Application of a Multivariable Adaptive Controller with PID Structure to a Wastewater Treatment Plant with D-N configuration.

A.M. Macarulla, J. Flórez*, J.X. Ostolaza, E. Ayesa**

Department of Electronics and Control Engineering
C.E.I.T. - Centro de Estudios e Investigaciones Técnicas de Guipuzcoa
P.O. Box 1555, 20009 San Sebastián, Spain

Abstract

In this paper a multivariable adaptive controller with PID structure based on the algorithm developed by Yusof and Omatu (1993) using the Minimum Variance strategy, has been applied to a model of a municipal Wastewater Treatment Plant (WWTP) with nitrification and denitrification processes (D-N configuration).

This plant presents a non-linear process with low controllability. The aim of the controller is to improve the overall performance of the system. The process must satisfy the quality requirements of the effluent water in terms of its concentrations of nitrates and ammonia. The manipulable variables are the Disolved Oxygen level in the oxic reactor and the Internal Recycle flow rate.

The results presented show the response of the plant and the controller under a seasonal variation of temperature. They prove that the operation of the plant is improved, and running costs are lower.

1. Introduction

Contemporary society is getting concerned about the importance of conserving the environment. In this context, growing urban sprawls generate wastewater, among other residues.

The authorities are conscious of this problem and are establishing regulations about wastewater quality (EU Council Directive 91/271/EEC). These regulations limit the maximum concentration of dangerous substances, especially organic matter, nitrogen and phosphorus in the effluent

Wastewater Treatment Plants (WWTP) are systems whose aim is to protect the water environment from the negative effects of wastewater.

The presence of nitrogen compounds in the plant effluent pollutes the receiving water (river, sea, etc.). Therefore, one of the most important objectives of the Wastewater Treatment Plant (WWTP) is to eliminate those compounds.

Biological wastewater treatment is an example of a successful large-scale process of biotechnology resulting from the coordinated application of engineering and microbiology.

The high costs of wastewater treatment together with its increasing importance justify the efforts to obtain optimum systems of design and operation for wastewater treatment plants, identifying those with maximum efficiency and minimum cost.

The introduction of advanced control strategies to wastewater treatment plants will allow management of the multiple processes involved, guaranteeing the effluent requirements and decreasing the running costs.

The great variety of mechanisms and processes that manage plant behaviour, the wide range of time response, and the uncertainty associated with the process are driving the control strategies toward hierarchical policies to apply **multivariable-adaptive** and **predictive-adaptive** controllers.

The type of controller used in this paper, developed by Yusof and Omatu (1993), with slight modifications (Macarulla, 1996), is a Multivariable Adaptive Controller with PID structure based on the Generalized Minimum Variance strategy and can be viewed as an extension to the multivariable self-tuning controller proposed by Koivo (1980), with the difference that this one uses the model-following polynomial matrix, in the form of a transfer function. Although the Minimum Variance strategy is for *linear* and *time-invariant* systems, the multivariable PID controller can be applied to *time-variant* and *non-linear* plants, as is shown in the application.

(*) Dr. J. Flórez is with the Department of Electrical, Electronic and Control Engineering, School of Engineering, University of Navarra, P.O. Box 1674, 20009 San Sebastián, Spain

(**) Dr. E. Ayesa is with the Environmental Engineering Section of CEIT.

2. The Process

The process considered here is a municipal Wastewater Treatment Plant (WWTP) with Nitrification and Denitrification (D-N configuration) whose diagram is represented in Figure 1.

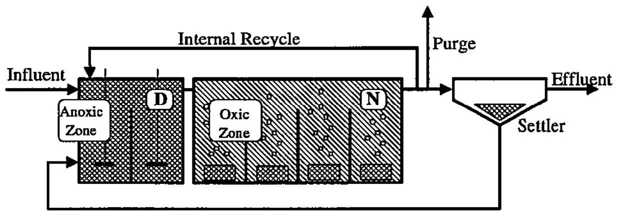

Figure 1. Wastewater plant diagram (D-N configuration)

Nitrification is the bacterial oxidation from ammonia to nitrates and nitrites by nitrificant bacteria. On the other hand, **Denitrification** is the process that reduces nitrates and nitrites to gas compounds of nitrogen by microorganisms which use these components instead of oxygen in the respiration process when oxygen falls short.

The first zone of the biological reactor (D zone), in Figure 1, has no aeration system. It must eliminate the organic material in the influent water using the nitrates as an oxidizing agent (Denitrification), the second one (N zone) is aerated and eliminates the rest of the organic material and the ammonia (Nitrification).

This plant configuration needs an internal recycle to support of the Denitrification process. This recycle supplies nitrates from the nitrification stage to the denitrification zone.

2.1 The Model of the Plant

The model of the plant used for the proposed application is based on the general model of activated single-sludge (Henze *et al.,* 1986) and the model of the dynamic response of the Settler (Urrutikoetxea and García de las Heras, 1994) (Appendix 1).

The most important aims of the control policy are, in this order: to maintain the plant operative, to comply with the requirements of effluent quality, and operate the process with maximum efficiency and minimum running costs.

The model and the controller have been implemented in *Simulink 1.3c* under *Matlab 4.2c*, and *Watcom C/C++ 10.0.*

2.2 Control Variables

The controller should be implemented in order to optimize the plant operation against the load profile. It must satisfy the quality requirements of the water and respond to perturbations such as seasonal variations, load disturbances, etc.

From a theoretical analysis of controllability, the control variables which are to be used in the present process are the **Dissolved Oxygen** level in the oxic reactor (N zone) and the **Internal Recycle** flow rate, which provides nitrates to the anoxic zone of the plant (D zone).

The DO level, is the most studied operable variable in the operation of wastewater treatment plants. The lower limit are the process requirements and the maintenance of the suspension of solids. On the other hand, an excess of ventilation is very expensive and does not raise substantially the level of dissolved oxygen because of the effect of the dissolved oxygen saturation in water.

Under typical operation, the desired DO level in the oxic reactor is fixed at a specific value. This level is the reference of a PI controller that evaluates the required air flow.

Proper operation of the Internal Recycle is based on the following points. On the one hand, its flow rate must be high enough to get the nitrate nitrogen concentration in the anoxic zone not to limit the denitrification. On the other hand, an excessive pumping can also inhibit the denitrification because of the contribution of oxygen from the oxic zone.

Under classical operation, Internal Recycle in D-N plants is set at a fixed value too.

In this manner, the controller considers the plant as a *2x2* system, where the output variables are the nitrate nitrogen (SNO) and ammonia nitrogen (SNH) concentrations at the outflow. The controller evaluates as input variables the desired Dissolved Oxygen (DO) level and the suitable Internal Recycle.

2.3 Operational Reference

The standard requirements of nitrates and ammonia concentrations of the plant effluent are daily averages and not instantaneous data. Moreover, the influent concentrations during the day have very different values.

The controller used in this paper needs an instantaneous reference signal to calculate the output error and to evaluate the control signal. Because of this, the average daily requirement must be reconverted to instantaneous values.

However, it is impossible to eliminate the variations of the daily cycle using the control effort. The aim must be to obtain the daily average values.

So, the strategy used to obtain an operational reference is to suppose that the system is near to its operation point, and the reference signal will be the output of the day before, measured at the same time, plus an adjustment factor obtained from the error in the output average of the day before. That is to say,

$$SNH(t) = SNH(t-24h) + K_{SNH}(\overline{SNH}_{ref} - \overline{SNH}_{yest}) \quad (1)$$

$$SNO(t) = SNO(t-24h) + K_{SNO}(\overline{SNO}_{ref} - \overline{SNO}_{yest}) \quad (2)$$

where,

$SNH(t)$: reference signal to the controller of ammonia nitrogen concentration evaluated at instant t

$SNO(t)$: reference signal to the controller of nitrate nitrogen concentration evaluated at instant t

$SNH(t\text{-}24h)$: output concentration of ammonia nitrogen measured the day before at the same time

$SNH(t\text{-}24h)$: output concentration of nitrate nitrogen measured the day before at the same time

$\overline{SNH}_{ref}$: required daily average of ammonia nitrogen concentration

$\overline{SNO}_{ref}$: required daily average of nitrate nitrogen concentration

$\overline{SNH}_{yest}$: output daily average of amonia nitrogen concentration of the day before

$\overline{SNO}_{yest}$: output daily average of nitrate nitrogen concentration of the day before

K_{SNH} : constant value (0.9)

K_{SNO} : constant value (0.9)

3. The Control Algorithm

This section is a summary of the algorithm. It is divided into three different parts: first, the multivariable adaptive controller is derived based upon Generalized Minimum Variance strategy; afterwards the PID structure is derived for the multivariable case, and in the last, the multivariable controller adopts the PID structure.

3.1 Multivariable Adaptive Controller

The CARMA model of the system is considered,

$$A(z^{-1})y(t)=z^{-k}B(z^{-1})u(t)+C(z^{-1})\xi(t) \quad (3)$$

where,

$y(t)$: measured output vector $(nx1)$

$u(t)$: control input vector $(nx1)$

$\xi(t)$: $(nx1)$ vector of uncorrelated random variables with zero mean and covariance $E\{\xi(t)\cdot\xi^T(t)\}=r_\xi$.

$$A(z^{-1})=I+A_1z^{-1}+A_2z^{-2}+...+A_{na}z^{-na} \qquad (nxn) \quad (4)$$

$$B(z^{-1})=B_1z^{-1}+B_2z^{-2}+...+B_{nb}z^{-nb}\ (B_1\neq[0]) \qquad (nxn) \quad (5)$$

$$C(z^{-1}) = I + C_1z^{-1} + C_2z^{-2} +...+C_{nc}z^{-nc} \qquad (nxn) \quad (6)$$

Using the Generalized Minimum Variance strategy (extended for multivariable systems) involves the control law that must minimize the next cost function,

$$\bar{I}_0 = E\left\{\left|P(z^{-1})y(t+k) - R(z^{-1})w(t)\right|^2 + \left|Q'(z^{-1})u(t)\right|^2\right\} \quad (7)$$

where,

$$P(z^{-1})=P_n(z^{-1})P_d^{-1}(z^{-1}) \quad (8)$$

and P, R and Q' are polynomial matrices (nxn) with the following form,

$$P(z^{-1})=I+P_1z^{-1}+P_2z^{-2}+...+P_kz^{-k} \quad (9)$$

$$R(z^{-1})=R_0+R_1z^{-1}+R_2z^{-2}+...+R_Nz^{-N} \quad (10)$$

$$Q'(z^{-1})=Q'_0+Q'_1z^{-1}+Q'_2z^{-2}+...+Q'_Mz^{-M} \quad (11)$$

At time t the control law must calculate $u(t)$ to minimize $\bar{I}_0$. The main obstacle of this minimization is that in the expression of $\bar{I}_0$ there are future terms of $y(t)$. If the model of the system (3) is used to predict the values of $y(t)$ at time $t+k$, the result is,

$$y(t+k)=A^{-1}(z^{-1})B(z^{-1})u(t)+A^{-1}(z^{-1})C(z^{-1})\xi(t+k) \quad (12)$$

Now the expression $A^{-1}(z^{-1})$ has infinite terms, this fact means that in the last term of equation (12) there were $\xi(t)$, $\xi(t-1)$,... these are not independent of $u(t-1)$, $u(t-2)$,..., thus, to eliminate these terms the next identity is considered,

$$A^{-1}(z^{-1})C(z^{-1})P_n(z^{-1})=P_d(z^{-1})E(z^{-1})+z^{-k}A^{-1}(z^{-1})F(z^{-1}) \quad (13)$$

The left part of equation (13) is separated into two terms: the first is relative to the future values of $\xi(t)$

and the second to past values. Therefore, the degree of polynomials $E(z^{-1})$ and $F(z^{-1})$ are $k-1$ and n_a+n_{pd}, respectively.

Equation (13) can be written too,

$$C(z^{-1})P_n(z^{-1})=A(z^{-1})P_d(z^{-1})E(z^{-1})+z^{-k}F(z^{-1}) \tag{14}$$

Thus, a pseudo-commutativity relationship is introduced in the derivation of the multivariable adaptive controller. In this case the next identity, parallel to expression (14), is introduced,

$$\tilde{C}(z^{-1})P_n(z^{-1})=\tilde{E}(z^{-1})A(z^{-1})P_d(z^{-1})+z^{-k}\tilde{F}(z^{-1}) \tag{15}$$

and it is defined,

$$\tilde{E}(z^{-1})F(z^{-1})=\tilde{F}(z^{-1})E(z^{-1}) \tag{16}$$

If (14) is premultiplied by $\tilde{E}(z^{-1})$ and (15) is postmultiplied by $E(z^{-1})$, using the identity (16), the next expression is,

$$\tilde{C}(z^{-1})P_n(z^{-1})E(z^{-1})=\tilde{E}(z^{-1})C(z^{-1})P_n(z^{-1}) \tag{17}$$

Now, if (15) is postmultiplied by $P_d(z^{-1})$ and this is substituted in the model of the system (3) and at this point, for simplicity, it is assumed that $C(z^{-1})$ is an identity matrix and therefore $\tilde{C}(z^{-1})$ is too. It subsequently (17) becomes,

$$P(z^{-1})y(t+k)=\tilde{F}(z^{-1})P_d^{-1}(z^{-1})y(t)+\tilde{E}(z^{-1})B(z^{-1})u(t)+\tilde{E}(z^{-1})\xi(t+k) \tag{18}$$

If $y^*(t+k/t)$ is defined as the predicted output at time $t+k$ based on the measurement outputs up to t, the prediction error can be written as,

$$e'(t+k)=P(z^{-1})[y(t+k)-y^*(t+k/t)] \tag{19}$$

or what amounts to the same thing,

$$e'(t+k)=\tilde{E}(z^{-1})\xi(t+k)=\xi(t+k)+\tilde{E}_1\xi(t+k-1)+\ldots+\tilde{E}_{k-1}\xi(t+1) \tag{20}$$

Since the future errors $-\xi(t+1),\ldots,\xi(t+k)-$ are uncorrelated with the past inputs and measured outputs $-u(t-1),u(t-2),\ldots,y(t),y(t-1),\ldots-$ they are therefore uncorrelated with the predicted output $y^*(t+k/t)$. So, the optimal k step ahead predictor can be written as,

$$P(z^{-1})y^*(t+k/t)=\tilde{F}'(z^{-1})P_d^{-1}(z^{-1})y(t)+\tilde{E}'(z^{-1})B(z^{-1})u(t) \tag{21}$$

Now, the cost function (7) can be written as,

$$\bar{I}_0=E\left\{\left|P(z^{-1})y^*(t+k/t)-R(z^{-1})w(t)\right|^2+\left|Q'(z^{-1})u(t)\right|^2\right\}+E\left\{\left|P(z^{-1})\tilde{E}(z^{-1})\xi(t+k)\right|^2\right\}=E\{I_1\} \tag{22}$$

To find the minimum cost with respect to $u(t)$, the differential of (22) with respect to $u(t)$ is set to zero. Hence,

$$B_1^T\left[P(z^{-1})y^*(t+k/t)-R(z^{-1})w(t)\right]+Q'(0)^TQ'(z^{-1})u(t)=0 \tag{23}$$

is obtained, where,

$$(B_1^T)^{-1}Q_0'^TQ'(z^{-1})=Q(z^{-1}) \tag{24}$$

Then,

$$P(z^{-1})y^*(t+k/t)-R(z^{-1})w(t)+Q(z^{-1})u(t)=0 \tag{25}$$

and substituting (21) into (25),

$$\tilde{F}(z^{-1})P_d^{-1}(z^{-1})y(t)+\tilde{E}(z^{-1})B(z^{-1})u(t)-R(z^{-1})w(t)+Q(z^{-1})u(t)=0 \tag{26}$$

renaming $\tilde{E}(z^{-1})B(z^{-1})=\tilde{G}(z^{-1})$ and $y_f(t)=P_d(z^{-1})y(t)$, equation (26) becomes,

$$\tilde{F}(z^{-1})y_f(t)+\tilde{G}(z^{-1})u(t)-R(z^{-1})w(t)+Q(z^{-1})u(t)=0 \tag{27}$$

Here $y_f(t)$ is the filtered output and $P_d(z^{-1})$ is the prefilter polynomial matrix, which is assumed to be diagonal. As the only components of the auxiliary output are those that depend on $y(t)$ it is defined,

$$\phi^*_y(t+k/t)=\tilde{F}(z^{-1})y_f(t)+\tilde{G}(z^{-1})u(t) \tag{28}$$

If the parameter of the plant model, $A(z^{-1})$ and $B(z^{-1})$, were known, the parameters of the controller $\tilde{F}(z^{-1})$ and $\tilde{G}(z^{-1})$, could be calculated using (14), (15) and (16).

However, in the case of unknown plant parameters, parameter estimation schemes, such as the recursive least-squares method, can be used to find the controller parameters. The inclusion of the recursive least-square estimator in the control loop makes the controller exhibit self-tuning characteristics.

To identify these parameters a recursive **least-squares estimator with UD factorization** (Bierman, 1977) has been used, where $x(t)$ is the known parameter vector and $\theta_i(t)$ is the estimated parameter vector corresponding to *i-th* variable, that are,

$$x(t)=[y_f^T(t),y_f^T(t-1),\ldots,u^T(t-1),u^T(t-2),\ldots]$$

$$\theta_i(t)=[f_{i0}^0(t),f_{i1}^0(t),\ldots,f_{im}^0(t),f_{i0}^1(t),f_{i1}^1(t),\ldots,f_{im}^1(t),\ldots,$$
$$g_{i0}^0(t),g_{i1}^0(t),\ldots,g_{im}^0(t),g_{i0}^1(t),g_{i1}^1(t),\ldots,g_{im}^1(t),\ldots] \quad (29)$$

The measurement data $\phi_i(t+k)$ are given by,

$$\phi_i(t+k)=x^T(t)\theta_i(t)+x_i(t+k) \quad (30)$$

The controller parameter identification is the highest computational cost of the controller. However, it is interesting to note that for all the transmission channels, the covariance matrix is the same, so, it is calculated once in each step, the first, and is used in the remaining n-1.

Once the covariance matrix has been obtained the parameter identification of each variable, the vectors $\theta_i(t)$, can be parallelized, dividing the time of calculation by n.

3.2 PID Structure

The velocity-type form of the PID controller scheme is,

$$\Delta u(t)=u(t)-u(t-1)=K_P[e(t)-e(t-1)]+K_I e(t-1)+$$
$$K_D[e(t)-2e(t-1)+e(t-2)] \quad (31)$$

Substituting $e(t)$ by its expression in (31),

$$\Delta u(t)=K_P[w(t)-y(t)-w(t-1)+y(t-1)]+$$
$$K_I[w(t-1)-y(t-1)]+ \quad (32)$$
$$K_D[w(t)-y(t)-2w(t-1)+2y(t-1)+w(t-2)-y(t-2)]$$

assuming that $w(t)=w(t-1)=w(t-2)$, and grouping terms,

$$\Delta u(t)=K_I w(t)-[K_P+K_D]y(t)-[K_I-K_P-2K_D]y(t-1)-K_D y(t-2) \quad (33)$$

is obtained and later, identifying the terms of the adaptive controller with the terms of the PID controller, the PID Multivariable Adaptive Controller will be obtained.

3.3 The Multivariable Adaptive Controller with PID Structure

The multivariable adaptive control law can be written as follows,

$$[\tilde{G}(z^{-1})+Q(z^{-1})]u(t)=-\tilde{F}(z^{-1})y_f(t)+R(z^{-1})w(t) \quad (34)$$

It is intended that expression (34) takes the form of (33), and to eliminate the steady-state error, in the expression of $y_f(t)$ based upon $y(t)$ it is supposed that $z=1$, that is to say,

$$y_f(t)=(I+P_d)^{-1}y(t) \quad (35)$$

To obtain this term identification it is necessary that the degree of the polynomial matrix $\tilde{F}(z^{-1})$ are two,

$$\tilde{F}(z^{-1})=\tilde{F}'_0+\tilde{F}'_1 z^{-1}+\tilde{F}'_2 z^{-2} \quad (36)$$

A further requirement is to introduce an integral control action into the control law. This can be done by letting,

$$[\tilde{G}(z^{-1})+Q(z^{-1})]u(t)=V'(1-z^{-1})u(t) \quad (37)$$

where V' is a diagonal matrix (nxn) of the form,

$$V'=\begin{bmatrix} V_1 & 0 & \ldots & 0 \\ 0 & V_2 & \ldots & 0 \\ & & \ldots & \\ 0 & 0 & \ldots & V_n \end{bmatrix} \quad (38)$$

The matrices obtained are,

$$K_P=V'^{-1}(\tilde{F}'_0-\tilde{F}'_2)(I+P_d)^{-1}$$
$$K_I=V'^{-1}(\tilde{F}'_0+\tilde{F}'_1+\tilde{F}'_2)(I+P_d)^{-1} \quad (39)$$
$$K_D=V'^{-1}\tilde{F}'_2(I+P_d)^{-1}$$

The algorithm can be summarized as follows,

i. Select the prefilter polynomial matrices $P_n(z^{-1})$ and $P_d(z^{-1})$, and also the gain matrix V'.

ii. Compute initial values of $y(t)$ and $y_f(t)$.

iii. Calculate the new values of $w(t)$, $y(t)$ and $y_f(t)$.

iv. Estimate the controller parameters $\tilde{F}'_0$, $\tilde{F}'_1$ and $\tilde{F}'_2$ using the U-D estimator.

v. Compute the control input.

vi. Set $t=t+1$.

vii. Go to *iii*.

4. Case Study

To show the performance behaviour of the controller some experiments have been carried out. The proposed control strategy has been implemented to the model of the municipal Wastewater Treatment Plant at Arazuri (Pamplona, Spain). The model of this plant has been developed by the Environmental Engineering Section of the CEIT.

The most relevant characteristics and operational variables of the plant are:

Influent flow rate (Q_{in}): 104,600 m^3/day
Oxic reactor volume: 38,135 m^3
Anoxic reactor volume: 16,344 m^3
Settler volume: 24,000 m^3

Sludge age (SRT): 18 days
Sludge recycle flow rate: $1.0 \cdot Q_{in}$
Internal recycle flow rate: 65,000 m^3/day
Dissolved Oxygen concentration (DO): 2 mg/l
Design temperature: 13°C

Some of the parameters of the model are shown in Appendix 2.

Since the dynamics involved in the process are certainly nonlinear, it has not been feasible to analyze the stability of the closed-loop system using the proposed control strategy. This stability test establishes limits to the two pre-filter and the gain diagonal matrices.

However, these matrices allow a wide range of values. Therefore, it has not been hard to define these matrices. After several tries, the diagonal values of the two pre-filter matrices have been fixed to -0.5, and the gain matrix ones to 200 and 100 respectively. These values have been validated in many other experiments, working at different operation points.

Water temperature supports seasonal variations. Biomass activity is generally favored by high temperature. For this reason, Wastewater Treatment Plants are designed to attain the desired effluent quality, even at the expected lowest water temperature.

The experiment presented in this paper consists of a gradual rise in temperature of the influent wastewater during spring time and a gradual decrease during fall, as shown in Figure 2.

First, the experiment has been carried out applying the traditional operation mode, that fixes control variables typically at their standard operation point -to 65,000 m^3/day of internal recycle, and to 2 mg/l of DO level- represented by the horizontal lines in Figure 3.

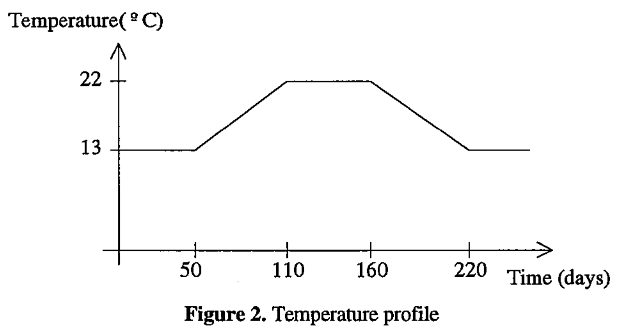

Figure 2. Temperature profile

With this regime, the nitrification rate increases as temperature does, and achieves an effluent ammonia concentration lower than the required level (1.5 mg/l). In the other way, the denitrification rate increases too, but it can not compensate the increase in nitrate nitrogen arising from the strengthened nitrification, so the daily average concentration of nitrate nitrogen is notably higher than the required level (8 mg/l). These results are shown by dotted lines in Figure 4.

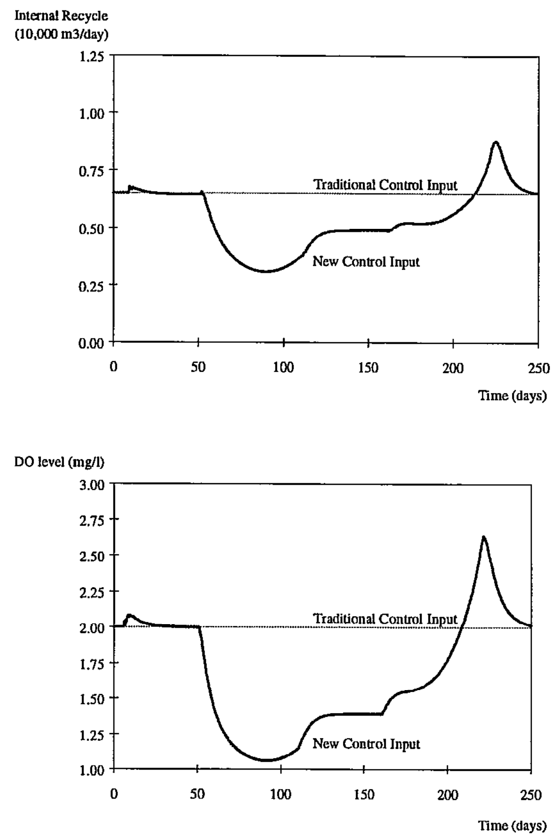

Figure 3. Profile of control variables (input)

Afterwards, the experiment has been carried out applying the multivariable controller, setting a sampling time of 0.01 days. The values of control variables evaluated by the controller are represented by the solid lines in Figure 3, and achieved levels of nitrate nitrogen (SNO) and ammonia nitrogen (SNH) daily average concentrations are represented by solid lines in Figure 4.

The results obtained with the proposed controller show that the operation of the plant is more efficient, mainly in two aspects. First, because obtained concentration levels are closer to those required ones. Second, on account of the pumping economy. Pumping energy, and particularly aeration energy represents one of the most significant costs in running a Wastewater Treatment Plant.

Several experiments have been carried out with the proposed controller. The results prove that about 15-20% pumping and aeration energy savings are possible with this regime.

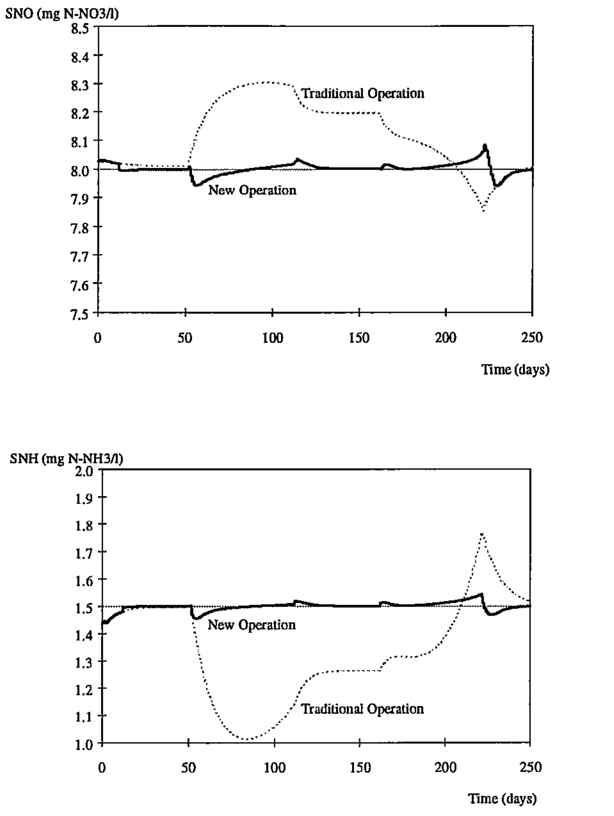

Figure 4. Profile of daily average effluent concentrations (output)

5. Conclusions

The results obtained using the proposed controller applied to a WWTP are satisfactory. It causes a better performance of the plant because the levels obtained are nearer to those required by environmental law and a notable reduction in the running costs is produced. Thus, the operation of the plant is notably more efficient.

At present, the application of those automatic adaptive controllers to these type of plants is in implementation phase. The results obtained here establish a step towards this objetive.

6. Acknowledgements

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8. Appendices

8.1 Appendix 1. Mathematical Model N. 1 of the IAWQ

This is the most widely used model to describe wastewater treatment systems. It was presented as a consequence of the "Task Group on Mathematical Modelling", established by the IAWQ in 1983.

The development of such a mathematical model allows the study of different configurations of Activated Sludge processes. The computer simulation of the model gives information about the suitability of different control strategies.

The kinetics of the model are shown in Figure 5.

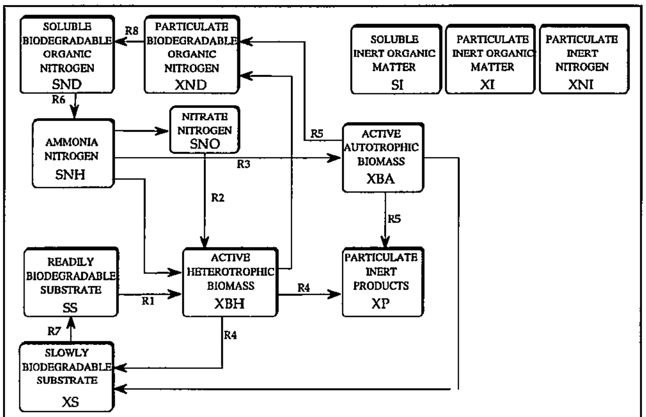

Figure 5. Reactions of the model N. 1

8.1.1 Equations of the Dynamic Model of Biodegradation by Activated Sludge

The concentrations of all the organic compounds are expressed in terms of Chemical Oxygen Demand (COD).

Growth of Heterotrophic Biomass

Corresponds to R1 and R2 reactions of aerobic and anoxic growth respectively.

$$\rho_1 = \left(\mu_H \frac{S_S}{K_S + S_S}\right)\left(\frac{S_O}{K_{OH} + S_O}\right)\left(\frac{S_{NH}}{K_{NHG} + S_{NH}}\right) X_{BH} \tag{A.1}$$

$$\rho_3 = \left(\mu_A \frac{S_{NH}}{K_{NH} + S_{NH}}\right)\left(\frac{S_O}{K_{OA} + S_O}\right) X_{BA} \tag{A.2}$$

Where $\mu_H \frac{S_S}{K_S + S_S}$ and $\mu_A \frac{S_{NH}}{K_{NH} + S_{NH}}$ are their specific growth-rate, and K_S and K_{NH} are their saturation constants.

The heterotrophic biomass can grow under anoxic conditions. It uses the oxygen from the nitrate nitrogen as a source of energy, in the process known as denitrification. The kinetics of this reaction is,

$$\rho_2 = \left(\mu_H \cdot \frac{S_S}{K_S + S_S}\right)\left(\frac{K_{OH}}{K_{OH} + S_O}\right)\left(\frac{S_{NO}}{K_{NO} + S_{NO}}\right) \cdot \eta_g \left(\frac{S_{NH}}{K_{NHG} + S_{NH}}\right) X_{BH} \tag{A.3}$$

Decay of Heterotrophic and Autotrophic Biomass

These are the reactions R4 and R5 where b_H and b_A are the specific decay-rate of heterotrophic and autotrophic biomass respectively.

$$\rho_4 = -b_H \cdot X_{BH} \tag{A.4}$$

$$\rho_5 = -b_A \cdot X_{AH} \tag{A.5}$$

Ammonification of the Soluble Organic Nitrogen

This reaction occurs because of the presence of heterotrophic bacteria and corresponds to R6, where K_A is the ammonification constant.

$$\rho_6 = -K_A \cdot S_{ND} \cdot X_{BH} \tag{A.6}$$

Hydrolysis of the Slowly Biodegradable Substrate (XS) and the Particulate Biodegradable Organic Nitrogen (XND)

These are R7 and R8 reactions respectively. K_H is the hydrolysis constant and K_X is its saturation constant.

$$\rho_7 = K_H \frac{\left(\frac{X_S}{X_{BH}}\right)}{K_X + \left(\frac{X_S}{X_{BH}}\right)} X_{BH} \cdot \left[\left(\frac{S_O}{K_{OH} + S_O}\right) + \eta_H \left(\frac{K_{OH}}{K_{OH} + S_O}\right)\left(\frac{S_{NO}}{K_{NO} + S_{NO}}\right)\right] X_{BH} \tag{A.7}$$

$$\rho_8 = \rho_7 \cdot \frac{X_{ND}}{X_S} \tag{A.8}$$

The stoichiometric coefficients that regulate the mass transference in the reactions are:

- Heterotrophic yield Y_H. Reaction R1 efficiency
- Autotrophic yield Y_A. Reaction R3 efficiency
- Inert fraction of biomass f_P
- Nitrogen/COD relation in heterotrophic biomass, i_{XB}

- Nitrogen/COD relation in inert mass, i_{XP}

8.1.2 Equations of the Model of the Dynamic Response of the Settler

This model allows the prediction of some variables in the process, the mass of solids, the concentration at the bottom and the sludge blanket level.

The mass per unit of stored area in the settler is calculated with the next equation,

$$M = \int_0^{h_c} c(h)\cdot dh \qquad (A.9)$$

where $c(h)$ is the concentration of the blanket at depth h.

Now, the law of variation of the concentration depending on h and the integration limits are,

$$(c - X_{AT})^n = (X_{RT} - X_{AT})^n \left(\frac{h_c - h}{h_c}\right) \quad 0<h<h_c \qquad (A.10)$$

$$X_{RT} = K_1 + (K_1 - X_{AT})\cdot e^{K_2 h_c} \qquad (A.11)$$

where,

X_{AT}: concentration of particulate mass in the effluent to the settler
X_{RT}: concentration in the bottom of the settler
h_c: sludge blanket depth
h: depth from the bottom of the settler
K_1, K_2 and n: constants of the model

To calculate the variation of the sludge blanket depth, the equation (A.9) must be resolved, and afterwards must be differentiated with respect to time. Then the next equation,

$$\dot{h}_c = \frac{\dfrac{Q_s\cdot X_{AT} - Q_r\cdot X_{RT}}{a_c}}{X_{AT} + \left(\dfrac{n}{n+1}\right)\cdot(X_{RT} - X_{AT})} \qquad (A.12)$$

is obtained, where a_c is the settler area.

Equations (A.11) and (A.12) represent the model of the settler.

8.2 Appendix 2. Coefficients of the Model

8.2.1 Units

The units used for the state variables and for the coefficients of the model are shown in table A.1.

8.2.2 Values of Coefficients

The values of the coefficients at T°C are obtained from their value at 20°C and from their parameter of variation with respect to temperature (table A.2). These coefficients obey the equation of Arrhenius (A.13).

$$Coef(T) = Coef(20°C)\cdot Coef\_var^{(T-20)} \qquad (A.13)$$

Table A.1. Units of coefficients and state variables

μ_H	day^{-1}
μ_A	day^{-1}
K_S	mg COD/litre
K_{NH}	mg N/litre
K_H	mg COD/(mg COD day^{-1})
K_X	mg COD/mg COD
K_A	1/(mg COD day^{-1})
b_H	day^{-1}
Y_H	mg COD/mg COD
Y_A	mg COD/mg N
b_A	day^{-1}
f_P	mg COD/mg COD
i_{XB}	mg N/mg COD
i_{XP}	mg N/mg COD
K_{NHG}	mg N/mg N
S_I	mg COD/litre
S_S	mg COD/litre
X_I	mg COD/litre
X_S	mg COD/litre
X_{BH}	mg COD/litre
X_{BA}	mg COD/litre
X_P	mg COD/litre
S_O	mg Oxygen/litre
S_{NO}	mg N/litre
S_{NH}	mg N/litre
S_{ND}	mg N/litre
X_{ND}	mg N/litre

Table A.2. Values of coefficients and their variation

Coefficient	Value at 20° C	Parameter of variation
μ_H	3,00	1,070
μ_A	0,55	1,103
K_S	5,00	1,000
K_{NH}	0,80	1,123
K_H	2,00	1,070
K_X	0,02	1,000
K_A	0,08	1,070
b_H	0,62	1,070
Y_H	0,67	1,000
Y_A	0,24	1,000
b_A	0,13	1,103
f_P	0,08	1,000
i_{XB}	0,08	1,000
i_{XP}	0,06	1,000
K_{NHG}	0,27	1,000