A Robust-Adaptive Locomotion Controller for 9-Link Bipeds with Rapidly Varying Unknown Parameters

S.G. Tzafestas(*), A.E. Krikochoritis(*) and C.S. Tzafestas(**)

(*) Intelligent Robotics and Automation Laboratory
Department of Electrical and Computer Engineering
National Technical University of Athens
15773, Zografou Campus, Athens, Greece

(**) Laboratoire Robotique de Paris-CRIIF
Univ. P. et M. Curie, 10-12 av. de l'Europe
78140 Velizy-Villacoublay, France

Abstract

In this paper a robust-adaptive control scheme is applied to a 9-link (8-degrees-of-freedom) biped robot under the assumption that the biped is subject to rapidly time-varying parameters. The eight degrees of freedom correspond to two hip, two knee, two ankle, and two metatarsal joints, while the motion is constrained to be on the sagittal plane. The robust-adaptive controller consists of two components; a parameter updating law based on the σ-modification principle, and a nonlinear control law which is designed so that to ensure that all signals involved remain bounded. Extensive simulation experiments were carried out which show the practicality and effectiveness of the proposed robust-adaptive controller for biped walking.

1 Introduction

One of the primary motivations for designing biped robots is to perform tasks in environments that are too dangerous for human beings. To be a satisfactory substitute for the human being, the robot must be able to enter a region originally designed for human access, and perfom tasks that are not already automated and normally require the capabilities of a person. One measure of the success of a biped design is how well it can emulate the agility of a human being. Therefore, a useful biped robot needs feet. It is not possible for a passive platform to stand in a single, stable position if it is supported on only two points. However, a dynamic system can balance on two points like stilts if the supporting points are allowed to move and are controlled by a sufficiently sophisticated control system. The stiff legged stilt biped must remain in a continuous state of motion to maintain balance.

In this paper a 9-link planar biped model is studied which includes not only the main links : legs, thighs, and trunk, but also a two segments foot. This biped has two hip, two knee, two ankle, and two metatarsal joints, with one d.o.f. each of them. The motion is constrained on the sagittal plane, and as a consequence, the total number of degrees of freedom is going to be limited enough, always depending on the phase of the walking being executed. This two dimensional motion can in fact be achieved in reality, as it was shown by the Kenkyaku-2 biped [1] which has a steel pipe attached to the lowest end of the leg in order to maintain the lateral balance.

The goal for the choice of such a model is the achievement of a more satisfactory substitute for the anthropomorphic gait, giving great attention to the model which describes the foot. Most of the previous biped studies consider the foot as one solid element. Here, each foot is composed of two rigid parts connected at the transverse tarsal joint. The calcancus and talus as a single unit form the proximal segment, and the remaining bones and joints of the foot the distal segment.

2 The Walking Pattern

The most popular analytical model of walking is the one based on the hypothesis that walking is performed so as to have the least expenditure of energy [2]. In our 9-link biped robot model, it is assumed that at the middle of the supporting leg period, that is when the swing leg moves before the suporting leg, a new phase of the gait exists. This is the *kick phase*, where an ankle motion of the supporting leg is achieved, so that a maximum of the vertical force just before the collision appears.

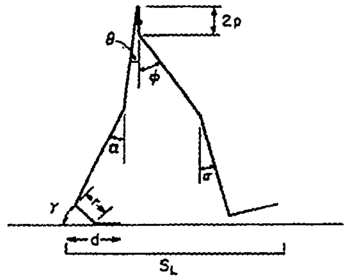


Fig.1 A ballistic walking model

Furthemore, it is assumed that the torque, applied to the knee joint is zero, the desired trajectories of the angles θ and γ are specified as a function of α, and in addition the reference signals are chosen such that to use the effects of the gravity in a way that increases the angular momentum during the single leg supporting phase (see Fig.1).

Therefore, trying to utilize the gravity effect skillfully, the following walking pattern is adopted in this study.

1. The body is always kept upright.

2. The knee of the supporting leg extends straight and as a result the first assumption is satisfied, since the

relation between the thigh angle θ and the shank angle α is $\theta=\alpha$.

3. The ankle and foot joint of the supporting leg is free except for the kick-phase.

4. The foot of the swing leg is kept parallel to the ground.

5. The leg-support-exchange is done in an instant (there is no double-legs-supporting phase).

6. At the touchdown, the knee joint of the swing leg is kept in bending state.

7. The touchdown of the swing leg is assumed to occur in two stages. Firstly, the toes of the swing leg take a collision with the ground and then the collision of the heel follows.

8. The same reference signals are supplied at each step repeatedly.

The reference signals shown in Fig.5 (thin line) describe the desire change of the angular position of the robot joints during the first two steps. It is obvious, that the reference signals during the first step are a little different, since the robot starts walking from the upright posture. The signals of the second step are recurrently used in every step.

3 Kinematic and Dynamic Model of a 9-Link Biped Robot

The kinematic and dynamic equations of the 9-link biped robot model can be found as was done for the 5-link biped robot in [3].

3.1 Kinematic Model

The 9-link biped under consideration is shown in Fig.2. It includes the trunk (link 5) and four links in each leg which represent the thigh (links 4 and 6), the shin (links 3 and 7), the heel (links 2 and 8) and the metatarsal (links 1 and 9). The links labeled l_i (i=1,...,9) are joined together at ideal pin joints. Hence, it has two hip joints (joints 4 and 5), two knee joints (joints 3 and 6), two ankle joints (joints 2 and 7) and two metatarsal joints (joints 1 and 8), which are assumed to be ideal (without friction) rotational joints (with one d.o.f each of them) driven by independent electric DC motors. At each joint, except the one which contacts the ground, there is an ideal torque τ_i. Since the motion of the biped robot is constrained to be on the sagittal plane, for a definite description, we use as generalized variables the set of the angles of each link i with the vertical, which is denoted as θ_i. The direction of the θ_i is as shown in Fig.2.

There are four parameters for each link : the mass of the link m_i, its moment of inertia about the c.o.g. I_i, the length of the link l_i, and the distance between the c.o.g. and the lower joint d_i. Fig.3 shows these parameters for the i-th link. For the heel the notation is somewhat diferent (Fig.4). The numerical values used for all these parameters have been taken from [4].

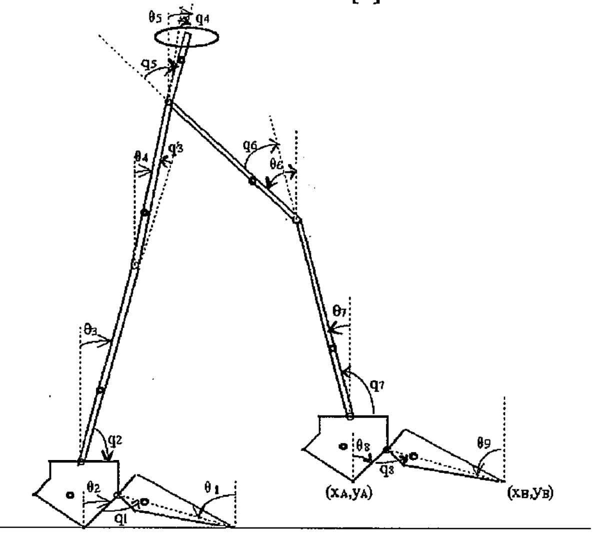
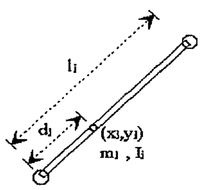


Fig.2 9-link planar biped robot model



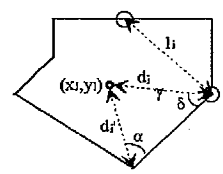


Fig.3 Parameters of the i-th link *Fig.4 Parameters of the heel*

The kinematic model which describes the relation between the velocity of the foot of the swing leg and the change of the generalized variables is given by the equations

$$V_A = \begin{bmatrix} \dot{x}_A \\ \dot{y}_A \end{bmatrix} = \begin{bmatrix} -l_1 \cos\theta_1 \\ -l_1 \sin\theta_1 \end{bmatrix}\dot{\theta}_1 + \begin{bmatrix} -l_2 \cos(\theta_2+\gamma) \\ l_2 \sin(\theta_2+\gamma) \end{bmatrix}\dot{\theta}_2 + \begin{bmatrix} l_3 \cos\theta_3 \\ -l_3 \sin\theta_3 \end{bmatrix}\dot{\theta}_3 + \begin{bmatrix} l_4 \cos\theta_4 \\ -l_4 \sin\theta_4 \end{bmatrix}\dot{\theta}_4$$

$$+ \begin{bmatrix} l_6 \cos\theta_6 \\ l_6 \sin\theta_6 \end{bmatrix}\dot{\theta}_6 + \begin{bmatrix} l_7 \cos\theta_7 \\ l_7 \sin\theta_7 \end{bmatrix}\dot{\theta}_7 + \begin{bmatrix} l_8 \cos(\theta_8+\gamma) - d_8 \cos(\theta_8+\delta) - d_8{}' \cos(\theta_8-\alpha) \\ -l_8 \sin(\theta_8+\gamma) - d_8 \sin(\theta_8+\delta) + d_8{}' \sin(\theta_8-\alpha) \end{bmatrix}\dot{\theta}_8$$

$$V_B = \begin{bmatrix} \dot{x}_B \\ \dot{y}_B \end{bmatrix} = V_A{}^{1-7} + \begin{bmatrix} l_8 \cos(\theta_8+\gamma) \\ -l_8 \sin(\theta_8+\gamma) \end{bmatrix}\dot{\theta}_8 + \begin{bmatrix} l_9 \cos\theta_9 \\ l_9 \sin\theta_9 \end{bmatrix}\dot{\theta}_9 \qquad (1)$$

3.2 Dynamic Model

Non-kick action in single-leg-supporting phase : Here, the dynamic equations are studied when the biped robot has one supporting leg and there is no raising of the heel (θ_1, θ_2 constant). Applying the Lagrange dynamic equation, the equations of motion take the following closed form, for the non-kick phase :

$$\mathbf{D}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{G}(\boldsymbol{\theta}) = \mathbf{T}_\theta \qquad (2)$$

where T_θ is the generalized torque which corresponds to the variable θ_i, $\mathbf{D}(\theta)$ is the positive symmetric 9x9 inertia matrix, $\mathbf{C}(\theta,\dot{\theta})$ is the 9x9 matrix (with zero diagonal terms) which includes terms from the centrifugal and Coriolis torques, and $\mathbf{G}(\theta)$ is the 9-dimensional vector which represents the gravitational torques. The form of these matrices is given in [5] and due to space limitation are not included here.

Kick action in single-leg-supporting phase : As mentioned before, since our biped robot has a 2-link foot we can adopt the biped locomotion with kick-action (only in the single leg support phase) which was firstly employed in Kenkyaku-2 [1]. However, as seen from the shape of the human foot, the ankle torque of the supporting leg can decrease the walking speed but cannot increase it. Since the reduction of the speed causes an energy loss, and according to the first assumption that the biped robot has to keep the properties of a balistic model (to keep its energy at a constant level), in the continuous walking of this study the ankle joint of the supporting leg is set to be free except for the kick phase, when raising of the heel exists. In this case, an additional variation of the angle θ_2 occurs, while the angle θ_1 keeps on a constant value.

Based again on the Lagrange dynamic model, the equations of motion, during the kick phase, take a form similar to (2) (the matrices **D**, **C**, **G** are similar to the corresponding matrices with some additional terms caused by the raising of the heel, (see [5]).

In the two previous phases, introducing the transformations

$$q_1 = \theta_1 + \theta_2 \qquad q_5 = \theta_5 + \theta_6$$
$$q_2 = -90^0 + \varepsilon + \theta_2 - \theta_3 \qquad q_6 = \theta_6 - \theta_7$$
$$q_3 = \theta_3 - \theta_4 \qquad q_7 = -90^0 + \varepsilon + \theta_8 + \theta_7$$
$$q_4 = \theta_4 - \theta_5 \qquad q_8 = \theta_8 + \theta_9$$
$$\tau_i = \sum_{j=1}^{9} T_{\theta j}(\partial\theta_j / \partial q_i) \quad i = 1,\dots,8$$

where q_i is the joint angular position, and τ_i is the real driving torque exerted by each independent actuator to each joint of the biped robot (the torque at the toes of the supporting leg is zero because of the existence of one unpowered d.o.f.), we get the following closed form of dynamic equations

$$\mathbf{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)} = \tau$$

This dynamic model will be used in the control part of the paper.

Leg-support-exchange : The walking pattern adopted here implies that the leg-support-exchange is done in an instant. In this way, the double-leg-supporting phase is omitted. Then, just after the touchdown of the swing leg, the exchange of the supporting leg occurs. Hence, we assume that the biped robot is instantly, just before the collision, on the air. As a result, at the time of the swing leg collision with the ground, the constraint $x_T = y_T =$constant, which exists during the single leg supporting phase, is lost. In this case, two more variables (the coordinates x_T, y_T of the supporting leg toes) are required for an exact description of the position of the biped robot. The elements of the inertial matrix $\mathbf{D_a}$ for this case can again be found in [5].

Collision of the swing leg with the ground : For a mobile robot, the collision with the environment is an ordinary affair and one of the effects of robot collision is the abrupt change of the joint angular velocities. As mentioned in the walking pattern, the collision with the ground occurs in two stages. Firstly, the toes (B) of the swing leg take a collision with the ground and then the collision of the heel (A) follows. Thus the velocity change is given by [6] :

$$\Delta\dot{\theta} = \mathbf{D_a}^{-1}\,{}^{B}\mathbf{J_a}^{T}\left({}^{B}\mathbf{J_a}\,\mathbf{D_a}^{-1}\,{}^{B}\mathbf{J_a}^{T}\right)^{-1}\Delta\dot{x}_B \tag{3}$$

where $\mathbf{D_a}$ is the inertia matrix of the robot model when it is instantly on the air and ${}^{B}\mathbf{J_a}$ is the associated Jacobian matrix. After the first collision, the velocity of the toes (B) of the swing leg vanishes, hence

$$\underset{\text{after}}{\dot{\theta}_B} = \underset{\text{before}}{\dot{\theta}_B} + \mathbf{D_a}^{-1}\,{}^{B}\mathbf{J_a}^{T}\left({}^{B}\mathbf{J_a}\,\mathbf{D_a}^{-1}\,{}^{B}\mathbf{J_a}^{T}\right)^{-1}(-\underset{\text{before}}{\dot{x}_B}) \tag{4}$$

Then, the collision of the heel (A) of the swing leg occurs. Hence, the general relation takes the form

$$\Delta\dot{\theta} = \mathbf{D_a}^{-1}\,{}^{A}\mathbf{J_a}^{T}\left({}^{A}\mathbf{J_a}\,\mathbf{D_a}^{-1}\,{}^{A}\mathbf{J_a}^{T}\right)^{-1}\Delta\dot{x}_A \tag{5}$$

Similar to the first case, after the second collision, the velocity of the heel (A) goes to zero. Hence

$$\underset{\text{after}}{\dot{\theta}_A} = \underset{\text{before}}{\dot{\theta}_A} + \mathbf{D_a}^{-1}\,{}^{A}\mathbf{J_a}^{T}\left({}^{A}\mathbf{J_a}\,\mathbf{D_a}^{-1}\,{}^{A}\mathbf{J_a}^{T}\right)^{-1}(-\underset{\text{before}}{\dot{x}_A}) \tag{6}$$

where $\underset{\text{before}}{\dot{\theta}_A}$ is equal to $\underset{\text{after}}{\dot{\theta}_B}$ which is computed from the first collision.

4 Robust-Adaptive Control of Biped Robots

The locomotion activity and gait,in particular, belongs to the class of highly automated motions. When a man is walking in a steady regime or in an environment imposing small disturbances, the central nervous system is not involved. When large disturbances occur, the system actions are directed only to the preservation of the system overall stability, i.e., towards preventing the system from falling down. This requirement is of primary importance in biped locomotion [7].

In this paper, we apply a robust-adaptive control scheme which aims at minimizing the sensitivity of the system performance under the presence of large and rapid time-variations in the robot parameters. This control scheme ensures that all signals of the robot system are bounded, and that the mean tracking error is of the order of the parameter variations which are not required to be small [8].

The dynamic equations of a biped robot whose parameters may explicitly depend on time, have the following form

$$\mathbf{D(q,t)\ddot{q}} + \frac{\partial\mathbf{D(q,t)}}{\partial t}\mathbf{\dot{q}} + \mathbf{C(q,\dot{q},t)\dot{q} + F(q,t)} = \tau \tag{7}$$

where $\mathbf{F(q,t)} = \dfrac{\partial\mathbf{P(q,t)}}{\partial t}$, and P(q,t) is the potential energy of the system. Here, because of parameter time-variations, the important skew-symmetry property of the matrix $\left(\dot{\mathbf{D}} - 2\mathbf{C}\right)$ is written in the form :

$$\dot{\mathbf{q}}^T\left[\left(\frac{\partial D_{ij}}{\partial \mathbf{q}}\right)^T \dot{\mathbf{q}} - 2\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},t)\right]\dot{\mathbf{q}} = 0 \ \forall \mathbf{q} \in \Re^n \tag{8}$$

where D_{ij} is the ij-th element of $\mathbf{D}(\mathbf{q},t)$.

Define now the new vector $\mathbf{s}$ as :

$$\mathbf{s} = \dot{\tilde{\mathbf{q}}} + \boldsymbol{\Lambda}\tilde{\mathbf{q}} \tag{9}$$

where $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$ is the n-vector of tracking errors and $\boldsymbol{\Lambda}$ is a symmetric positive definite matrix, or more generally a matrix such that $\boldsymbol{\Lambda}$ is Hurwitz. Furthemore, we may interpret $\mathbf{s}$ as a 'velocity error' term

$$\mathbf{s} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r \quad \text{where} \quad \dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \boldsymbol{\Lambda}\tilde{\mathbf{q}} \tag{10}$$

The control objective is : *For a given reference signal* $\mathbf{q}_d(t)$, *generate the applied torque* τ *for the biped robot* (7) *with unknown and time-varying parameters so that all signals in the robot system are bounded and the joint position* $\mathbf{q}$ *tracks* $\mathbf{q}_d$ *as close as possible.*

To achieve such an objective the biped model (7) is first parameterized as :

$$\begin{aligned} D(q,t)\dot{s} + C(q,\dot{q},t)s &= \tau - D(q,t)\ddot{q}_r - C(q,\dot{q},t)\dot{q}_r - F(q,t) - \frac{\partial D(q,t)}{\partial t}\dot{q} \\ &= \tau - D_K\ddot{q}_r - C_K\dot{q}_r - F_K - Y(q,\dot{q},\dot{q}_r,\ddot{q}_r,t)\,a(t) - \frac{\partial D(q,t)}{\partial t}\dot{q} \end{aligned} \tag{11}$$

where $(-\mathbf{D}_K\ddot{\mathbf{q}}_r - \mathbf{C}_K\dot{\mathbf{q}}_r - \mathbf{F}_K)$ is the term of the model with unknown parameters, $\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,t)$ is a nxr matrix of known functions for some r>0, and $\mathbf{a}(t) \in \Re^r$ contains parameters which may be time-varying. In (11) the regressor $\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,t)$ is bounded for bounded $\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r$, and $\ddot{\mathbf{q}}_r$.

The following assumptions are made about the biped robot model (7) :

(i) $\|\mathbf{a}(t)\| \le \rho_0$, $\|\dot{\mathbf{a}}(t)\| \le \rho$ for some constants $\rho_0 > 0$, $\rho > 0$.

(ii) $\left\|\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\right\| \le \gamma\, f(\mathbf{q})$ for some constant $\gamma>0$ and known bounded $f(\mathbf{q})$ for bounded $\mathbf{q}$.

(iii) $\mathbf{q}_d(t)$, $\dot{\mathbf{q}}_d(t)$, $\ddot{\mathbf{q}}_d(t)$ are bounded.

The new characteristics of this biped robot model are: (i) the presence of the unknown and time-varying vector $\mathbf{a}(t)$ (which is going to be estimated), and (ii) at the same time the existence of the unknown term $\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\dot{\mathbf{q}}$. If the parmeters of the terms $\mathbf{D}(\mathbf{q},t)$ and $P(\mathbf{q},t)$ were known, then $\mathbf{a}(t)$ and $\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\dot{\mathbf{q}}$ could be calculated so that the control law

$$\tau(t) = \mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,t)\,\mathbf{a}(t) + \frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\dot{\mathbf{q}} - \mathbf{K}_D\,\mathbf{s} \tag{12}$$

could be implemented (like the computed torque methodology), which guarantees global stability and asymptotic tracking. However, for unknown $\mathbf{D}(\mathbf{q},t)$ and $P(\mathbf{q},t)$ we have to develop an adaptive control scheme (control law - update law) which is robust with respect to the time-variation of $\mathbf{a}(t)$ and $\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\dot{\mathbf{q}}$.

A first feedback controller suggested by (12) could include a 'feedforward' term $\mathbf{D}_K\ddot{\mathbf{q}}_r + \mathbf{C}_K\dot{\mathbf{q}}_r + \mathbf{Y}\hat{\mathbf{a}}$, a simple PD term $\mathbf{K}_D\,\mathbf{s}$, and a term which causes robustness with respect to the time-variations of $\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\dot{\mathbf{q}}$.

A first type of the parameter update law could include a term $\dot{\hat{\mathbf{a}}} = -\boldsymbol{\Gamma}^{-1}\mathbf{Y}^T\mathbf{s}$, and at the same time a term $-\boldsymbol{\Gamma}^{-1}\sigma(t)\hat{\mathbf{a}}(t)$ suggested by the σ-modification technique which takes care of the robustness to the time-variations of $\mathbf{a}(t)$.

Taking into account the above considerations, the following controller and update law structures are proposed :

Controller:

$$\tau(t) = D_K(q,t)\ddot{q}_r(t) + C_K(q,\dot{q},t)\dot{q}_r(t) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r,t)\hat{a}(t) + \tau_R(t) - K_D s(t) \tag{13}$$

where $\tau_R(t)$ is the term which will ensure the robustness with respect to $\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\dot{\mathbf{q}}$.

Update law :

$$\dot{\hat{\mathbf{a}}} = -\Gamma^{-1}\mathbf{Y}^T(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,t)\mathbf{s}(t) - \Gamma^{-1}\mathbf{s}(t)\hat{\mathbf{a}}(t) \tag{14}$$

where $\sigma(t)$ is the switching signal :

$$\sigma(t) = \begin{cases} 0 & \text{if } \|\hat{a}(t)\| < M \\ \sigma_0\left(\frac{\|\hat{a}(t)\|}{M} - 1\right) & \text{if } M \le \|\hat{a}(t)\| < 2M \ (\sigma_0 > 0) \\ \sigma_0 & \text{if } \|\hat{a}(t)\| \ge 2M \end{cases} \tag{15}$$

which uses the a priori information that $\sup\|\mathbf{a}(t)\|$ is upper bounded by M.

Let us now consider the positive definite function

$$V(\mathbf{s},\tilde{\mathbf{a}}) = (1/2)\left(\mathbf{s}^T\mathbf{D}\mathbf{s} + \tilde{\mathbf{a}}^T\boldsymbol{\Gamma}\tilde{\mathbf{a}}\right)$$

where $\mathbf{D}=\mathbf{D}(\mathbf{q}(t),t)$ and $\tilde{\mathbf{a}}(t) = \hat{\mathbf{a}}(t) - \mathbf{a}(t)$.

Differentiating, and using (8) yields :

$$\begin{aligned} \dot{V}(s,\tilde{a},t) &= -\frac{1}{2}s^T\frac{\partial D(q,t)}{\partial t}\left(\dot{q}(t) + \dot{q}_r(t)\right) + s^T\tau - s^T\left[D_K\ddot{q}_r(t) + C_K\dot{q}_r(t) + Ya(t)\right] \\ &\quad + \tilde{a}^T(t)\Gamma\dot{\hat{a}}(t) - \tilde{a}^T(t)\Gamma\dot{a}(t) \\ &= -\frac{1}{2}s^T\frac{\partial D(q,t)}{\partial t}\left(\dot{q}(t) + \dot{q}_r(t)\right) + s^T\tau_R(t) - s^TK_Ds - \tilde{a}^T(t)\sigma(t)a(t) - \tilde{a}^T(t)\Gamma\dot{a}(t) \\ &\le \frac{1}{2}\left\|s^T(t)\right\|\left\|\frac{\partial D(q,t)}{\partial t}\right\|\left\|(\dot{q}(t) + \dot{q}_r(t))\right\| + s^T\tau_R(t) - s^TK_Ds - \tilde{a}^T(t)\sigma(t)a(t) - \tilde{a}^T(t)\Gamma\dot{a}(t) \end{aligned} \tag{16}$$

Then choosing :

$$\tau_R(t) = -\left(k_0\left\|(\dot{\mathbf{q}}(t) + \dot{\mathbf{q}}_r(t))\right\|f(\mathbf{q})\right)^2\mathbf{s}(t)$$

the inequality (16), becomes :

$$\dot{V} \le -\left(m(t)\|s(t)\| - \frac{\gamma}{4k_0}\right)^2 + \frac{\gamma^2}{16k_0^2} - s^TK_Ds - \tilde{a}^T(t)\sigma(t)a(t) - \tilde{a}^T(t)\Gamma\dot{a}(t) \tag{17}$$

where $m(t) = k_0\left\|\dot{\mathbf{q}}(t) + \dot{\mathbf{q}}_r(t)\right\|f(\mathbf{q})$ and $\tau_R(t) = -m^2(t)\mathbf{s}(t)$.

Finally, using (15) and the assumption (i) (Sec. 2) of the biped, one gets :

$$\dot{V} \le -\left(m(t)\|s(t)\| - \frac{\gamma}{4k_0}\right)^2 + \frac{\gamma^2}{16k_0^2} - s^T K_D s$$
$$-\left(\sigma(t)-\sigma_0\right)\tilde{a}^T\hat{a} - \sigma_0\left[\|\hat{a}(t)\| - \left(\frac{\rho_0}{2} + \frac{\|\Gamma\|\rho}{2\sigma_0}\right)\right]^2 + \left(\frac{\rho_0}{2} + \frac{\|\Gamma\|\rho}{2\sigma_0}\right)^2 \sigma_0 + \rho\rho_0\|\Gamma\| \tag{18}$$

In the inequality (18) we observe that since γ, ρ and ρ_0 are positive constants, then the first, third and fifth terms are negative definite functions and as a result have a negative contribution to $\dot{V}$, while the other terms are positive definite. Moreover, since $\sigma(t)$ defined in (15) satisfies the inequality :

$$\left|\left(\sigma(t)-\sigma_0\right)\tilde{a}^T(t)\hat{a}(t)\right| \le 12\sigma_0 M^2 \tag{19}$$

it follows that $\dot{V} \le 0$ in a region $V \ge V_0$, for $\hat{a}(t)$, $s(t)$ outside a certain bounded set so that the first, third and fifth terms predominate (contributing negatively) in this inequality. The bound V_0 is dependent on the other positive definite terms. Therefore, this robust-adaptive control scheme causes the boundedness of $V(s,\tilde{a},t)$ and as a consequence the signals $s(t)$ and $\hat{a}(t)$ are bounded, which, in view of (9), (10) and (13) implies that $q(t)$, $\dot{q}(t)$ and $\tau(t)$ are also bounded.

Now the tracking error $\tilde{q}(t)$ will be computed. Working, in a similar way on $s(t) = \dot{\tilde{q}}(t) + \Lambda\tilde{q}(t)$, one can derive the following inequality for the tracking error $\tilde{q}(t)$ [5] :

$$\int_{t1}^{t2} \|\tilde{q}(t)\|^2 dt \le \alpha_0\left(\frac{\gamma^2}{k_0^2} + \rho\right)(t2 - t1) + \beta_0 \tag{20}$$

for some constants $\alpha_0 > 0$, $\beta_0 > 0$ and $t2 > t1 \ge 0$.

To implement the controller (13), one needs the knowledge of f(**q**) to generate the bounding signal m(t). A more sophisticated choice of f(**q**) admits a wider class of $\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}$, but it may make the implementation of m(t) more complicated. Note also that the above design does not need the knowledge of the bounds γ and ρ.

Furthermore, for a chosen f(**q**), different choices of k_0 in generating m(t) may have different effects on the tracking performance, while increasing k_0 may reduce the effect of γ in the mean error.

Finally, let us note that for the signal boundedness and the mean tracking error (20), the parameter variations characterized by γ and ρ are not required to be small.

For the 9-link biped robot, we have assumed the presence of time variations in the trunk parameters of the robot (mass m_5, rotational inertia I_5, distance d_5 from the hip joint to the mass center of the trunk). This is a very reasonable assumption. Therefore, here, the vector $\mathbf{a}(t) \in \Re^4$, which contains all the unknown and rapidly time-varying trunk parameters, takes the form

$$\mathbf{a}(t) = [I_5,\ m_5 d_5,\ m_5 d_5^2,\ m_5]$$

which results in a specific function $\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,t)$.

Clearly, the only thing one has to know about the time-varying parameters are the upper bounds of the variations, so that he has an estimation of M in (15). Here, the variation (which is not really known) is considered such that the parameters m_5, d_5, and I_5 take values in the region (26.95kg, 71.05kg), (0.252m, 71.05m), and (1.2925kgm, 3.4075kgm), respectively, i.e., the mass parameters m_5 and I_5 vary 45% around the constant value that they would have if no time-variation existed, and the parameter d_5 varies 10% around the corresponding value. Thus

$$|a_1(t)| = |I_5| \le 3.4075 = \varphi_1$$
$$|a_2(t)| = |m_5 d_5| \le 71.05 \cdot 0.308 = 21.88 = \varphi_2$$
$$|a_3(t)| = |m_5 d_5^2| \le 6.74 = \varphi_3$$
$$|a_4(t)| = |m_5| \le 71.05 = \varphi_4$$
$$M = \sup\|a(t)\| = \sup\sqrt{\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2} = 74.72 \approx 75$$

Furthermore, there exist constants $\gamma>0$, $\rho>0$ such that

$$\|\dot{a}(t)\| \le \rho \qquad \left\|\frac{\partial \mathbf{D}(\mathbf{q},t)}{\partial t}\right\| \le \gamma\, f(\mathbf{q})$$

As mentioned before, these constants affect only the mean tracking error and have no influence on the design of control and update law. Hence

$$|\dot{a}_1(t)| = |\dot{I}_5| = 1.057dt = \rho_1$$
$$|\dot{a}_2(t)| = |\dot{m}_5 d_5 + m_5 \dot{d}_5| \le 8.78dt = \rho_2$$
$$|\dot{a}_3(t)| = |\dot{m}_5 d_5^2 + m_5 2 d_5 \dot{d}_5| \le 3.32dt = \rho_3$$
$$|\dot{a}_4(t)| = |\dot{m}_5| = 22.05dt = \rho_4$$
$$\rho = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2 + \rho_4^2} \approx 24dt$$

where dt is the sampling rate.

Here, f(**q**) is a known function which contains trigonometrical terms of the angular position q_i, and as a result it is bounded for bounded **q**. In particular, an upper bound is f(**q**)=1.

For the exact application of the control, one has to compute the positive definite matrices $\mathbf{\Lambda}, \mathbf{K}_D, \mathbf{\Gamma}$. The matrices $\mathbf{\Lambda}, \mathbf{K}_D$ are chosen to take values similar to those we should have chosen in the computed torque control case, in order to achieve the ideal convergence. It is remarked that this approach does not necessarily estimate the unknown and time-varying parameters exactly, but simply generates values that allow the desired task (bounded tracking error) to be achieved. 'Sufficient richness' conditions on the desired trajectory indicate how demanding the desired trajectory should be for tracking convergence to necessarily require parameter convergence. For example, in case of a constant desired trajectory, it would be more difficult to achieve parameter convergence. Hence, using our a priori knowledge of the desired trajectories of the robot joints, we choose different values for the terms of $\mathbf{\Gamma}$ for the cases that these signals don't include enough information. Also, the tracking error does not merely tend asymptotically to zero, but for all practical purposes, converges within finite time constants determined for a given trajectory by the values of the gain matrices $\mathbf{\Lambda}, \mathbf{K}_D$ and $\mathbf{\Gamma}$, themselves limited by the presence of high-frequency unmodeled dynamics and measurement noise.

5 Simulation Experiments

The 9-link biped robot, initially at upright posture (assuming the time-variation of the parameters m_5, d_5, I_5), is commanded a desired trajectory similar to that synthesized by the reference signals adopted in the walking pattern. The corresponding angular positions and position errors, during the first two steps (in a 3.5-sec interval), are plotted in Figures 5 and 6, respectively. These diagrams show clearly the very good tracking of the desired reference signals despite the presence of the uncertainty. Something that is also obvious from the fact that the average tracking error for the first and second step is 0.037 rads and 0.09 rads, respectively.

6 Conclusions

In this paper the effectiveness of a robust-adaptive control scheme applied to a 9-link biped robot was studied. The biped robot was assumed to have rapidly time-varying unknown parameters. The eight degrees of freedom correspond to two hip, two knee, two ankle, and two metatarsal joints, while the motion is constrained to be on the sagittal plane. The robust-adaptive control scheme involves a parameter updating law designed using the σ-modification technique, and a nonlinear control law, and ensures that all signals of the biped system are bounded, while the mean tracking error is of the order of the parameter variations which are not required to be small. A set of simulation experiments were performed under the assumption that there are time variations in the trunk parameters of the biped. These experiments have demonstrated the strong capabilities of the proposed gait control technique which is a good candidate for practical application.

7 References

[1] M.Yamada, J.Furusho and A.Sano, "Dynamic Control of Walking Robot with Kick-Action", *Proc. 1985 Intl. Conf. on Advanced Robotics (ICAR'85)*, pp. 405-412, Tokyo, 1985.

[2] S.Mochon, "A mathematical model of human walking", *Lectures on Mathematics in Life Sciences*, vol.14, Amer. Math. Soc., New York, 1981.

[3] S.Tzafestas, M.Raibert and C.Tzafestas, "Robust Sliding-mode Control Applied to a 5-Link Biped Robot", *J. Intel. Robotic Syst.*, vol.15, pp.67-133, 1996.

[4] H.Hemami, Y.F.Zheng and M.Hines, "Initiation of Walk and Tiptoe of Planar Nine-Link Biped", *Mathem. Biosci.*, vol.61, pp.163-189, 1982.

[5] S.G.Tzafestas, A.E.Krikochoritis and C.S.Tzafestas, "Robust Adaptive Gait Control of a 9-Link Biped Robot", *Tech. Report*, IRAL, NTUA,Greece(Dec. 1996).

[6] Y.F.Zheng and H.Hemami, "Impact Effects of Biped Contact with the Environment", *IEEE Trans. Syst., Man and Cybern.*, vol.14, No.3, pp.437-443, 1984.

[7] M.Vukobratovic, B.Borovac, D.Surla and D.Stokic, "*Scientific Fundamentals of Robotics 7-Biped Locomotion*", Springer, Berlin, 1990.

[8] Gang Tao, "On Robust Adaptive Control of Robot Manipulators", *Automatica*, vol.28, No.4, pp.803-807, 1992.

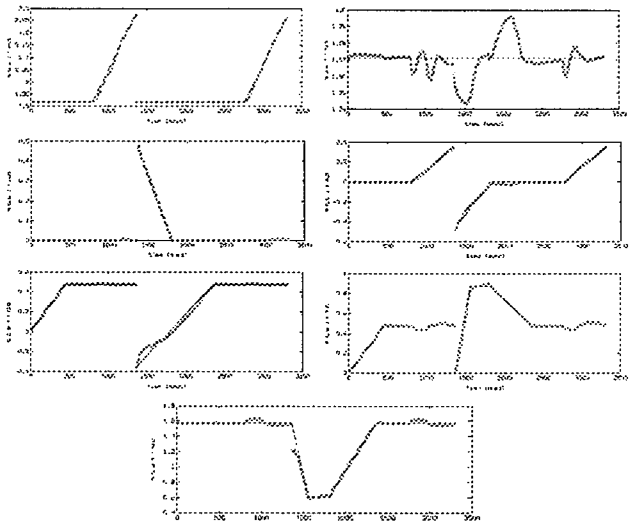


Fig.5 Angle displacements and reference signals of the 9-link, human-sized biped robot

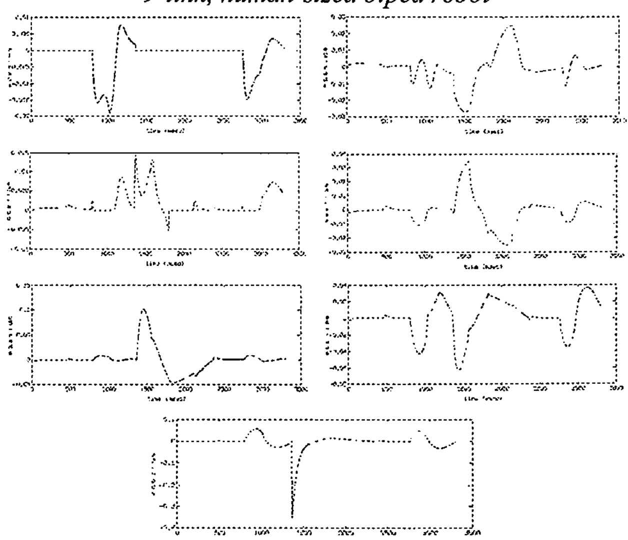


Fig.6 Angle tracking errors of the 9-link, human-sized biped robot