Control synthesis using the state equations and the "ARMA" model in Timed Event Graphs

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ABSTRACT

An important aim is to make control synthesis of processes modelled by timed event graphs. Using the state equations, we solve the optimal tracking problem for any past evolution of the system. However, the model is only a picture of the reality and modelling errors and faults generate misestimation of the state vector. In the case of an unknown state vector, we propose to compensate this information loss by the use of the "ARMA" model in control field .

Keywords : Discrete Event Systems, Linear systems, Process control, Timed Event Graphs, Dioïd Algebra.

1. INTRODUCTION

Discrete Event Dynamic Systems represent a large amount of systems such as flexible manufacturing systems, multiprocessor systems, and transportation networks implying synchronisation, parallelism or concurrence. Among formalisms used to represent DEDS, Timed Petri Nets allow to explicitly integrate time. The subclass of Timed Event Graphs plays an important role by its deterministic behaviour. Its evolution is described by linear systems defined on a dioïd. The interpretation of each variable is, for example of dater type for (max,+) algebra : each function $x_i(k)$ represents the date of the kth firing of the transition x_i. $\oplus$ stands for the max. operation while the usual addition plays the role of the multiplication, denoted $\otimes$.

For the (max,+) algebra, state equations are :
$x(k+1) = A\otimes x(k) \oplus B\otimes u(k+1)$
$y(k) = C\otimes x(k)$
where the state x , the output y and the control u are defined on $\Re\cup\{-\infty\}$ with the dimensions n, p and q, respectively.

An important objective is to make control synthesis of systems described by Timed Event Graphs. The problem is to compute the latest firing dates of the input transitions in such a way that the output events occur at the latest before the desired dates. However, the process undergoes unavoidable model errors and failures, and the model presents ruptures in its description which generate misestimation of the state vector.

Thus, the knowledge of model and initial condition makes possible to characterise the state vector by state equation iteration. But, this solution disregards unavoidable model errors and must start from a known state. The development of a similar technique for observers appears at first sight necessary not only to know the system state but also for every control system which will use it. Unfortunately, the simple transposition of the classical structure gives a non-linear closed-loop system which raises the problem of convergence.

In order to overcome these difficulties, we propose to use a different model composed of equations called "ARMA" by analogy with ARMA equations used in classical control systems. The proposed method aims at avoiding the need to know the state vector rather than estimating it. Equally, the "ARMA" model allows us to take a general initial condition contrary to the transfer function model and it operates on a finite horizon.

In this paper, we solve the optimal tracking problem for any past evolution of the system. Equally, we show the possibility of using "ARMA" model to make a temporal control synthesis without knowing the state vector. The principle of the approach is based on "predictability" and "commandability" conditions : as the addition $\oplus$ does not have the property of symmetry, we cannot express the output from the input as usual and these concepts make possible to reduce the "ARMA" equation structure.

The paper is organised as follows. In section 2, we give notations and background concerning dioïds. The section 3 presents the control synthesis using the state equations. We introduce then the "predictability" and "commandability" concepts which enable us to propose control synthesis based on the "ARMA" model.

2. PRELIMINARY

One of the tools used in this paper is (max,+) algebra, which is a particular example of the algebraic structure generally called dioïd. In this introduction, we shall limit ourselves to present notations and main concepts. A complete description may be found in [1] [4].

A semi-ring S is a triplet $(S,\oplus,\otimes)$ where $(S,\oplus)$ and $(S,\otimes)$ are monoïds, $\oplus$ is commutative, $\otimes$ is distributive with respect to $\oplus$ and the zero element of $\oplus$ is the absorbing element of $\otimes$. A dioïd D is an idempotent semi-ring. Set $\mathbb{R}\cup\{-\infty\}$ with max. denoted $\oplus$ and with addition noted $\otimes$ is usually called (max,+) algebra and is an example of dioïd.

We have : $\mathbb{R}_{max.} = (\mathbb{R}\cup\{-\infty\},\oplus,\otimes)$ with

$a \oplus b = \max.(a,b)$; $\varepsilon = -\infty$ is the zero element of $\oplus$

$a \otimes b = a+b$; $e = 0$ is the identity element of $\otimes$

$a \oplus a = a$ (idempotency of $\oplus$)

$a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon$ (absorbing element ε)

We denote $a\backslash b = \max\{x \text{ such that } ax \le b\}$ the left residuated of b by a, also called the subsolution of equality ax=b.

The function $x \longrightarrow a\backslash x$ (respectively $x \longrightarrow x\backslash b$) is increasing (respectively decreasing).

We denote $A\backslash B = \max. \{x \mid Ax \le B\}$ with $A \in \mathfrak{R}^{m.n}_{\max}$, $B \in \mathfrak{R}^{m}_{\max}$, $x \in \mathfrak{R}^{n}_{\max}$ and $A\backslash B = A^t \odot B$ where $\odot$ is a matrix product where operation $\oplus$ and $\otimes$ of $\mathfrak{R}_{\max}$ are replaced respectively by $\wedge$ (minimum) and $\backslash$ of $\mathfrak{R}_{\max}$.

3. CONTROL SYNTHESIS

3.1 Introduction

We note $Y^{k_1}_{k_2} = \begin{pmatrix} y(k_1) \\ y(k_1+1) \\ . \\ . \\ y(k_2) \end{pmatrix}$ $U^{k_1}_{k_2} = \begin{pmatrix} u(k_1) \\ u(k_1+1) \\ . \\ . \\ u(k_2) \end{pmatrix}$. To lighten the notations, we write equally Y for $Y^{k_1}_{k_2}$ and U for $U^{k_1}_{k_2}$ if the context specifies the vectors without ambiguity. From state equation, we deduce

$$y(k+h) = C A^h x(k) \oplus \sum_{j=0}^{h-1} CA^j B\, u(k+h-j)$$

and $Y^{k+1}_{k+h} = \mathbb{O}\, x(k) \oplus \mathbb{C}\, U^{k+1}_{k+h}$

with

$$\mathbb{O} = \begin{pmatrix} CA \\ CA^2 \\ . \\ . \\ . \\ CA^{h-1} \\ CA^h \end{pmatrix} \text{ and } \mathbb{C} = \begin{pmatrix} CB & \varepsilon & \cdots & \varepsilon & \varepsilon \\ CAB & CB & \cdots & \varepsilon & \varepsilon \\ . & . & & . & . \\ . & . & & . & . \\ . & . & & . & . \\ CA^{h-2}B & CA^{h-3}B & \cdots & CB & \\ CA^{h-1}B & CA^{h-2}B & \cdots & CAB & CB \end{pmatrix}$$

Let the matrix A of dimensions m.n . $A_{i,.}$ (respectively $A_{.,j}$) is the ith row (respectively the jth column) of the matrix A. Let $I=\{i_1,i_2,\ldots,i_k\}$ and $J= \{j_1,j_2,\ldots,j_r\}$ with $k \le m$ and $r \le n$. We note $A_{I,J}$ the matrix composed of the rows $i_1,i_2,\ldots,i_k$ and the columns $j_1,j_2,\ldots,j_r$ of A .

Problem

In this part, we generalise the classical tracking problem as follows : let's suppose that the control and output dates are known, the number of events being inferior or equal to k_0 ; we want that output follows a desired output trajectory starting from any k_0. A complete formulation can be :

Let the dates' values of the control and the output, the number of events being inferior or equal to k_0 and let a sequence of desired output $z(k)$, k ranging from $k_s = k_0+1$ to K_f. The problem is to determine the greatest control sequence u(k) *if it exists* such that the output trajectory under the control effect satisfies $\forall\, k \in [k_s, k_f]\ y(k) \le z(k)$ and y(k) maximum

3.2 Control synthesis using state equations

Now, we solve the optimal tracking problem for a known state vector.

Property 1

The two following equations are equivalent :

(1) $y(k_0+j) = \mathbb{O}_{j,.}\ x(k_0) \oplus \mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} \leq z(k_0+j)$

(2) $\mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} \leq z(k_0+j) \oplus \mathbb{O}_{j,.}\ x(k_0)$

with $\mathbb{O}_{j,.} = CA^j$

$\mathbb{C}_{j,.} = (CA^{j-1}B, \ldots., CB, \varepsilon, \ldots, \varepsilon)$

and $U_{k_0+h}^{k_0+1}$ the greatest control

Proof

Indeed, for the equation $y(k_0+j) = \mathbb{O}_{j,.}\ x(k_0) \oplus \mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1}$, we have two cases.

1) $\mathbb{O}_{j,.}\ x(k_0) \leq z(k_0+j)$

We can take $U_{k_0+h}^{k_0+1}$ such that $\mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} \leq z(k_0+j)$.

So, $\mathbb{O}_{j,.}\ x(k_0) \oplus \mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} = y(k_0+j) \leq z(k_0+j)$

We take the greatest control $U_{k_0+h}^{k_0+1}$ such that $\mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} \leq z(k_0+j) = z(k_0+j) \oplus \mathbb{O}_{j,.}\ x(k_0)$.

2) $\mathbb{O}_{j,.}\ x(k_0) > z(k_0+j)$

In this case, for any $U_{k_0+h}^{k_0+1}$ $\mathbb{O}_{j,.}\ x(k_0) \oplus \mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} = y(k_0+j) > z(k_0+j)$

We take the greatest control $U_{k_0+h}^{k_0+1}$ such that $\mathbb{C}_j\ U_{k_0+h}^{k_0+1} \leq \mathbb{O}_j\ x(k_0) = z(k_0+j) \oplus \mathbb{O}_j\ x(k_0)$ □

The following theorem generalises the results of [2] [8] : it makes possible to compensate the effects of unfavourable past evolution of the system for any horizon h of the desired output trajectory.

Theorem 1

In the Multi-input Multi-output case, the greatest control so that each output y(k) occurs at the latest before z(k), otherwise at the earliest after this time is given by :

for j=1 to h and i=1 to q

$v(k_0+j) = \mathbb{C}_{.,J} \setminus [\ Z_{k_0+h}^{k_0+1} \oplus \mathbb{O}\ x(k_0)\ ]$

with $J=\{(j-1)q+1,(j-1)q+2,\ldots, (j-1)q+q\}$

$u_i(k_0+j)=v_i(k_0+j) \wedge u_i(k_0+j+1)$

under the conditions $u_i(k_0+j) \geqslant \sum_{i=1}^{n} x_i(k_0)$

and $u_i(k_0+1) \geqslant u_i(k_0)$

Proof

We want to calculate the greatest control such that $Y_{k_0+h}^{k_0+1} \leq Z_{k_0+h}^{k_0+1}$. The model is

$Y_{k_0+h}^{k_0+1} = \mathbb{O}\ x(k_0) \oplus \mathbb{C}\ U_{k_0+h}^{k_0+1}$

If $\mathbb{O}\ x(k_0) \leq Z_{k_0+h}^{k_0+1}$, the greatest control is $\mathbb{C} \setminus Z_{k_0+h}^{k_0+1}$. In this case, $Y_{k_0+h}^{k_0+1} = \mathbb{O}\ x(k_0) \oplus \mathbb{C}\ (\mathbb{C} \setminus Z_{k_0+h}^{k_0+1})$ is maximum and $Y_{k_0+h}^{k_0+1} \leqslant Z_{k_0+h}^{k_0+1}$.

But, the proposition $\mathbb{O}\ x(k_0) \leq Z_{k_0+h}^{k_0+1}$ can be false because $x(k_0)$ is function of the past control and the initial condition. To obtain the latest control, we consider each row j of the equation set which is equivalent to

$\mathbb{C}_{j,.}\ U_{k_0+h}^{k_0+1} \leq z(k_0+j) \oplus \mathbb{O}_{j,.}\ x(k_0)$ from the property 1.

If j = 1..h, we obtain $\mathbb{C}\ U_{k_0+h}^{k_0+1} \leq Z_{k_0+j}^{k_0+1} \oplus \mathbb{O}\ x(k_0)$; the formulation is identical in the Multi-output case.

However, the control is non-decreasing and the greatest solution is obtained by :

$v(k_0+j) = \mathbb{C}_{.,J} \setminus [\ Z_{k_0+j}^{k_0+1} \oplus \mathbb{O}(C,A)\ x(k_0)\ ]$ with $J=\{1+(j-1)q, 2+(j-1)q,\ldots, q+(j-1)q\}$ in the Multi-input Multi-output case and

$u_i(k_0+j)=v_i(k_0+j) \wedge u_i(k_0+j+1)$.

Equally, $x(k_0)$ must be known before the application of the first control $u(k_0+1)$; otherwise we must take $\sum_{i=1}^{n} x_i(k_0)$ for the corresponding input transition □

Remark

The results presented in [8] correspond to a similar development in (min,+) algebra but restricted to the case $\mathbb{O}\ x(k_0) \leq Z_{k_0+h}^{k_0+1}$. In this case and in (max,+) algebra, the left hand term $u(k_0+j-1)$ is useless because the calculus of $v(k_0+j)$ gives directly a non-decreasing input sequence.

3.3. MULTI-STEP CONTROL SYNTHESIS USING "ARMA" MODEL

In this section, we take the hypothesis of an unknown vector state.

3.3.1 Cyclicity and model

Definition

Let λ be the maximum mean value of circuit weight of a graph associated with a general matrix A. λ is also the maximum proper value of this matrix. We say that a matrix A is cyclic if there are d and m such that : $\forall l \geq m,\ A^{l+d} = \lambda^d A^l$

d is called matrix A cyclicity and we say that A is d-cyclic.

Theorem 2

Every irreducible A matrix is d-cyclic □

While residuation is well adapted to the resolution of inequalities of the type Ax⩽ B, the (max,+) algebra extension S_{max}. [6] [7] is an interesting approach that facilitates analogies with classical algebra. Based on this theory, in [5], we deduce from the state equations the following ARMA model for the important class of systems presenting the cyclicity property, like the strongly connected Petri Nets.

$$y(k) \oplus \sum_{i=d}^{m+d-1} \beta_i \lambda^d g_{i-d} u(k-i) = \lambda^d y(k-d) \oplus \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i) \qquad (1)$$

with $g_i = CA^iB$ and for i from d to m+d-1

$(\alpha_i, \beta_i) = (\varepsilon, \varepsilon)$ if $g_i = \lambda^d g_{i-d}$

$(\alpha_i, \beta_i) = (e, \varepsilon)$ if $g_i > \lambda^d g_{i-d}$

$(\alpha_i, \beta_i) = (\varepsilon, e)$ if $g_i < \lambda^d g_{i-d}$

Each term contains once the output and a function of the control. However, as the addition does not have the property of symmetry, we cannot express the output y(k) from the other terms. An objective will be to reduce this form. In this aim, we introduce the following initial concepts of the control synthesis :

Note : if the Petri Net is without loop, we obtain the equation : $y(k) = \sum_{i=0}^{n-1} g_i\, u(k-i)$. We can deduce this structure from (1) if we put $\lambda^d = \varepsilon$ and $m+d=n$ and consequently, we can easily transpose the next results.

3.3.2 "Predictable" and "Commandable" concepts

Control conditions

The three conditions such that the control can determine the output are :

a) Deterministic model

The ARMA model must be the most representative and the equation (1) must be compatible or in other words, the equality is verified.

b) Predictable or "observable" behaviour

If we apply any control, we must anticipate its effects on the output. It is the case if

$$y(k) \geq \sum_{i=d}^{m+d-1} \beta_i \lambda^d g_{i-d} u(k-i) \quad (\text{condition } C_1)$$

So, the condition C_1 makes possible to "observe" the output, knowing the past values of the input and output. So, we have, under condition C_1

$$y(k) = \lambda^d y(k-d) \oplus \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

The first right hand term represents the internal periodic behaviour of the system as the other term gives the effects of the external control on the output.

c) "Commandable" behaviour

The preceding form shows that the control can only delay the output y(k) relatively to λ^dy(k-d). So, a condition such that the control have an effect on the output is

$$\lambda^d y(k-d) \leq \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i) \quad (\text{condition } C_2)$$

Under the three previous conditions, we obtain finally

$$y(k) = \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

The following definition expresses the output trajectory characteristic. It can be applied equally to the control or to the desired output after a past evolution of the process.

Definition

d-cyclic trajectory : Output y follows a d-cyclic trajectory starting from $k=k_s$ to k_f if
$y(k) \geq \lambda^d y(k-d)$ with $k_s \leqslant k \leqslant k_f$
Notice that a d-cyclic desired output follows the production capacity of the process.

Equation set

If we vary the number of events k in the ARMA equation (1) , we obtain the following equation set :

$$\lambda^d \begin{pmatrix} y(k-d) \\ y(k-d+1) \\ \vdots \\ y(k+m-1) \end{pmatrix} \oplus \begin{pmatrix} Q & R & S \end{pmatrix} \begin{pmatrix} U_{k-1}^{k-m-d+1} \\ U_k \\ U_{k+m+d-1}^{k+1} \end{pmatrix} = \begin{pmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+m+d-1) \end{pmatrix} \oplus \begin{pmatrix} Q' & R' & S' \end{pmatrix} \begin{pmatrix} U_{k-1}^{k-m-d+1} \\ U_k \\ U_{k+m+d-1}^{k+1} \end{pmatrix} \quad (2)$$

with U_k=u(k)

$$Q=\begin{pmatrix} a_{m+d-1} & a_{m+d} & \cdots & a_{d+1} & a_d & a_{d-1} & \cdots & a_1 \\ \varepsilon & a_{m+d-1} & \cdots & a_{d+2} & a_{d+1} & a_d & \cdots & a_2 \\ \varepsilon & \varepsilon & \cdots & a_{d+3} & a_{d+2} & a_{d+1} & \cdots & a_3 \\ & & & & & & & \\ \varepsilon & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon & \cdots & \varepsilon \end{pmatrix}$$

$$R=\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ . \\ a_{m+d-1} \end{pmatrix} \quad S=\begin{pmatrix} \varepsilon & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon \\ a_0 & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon \\ a_1 & a_0 & \cdots & \varepsilon & \varepsilon & \varepsilon \\ & & & & & \\ a_{m+d} & a_{m+d-1} & \cdots & a_2 & a_1 & a_0 \end{pmatrix}$$

$$Q'=\begin{pmatrix} b_{m+d-1} & b_{m+d} & \cdots & b_{d+1} & b_d & \varepsilon & \cdots \\ \varepsilon & b_{m+d-1} & \cdots & b_{d+2} & b_{d+1} & b_d & \cdots \\ \varepsilon & \varepsilon & \cdots & b_{d+3} & b_{d+2} & b_{d+1} & \cdots \\ & & & & & & \\ \varepsilon & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon & \cdots \end{pmatrix}$$

$$R'=\begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ b_d. \\ . \\ . \\ b_{m+d-1} \end{pmatrix} \quad S'=\begin{pmatrix} \varepsilon & \cdots & \varepsilon & \varepsilon & \cdots & \varepsilon \\ . & & . & . & & . \\ \varepsilon & \cdots & \varepsilon & \varepsilon & \cdots & \varepsilon \\ b_d & \cdots & \varepsilon & \varepsilon & \cdots & \varepsilon \\ . & & . & . & & . \\ b_{m+d-1} & \cdots & b_d & \varepsilon & \cdots & \varepsilon \\ b_{m+d} & \cdots & b_{d+1} & b_d & \cdots & \varepsilon \end{pmatrix}$$

$a_i = \alpha_i CA^i B$; $b_i = \beta_i \lambda^d CA^{i-d} B$

R, R' are two column matrices of dimension m+d.1 . They are respectively defined by :
$R_i = \alpha_i\, g_i$ for i equals 0 to m+d-1
$R'_i = \varepsilon$ for i equals 0 to d and $\beta_i \lambda^d g_{i-d}$ for i equals d to m+d-1
Q,Q' two upper triangular matrices of dimension m+d. m+d-1. So, by construction, the last line of both Q and Q' is zero.

We introduce the two following lemmas that give conditions of a "predictable" and "commandable" behaviour of a trajectory. The proof of the lemma 1 is given in [5]. The theorem 2 that summarises the results on the concepts of the part 3.3, enables us to present the control synthesis approach in four steps.

Lemma 1
If the trajectory is d-cyclic and verifies $Y^{k_s}_{ks+m+d-1} \geq Q' U^{k_s-m-d+1}_{k_s-1}$ and if $u(k) = R \backslash Y^{k}_{k+m+d-1}$ for $k \geq k_s$ then the output trajectory is predictable (C_1)

Lemma 2
Let consider an output trajectory and an input vector such that the equation set (2) is compatible. If the output trajectory is predictable and d-cyclic then the output are "commandable" (C_2)

Proof
Under the condition of exact model and predictable behaviour, we obtain

$$y(k) = \lambda^d y(k-d) \oplus \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

If the trajectory is d-cyclic, the equation is reduced to

$$y(k) = \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

As $\lambda^d y(k-d) \leq y(k)$, a consequence is

$$\lambda^d y(k-d) \leq \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

which is the condition C_2 □

Theorem 2
The two following propositions are equivalent :
- the equation set (2) is compatible. The output trajectory is predictable (C_1) and "commandable" (C_2)
- the trajectory is d-cyclic and verifies

$$Y^{k_s}_{k_s+m+d-1} \geq Q' U^{k_s-m-d+1}_{k_s-1}$$

and if

$$u(k) = R \backslash Y^{k}_{k+m+d-1} \text{ for } k \geq k_s$$

$$y(k) = \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

Proof
* Suppose that the trajectory is d-cyclic (H_1) and verifies

$$Y^{k_0+1}_{k_0+m+d} \geq Q' U^{k_0-m-d+2}_{k_0} \quad (H_2)$$

$$u(k+1) = R \lhd Y^{k+1}_{k+m+d} \text{ for } k \geq k_0 \ (H_3)$$

$$y(k) = \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i) \ (H_4) \quad .$$

- H_1,H_2 and H_3 ⇒ predictable condition (lemma 1)
- H_1 and H_4⇒

$$y(k) = \lambda^d y(k-d) \oplus \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$$

Moreover,

$$y(k) \geq \sum_{i=d}^{m+d-1} \beta_i \lambda^d g_{i-d} u(k-i) \quad (\text{condition } C_1) \ .$$

So, the equation

$$y(k) \oplus \sum_{i=d}^{m+d-1} \beta_i \bar{\lambda}^d g_{i-d} u(k-i) =$$

$$\bar{\lambda}^d y(k-d) \oplus \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i) \text{ is compatible.}$$

- Compatibility , predictable condition and $H_1 \Rightarrow$ "commandability" condition by the lemma 2.

* Reciprocally, suppose that the equation set (2) is compatible (H_5), the output trajectory is predictable (H_6) and "commandable" (H_7). We have :

- H_5, $H_6 \Rightarrow H_1$ (d-cyclicity)
- H_6 entails directly H_2 which is included in H_6 $\left(Y_{k_0+m+d}^{k_0+1} \geq Q' U_{k_0}^{k_0-m-d+2}\right)$
- H_5, H_6 and $H_7 \Rightarrow H_4$ $\left(y(k) = \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)\right)$
- $H_4 \Rightarrow u(k+1) = R \setminus Y_{k+m+d}^{k+1}$ for $k \geq k_0$ which verifies the equality □

The four steps of the control synthesis

The solution is given by the four following steps, where the three first ones are constructive and the last one is a verification.

a) d-cyclic trajectory

$$y(k) \geq \bar{\lambda}^d y(k-d) \quad \textit{for}\ k_s + d \leq k \leq k_f$$

We deduce it from

$y(k) = z(k) \wedge \bar{\lambda}^d \setminus y(k+d)$ for

$k_s \leq k \leq k_f$ with $y(k) = +\infty$ for $k > k_f$

b) Control

Control is deduced as above by

$$u(k) = R \setminus Y_{k+m+d-1}^{k} \text{ for } k_s \leqslant k \leqslant k_f$$

c) Compatible trajectory

We deduce a compatible trajectory y with

$$\begin{pmatrix} R & S \end{pmatrix} U_{k_s+m+d-1}^{k_s} = Y_{k_s+m+d-1}^{k_s}$$

and $y(k) = \sum_{i=0}^{m+d-1} \alpha_i g_i u(k-i)$ for $k \geqslant k_s+m+d$

d) Initial constraint

Trajectories must verify the following inequalities :

$$Y_{k_s+m+d-1}^{k_s} \geq (Q \oplus Q') U_{k_s-1}^{k_s-m-d+1} \oplus \bar{\lambda}^d \begin{pmatrix} Y_{k_s-1}^{k_s-d} \\ \varepsilon_m \end{pmatrix}$$

and $u(k_s) \geq u(k_0) \oplus y(k_0)$ (causality condition in single-input single-output case)

4. CONCLUSION

In this paper, we introduced two control synthesis approaches which use state equations and ARMA model, respectively. The first one needs the state vector knowledge, contrary to the second approach. In the two cases, we considered a past evolution of the system and the two approaches authorise changes of desired output and of the production rate. Moreover, the control expression shows some structure similarities and some natural relations can be realised. Future work will specify the connections between the different approaches and will define the consequences of an unknown vector state.

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