Modelling and Control of Sampled-Data Systems in Infinite Dimensions

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Abstract

The H_2 and H_∞ control problems for distributed parameter systems with zero-order hold and sampled observation are considered. Distributed parameter systems are described by C_0-semigroup in a Hilbert space. It is shown that the abstract sampled-data system can be rewritten as an extended semigroup model with jumps in the state. First the H_2 and H_∞ results are given for this model and then they are interpreted for the original sampled-data system. As basic examples a heat equation and a delay differential equation are considered and the H_∞ Riccati equation are derived.

1 Introduction

The H_2 and H_∞ theories for sampled-data systems are now well-known and by a certain transformation they can be reduced to those of discrete-time systems ([1]). It is also possible to establish the theories using systems with jumps ([4], [6]). Systems with jumps in the state were first introduced by Sun, Nagpal and Khargonekar [7] when they considered the H_∞ control and filtering problems for continuous time systems with sampled observation.

In this paper we take distributed parameter systems with zero-order hold and sampled observation and consider the H_2 and H_∞ control problems. Our mathematical model is an infinite dimensional system which is described by the infinitesimal generator of a C_0-semigroup in a Hilbert space. As in finite dimensions we can rewrite the sampled-data systems as a semigroup model with jumps in the state. We first give the generalization of H_2 and H_∞ results to the infinite dimensional system with jumps. We then obtain the solutions of the H_2 and H_∞ problems for our sampled-data system. As basic examples covered by our model we take a heat equation and a delay differential equation and derive the H_∞ Riccati equations.

Our semigroup model with jumps can express systems with impulsive inputs and sampled-data systems with first-order hold. In finite dimensions we have already considered the H_2 and H_∞ problems for the sampled-data systems with first-order hold ([3]) but our model also covers the infinite dimensional case as well.

2 The Infinite Dimensional System with Jumps

2.1 The H_2 and H_∞ Norms

Consider the system G:

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}w, \\ & \quad ih < t < (i+1)h,\ h > 0, \\ x(ih^+) &= \mathbf{A}_d x(ih) + \mathbf{B}_d w_d(i), \\ z_c &= \mathbf{C}x, \\ z_d(i) &= \mathbf{C}_d x(ih) + \mathbf{D}_d w_d(i), \end{aligned} \tag{1}$$

where $x \in \mathbf{H}$, a real separable Hilbert space, $\mathbf{A}$ is the infinitesimal generator of a strongly continuous semigroup $\mathbf{S}(t)$ in $\mathbf{H}$, $\mathbf{A}_d \in \mathcal{L}(\mathbf{H})$, the space of bounded linear operators on $\mathbf{H}$, $x(ih^+)$ is the right limit at $t = ih$, $w \in \mathbf{W}$, $w_d \in \mathbf{W}_d$, $z_c \in \mathbf{Z}_c$, $z_d \in \mathbf{Z}_d$, input and output spaces $\mathbf{W}$, $\mathbf{W}_d$, $\mathbf{Z}_c$, $\mathbf{Z}_d$ are all real Hilbert spaces and all operators $\mathbf{B}$, $\mathbf{B}_d$, $\mathbf{C}$, $\mathbf{C}_d$ and $\mathbf{D}_d$ are linear bounded operators in appropriate spaces (i.e., $\mathbf{B} \in \mathcal{L}(\mathbf{W}, \mathbf{H})$ etc). The solution of (1) for a given initial condition and a locally integrable w is defined in a piecewise manner as follows:

$$\begin{aligned} x(t) &= \mathbf{S}(t)x(ih^+) + \textstyle\int_{ih}^{t} \mathbf{S}(t-r)\mathbf{B}w(r)dr, \\ & \qquad ih < t \le (i+1)h. \end{aligned}$$

It is continuous on $(ih, (i+1)h)$, left-continuous at ih and has jumps at ih according to the second equation of (1).

Assume that the system (1) $((\mathbf{A}, \mathbf{A}_d))$ is exponentially stable. Then G: $(w, w_d) \in L^2(0, \infty; \mathbf{W}) \times l^2(0, \infty; \mathbf{W}_d) \to (z, z_d) \in L^2(0, \infty; \mathbf{Z}_c) \times l^2(0, \infty; \mathbf{Z}_d)$ is a bounded linear operator. We denote the norm by $\| G \|_\infty$ and call it the H_∞-norm of G.

Let T_{zw} (respectively T_{zw_d}) be the operators from $w \in L^2(0, \infty; \mathbf{W})$ (respectively $w_d \in l^2(0, \infty; \mathbf{W}_d)$) to $z = (z_c, z_d) \in L^2(0, \infty; \mathbf{Z}_c) \times l^2(0, \infty; \mathbf{Z}_d)$. Consider the impulse response $T_{zw}\delta(t-\tau)e_i$ corresponding to $w(t) = \delta(t-\tau)e_i$, $0 \le \tau \le h$ where $\{e_i\}$ is an orthonormal system in $\mathbf{W}$. Let $T_{zw_d}\delta_{k0}f_j$ be the response to $w_d(0) = f_j$ and $w_d(k) = 0$, $k \ge 1$ where $\{f_j\}$ is an orthonormal system in $\mathbf{W}_d$. We assume one of the following conditions:

(i) $\mathbf{B}$, $\mathbf{B}_d$ and $\mathbf{D}_d$ are Hilbert-Schmidt operators.
(ii) $\mathbf{C}$, $\mathbf{C}_d$ and $\mathbf{D}_d$ are Hilbert-Schmidt operators.

Then the following norm is well-defined:

$$\begin{aligned} \| G \|_2^2 &= \textstyle\sum_i \frac{1}{h} \int_0^h \| T_{zw}\delta(\cdot - \tau)e_i \|^2_{L^2 \times l^2}\, d\tau \\ &\quad + \textstyle\sum_j \| T_{zw_d}\delta_{k0} f_j \|^2_{L^2\times l^2} \end{aligned}$$

and is called the H$_2$-norm of G. Consider

$$\begin{aligned} -\dot{\mathbf{L}}_o &= \mathbf{A}^*\mathbf{L}_o + \mathbf{L}_o\mathbf{A} + \mathbf{C}^*\mathbf{C}, \\ &\quad ih < t < (i+1)h, \\ \mathbf{L}_o(ih^-) &= \mathbf{A}_d^*\mathbf{L}_o(ih)\mathbf{A}_d + \mathbf{C}_d^*\mathbf{C}_d. \end{aligned} \tag{2}$$

The operator $\mathbf{L}_o$ is called a mild solution of (2) if it is right-continuous and satisfies

$$\begin{aligned} \mathbf{L}_o(t)x &= \textstyle\int_t^s \mathbf{S}^*(r-t)\mathbf{C}^*\mathbf{C}\mathbf{S}(r-t)x dr \\ &\quad + \mathbf{S}^*(s-t)\mathbf{L}_o(s)\mathbf{S}(s-t)x, \end{aligned}$$

$ih \le t \le s < (i+1)h$ and the jump conditions in (2). Since $(\mathbf{A}, \mathbf{A}_d)$ is stable, there exists a unique nonnegative h-periodic solution to (2). It is called the observability gramian. We can write the H$_2$-norm $\| G \|_2$ in terms of $\mathbf{L}_o$. Modifying the arguments in finite dimensions as in [6] we have the following:

Lemma 2.1.

$$\begin{aligned} \| G \|_2^2 &= \frac{1}{h}\int_0^h tr.(\mathbf{B}^*\mathbf{L}_o(\tau)\mathbf{B})d\tau \\ &\quad + tr.(\mathbf{B}_d^*\mathbf{L}_o(0)\mathbf{B}_d + \mathbf{D}_d^*\mathbf{D}_d). \end{aligned}$$

For the H$_\infty$-norm of G we have:

Lemma 2.2 $\| G \|_\infty < \gamma$ if and only if the Riccati equation below has a nonnegative h-periodic solution $\mathbf{X}$ such that $(\mathbf{A} + \frac{1}{\gamma^2}\mathbf{B}\mathbf{B}^*\mathbf{X}, \mathbf{A}_d + \mathbf{B}_d\mathbf{F})$ is exponentially stable:

$$\begin{aligned} -\dot{\mathbf{X}} &= \mathbf{A}^*\mathbf{X} + \mathbf{X}\mathbf{A} + \tfrac{1}{\gamma^2}\mathbf{X}\mathbf{B}\mathbf{B}^*\mathbf{X} + \mathbf{C}^*\mathbf{C}, \\ &\quad ih < t < (i+1)h, \\ \mathbf{X}(ih^-) &= \mathbf{A}_d^*\mathbf{X}(ih)\mathbf{A}_d + (\mathbf{F}^*\mathbf{T}\mathbf{F})(i), \end{aligned}$$

where $\mathbf{T}(i) = \gamma^2 I - \mathbf{D}_d^*\mathbf{D}_d - \mathbf{B}_d^*\mathbf{X}(ih)\mathbf{B}_d > 0$, $\mathbf{F}(i) = \mathbf{T}^{-1}(\mathbf{B}_d^*\mathbf{X}(ih)\mathbf{A}_d + \mathbf{D}_d^*\mathbf{C}_d)$.

Definition 2.1. (a) The pair $([\mathbf{A}, \mathbf{A}_d], [\mathbf{B}, \mathbf{B}_d])$ is said to be stabilizable if there exist bounded linear operators $\mathbf{K}$ and $\mathbf{K}_d$ such that $(\mathbf{A}+\mathbf{B}\mathbf{K}, \mathbf{A}_d+\mathbf{B}_d\mathbf{K}_d)$ is exponentially stable.
(b) The pair $([\mathbf{C}, \mathbf{C}_d], [\mathbf{A}, \mathbf{A}_d])$ is detectable if there exist bounded linear operators $\mathbf{J}$ and $\mathbf{J}_d$ such that $(\mathbf{A} + \mathbf{J}\mathbf{C}, \mathbf{A}_d + \mathbf{J}_d\mathbf{C}_d)$ is exponentially stable.

Now we consider the H$_2$ and H$_\infty$ problems for the system G_j:

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}_1 w,\ ih < t < (i+1)h, \\ x(ih^+) &= \mathbf{A}_d x(ih) + \mathbf{B}_2 u(i), \\ z_c &= \mathbf{C}_1 x(t), \\ z_d(i) &= \mathbf{D}_{12}u(i), \\ y(i) &= \mathbf{C}_2 x(ih) + \mathbf{D}_{21}w_d(i) \end{aligned} \tag{3}$$

where x, w, w_d, z, z_d and $\mathbf{A}$, $\mathbf{A}_d$ are given as for (1), $u \in \mathbf{U}$, $y \in \mathbf{Y}_d$, $\mathbf{U}$, $\mathbf{Y}_d$ are real Hilbert spaces and all other operators are bounded and linear. We assume the following:

$$\begin{aligned} &\text{(i)} && \mathbf{D}_{12}^*\mathbf{D}_{12} = d_1 I,\ \mathbf{D}_{21}\mathbf{D}_{21}^* = d_2 I,\ d_i > 0. \\ &\text{(ii)} && ([\mathbf{A}, \mathbf{A}_d], [\mathbf{B}_1, 0]) \text{ is stabilizable,} \\ &&& ([\mathbf{C}_1, 0], [\mathbf{A}, \mathbf{A}_d]) \text{ is detectable,} \\ &&& ([\mathbf{A}, \mathbf{A}_d], [0, \mathbf{B}_2]) \text{ is stabilizable,} \\ &&& ([0, \mathbf{C}_2], [\mathbf{A}, \mathbf{A}_d]) \text{ is detectable.} \end{aligned} \tag{4}$$

For G_j we allow for feedback controllers $u = Ky$ of the form

$$\begin{aligned} \dot{p} &= \hat{\mathbf{A}}p,\ t \neq ih, \\ p(ih^+) &= \hat{\mathbf{A}}_d p(ih) + \hat{\mathbf{B}}y(i), \\ u(i) &= \hat{\mathbf{C}}p(ih) + \hat{\mathbf{D}}y(i), \end{aligned} \tag{5}$$

where $\hat{\mathbf{A}}$ is the infinitesimal generator of a semigroup in a Hilbert space $\hat{\mathbf{H}}$ and $\hat{\mathbf{A}}_d$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{D}}$ are bounded linear operators.

2.2 H$_2$ Control

To formulate the H$_2$-problem for G_j, we introduce the following set of controllers.

$$\begin{aligned} \mathbf{K} &= \{K : K \text{ is of the form (5) and} \\ &\qquad\qquad \text{internally stabilizes } G_j\}. \end{aligned}$$

For simplicity we assume that $\mathbf{W}$ and $\mathbf{W}_d$ are finite dimensional. The H$_2$-problem is then to find a controller $K \in \mathbf{K}$ which minimizes $\| G \|_2$ where G is the input-output operator of the closed loop system G_j with $u = Ky$. To give the solution of this problem, we need the following result:

Lemma 2.3. (a) Suppose $([\mathbf{A}, \mathbf{A}_d], [0, \mathbf{B}_2])$ is stabilizable and $([\mathbf{C}_1, 0], [\mathbf{A}, \mathbf{A}_d])$ is detectable. Then there exists a unique h-periodic nonnegative (mild) solution of the Riccati equation with jumps:

$$\begin{aligned} -\dot{\mathbf{X}} &= \mathbf{A}^*\mathbf{X} + \mathbf{X}\mathbf{A} + \mathbf{C}_1^*\mathbf{C}_1, \\ &\quad ih \le t < (i+1)h, \\ \mathbf{X}(ih^-) &= \mathbf{A}_d^*\mathbf{X}(ih)\mathbf{A}_d - (\mathbf{H}^*\mathbf{E}^{-1}\mathbf{H})(i), \end{aligned} \tag{6}$$

which is stable, i.e., $(\mathbf{A}, \mathbf{A}_d + \mathbf{B}_2\mathbf{F})$ is stable, $\mathbf{F} = -\mathbf{E}^{-1}\mathbf{H}(i)$, $\mathbf{E}(i) = d_1 I + \mathbf{B}_2^*\mathbf{X}(ih)\mathbf{B}_2$, $\mathbf{H}(i) = \mathbf{B}_2^*\mathbf{X}(ih)\mathbf{A}_d$.
(b) If $([\mathbf{A}, \mathbf{A}_d], [\mathbf{B}_1, 0])$ is stabilizable and $([0, \mathbf{C}_2], [\mathbf{A}, \mathbf{A}_d])$ is detectable, there exists a unique h-periodic nonnegative (mild) solution of the Riccati equation with jumps:

$$\begin{aligned} \dot{\mathbf{Y}} &= \mathbf{A}\mathbf{Y} + \mathbf{Y}\mathbf{A}^* + \tfrac{1}{h}\mathbf{B}_1\mathbf{B}_1^*, \\ &\quad ih < t \le (i+1)h, \\ \mathbf{Y}(ih^+) &= \mathbf{A}_d\mathbf{Y}(ih)\mathbf{A}_d^* - (\hat{\mathbf{H}}^*\hat{\mathbf{E}}^{-1}\hat{\mathbf{H}})(i) \end{aligned} \tag{7}$$

which is stable, i.e., $(\mathbf{A}, \mathbf{A}_d + \mathbf{J}\mathbf{C}_2)$ is stable, $\mathbf{J} = -(\hat{\mathbf{H}}^*\hat{\mathbf{E}}^{-1})(i)$, $\hat{\mathbf{E}}(i) = d_2 I + \mathbf{C}_2\mathbf{Y}(ih)\mathbf{C}_2^*$, $\hat{\mathbf{H}}(i) = \mathbf{C}_2\mathbf{Y}(ih)\mathbf{A}_d^*$.

Consider the stabilizing controller based on the feedback gain $\mathbf{F}$ and the observer gain $\mathbf{J}$:

$$\begin{aligned} \dot{p} &= \mathbf{A}p,\ ih < t < (i+1)h, \\ p(ih^+) &= (\mathbf{A}_d + \mathbf{B}_2\mathbf{F} + \mathbf{J}\mathbf{C}_2)p(ih) \\ &\quad -\mathbf{J}y(i), \\ u(i) &= \mathbf{F}p(ih). \end{aligned} \tag{8}$$

The solution of the H_2 problem is given as follows:

Theorem 2.1. Consider the H_2-problem for G_j. The controller (8) is optimal and

$$\begin{aligned} \min_{K\in\mathbf{K}} \| G \|_2^2 &= \tfrac{1}{h}\int_0^h tr.(\mathbf{B}_1^*\mathbf{X}(\tau)\mathbf{B}_1)d\tau \\ &\quad + tr.[\mathbf{F}\mathbf{Y}(0)\mathbf{F}^*\mathbf{E}(0)]. \end{aligned}$$

2.3 H_∞-Control

Consider the system G_j and an internally stabilizing controller $u = Ky$. Here we allow for time-varying coefficients in (5). We assume that $\hat{\mathbf{A}}$ generates an evolution operator ([2]). Other operators can depend on i but are assumed to be uniformly bounded. Define the input-output operator of the closed loop system by $G(w, w_d) = (z_c, z_d)$. The H_∞-problem is to find necessary and sufficient conditions for the existence of an internally stabilizing controller $u = Ky$ such that $\| G \|_\infty < \gamma$. Such a controller is called γ-suboptimal.

To give the solution of this problem, we need the Riccati equations:

$$\begin{aligned} -\dot{\mathbf{X}} &= \mathbf{A}^*\mathbf{X} + \mathbf{X}\mathbf{A} + \tfrac{1}{\gamma^2}\mathbf{X}\mathbf{B}_1\mathbf{B}_1^*\mathbf{X} \\ &\quad + \mathbf{C}_1^*\mathbf{C}_1,\ ih < t < (i+1)h, \\ \mathbf{X}(ih^-) &= \mathbf{A}_d^*\mathbf{X}(ih)\mathbf{A}_d - (\mathbf{H}^*\mathbf{E}^{-1}\mathbf{H})(i), \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{\mathbf{Y}} &= \mathbf{A}\mathbf{Y} + \mathbf{Y}\mathbf{A}^* + \tfrac{1}{\gamma^2}\mathbf{Y}\mathbf{C_1}^*\mathbf{C}_1\mathbf{Y} \\ &\quad + \mathbf{B}_1\mathbf{B}_1^*,\ ih < t < (i+1)h, \\ \mathbf{Y}(ih^+) &= \mathbf{A}_d\mathbf{Y}(ih)\mathbf{A}_d^* - (\hat{\mathbf{H}}^*\hat{\mathbf{E}}^{-1}\hat{\mathbf{H}})(i), \end{aligned} \tag{10}$$

$$\mathbf{Y}(0) = 0. \tag{11}$$

Define the set of linear causal controllers

$$\begin{aligned} Q_\gamma = \{&\mathbf{Q} \in \mathcal{L}(l^2(0,\infty;\mathbf{Y}_d); l^2(0,\infty;\mathbf{U})) : \\ &\mathbf{Q} \text{ is of the form (5) and internally} \\ &\text{stable with } \| \mathbf{Q} \|_\infty < \gamma\}. \end{aligned}$$

A bounded nonnegative solution $\mathbf{X}$ of (9) is called stabilizing if $(\mathbf{A}_{tmp}, \mathbf{A}_d - \mathbf{B}_2\mathbf{E}^{-1}\mathbf{H})$, is exponentially stable, $\mathbf{A}_{tmp} = \mathbf{A} + \frac{1}{\gamma^2}\mathbf{B}_1\mathbf{B}_1^*\mathbf{X}$. A bounded nonnegative solution $\mathbf{Y}$ of (10) is called a stabilizing solution if $(\mathbf{A} + \frac{1}{\gamma^2}\mathbf{Y}\mathbf{C}_1^*\mathbf{C}_1, \mathbf{A}_d + \hat{\mathbf{H}}^*\hat{\mathbf{E}}^{-1}\mathbf{C}_2)$ is exponentially stable. Following [4], [5] we obtain the solution of the H_∞-problem for G_j.

Theorem 2.2. Assume the condition (4).
(a) There exists an internally stabilizing controller $u = Ky$ for G_j such that $\| G \|_\infty < \gamma$ if and only if the following hold:
(i) There exists an h-periodic nonnegative stabilizing mild solution $\mathbf{X}$ to the Riccati equation (9).
(ii) There exists a bounded nonnegative stabilizing mild solution $\mathbf{Y}$ to the Riccati equation (10) and (11).
(iii) $\rho(\mathbf{X}\mathbf{Y}) < \gamma^2 - \epsilon$, $^\forall t \geq 0$, for some $\epsilon > 0$, where ρ is the spectral radius of an operator.
(b) In this case the set of all γ-suboptimal controllers is given by

$$\begin{aligned} \dot{\hat{x}} &= \mathbf{A}_{tmp}\hat{x},\ ih < t \leq (i+1)h, \\ \hat{x}(ih^+) &= \mathbf{M}_1\hat{x}(ih) + \mathbf{M}_2 y(i) + \mathbf{M}_3 s(i), \\ u(i) &= \mathbf{N}_1\hat{x}(ih) + \mathbf{N}_2 y(i) + \mathbf{N}_3 s(i), \\ g(i) &= \mathbf{T}_2^{-\frac{1}{2}}(i)[-\mathbf{C}_2\hat{x}(ih) + y(i)], \\ s &= Qg,\ Q \in Q_\gamma. \end{aligned}$$

where

$\mathbf{M}_1(i) = (\mathbf{A}_d - \mathbf{B}_2\mathbf{E}^{-1}\mathbf{H})\mathbf{\Psi}(i)$,
$\mathbf{M}_2(i) = \mathbf{M}_1(i)\mathbf{Z}(ih)\mathbf{C}_2^*$,
$\mathbf{M}_3(i) = \frac{1}{\gamma}[(\mathbf{F}^* + \mathbf{B}_2\mathbf{E}^{-\frac{1}{2}})\mathbf{V}^{\frac{1}{2}}](i)$,
$\mathbf{N}_1(i) = -\mathbf{E}^{-1}\mathbf{H}\mathbf{\Psi}(i)$,
$\mathbf{N}_2(i) = \mathbf{N}_1(i)\mathbf{Z}(ih)\mathbf{C}_2^*$, $\mathbf{N}_3(i) = \frac{1}{\gamma}[\mathbf{E}^{-\frac{1}{2}}\mathbf{V}^{\frac{1}{2}}](i)$,
$\mathbf{Z}(ih) = (I - \frac{1}{\gamma^2}\mathbf{Y}(ih)\mathbf{X}(0))^{-1}\mathbf{Y}(ih)$,
$\mathbf{\Psi}(i) = I - \mathbf{Z}(ih)\mathbf{C}_2^*\mathbf{T}_2^{-1}(i)\mathbf{C}_2$
$\mathbf{T}_1(i) = \gamma^2 I - \mathbf{E}^{-\frac{1}{2}}\mathbf{H}\mathbf{Z}(ih)\mathbf{H}^*\mathbf{E}^{-\frac{1}{2}}$,
$\mathbf{T}_2(i) = d_2 I + \mathbf{C}_2\mathbf{Z}(ih)\mathbf{C}_2^*$,
$\mathbf{R}_1(i) = \mathbf{E}^{-\frac{1}{2}}\mathbf{H}\mathbf{Z}(ih)\mathbf{A}_d^*$,
$\mathbf{R}_2(i) = \mathbf{C}_2\mathbf{Z}(ih)\mathbf{A}_d^*$,
$\mathbf{S}(i) = \mathbf{C}_2\mathbf{Z}(ih)\mathbf{H}^*\mathbf{E}^{-\frac{1}{2}}$,
$\mathbf{V}(i) = [\mathbf{T}_1 + \mathbf{S}^*\mathbf{T}_2^{-1}\mathbf{S}](i)$,
$\mathbf{F}(i) = [\mathbf{V}^{-1}(\mathbf{R} - \mathbf{S}^*\mathbf{T}_2^{-1}\mathbf{R}_2)](i)$.

3 The Sampled-Data System

Consider the sampled-data system G_s:

$$\begin{aligned} \dot{x} &= Ax(t) + B_1 w(t) + B_2\tilde{u}(t), \\ z(t) &= \begin{bmatrix} C_1 x(t) \\ D_{12}\tilde{u}(t) \end{bmatrix}, \\ y(i) &= C_2 x(ih) + D_{21} w_d(i), \end{aligned}$$

where A is the infinitesimal generator of a C_0-semigroup $S(t)$ in a real separable Hilbert space H, $w \in W$, $\tilde{u} \in U$, $z \in Z \times Z_d$ and $y \in Y_d$, W, U, Z_c, Z_d and Y_d are real separable Hilbert spaces and all other operators in G_s are bounded linear. We assume that $\tilde{u}$ is realized through zero-order hold so that

$$\tilde{u}(t) = u(i),\ ih < t \leq (i+1)h.$$

Introduce a dynamics for $\tilde{u}$ regarding $u(i)$ as inputs:

$$\begin{aligned} \dot{p} &= 0,\ ih < t < (i+1)h, \\ p(0) &= 0, \\ p(ih^+) &= u(i). \end{aligned}$$

Then $\tilde{u}(t) = p(t)$ and G_s can be rewritten as

$$\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w(t), \\
\begin{bmatrix} x(ih^+) \\ p(ih^+) \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(ih) \\ p(ih) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u(i), \\
z_c &= \begin{bmatrix} C_1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix}, \qquad (12) \\
z_d(i) &= \sqrt{h} D_{12} u(i), \\
y(i) &= \begin{bmatrix} C_2 & 0 \end{bmatrix} \begin{bmatrix} x(ih) \\ p(ih) \end{bmatrix} + D_{21} w_d(i).
\end{aligned}$$

The system (12) is a special case of the system (3). We assume $D_{12}^* D_{12} = I$, $D_{21} D_{21}^* = I$ and the condition (4) for (12).

3.1 H$_2$ Control

We assume that W and W_d are finite dimensional and consider the H$_2$ problem for (12). We now apply Theorem 2.1 to (12). Let $\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ be the h-periodic stabilizing solutions of (6) and (7) respectively, where $X_{11}, Y_{11} \in \mathcal{L}(H)$, X_{12}, $Y_{12} \in \mathcal{L}(U, H)$ and X_{22}, $Y_{22} \in \mathcal{L}(U)$. Then from (6), we obtain

$$\begin{aligned}
-\dot{X}_{11} &= A^* X_{11} + X_{11} A + C_1^* C_1, \\
-\dot{X}_{12} &= A^* X_{12} + X_{11} B_2, \\
-\dot{X}_{22} &= B_2^* X_{12} + X_{12} B_2, \\
&\qquad ih < t < (i+1)h, \\
X_{11}(ih^-) &= X_{11}(ih) - X_{12}(ih) \\
&\quad \times [hI + X_{22}(ih)]^{-1} X_{12}^*(ih) \\
X_{12}(ih^-) &= 0, \;\; X_{22}(ih^-) = 0
\end{aligned}$$

The equation (7) yields

$$\begin{aligned}
\dot{Y}_{11} &= AY_{11} + Y_{11} A^* + \tfrac{1}{h} B_1 B_1^* \\
&\quad + B_2 Y_{12}^* + Y_{12}^* B_2, \\
\dot{Y}_{12} &= AY_{12} + B_2 Y_{22}^*, \\
\dot{Y}_{22} &= 0, \qquad ih < t < (i+1)h, \\
Y_{11}(ih^+) &= Y_{11}(ih) - Y_{11}(ih) C_2^* \\
&\quad \times (I + C_2 Y_{11}(ih) C_2^*)^{-1} C_2 Y_{11}(ih) \\
Y_{12}(ih^+) &= 0, \;\; Y_{22}(ih^+) = 0
\end{aligned}$$

Note that $\mathbf{Y}(t) = \lim_{n\to\infty} \hat{\mathbf{Y}}(t + nh)$ where $\hat{\mathbf{Y}}$ is the bounded stabilizing solution (7) with $\hat{\mathbf{Y}}(0) = 0$. Since $\dot{\hat{Y}}_{22} = 0$, $\hat{Y}_{22}(ih) = 0$ and $\hat{Y}_{22}(0) = 0$, we conclude $\hat{Y}_{22}(t) = 0$ for all $t \geq 0$. This together with $\hat{Y}_{12}(ih^+) = 0$ and $\hat{Y}_{12}(0) = 0$ gives $\hat{Y}_{12}(t) = 0$ for all $t \geq 0$. Hence $Y_{12}(t) = 0$, $Y_{22}(t) = 0$ and $\mathbf{Y}$ is of the form $\begin{bmatrix} Y & 0 \\ 0 & 0 \end{bmatrix}$ where $Y \in \mathcal{L}(H)$ is the solution of

$$\begin{aligned}
\dot{Y} &= AY + YA^* + \tfrac{1}{h} B_1 B_1^*, \\
&\qquad ih < t < (i+1)h, \\
Y(ih^+) &= Y(ih) - Y(ih) C_2^* \\
&\quad \times (I + C_2 Y(ih) C_2^*)^{-1} C_2 Y(ih).
\end{aligned}$$

We can write the optimal controller (8) as

$$\begin{aligned}
\dot{p} &= Ap + B_2 \tilde{v}(t), \\
&\qquad ih < t < (i+1)h, \\
p(ih^+) &= (I + JC_2) p(ih) - Jy(i), \\
u(i) &= Fp(ih),
\end{aligned} \qquad (13)$$

where $F = -(hI + X_{22}(0))^{-1} X_{21}(0)$, $J = -Y(0) C_2^* (I + C_2 Y(0) C_2^*)^{-1}$ and $\tilde{v}(t) = Fp(ih)$, $ih < t \leq (i+1)h$. The optimal value is given by

$$\begin{aligned}
&\min_{K \in \mathbf{K}} \| G \|_2^2 \\
&= \tfrac{1}{h} \int_0^h tr(B_1^* X_1(\tau) B_1) d\tau \qquad (14) \\
&\qquad + tr(FY(0)F^*(hI + X_{22}(0))).
\end{aligned}$$

Summing up we have:

Theorem 3.1. Consider the H$_2$-problem for G_s. The controller (13) is optimal and the minimum H$_2$ norm is given by (14).

3.2 H$_\infty$-Problem for G_s.

Now we consider the H$_\infty$-problem for G_s. Let $\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ be the stabilizing solutions of the Riccati equations (9) and (10), respectively. Then from (9) we obtain

$$\begin{aligned}
-\dot{X}_{11} &= A^* X_{11} + X_{11} A + \frac{1}{\gamma^2} X_{11} B_1 B_1^* X_{11} \\
&\quad + C_1^* C_1, \\
-\dot{X}_{12} &= A^* X_{12} + X_{11} B_2 + \frac{1}{\gamma^2} X_{11} B_1 B_1^* X_{12}, \\
-\dot{X}_{22} &= B_2^* X_{12} + X_{12}^* B_2 + \frac{1}{\gamma^2} X_{12}^* B_1 B_1^* X_{12}, \\
&\qquad ih < t < (i+1)h, \qquad (15) \\
X_{11}(ih^-) &= X_{11}(ih) - X_{12}(ih) \\
&\quad \times [hI + X_{22}(ih)]^{-1} X_{21}(ih) \\
X_{12}(ih^-) &= 0, \;\; X_{22}(ih^-) = 0.
\end{aligned}$$

As in the H$_2$ case $\mathbf{Y}$ is of the form $\begin{bmatrix} Y & 0 \\ 0 & 0 \end{bmatrix}$, where $Y \in \mathcal{L}(H)$ is the solution of

$$\begin{aligned}
\dot{Y} &= AY + YA^* + \tfrac{1}{\gamma^2} YC_1^* C_1 Y \\
&\quad + B_1 B_1^*, \;\; ih < t < (i+1)h, \\
Y(ih^+) &= Y(ih) - Y(ih) C_2^* \qquad (16) \\
&\quad \times (I + C_2 Y(ih) C_2^*)^{-1} C_2 Y(ih), \\
Y(0) &= 0.
\end{aligned}$$

The set of all γ-suboptimal controllers is given by

$$\begin{array}{rcl} \dot{p} &=& A_c p + B_c \tilde{v}(t), \\ && ih < t < (i+1)h, \\ p(ih^+) &=& \hat{A}p(ih) + \hat{B}_1 y(i) + \hat{B}_2 s(i), \\ u(i) &=& \hat{C}_1 p(ih) + \hat{D}_{11} y(i) + \hat{D}_{12} s(i), \\ g(i) &=& \Psi(i)[-C_2 p(ih) + y(i)], \\ s &=& Qg,\ Q \in Q_\gamma, \end{array} \tag{17}$$

where $A_c(t) = A + \frac{1}{\gamma^2} B_1 B_1^* X_{11}(t)$, $B_c(t) = B_2 + \frac{1}{\gamma^2} B_1 B_1^* X_{12}(t)$

$$\begin{array}{rcl} \hat{A}(i) &=& (I + Z(i)C_2^* C_2)^{-1}, \\ \hat{B}_1(i) &=& \hat{A}(i)Z(i)C_2^*, \\ \hat{B}_2(i) &=& \frac{1}{\gamma^2}\hat{A}(i)Z(i)X_{12}(0)E^{-\frac{1}{2}}\Xi^{-\frac{1}{2}}(i), \\ \hat{C}_1(i) &=& -E^{-1}X_{21}(0)\hat{A}(i), \\ \hat{D}_{11}(i) &=& \hat{C}_1(i)Z(i)C_2^*, \\ \hat{D}_{12}(i) &=& \frac{1}{\gamma}E^{-\frac{1}{2}}\Xi^{\frac{1}{2}}(i), \\ \Psi(i) &=& (I + C_2 Z(i) C_2^*)^{-\frac{1}{2}}, \\ Z(i) &=& (I - \frac{1}{\gamma^2}Y(ih)X_{11}(0))^{-1}Y(ih), \\ \Xi(i) &=& \gamma^2 I - E^{-\frac{1}{2}}X_{21}(0)\hat{A}(i)Z(i)X_{12}(0)E^{-\frac{1}{2}}, \\ E &=& hI + X_{22}(0), \\ Q_\gamma &=& \{Q \in \mathcal{L}(l^2(0,\infty;\mathbf{R}^q); l^2(0,\infty;\mathbf{R}^s)) : \\ && \text{Q is of the form (5) and internally} \\ && \text{stable with } \| Q \| < \gamma\} \end{array}$$

and $\tilde{v} = \hat{C}_1 p(ih) + \hat{D}_{11} y(i) + \hat{D}_{12} s(i)$, $ih < t \le (i+1)h$.

Summing up we have:

Theorem 3.2. Assume the condition (4).
(a) There exists an internally stabilizing controller $u = Ky$ for G_s such that $\| G \| < \gamma$ if and only if the following hold:
(i) There exists an h-periodic nonnegative stabilizing solution $\mathbf{X}$ to the Riccati equation (15).
(ii) There exists a bounded nonnegative stabilizing solution Y to the Riccati equation (16).
(iii) $\rho(\begin{bmatrix} X_{11}Y \\ X_{12}Y \end{bmatrix}) < \gamma^2 - \epsilon$, $^\forall t \ge 0$ for some $\epsilon > 0$.
(b) Under the condition (a), the set of all γ-suboptimal controllers is given by (17).

4 Examples

We give two simple examples covered by the system G_s and derive the H_∞ Riccati equations for them.

Example 4.1. Consider the heat equation

$$\begin{array}{rcl} \frac{\partial x}{\partial t} &=& \frac{\partial^2 x}{\partial s^2} + b_1(s)w(t) + b_2(s)\tilde{u}(t), \\ && 0 < s < 1, \\ \frac{\partial x}{\partial s}(t,0) &=& \frac{\partial x}{\partial s}(t,1) = 0, \\ z &=& \begin{bmatrix} \int_0^1 c_1(s)x(t,s)ds \\ \tilde{u}(t) \end{bmatrix}, \\ y(i) &=& \int_0^1 c_2(s)x(ih,s)ds + w_d(i), \end{array}$$

where b_i, $c_i \in L^2(0,1)$ and w, $\tilde{u} \in \mathbf{R}^1$. For this example we take $H = L^2(0,1)$, $W = U = Z_c = Y_d = \mathbf{R}^1$ and the generator A ([2]) defined by

$$\begin{array}{rcl} Ax &=& \frac{d^2 x}{ds^2}, \\ \mathcal{D}(A) &=& \{x \in L^2(0,1) :\ x,\ \frac{dx}{ds} \text{ are absolutely} \\ && \text{continuous with } \frac{d^2x}{ds^2} \in L^2(0,1), \\ && \frac{dx}{ds}(0) = \frac{dx}{ds}(1) = 0\}. \end{array}$$

Setting $[X_{11}(t)g](s) = \int_0^1 X_{11}(t,s,r)g(r)dr$, $g \in L^2(0,1)$ etc in (15) and using the definition of A we obtain:

$$\begin{array}{rcl} -\frac{\partial}{\partial t}X_{11}(t,s,r) &=& (\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial r^2})X_{11}(t,s,r) \\ && +\frac{1}{\gamma^2}\int_0^1 X_{11}(t,s,r)b_1(r)dr \\ && \times \int_0^1 b_1(s)X_{11}(t,s,r)ds, \\ && +c_1(s)c_1(r), \\ \frac{\partial}{\partial s}X_{21}(t,0,r) &=& \frac{\partial}{\partial s}X_{11}(t,1,r) \\ &=& \frac{\partial}{\partial r}X_{11}(t,s,0) \\ &=& \frac{\partial}{\partial r}X_{11}(t,s,1) = 0, \\ X_{11}(ih^-,s,r) &=& X_{11}(ih,s,r) - X_{12}(ih,s) \\ && \times[h + X_{22}(ih)]^{-1}X_{21}(ih,r) \\ -\frac{\partial}{\partial t}X_{12}(t,s) &=& \frac{\partial^2}{\partial s^2}X_{12}(t,s) \\ && + \int_0^1 X_{11}(t,s,r)b_2(r)dr, \\ && +\frac{1}{\gamma^2}\int_0^1 X_{11}(t,s,r)b_1(r)dr \\ && \times \int_0^1 b_1(s)X_{12}(t,s)ds, \\ \frac{\partial}{\partial s}X_{12}(t,0) &=& \frac{\partial}{\partial s}X_{12}(t,1) = 0, \\ X_{12}(ih^-,s) &=& 0, \\ -\dot{X}_{22}(t) &=& \int_0^1 b_2(s)X_{12}(t,s)ds \\ && + \int_0^1 X_{21}(t,s)b_2(s)ds, \\ && +\frac{1}{\gamma^2}\int_0^1 X_{21}(t,s)b_1(s)ds \\ && \times \int_0^1 b_1(s)X_{12}(t,s)ds \\ X_{22}(ih^-) &=& 0. \end{array}$$

Similarly from (16) we obtain

$$\begin{array}{rcl} \frac{\partial}{\partial t}Y(t,s,r) &=& (\frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial r^2})Y(t,s,r) \\ && +\frac{1}{\gamma^2}\int_0^1 Y(t,s,r)c_1(r)dr \\ && \times \int_0^1 c_1(s)Y(t,s,r)ds, \\ && +b_1(s)b_1(r), \\ \frac{\partial}{\partial s}Y(t,0,r) &=& \frac{\partial}{\partial s}Y(t,1,r) \\ &=& \frac{\partial}{\partial r}Y(t,s,0) \\ &=& \frac{\partial}{\partial r}Y(t,s,1) = 0, \\ Y(ih^+,s,r) &=& Y(ih,s,r) \\ && -\int_0^1 Y(ih,s,r)c_2(r)dr \\ && \times(1 + \int_0^1\int_0^1 Y(ih,s,r) \\ && c_2(s)c_2(r)dsdr)^{-1} \\ && \times \int_0^1 c_2(s)Y(ih,s,r)ds. \end{array}$$

Example 4.2. Consider the delay system

$$\begin{array}{rcl} \dot{x}_1 &=& A_0 x_1(t) + A_1 x_1(t-a) \\ && +B_1 w(t) + B_2\tilde{u}(t),\ a > 0, \\ z &=& \begin{bmatrix} C_1 x_1 \\ \tilde{u} \end{bmatrix}, \\ y(i) &=& C_2 x_1(ih) + w_d(i), \end{array}$$

where $x_1 \in \mathbf{R}^n$, $w \in \mathbf{R}^{m_1}$, $\tilde{u} \in \mathbf{R}^{m_2}$, $y \in \mathbf{R}^{p_2}$and $C_1 \in \mathbf{R}^{p_1 \times n}$. For this example we take the state $x = (x_1(t),\ x_1(t+s))$ in $H = \mathbf{R}^n \times L^2(-a, o; \mathbf{R}^n)$ and the generator A ([2]) given by

$$\begin{aligned} A\begin{bmatrix} x(0) \\ x(\cdot) \end{bmatrix} &= \begin{bmatrix} A_0 x_0 + A_1 x(-a) \\ \frac{dx}{ds} \end{bmatrix}, \\ \mathcal{D}(A) &= \{\begin{bmatrix} x(0) \\ x(\cdot) \end{bmatrix} : x(\cdot) \text{ is absolutely} \\ &\quad \text{continuous, } \tfrac{dx}{ds} \in L_2(-a, 0; R^n)\}. \end{aligned}$$

Now set $\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}$, then $X_{11} \in \mathbf{R}^{n \times n}$, $X_{12} \in \mathcal{L}(L^2(-a, 0; \mathbf{R}^n); \mathbf{R}^n)$, $X_{22} \in \mathcal{L}(L^2(-a, 0; \mathbf{R}^n))$ $X_{13} \in \mathbf{R}^{n \times m_2}$, $X_{23} \in \mathcal{L}(\mathbf{R}^{m_2}; L^2(-a, 0; \mathbf{R}^n))$ and $X_{33} \in \mathbf{R}^{m_2 \times m_2}$. Using the definition of A we obtain from (15) the following:

$$\begin{aligned}
-\dot{X}_{11} &= A_0' X_{11} + X_{11} A_0 + X_{12}(t, 0) \\
&\quad + X_{21}(t, 0) + C_1' C_1, \\
&\quad + \tfrac{1}{\gamma^2} X_{11} B_1 B_1' X_{11}, \\
X_{11}(ih^-) &= X_{11}(ih) - X_{13}(ih) \\
&\quad \times [hI + X_{33}(ih)]^{-1} X_{31}(ih), \\
-\tfrac{\partial}{\partial t} X_{12}(t, s) &= -\tfrac{\partial}{\partial s} X_{12}(t, s) + A_0' X_{12}(t, s) \\
&\quad + X_{22}(t, 0, s) \\
&\quad + \tfrac{1}{\gamma^2} X_{11} B_1 B_1' X_{12}(t, s), \\
X_{12}(t, -h) &= X_{11}(t) A_1, \\
X_{12}(ih^-) &= X_{12}(ih) - X_{13}(ih) \\
&\quad \times [hI + X_{33}(ih)]^{-1} X_{32}(ih), \\
-\tfrac{\partial}{\partial t} X_{22}(t, s, r) &= -(\tfrac{\partial}{\partial s} + \tfrac{\partial}{\partial r}) X_{22}(t, s, r) \\
&\quad + \tfrac{1}{\gamma^2} X_{21}(t, s) B_1 B_1' X_{12}(t, r), \\
X_{22}(t, s, -h) &= X_{22}(t, -h, r) = 0, \\
X_{22}(ih^-, s, r) &= X_{22}(ih, s, r) - X_{23}(ih) \\
&\quad \times [hI + X_{22}(ih)]^{-1} X_{32}(ih), \\
-\dot{X}_{13}(t) &= A_0' X_{13} + X_{23}(t, 0) + X_{11} B_2 \\
&\quad + \tfrac{1}{\gamma^2} X_{11}(t, s) B_1 B_1' X_{13}, \\
X_{13}(ih^-) &= 0, \\
-\tfrac{\partial}{\partial t} X_{23}(t, s) &= -\tfrac{\partial}{\partial s} X_{23}(t, s) + A_0' X_{13}(t, s) \\
&\quad + X_{21}(t, s) B_2 \\
&\quad + \tfrac{1}{\gamma^2} X_{21}(t, s) B_1 B_1' X_{13}, \\
X_{23}(ih^-) &= 0, \\
-\dot{X}_{33}(t) &= B_2' X_{13}(t) + X_{31}(t) B_2 \\
&\quad + \tfrac{1}{\gamma^2} X_{31} B_1 B_1' X_{13}, \\
X_{33}(ih^-) &= 0.
\end{aligned}$$

Similarly from (16) we obtain

$$\begin{aligned}
\dot{Y}_{11} &= A_0 Y_{11}(t) + Y_{11}(t) A_0' \\
&\quad + A_1 Y_{21}(t, -h) + B_1 B_1' \\
&\quad + \tfrac{1}{\gamma^2} Y_{11}(t) C_1' C_1 Y_{11}(t), \\
Y_{11}(ih^+) &= Y_{11}(ih) - Y_{11}(ih) C_2' \\
&\quad \times (I + C_2 Y_{11}(ih) C_2')^{-1} \\
&\qquad \times C_2 Y_{11}(ih), \\
\tfrac{\partial}{\partial t} Y_{12}(t, s) &= \tfrac{\partial}{\partial s} Y_{12}(t, s) + A_0' Y_{12}(t, s) \\
&\quad + A_1 Y_{22}(t, -h, s) \\
&\quad + \tfrac{1}{\gamma^2} Y_{11}(t) C_1' C_1 Y_{12}(t, s), \\
Y_{12}(t, 0) &= Y_{11}(t), \\
Y_{12}(ih^+, s) &= Y_{12}(ih, s) - Y_{11}(ih) C_2' \\
&\quad \times (I + C_2 Y_{11}(ih) C_2')^{-1} \\
&\qquad \times C_2 Y_{12}(ih, s), \\
\tfrac{\partial}{\partial t} Y_{22}(t, s, r) &= (\tfrac{\partial}{\partial s} + \tfrac{\partial}{\partial r}) Y_{22}(t, s, r), \\
&\quad + \tfrac{1}{\gamma^2} Y_{21}(t, s) C_1' C_1 Y_{12}(t, r), \\
Y_{22}(t, s, 0) &= Y_{21}(t, s), \\
Y_{22}(t, 0, r) &= Y_{12}(t, r), \\
Y_{22}(ih^+, s, r) &= Y_{22}(ih, s, r) - Y_{21}(ih, s) C_2' \\
&\quad \times (I + C_2 Y_{11}(ih) C_2')^{-1} \\
&\qquad \times C_2 Y_{12}(ih, r).
\end{aligned}$$

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