Inverse Problems For Nonautonomous Nonlinear Distributed Parameter Systems

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1 Introduction

The goal of this note is to announce a general convergence and stability theory for Galerkin approximations of inverse problems involving the identification of time dependent, nonlinear, distributed parameter systems. The work we discuss here is an extension of the general theory presented in [2]. Our treatment of this problem depends heavily on the results established in that paper, as well as on the theory of monotone nonlinear operators in Banach spaces given by Barbu in [3].

2 The Problem

We will next define the class of nonlinear evolution systems and identification problems on which we focus our attention. To this end, let $\mathcal{D}$ be a metric space with the admissible parameter set Q a compact subset of $\mathcal{D}$, and let the observation space Z be a normed linear space with norm $|\cdot|_Z$. Let H be a Hilbert space with inner product $\langle \cdot , \cdot \rangle$ and corresponding norm $|\cdot|$. Finally, let V be a Hilbert space that is densely and continuously embedded in H, with norm $\|\cdot\|$ and embedding constant K : For each $\phi \in V$, we have $|\phi| \leq K\|\phi\|$. We use these spaces to form a Gelfand triple structure $V \hookrightarrow H = H^* \hookrightarrow V^*$. Consider now the following abstract identification problem:

(ID) Given observations $z \in Z$, find parameters $\overline{q} \in Q$ which minimize the performance index

$$J(q) = \Phi(u(\cdot, q); z),$$

where $u(\cdot, q)$ is the solution to the initial value problem

$$\begin{cases} \dot{u}(t) + A(t,q)u(t) = F(t, u(t), q) \\ u(0) = u_0(q). \end{cases} \tag{2.1}$$

Here each $A(t,q)$ is a hemicontinuous, nonlinear, operator defined on all of V with range in V^*. We note that the above identification problem was studied in [2] for the case when the function F did not depend on u, i.e. $F = F(t,q)$.

3 Main Results

We begin our discussion by presenting several results concerning the existence, uniqueness and regularity of solutions to the initial value (2.1), under reasonable and easily verifiable assumptions on the operators $A(t,q)$.

Our main goal in this paper is a convergence theory for least squares based parameter estimation. Toward that end, we consider an approximation method based on a sequence of Hilbert spaces H^N, $N = 1, 2, \ldots$, with orthogonal projections $P^N : H \to H^N$. The Galerkin approach to approximation involves restricting $A(t,q)$ to H^N, yielding operators $A^N(t,q)$ satisfying the following: For any $\phi^N \in H^N$ we have $A^N(t,q)\phi^N = \psi^N$ where ψ^N is that element in H^N which satisfies $\langle A(t,q)\phi^N, \xi^N \rangle = \langle \psi^N, \xi^N \rangle \; \forall \, \xi^N \in H^N$.

Next, based on some assumptions concerning the subspaces H^N and the orthogonal projections P^N, we show that the Galerkin approximation $u^N(\cdot, q^N) \to u(\cdot, q)$ in $C(0,T;H)$ and in $L^2(0,T;V)$, when $q^N \to q$ in Q. Such continuous dependence results indicate that a minimizer of our identification problem (ID) exists within the compact set Q. In addition, the approximation u^N of the state variable u leads to a performance index

$$J^N(q) = \Phi(u^N(\cdot, q); z)$$

to be minimized. The convergence results we discuss guarantee that if $q^N \to q$, then $J^N(q^N) \to J(q)$, which will give us (as in [1]) subsequential convergence of minimizers of the finite-dimensional identification problem to a minimizer of the infinite-dimensional problem (ID). Finally, to illustrate the feasibility of these techniques we apply the above theory to an example. Full details of our results are expected to appear elsewhere.

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