Feedback Linearization of a Multi-Input SI-Engine System for Idle Speed Control

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Abstract

The idle speed problem is a classical example of an automotive control application. The set-up corresponds to a disturbance rejection problem where the main plant output (engine speed) has to be maintained at a (low) constant value despite the torque disturbances acting on the engine crank-shaft (servo-steering pump, air-conditioning, etc.). The relevance (comfort, fuel consumption, etc.) and the technical challenges (nonlinear plant with large time delays) of this control problem have led to many different control strategies. PID [7], LQ [6], $\mathcal{H}_\infty$ [3, 11], ℓ_1 [2], fuzzy control [1], adaptive control [8], sliding mode [5] and neural networks [10] are some of the frameworks used to tackle this problem.
Feedback linearization was also investigated in some papers [5], but the engine's induction to power stroke delay was neglected. Unfortunately, this effect, that depends also on the engine speed, is often the limiting factor for the controller design. For this reason, in the here presented work, this delay is approximated with first order low pass elements, which have an engine-speed dependent time constant.
The resulting nonlinear plant with two inputs (air-bypass valve and spark-advance) is not affine in the inputs. Nevertheless, by introducing additional static compensations the plant is shown to be exactly feedback linearizable. The linearized plant permits the application of well-known linear control design methods. In this paper the different bandwidths of the two input-channels are used in a setting similar to the one presented by [3, 11] to guarantee an optimal engine operation, both under transient and steady-state conditions.

1. Notation

The following notation is used in this paper

θ	:	first input, air-bypass valve
$\dot{m}$	:	air-bypass valve mass flow rate
P	:	intake manifold pressure
P_a	:	atmospheric pressure
T	:	intake manifold air temperature
R	:	air gas constant
V_m	:	intake manifold volume
$\dot{M}$	:	cylinder air mass flow rate
N	:	engine speed
T_e	:	net engine torque
T_l	:	engine load torque
K_τ	:	delay parameter
τ	:	induction to power stroke delay
δ	:	second input, spark-advance
K_i	:	regular load torque parameter
J_e	:	effective engine rotational inertia.

2. Nonlinear SI-Engine Model

The nonlinear engine model is based on a work of Powell and Cook [9]. A block diagram of it is presented in Fig. 1.

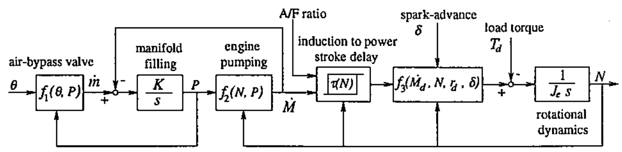

Figure 1: Nonlinear engine model

Assumptions: A constant intake manifold temperature, no intake manifold leaks and constant stoichiometric air/fuel-ratio is assumed in this work.

Under these assumptions the model of the analyzed plant is described by the following set of equations. For the throttle-plate behavior

$$\dot{m} = f_1(\theta, P) = f_\theta(\theta) f_P(P) \tag{1}$$

where

$$f_\theta(\theta) = \beta_0 + \beta_1\theta + \beta_2\theta^2$$

and

$$f_P(P) = \begin{cases} 1 & , \quad P \leq P_a/2 \\ \frac{2}{P_a}\sqrt{PP_a - P^2} & , \quad P > P_a/2, \end{cases}$$

for the manifold air-mass-balance

$$\dot{P} = K(\dot{m} - \dot{M}), \text{ where } K = R\,T/V_m, \tag{2}$$

for the engine pumping behavior

$$\dot{M} = f_2(N, P) = \alpha_0 NP + \alpha_1 NP^2, \tag{3}$$

for the induction to powerstroke delay

$$\tau = K_\tau/N, \tag{4}$$

for the engine torque output

$$\begin{aligned} T_e \;\; &= \;\; \varphi_0 + \varphi_1\dot{M}(t-\tau) + \varphi_2\delta + \varphi_3\delta^2 + \\ &\quad\;\; \varphi_4\delta N + \varphi_5 N + \varphi_6 N^2, \end{aligned} \tag{5}$$

for the load torque

$$T_l = N^2/K_i^2 + T_d \tag{6}$$

and finally, for the engine's rotational dynamics

$$\dot{N} = \frac{1}{J_e}(T_e - T_l). \tag{7}$$

Equations (1-7) are the mathematical description of the nonlinear engine model. They are the basis for the following synthesis of the feedback linearization, the linear feedback controller and for all the simulations. The disturbance torque T_d is assumed to be unmeasurable and unpredictable, i.e., all disturbances that are measurable or predictable are assumed to be compensated by a feed-forward controller (not discussed in this paper).

One important aspect of the controller synthesis is the following fact. The control of the engine-speed using the spark-advance path is much faster than using the air-bypass channel. Therefore, a typical idle speed control-transient must split up in two parts. In the first part, the controller uses the spark-advance as main input and after that, the engine speed becomes controlled by acting on the air-bypass and the spark-advance returns to its nominal value.

Of course, the first phase should be as short as possible, since during that period a non-ideal combustion takes place. It is the specific contribution of this paper to investigate an approach that minimizes these effects by enhancing the response characteristics of the slower control-channel using nonlinear methods.

3. Delay Approximation

The induction to power stroke delay can not be described by a finite dimensional ODE, and therefore it is often approximated by rational transfer functions for the controller design. Moreover, it is obvious, that the induction to power stroke delay is engine speed dependent. For these reasons it is approximated by a first order element whose "time-constant" depends on the inverse engine speed (this corresponds, as will become clearer below, to a bilinear system).

$$\dot{y}(t) = \tilde{\tau}(t)^{-1}(-y(t) + u(t)) \tag{8}$$

This form of the approximation (no finite zeros) is necessary to guarantee that the relative degree of the complete system will be equal to its order and will therefore contain no zero dynamics [4]. Higher order approximations (several elements (8) in series-connection) are also possible, although with the technique used below, this would lead to some form of state-extension (additional integrators at the input). The variable $\tilde{\tau}$ is chosen to minimize the error area between the step response $h(t)$ of a linear reference system

$$G(s) = e^{-s\tau(N_0)}$$

and the step-response $\tilde{h}(t)$ of (8) for a fixed engine speed $N = N_0$, i.e.,

$$\int_0^\infty |h(t) - \tilde{h}(t)|\, dt \overset{!}{=} \min. \tag{9}$$

It turns out that the best choice for $\tilde{\tau}$ in the sense of (9) is given by

$$\tilde{\tau} = \frac{\tau(N_0)}{1.678346\ldots}. \tag{10}$$

Remark 1: Notice that the engine speed does not influence the weighting factor 1.678346... and that therefore the choice (10) is generally applicable, i.e., that

$$\tilde{\tau}(t) = \frac{\tau(t)}{1.678346\ldots} = \frac{K_\tau}{N(t)\;1.678346\ldots} \tag{11}$$

is a point-wise optimal solution.

Defining a new state-variable $y = x_2$ and the input $u = \dot{M}$, the description of the powerstroke delay approximation used below is then given by

$$\dot{x}_2 = \frac{1.678346\ldots}{K_\tau} N(t)(\dot{M} - x_2) \tag{12}$$

Remark 2: The proposed approximation works well only if the dynamics of the engine speed $N(t)$ is substantially slower than the one of the cylinder air mass flow rate $\dot{M}(t)$. Fortunately, in typical engine settings this is the case (the manifold pressure P varies much faster than the engine speed N).

4. Feedback Linearization

Before discussion the main issue of this section, a slight technical difficulty has to be surmounted. The plant description as introduced in (1-7) does not fit completely into the usual framework, i.e., the system's equations are not affine in the two inputs. By introducing two fictitious new inputs u_1 and u_2 and solving the following two quadratic equations

$$u_1 = \frac{1}{J_e}\left(\varphi_2 + \varphi_4 N + \varphi_3\,\delta\right)\delta \tag{13}$$

$$u_2 = \beta_0 + \beta_1\theta + \beta_2\theta^2 \tag{14}$$

(which corresponds to a static nonlinear transformation in each input-channel) the problem can be transformed to its standard form.

With the above modifications of the system description and the static compensation of the input nonlinearities, the system can be written as follows

$$\begin{aligned}
\dot{x}_1 &= a_0 + a_1x_1 + a_2{x_1}^2 + a_3x_2 + u_1 - a_dT_d \\
\dot{x}_2 &= a_4x_1x_2 + a_5x_1^2x_3 + a_6x_1^2x_3^2 \\
\dot{x}_3 &= a_7x_1x_3 + a_8x_1x_3^2 + a_9f_P(x_3)u_2
\end{aligned} \tag{15}$$

where $x_1 = N$ and $x_3 = P$ and the coefficients a_i follow directly from the "physical" parameters of the model (1-7).

The special structure of this system will play a crucial role in the following considerations. Instead of pursuing a "regular" square MIMO-system feedback linearization [4], a cascade-like approach is chosen. As a first step the fast spark-channel in (15) is linearized by a pre-compensation involving the engine speed only

$$u_1 = v_1 - (a_0 + a_1x_1 + a_2{x_1}^2) \tag{16}$$

where v_1 is the new spark-channel input. Notice that the link to the (slower) air-channel (represented by the term a_3x_2) is not canceled. Beside the fact that this is not needed (the link is already linear) this would also make little sense for the control-problem at hand.

To formalize this step a first coordinate transformation is introduced

$$z_1(t) = x_1(t) \tag{17}$$

and by construction

$$\dot{z}_1 = a_3x_2 + v_1 - a_dT_d. \tag{18}$$

An obvious choice for a second coordinate transformation is

$$\begin{aligned}
z_2(t) &= x_2(t) \\
z_3(t) &= a_4x_1x_2 + a_5x_1^2x_3 + a_6x_1^2x_3^2.
\end{aligned} \tag{19}$$

The resulting dynamic equations are

$$\begin{aligned}
\dot{z}_2(t) &= z_3 \\
\dot{z}_3(t) &= \varphi(x, v_1) + \psi(x)u_2 - \xi(x)T_d
\end{aligned} \tag{20}$$

where $x = [x_1, x_2, x_3]'$ and (all time dependencies have been omitted for space reasons)

$$\begin{aligned}
\varphi &= (v_1 + a_3x_2)\left(a_4x_2 + x_1x_3\left(2a_5 + 2a_6x_3\right)\right) \\
&\quad + x_1^3x_3^2\left(a_4a_6 + a_5a_8 + 2a_6a_7 + 2a_6a_8x_3\right) \\
&\quad + a_4^2x_1^2x_2 + x_1^3x_3a_5\left(a_4 + a_7\right) \\
\psi &= a_9x_1^2f_P(x_3)\left(a_5 + 2a_6x_3\right) \\
\xi &= a_d\left(a_4x_2 + 2a_5x_1x_3 + 2a_6x_1x_3^2\right).
\end{aligned} \tag{21}$$

Choosing the air-bypass control input as follows

$$u_2(t) = \psi(x)^{-1}\left[v_2 - \varphi(x, v_1)\right] \tag{22}$$

produces a linear system whose structure is depicted in Fig. 2. The function $\tilde{\xi}(.)$ is defined by

$$\tilde{\xi}(z) = \xi \circ \Phi(z) \tag{23}$$

where $\Phi(z)$ is the inverse coordinate transformation

$$x = \Phi(z) = \begin{bmatrix} z_1 \\ z_2 \\ \frac{-a_5}{2a_6} + \sqrt{\frac{a_5^2}{4a_6^2} - \frac{a_4z_1z_2 - z_3}{a_6z_1^2}} \end{bmatrix} \tag{24}$$

(the ambiguity arising from the solution of the involved quadratic equation can be resolved by physical arguments).

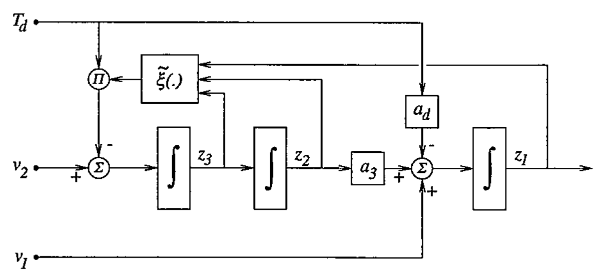

Figure 2: Structure of the feedback linearized system

Remark: The control (22) is singular for all points on the set defined by $\psi(x) = 0$. However, this set is not relevant for physically meaningful values of the two variables $x_1 = N > 0$ and $x_3 = P > 0$ (the three parameters a_5, a_6 and a_9 are all positive and in idle conditions the manifold pressure P remains always below the ambient pressure P_a).

5. Linear Controller

After having compensated all nonlinearities the next step is to design a linear feedback controller that satisfies all specifications and limitations of the idle-speed problem. This design-step is performed here for illustration purposes using a cascade-like LQR-approach. The control structure of the complete system with the linear controller is shown in Fig. 3.

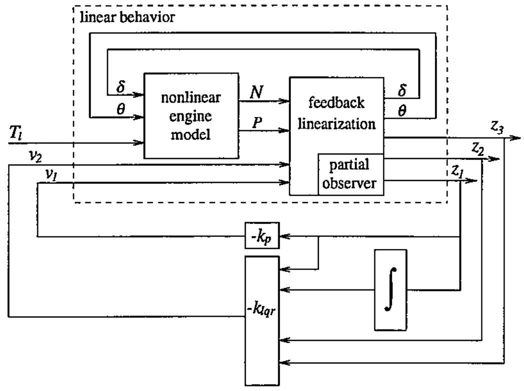

Figure 3: Control structure of the complete system

The controls are assumed to have the following bounds

$$\delta \in [-10°, 30°], \quad \theta \in [0°, 90°]. \tag{25}$$

The controller is designed in a hierarchical manner: in a first step the v_1-loop is closed by a proportional feedback k_p, and in a second step the v_2-loop is extended by an additional integrator on the engine speed (to eliminate constant disturbances) and a LQR-regulator k_{lqr} is designed for the remaining SISO plant. Notice that the controller utilizes 4 states of which only three (engine speed, integrated engine speed and manifold pressure) are available. The state x_2 has to be estimated using a (partial) observer. The resulting linear controller is of order 2 (one for the partial observer, one for the integral action).
The simulations shown in Fig. 4 and 5 were performed on the original nonlinear plant, i.e., with the speed-dependent induction-to-powerstroke delay. The achievable responses with the limitations (25) are comparable with those published in previous papers. The main benefit of the proposed approach is that the different speed requirements for the two input channels can be easily satisfied. In fact the v_1 channel is closely (but not completely) linked to the spark-action. For this reason manipulating this control influences substantially the dynamics of the spark action. Since this channel is driven by the speed error only, in steady-state no spark action is automatically guaranteed.

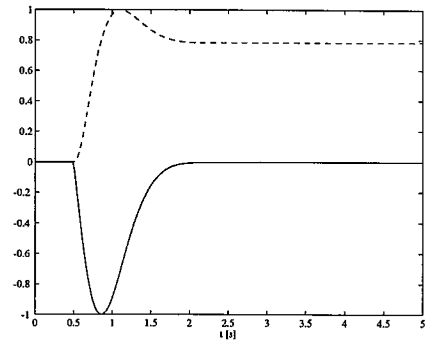

Figure 4: Normalized state variables: engine speed (solid, nominal value = 740 rpm, range=60 rpm), manifold pressure (dashed, nominal value = 0.28 bar, range = 0.45 bar) for a 5 Nm disturbance step

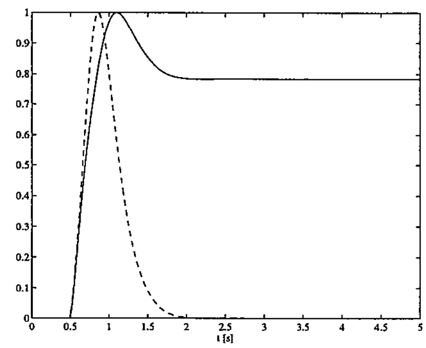

Figure 5: Normalized control actions: spark (dashed, nominal value = 5°, range = 7°), air-bypass (solid, nominal value = 15°, range = 24°) for a 5 Nm disturbance step

The slower air-bypass channel is then designed to compensate for constant load torques and its dynamics can be as slow as necessary to satisfy robustness and actuator requirements.

6. Conclusion

In this paper it has been shown that the idle-speed system is feedback linearizable even when the nonlinear powerstroke delay is taken into consideration.
For the outer linear controller an intuitive design approach, which is based on physical information, remains possible despite the multivariable structure of the plant. The design approach is cascade-like and the physical intuition is not completely lost due to the nonlinear transformations.

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