Dynamic Output Feedback Stabilization For A Class of Saturated Linear Systems

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Abstract

This paper addresses the problem of the global stabilization of a linear system with saturating controls by means of a dynamic output feedback using a saturated linear controller built from an observer. It is shown that a simple linear control law of an optimal-like type always globally stabilizes the closed-loop system when the linear system to be controlled is asymptotically or critically stable.

Key words

Continuous-time system, saturated system, dynamic output feedback, observer, compensator, global stability.

1 Introduction

This paper deals with the problem of the global stabilization of a linear systems with saturable controls by means of a dynamic output feedback using a full observer. It is well-known that this problem is closely related to that of the determination of a state feedback globally stabilizing a linear system (here original system + observer) with saturating controls. Feedback control of a linear system subject to magnitude constraints on the input has often been formulated in terms of an optimal control problem. Obtaining these control law is cumbersome and it is well-known that their subsequent implementation is difficult [6]. By using Lyapunov's functions [10] and frequency domain technics [12] some sufficient conditions have been established to acheive global stability, i.e., Popov's stability criteria. Those latter results were obtained for any nonlinearity contained in a conic sector where saturation was included. By addressing directly the saturation issue, one should expect to obtain less restrictive stability conditions [15].

An important theoritical result has been established in [13], [18]. It is shown that : an (A, B)-stabilizable system with constrained controls can be globally stabilized by means of a *nonlinear state feedback* if the open-loop system is not strictly unstable (see definition below). Unfortunately this *existence* result is not constructive in the sense that its proof do not furnish any methodology allowing the determination of this feedback. But this result has the advantage to confirm the intuitive result which states that a strictly unstable open-loop system with constrained control can not be globally stabilized by any nonlinear state feedback and consequently *a priori* by any linear static state feedback.

Clearly this result do not bring an answer to the problem to know if there exists a *static state feedback* which allows the global stabilization when the open-loop system is asymptotically stable, critically stable or critically unstable. In the latter case (critically unstable case) a counterexample furnished by [3] and next by [14] brings a negative answer. For the two remaining cases (asymptotically or critically stable open-loop system) it has been shown the following important result [2], [16] : if the open-loop system is (A, B)-stabilizable, there exist a *static state feedback* matrix of the optimal-like form globally stabilizing the closed-loop system. Furthermore, this matrix is obtained from any solution of the Lyapunov equation of the open-loop system and can be easily computed. This is an important practical result.

The problem of the global stabilization by using a *static saturable feedback* has been considered in [8]. This problem conserves, at least, the difficulties encountred in the determination of a static output feedback for the unsaturated case.

The global stabilization by means of a *saturable dynamic output feedback* using a full-order observer has been also considered. Under the stabilizability and detectability assumptions, and like in the previous discussion, the existence of a *nonlinear feedback* globally stabilizing the composite system, provided that the open-loop system is not strictly unstable, is proven in [13], [18]. This result suffers the same drawbacks as above mentionned and cannot be used to answer the question of the global stabilization by using a *static linear feedback* law.

Thus, the objective of this paper is to show that a *linear static state feedback* of the optimal-like type can globally stabilize the composite system (original system + observer or compensator) when the open-loop system is asymptotically or critically stable. This feedback law is simply derived from any solution of the Lyapunov equation of the open-loop system, and then can be easily computed.

Throughout this paper the following notations are used. If x is a vector of $\Re^n$, x^t denotes its transpose, and $||x||$ its norm ; matrix A^t denotes the transpose of matrix A, $||A||, \lambda_i(A)$ are the induced norm and the ith eigenvalue, respectively. Further, a symmetric and positive definite (resp. semi-definite) matrix M is denoted by $M > 0$ (resp. $M \geq 0$). For a system $\dot{x}(t) = Ax(t)$, $t \geq 0$, a set $D \subset \Re^n$ is said to be positively invariant if $exp(tA)D \subseteq D$. The equilibrium $x = 0$ of linear autonomous system $\dot{x} = Ax$ is said to be asymptotically stable if $Re(\lambda_i(A)) < 0$, $\forall i$. If $Re(\lambda_i(A)) \leq 0$, with for some index i, $Re(\lambda_i(A)) = 0$, then it is said to be critically stable if the algebraic multiplicity of each critical eigenvalue is equal to its geometrical multiplicity ; else it is said to be critically unstable. If there exists at least an index i for which $Re(\lambda_i(A)) > 0$, the equilibrium $x = 0$ is strictly unstable (or exponentially unstable).

2 Preliminaries

Let us consider the following continuous-time system described by:

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{array} \right. \tag{1}$$

where matrices $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{l\times n}$, and controls $u(t)$ can saturate, that is, $u(t)$ belongs to a compact set $\Omega \subset \Re^m$ defined by

$$\begin{array}{rl} \Omega = & \{u(t) \in \Re^m | -u_m^i \leq u^i(t) \leq u_M^i \ ; \\ & u_m^i,\ u_M^i,\ \forall i = 1, ..., m\} \end{array} \tag{2}$$

It is assumed that system (1) is (A, B)-stabilizable and (A, C)-detectable in Lasalle's sense [9]. As in the classical case of stabilization we introduce the following observer described by

$$\bar{x}(t) = (A + KC)\bar{x}(t) + Bsat(F\bar{x}(t)) - KCx(t) \tag{3}$$

in which matrices $F \in \Re^{m\times n}$, $K \in \Re^{n\times l}$, and the term $sat(F\bar{x}(t))$ is defined, for $i = 1, 2, ..., m$, by

$$sat(F\bar{x}(t))^i = \left\{ \begin{array}{ll} u_M^i & \text{if } (F\bar{x}(t))^i > u_M^i \\ (F\bar{x}(t))^i & \text{if } -u_m^i \leq (F\bar{x}(t))^i \leq u_M^i \\ -u_m^i & \text{if } (F\bar{x}(t))^i < -u_m^i \end{array} \right. \tag{4}$$

Finally the problem to be solved consists in finding the suitable feedback matrix F which globally stabilizes (in the classical sense used by [2], [8], [13]) the following composite system given by

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bsat(F\bar{x}(t)) \\ \bar{x}(t) = (A + KC)\bar{x}(t) + Bsat(F\bar{x}(t)) - KCx(t) \end{array} \right. \tag{5}$$

for which the error $e(t) = \bar{x}(t) - x(t)$ satisfies

$$\dot{e}(t) = (A + KC)e(t) \tag{6}$$

Clearly, since (A, C) is assumed to be detectable, matrix K can be picked so that matrix $(A + KC)$ is Hurwitz, that is, $Re(\lambda_i(A + KC)) < 0, \forall i$.
Note that in the case of the global stability, $x = 0$ is then the only equilibrium point in $\Re^n$ and the domain of attraction of origin is $\Re^n$.
At the opposite $x = 0$ being the only equilibrium point for (5) does not imply the global asymptotic stability of (5) ; concerning the difficult problem of equilibrium points and domain of attraction, also called stability region, see for instance [1], [15], and references of this paper.

In order to develop our results the term $sat(F\bar{x}(t))$ is written as

$$sat(F\bar{x}(t)) = D(\alpha(\bar{x}(t)))F\bar{x}(t) \tag{7}$$

whose entries $\alpha^i(\bar{x}(t))$ of the diagonal matrix D are defined for $i = 1, 2, ..., m$, by

$$\alpha^i(\bar{x}(t)) = \left\{ \begin{array}{ll} \frac{(u_M)^i}{(F\bar{x}(t))^i} & \text{if } (F\bar{x}(t))^i > u_M^i \\ 1 & \text{if } -u_m^i \leq (F\bar{x}(t))^i \leq u_M^i \\ -\frac{(u_m)^i}{(F\bar{x}(t))^i} & \text{if } (F\bar{x}(t))^i < -u_m^i \end{array} \right. \tag{8}$$

and satisfy

$$0 < \alpha^i(\bar{x}(t)) \leq 1 \tag{9}$$

Note also that, since matrix $(A + KC)$ is Hurwitz and by using a well-known Kalman's result [4], there always exists a matrix $P_0 > 0$ solution of

$$(A + KC)^t P_0 + P_0(A + KC) = -Q_0 \tag{10}$$

where $Q_0 > 0$. Analogously the Lyapunov's equation of the autonomous system $\dot{x}(t) = Ax(t)$, given by

$$A^t P + PA = -Q \tag{11}$$

admits also a solution $P > 0$ corresponding either to any matrix $Q > 0$, if matrix A is asymptotically stable (A.S), or to a suitable $Q \geq 0$ if matrix A is critically stable (C.S).

3 Main results

The main result of this paper is given by the following theorem.

Theorem 3.1 : *Under the assumptions of (A,B)-stabilizability and (A,C)-detectability of system (1), and provided that matrix A is stable (asymptotically or critically), the composite system (5), is globally* *asymptotically stabilizable by means of the feedback matrix F given by*

$$F = -\Lambda(\gamma)B^t P \tag{12}$$

where $\Lambda(\gamma)$ is any diagonal matrix with positive elements $\gamma_i, i = 1, ..., m$, and matrix $P > 0$ is a solution of the Lyapunov equation (11).

This result follows from the next Lemma which consider separately the cases, when matrix A is asymptotically stable and critically stable.

Lemma 3.1 : *Assume system (1) is (A,B)- stabilizable and (A,C)-detectable, then the feedback matrix (12), globally stabilizes the composite system (5) provided that matrix $P > 0$ is a solution of (11) and γ satisfies:*

$$i)\ 0 < \gamma_i < \frac{\sqrt{\lambda_{min}(Q)\lambda_{min}(Q_0)}}{||PB||^2}, \ \forall i = 1, ..., m, \tag{13}$$

in the case when matrix A is asymptotically stable, and

$$ii)\ 0 < \gamma_i \leq \frac{\gamma_{max}^2}{\gamma_{min}} < \frac{2\lambda_{min}(Q_0)}{||PB||^2}, \ \forall i = 1, ..., m, \tag{14}$$

if matrix A is critically stable, where $\gamma_{max} = \lambda_{max}(\Lambda(\gamma))$ and $\gamma_{min} = \lambda_{min}(\Lambda(\gamma))$.

The Proof of this Lemma, when matrix A is critically stable, needs the use of the following result.

Proposition 3.1 : *[16] The subspace*

$$[Ker(Q) \cap Ker(B^t P)] \backslash \{0\} \neq \emptyset$$

or one of its subspaces, is not positively invariant with respect to system $\dot{x}(t) = Ax(t)$ if and only if system (1) is (A, B)-stabilizable.

Proof of Lemma 3.1 : Since matrix $(A + KC)$ is Hurwitz, then from (6) $e(t) \to 0$ as $t \to \infty$, $\forall e_0 \in \Re^n$. With $\bar{x}(t) = x(t) + e(t)$ and using (7), system (5) becomes

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BD(\alpha)F(x(t) + e(t)) \\ \dot{e}(t) &= (A + KC)e(t) \end{aligned} \tag{15}$$

Consider the following Lyapunov function, candidate for system (15):

$$V(x, e) = x^t Px + e^t P_0 e \tag{16}$$

where $P > 0$ and $P_0 > 0$ are solution of (11) and (10), respectively. Compute the time derivative of this function along the trajectories of (5) ; using (15) and (16) we get:

$$\dot{V}(x, e) = -x^t Qx - e^t Q_0 e + 2x^t PBD(\alpha)F(x+e) \tag{17}$$

Substituting F, given by (12), into (17) yields:

$$\begin{aligned} \dot{V}(x, e) = \ & -x^t Qx - e^t Q_0 e - 2x^t PBD(\alpha)\Lambda(\gamma)B^t Px - \\ & 2x^t PBD(\alpha)\Lambda(\gamma)B^t Pe \end{aligned} \tag{18}$$

Part (i): Taking into acount that $||D(\alpha)|| \leq 1$, $||\Lambda(\gamma)|| = \gamma_{max} = \max_i(\gamma_i)$ and A is asymptotically stable, it follows from (18):

$$\begin{aligned} \dot{V}(x, e) \leq \ & -\lambda_{min}(Q)||x||^2 - \lambda_{min}(Q_0)\ ||e||^2 + \\ & 2\gamma_{max}||PB||^2\ ||x||\ ||e|| \end{aligned} \tag{19}$$

or

$$\dot{V}(x, e) \leq -z^t(t)Mz(t) \tag{20}$$

where $z^t(t) = [||x||\ \ ||e||]$ and

$$M = \begin{bmatrix} \lambda_{min}(Q) & -\gamma_{max}||PB||^2 \\ -\gamma_{max}||PB||^2 & \lambda_{min}(Q_0) \end{bmatrix}$$

Clearly $\dot{V}(x, e) < 0$, $\forall x, e \neq 0$ if the above matrix in (20) is positive definite, that is, since $\lambda_{min}(Q) > 0$ and $\lambda_{min}(Q_0) > 0$, if condition (i) of Lemma 3.1 holds. Indeed since $e(t) \to 0$, as $t \to \infty$, then from (20), $\dot{V}(x, e) < 0$ implies that $x(t) \to 0$ as $t \to \infty$.

Part (ii): Write $D(\alpha)$ as $H^t(\alpha)H(\alpha)$ with $H(\alpha)$ a diagonal matrix which obviously satisfies $||H(\alpha)|| \leq 1$. From (18) we get

$$\begin{aligned} \dot{V}(x, e) \leq \ & -x^t Qx - \lambda_{min}(Q_0)||e||^2 - \\ & 2x^t PBH(\alpha)H^t(\alpha)\Lambda(\gamma)B^t Px - \\ & 2x^t PBH(\alpha)H^t(\alpha)\Lambda(\gamma)B^t Pe \end{aligned} \tag{21}$$

which becomes, with $\xi = H^t(\alpha)B^t Px$,

$$\begin{aligned} \dot{V}(x, e) \leq \ & -x^t Qx - \lambda_{min}(Q_0)||e||^2 - 2\gamma_{min}||\xi||^2 + \\ & 2\gamma_{max}||\xi||\ ||PB||\ ||e|| \end{aligned} \tag{22}$$

or

$$\dot{V}(x, e) \leq -x^t Qx - w^t \Delta w \tag{23}$$

where $w^t = [||e||\ \ ||\xi||]$ and

$$\Delta = \begin{bmatrix} \lambda_{min}(Q_0) & -\gamma_{max}||PB|| \\ -\gamma_{max}||PB|| & 2\gamma_{min} \end{bmatrix}$$

Hence, if condition (ii) of Lemma 3.1 holds, then the term $w^t \Delta w$ is positive definite and $\dot{V}(x, e) < 0$ except perhaps for $z = 0$, that is, $e = 0, \xi = 0$ or equivalently for $e = 0$ and $x \in Ker(B^t P)$.

Clearly in this case, since $-x^t Qx \leq 0$ we may obtain $\dot{V}(x, 0) = 0$ if $x \in (Ker(Q) \cap Ker(B^t P))$. If $[Ker(Q) \cap Ker(B^t P)] \backslash \{0\} = \emptyset$ then $\dot{V}(x, e) < 0$,

$\forall x \in \Re^n\backslash\{0\}$ and $e \in \Re^n\backslash\{0\}$. Nevertheless when $[Ker(Q) \cap Ker(B^tP)]\backslash\{0\} \neq \emptyset$, under the (A, B)-stabilizability property and from Proposition 3.1, using Lasalle's invariance principle, it must be proven that for $e = 0$ and an initial value $x(t_0) \in (Ker(Q) \cap Ker(B^tP))$ it is impossible to get:

$$\dot{V}(x(t; x(t_0), 0) = 0, \forall t \geq t_0$$

For $e = 0$, system (14) is reduced to system

$$\dot{x} = Ax(t) + BD(\alpha)Fx(t)$$

In this case, Proposition 3.1 implies that if there exists for t_0, $x(t_0) \in (Ker(Q) \cap Ker(B^tP))$ then, there exist an instant $t_1 > t_0$ such that:

$$x(t; x(t_0)) \notin (Ker(Q) \cap Ker(B^tP)), \forall t \geq t_1$$

Then for $t \geq t_1$ we get $\dot{V}(x(t, x(t_0)), 0) < 0$. From Lasalle's result follows the same conclusion as in the above part (i). ■

Remark 3.1 : *If matrix A of (1) is asymptotically stable, a mathematical solution stabilizing (5) consists to take $F = 0$ (even $K = 0$). Nevertheless in practice such a solution is not wished, since taking $F \neq 0$, $K \neq 0$ may be offers the possibility to improve the decay rate of the solutions. This can be showed, by using the results developed in [2], [7], that display in what conditions such control law gives a dynamically faster regulator than the open-loop system in term of decrease of the Lyapunov function $V(x, e)$.*

Proof of Theorem 3.1 : Conditions (i) and (ii) of Lemma 3.1 clearly depend only on the choice of $\lambda_{min}(Q_0)$ which can be arbitrary for any choice of a suitable matrix K. Consequently these conditions can be satisfied for any choice of the diagonal matrix $\Lambda(\gamma)$ and any matrix P solution of (11). ■

Remark 3.2 : *When matrix A is stable (asymptotically or critically) the previous results show that the global stabilization of the composite system (5) is possible by using the static linear state feedback given by (12). From [2] and [11] it is obvious that the case corresponding to a matrix A strictly unstable cannot be globally stabilized by feedback. But when matrix A is critically unstable it is possible that the semi-global stabilization of system (5) can be acheived by using a static linear state feedback. For the moment, when matrix A is unstable (strictly or critically) a positively invariant and asymptotically stable local domain can always be determined [2], [17].*

Dynamic output compensator

Consider the dynamic state feedback compensator of the form:

$$\left\{ \begin{array}{l} \dot{w}(t) = A_r\, w(t) + B_r\, y(t) \\ v(t) = M_r\, w(t) + N_r\, y(t) \end{array} \right. \tag{24}$$

where $w(t) \in \Re^r$ and A_r, B_r, M_r, N_r are constant matrices of appropriate dimensions. The order "r" is chosen such that $(m+r)(l+r) \geq (n+r)$ or $(m+l+2r) > (n+r)$.

The objective consists in finding matrices A_r, B_r, M_r, N_r such that the following composite system:

$$\left\{ \begin{array}{l} \dot{x}(t) = A\, x(t) + B\, sat(v(t)) \\ \dot{w}(t) = A_r\, w(t) + B_r\, y(t) \end{array} \right. \tag{25}$$

will be globally asymptotically stable.

Letting $z^t = [x^t, w^t]$ and $y_c^t = [y^t,\ w^t]$, system (25) can be expressed by:

$$\left\{ \begin{array}{l} \dot{z}(t) = A_c\, z(t) + B_c U(t) \\ y_c(t) = C_c\, z(t) \end{array} \right. \tag{26}$$

where

$$A_c = \left[\begin{array}{cc} A & 0 \\ 0 & 0_{r\times r} \end{array} \right],\ B_c = \left[\begin{array}{cc} B & 0 \\ 0 & I_{r\times r} \end{array} \right],$$

$$C_c = \left[\begin{array}{cc} C & 0 \\ 0 & I_{r\times r} \end{array} \right],\ U(t) = \left[\begin{array}{c} sat(v(t)) \\ \dot{w}(t) \end{array} \right]$$

By using the equivalent form of the saturation term the new control vector $U(t)$ can be written as:

$$U(t) = \ \Phi_c(\alpha)\ \ K_c C_c\, z(t)$$

where

$$\Phi_c(\alpha) = \left[\begin{array}{cc} \Phi(\alpha) & 0 \\ 0 & I_{r\times r} \end{array} \right],\ K_c = \left[\begin{array}{cc} N_r & M_r \\ B_r & A_r \end{array} \right]$$

Similarly, system (25) can be described by:

$$\dot{z}(t) = \bar{A}_c\, z(t) + \bar{B}_c\ sat(\bar{K}_c\ y_c(t)) \tag{27}$$

where

$$\bar{A}_c = \left[\begin{array}{cc} A & 0 \\ B_rC & A_r \end{array} \right],\ \bar{B}_c = \left[\begin{array}{c} B \\ 0_{r\times m} \end{array} \right],\ \bar{K}_c = [N_r \quad M_r]$$

This shows that the design of dynamical compensator always returns to a problem of static output feedback of high order. Thus, a possible solution for the global stabilization problem, via dynamic compensator, is to use the approach developed in [7] and [8].

4 Application

Consider the linearized model of the plane "GEF 404", chosen in 35ft altitude condition, described by [5]:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -1.46 & 2.35 & 0 \\ 0.321 & -2.23 & 0 \\ 0.175 & -0.39 & 0 \end{bmatrix} x(t) + \\ & \begin{bmatrix} 0.476 & 4.9 \\ 0.708 & 0.027 \\ 0.471 & -1.24 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) \end{aligned} \tag{28}$$

The control vector is constrained by the following limits:

$$\begin{bmatrix} -3 \\ -1 \end{bmatrix} \leq u(t) \leq \begin{bmatrix} 3 \\ 3 \end{bmatrix} \tag{29}$$

Indeed system (1) is (A, B)-controllable and (A, C)-observable whereas matrix A is critically stable, since $\sigma(A) = \{0; -2.8; -0.9\}$.

The aim of the design is to determine a dynamic saturated control law so that the closed-loop system is globally asymptotically stable.

To determine a suitable dynamic stabilizer, we must compute the state feedback matrix F_0 given in (12). Thus for the following choice of matrix Q:

$$Q = \begin{bmatrix} 1.6059 & -1.1028 & 0 \\ -1.1028 & 14.4015 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{30}$$

we obtain as solution of equation (11) the following matrix :

$$P = \begin{bmatrix} 0.6563 & 0.4258 & 0.1060 \\ 0.4258 & 3.6888 & -0.0632 \\ 0.106 & -0.0632 & 1 \end{bmatrix} \tag{31}$$

For the choice $\gamma = 1$ matrix F_0 in (12) is given by:

$$F_0 = \begin{bmatrix} -0.6638 & -2.7846 & -0.4767 \\ -3.0959 & -2.2644 & 0.7223 \end{bmatrix} \tag{32}$$

Then it remains to determine matrix K so that matrix $(A + KC)$ is Hurwitz ; this can be done for example, with:

$$K = \begin{bmatrix} -7.31 & -0.175 \\ -34.88 & 0.39 \\ 0 & -7 \end{bmatrix} \tag{33}$$

From matrices F_0 and K previously computed, the composite system (5) has been simulated and its controls behaviors and transient responses are respectively presented in Figure 1 and Figure 2 for the initial conditions $x(0) = [150 \quad 200 \quad 70]^t$ and $\bar{x}(0) = [0 \quad 0 \quad -10]^t$.

5 Conclusion

In this paper it has been shown that a stable (asymptotically, critically) linear system with saturating controls can always be globally stabilized by using a dynamic output feedback which is a saturated linear controller, built from an n-dimensional observer. The suitable control law is derived easily from a Lyapunov function of the open-loop system and it is of the optimal-like type. The case when the linear system is unstable (critically or not) has been evoked in the remark 3.2. The case of dynamic output compensator has been briefly described.

References

[1] J.Alvarez, R.Suárez and J.Alvarez: *Planar linear systems with single saturated feedback*, Systems and Control Letters, 20, pp.319-326, 1993.

[2] C.Burgat, S.Tarbouriech and M.Klaï: *Continuous time saturated state feedback regulators-Theory and design*, Int.J.Syst.Science, vol.25, pp.315-336, 1994.

[3] A.T.Fuller: *In the large stability of relay and saturated control systems with linear controllers*, Int.J.Control 1969, Vol 10, pp. 457-480.

[4] R.E.Kalman and J.E.Bertram: *Control system analysis and design via "the second method" of Lyapunov*, part II, J. of Basic Eng. Trans of A.S.M.E, pp.394-400, 1960.

[5] P.Kapasouris and M.Athans : *Multivariable control systems with saturating actuators antireset strategies*, Proc. of ACC, vol.3, pp.1579-1584, 1985.

[6] P.Kapasouris, M.Athans and G.Stein : *Design of feedback control systems for stable plants with saturating actuators*, Proc. of 27^{th} CDC-IEEE, pp.469-479, 1988.

[7] M.Klaï : *Stabilisation des systèmes linéaires continus contraints sur la commande par retour d'état et de sortie saturé*, PhD thesis of the University Paul Sabatier, Toulouse (France), LAAS Report no.94323, 1994.

[8] M.Klaï, S.Tarbouriech and C.Burgat: *Some results on output stabilization of saturated linear systems*, ASME-WAM, Advances in robust nonlinear control systems, D.S.C., vol.53, pp.67-70, New-Orleans (U.S.A), November 1993.

[9] J.P.Lasalle: *The stability and control of discrete process*, App. Math. Sci, 62, Springer-Verlag, 1986.

[10] S.Lefschetz : *Stability of nonlinear control systems*, Academic Press, New-York, 1965.

[11] C.C.H.Ma : *Unstabilizability of linear unstable systems with inputs limits*, J. of Dynamic systems, Measurement and Control, ASME, vol.113, pp.742-744, 1991.

[12] K.S.Narendra and J.H.Taylor : *Frequency domain criteria for absolute stability*, Academic Press, New-York, 1973.

[13] E.D.Sontag and H.J.Sussmann: *Output feedback design for linear systems with saturating controls*, Proc. of the 29th. C.D.C,pp.3414-3417, Hawaii, December 1990.

[14] H.J.Sussmann and Y.Yang: *On the stabilizability of multiple integrators by means of bounded feedback controls*, Proc. of 30th. C.D.C, Brighton, England, Vol 1, pp.70-72, December 1991.

[15] R.Suárez, J.Alvarez and J.Alvarez :*Linear systems with single saturated input: stability analysis*, Proc. of 30th IEEE-CDC, pp.223-228, Brighton, England, December 1991.

[16] S.Tarbouriech and C.Burgat : (A, B)-*Stabilizability conditions with respect to certain Lyapunov functions*, Proc. of IEEE/SMC, vol.1, pp.319-324, Le Touquet, France, October 1993.

[17] S.Tarbouriech, M.Klaï and C.Burgat : *Some stability domains for linear systems with saturating controls and observer*, IFAC Symposium on Robust Control Design, vol.1, pp.291-296, Rio, brazil, 14-16 September 1994.

[18] Y.Yang, H.J.Sussmann and E.D.Sontag: *Stabilization of linear systems with bounded controls*, Proc. NOLCOS'92, pp.15-20, Bordeaux, France, June 1992.

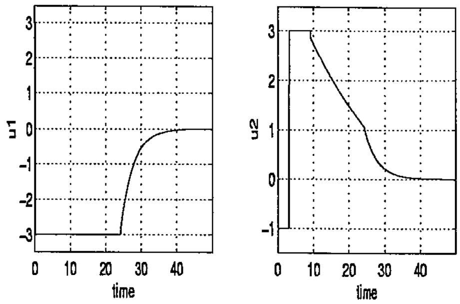

Figure 1: Controls behaviors.

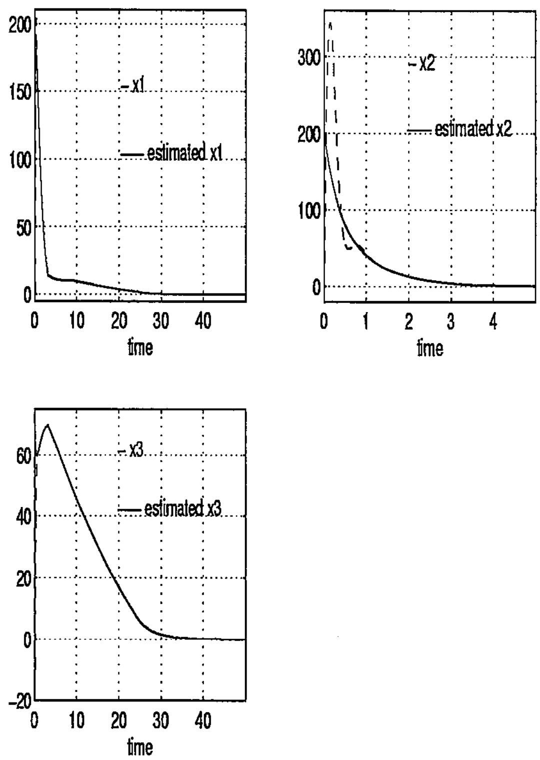

Figure 2: Behavior of: states and its estimates.