N)^*G^N = 0 \tag{13}$$

and $\bar{r}^N(t)$, the coefficient vector for the finite-dimensional *tracking variable*, satisfies

$$\bar{r}^N = -[i\omega I + (\mathcal{A}^N)^* - \theta^{-1}G^N\mathcal{B}^N(\mathcal{B}^N)^*]^{-1}Q^N\bar{\mathbf{x}}_1^N. \tag{14}$$

In (13) and (14) the matrix Q^N is defined as DW^N where D is the diagonal matrix

$$D = diag\,[d_1\, I,\, d_2\, I],$$

where I is the $N \times N$ identity matrix and and d_i's are parameters that are to be chosen for improving the performance of the control system. The vector $\bar{\mathbf{x}}_1^N$ is the coefficient vector of the approximate offending noise. To obtain an expression for this vector, we first expand the components of the approximate offending noise function

$$\mathbf{x}_1^N(x,t) = [\hat{u}_1^N(x), \hat{p}_1^N(x)]^T e^{i\omega t}$$

in terms of Legendre polynomials as

$$\hat{u}_1^N = \sum_{i=1}^{N} u_{1,i}^N \hat{L}_i, \quad \hat{p}_1^N = \sum_{i=1}^{N} p_{1,i}^N \hat{L}_i.$$

The coefficients $\bar{u}_1^N = (u_{1,1}^N, \ldots, u_{1,N}^N)$ and $\bar{p}_1^N = (p_{1,1}^N, \ldots, p_{1,N}^N)$ are given by

$$(\bar{u}_1^N)_i = (K^N)_{i,j}^{-1} \langle \hat{L}_j, \hat{u}_1(x) \rangle_1$$
$$(\bar{p}_1^N)_i = (M^N)_{i,j}^{-1} \langle \hat{L}_j, \hat{p}_1(x) \rangle_{L^2},$$

where $\bar{\mathbf{x}}_1^N = [\bar{u}_1^N, \bar{p}_1^N]^T$.

From the above, we can obtain the following equation for the coefficient vector of the finite-dimensional optimal state

$$\bar{\mathbf{x}}_2^N = -[i\omega I - (\mathcal{A}^N - \theta^{-1}\mathcal{B}^N(\mathcal{B}^N)^* G^N)]^{-1} \theta^{-1} \mathcal{B}^N (\mathcal{B}^N)^* \bar{r}^N.$$

6 Numerical Results

For the numerical optimization, we optimize the following finite-dimensional cost function evaluated at the optimal finite-dimensional control (12) with respect to the location of the center of the control, x_c :

$$\mathrm{J}^{\mathrm{N}}(\mathrm{F}_{\mathrm{opt}}^{\mathrm{N}}) = \int_0^T \{(\bar{\mathrm{x}}_1^{\mathrm{N}})^* \mathrm{Q}^{\mathrm{N}} \bar{\mathrm{x}}_1^{\mathrm{N}} - \theta^{-1} (\bar{\mathrm{r}}^{\mathrm{N}})^* \mathcal{B}^{\mathrm{N}} (\mathcal{B}^{\mathrm{N}})^* \bar{\mathrm{r}}^{\mathrm{N}}\} \, \mathrm{dt}. \tag{15}$$

For the offending noise modeled as a simple harmonic wave, we consider three different frequencies: $f = \omega/2\pi = 10\,Hz$, $f = 173\,Hz$, the second harmonic resonance frequency of the one dimensional cavity, and $f = 346\,Hz$, the third resonance frequency. In each case, we calculate the optimal location of the center of the control, with a radius, $a = 0.1$, and graph the norm of the overall reduced noise field versus the norm of the offending noise with the control situated at the optimal location. The degree of approximation is equal to 16, and all these calculations are performed using the MATLAB Optimization Toolbox. The other parameters used in the calculations are set as follows: $c = 346\,\frac{m}{s}$, $\theta = 10^{-6}$, $\rho = 1.21\,\frac{kg}{m^3}$, $d1 = 1000$, $d2 = 10^5$, $\zeta = 29 + i0.07$, and the amplitude of the offending pressure field is chosen to have the value of $2\,Pa$. The following graphs summarize our simulations.

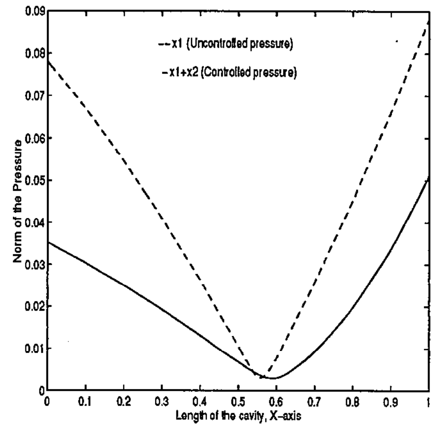

Figure 1: Norm of the pressure fields vs x, with control located optimally at $x = 0.9$ for frequency =10. Total reduction=6 dB.

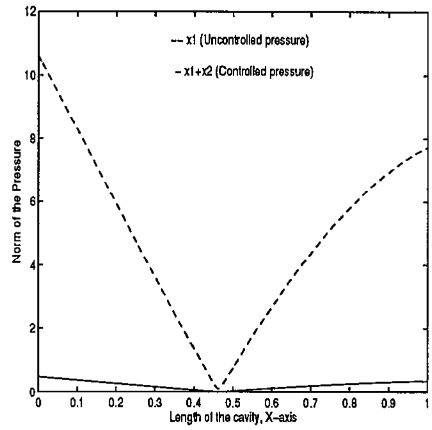

Figure 2: Control located optimally at $x = 0.1$ for frequency=173. Total reduction=26 dB.

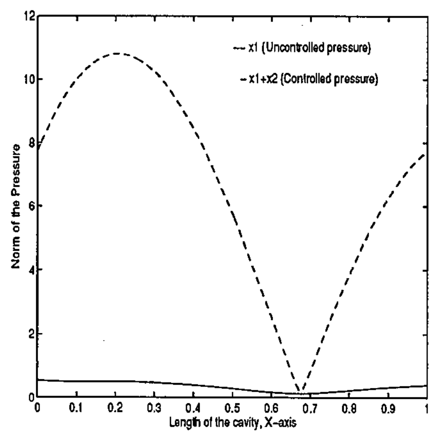

Figure 3: Control located optimally at $x = 0.5039$ for frequency= 346. Total reduction= 24 dB.

From these graphs we see that for the first two cases, the optimal location of the control is at one of the two ends of the cavity where the offending noise has the largest norm, and for the last case the optimal location is at the middle which is a good compromise for attenuating the two peaks of the offending noise. To see the effectiveness of putting the control at the optimal location for $f = 173$, we calculate the overall noise field for the control located at $x = 0.55$, which is a non-optimal location, and compare the results to the optimal case.

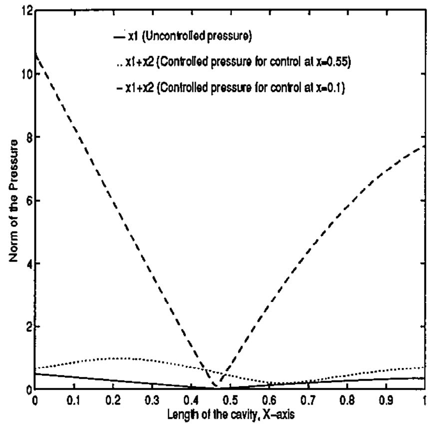

Figure 4: Comparison of optimal and non-optimal locations in reducing the pressure field for frequency =173.

While the overall reduction of noise for the optimal location is 26 dB, for the non-optimal location the reduction level is 21 dB. Also, from the graph one can see that in the case of the control at $x = 0.55$, the noise can locally increase (see the middle region), while for the optimally located control, the noise is reduced everywhere.

7 Conclusions

In this paper, we have considered the problem of finding the optimal location of controls for an active noise control problem. We have formulated a control strategy and an appropriate optimization problem, and have developed a numerical scheme based on the Legendre-Galerkin method to calculate the feedback control and the optimal location for the control. Our numerical results indicate that our control strategy is successful at attenuating the offending harmonic noise and situating the control at its optimal location offers much improved performance over an arbitrarily located control. The future goals involve extending the theoretical results to the problem of finding the optimal location of sensors that are used to estimate the unknown offending noise.

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