Invariant Line Matching of Consecutive Images

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Abstract

This paper presents a general line matching method based on geometrical invariants. A line segment is characterized locally by invariant parameters under the group of displacements within an image and the scale changes. Matching is achieved through two steps, features clustering and hypotheses verification. Experiments have shown an high rate matching on different types of images. This is due to the stability of the used line invariants which leads to few hypotheses of matching, as well as the verification using geometrical constraints. Tests conducted on different sequences have shown efficient matching of images and parts of scenes.

1 Invariant matching technique

Matching an image to a model, or to another image for the purpose of object recognition, motion estimation or viewer localization is a common and important task in computer vision. The problem of matching was widely treated [5, 6, 7]. The invariants are the properties of geometric configurations which remain unchanged under an appropriate class of transformations such as rigid, affine and projective transformations, and therefore, they are of the major importance in various problems in matching tasks. A lot of work in object recognition and matching based on the invariant scene description has been already reported [1, 3, 4]. Most of these invariants are developed for planar objects, and are often applied to images with invariants computed from point sets and so, they are inherently error-prone. In the proposed matching approach, invariants features are directly computed from sets of lines. This type of invariants are less sensitive to the image noise. The essential philosophy of our approach is that the primitives in the images are to be described in terms of such invariants and that these invariants provide all the information about shape and configuration required to carry out the matching tasks. The article is organized as follows. In section 2, we give a brief review of the most important geometrical relations and some useful line representations in images. The section 3 describes the analytical method to compute the invariant line features from corresponding projective bases. The section 4 is a summary of experimental results realized on different types of computer generated and real data. We conclude with some future directions and applications of this work.

2 Line coordinates in (θ, r) space

We deliberately use the projective representation [9] instead of polar representation [2] due to the fact that the former is homogeneous and can easily be subject to the matrix operations. However, with a view to the computation of the invariants, we may wish to have an unique representation of line (u and λu represent the same line). The representation (θ, r) is a good candidate for our purpose; in addition, it can be related to the projective parameterization by the function F which maps the polar representation (θ, r) to the projective representation (u_1, u_2, u_3) :

$$F : \Re^2 \longmapsto \Re^3$$
$$F(\theta, r) = (\cos\theta, \sin\theta, -r) = (u_1, u_2, u_3)$$

F maps the vector $\omega = (\theta, r)$ representing a line in the (θ, r) space into the parameters vector $u = (u_1, u_2, u_3)$ representing the same line in the projective plane :

$$F(\omega) = u$$

If $\theta \in [0, \pi)$, then F^{-1} exists and is defined by the mapping G :

$$G : \Re^3 \longmapsto \Re^2$$

$$G(u_1, u_2, u_3) = \begin{cases} (tan^{-1}(\frac{u_2}{u_1}), \frac{-u_3}{\sqrt{u_1^2+u_2^2}}) & u_2 \geq 0 \\ (0, \frac{-u_3}{u_1}) & u_2 = 0 \\ (tan^{-1}(\frac{-u_2}{-u_1}), \frac{u_3}{\sqrt{u_1^2+u_2^2}}) & u_2 \leq 0 \end{cases} \quad (1)$$

G maps the vector u into $G(u) = \omega$ such that the followings equations define the same line :

$$\begin{cases} u_1x_1 + u_2x_2 + u_3x_3 = 0 \\ \cos\theta x_1 + \sin\theta x_2 - rx_3 = 0 \end{cases} \quad (2)$$

As λu and u define the same line, they will be mapped by G to the same ω :

$$G(\lambda u) = \omega$$

In the projective plane, let P be a mapping :

$$u^I = P(u) = P(F(\omega)) \tag{3}$$

Note that to the vector u^I we associate the vector $\omega^I = (\theta^I, r^I)$ in the parameter space. In the projective plane, the mapping P transforms u into u^I; H^I is the corresponding mapping which transforms ω into ω^I in the parameter space. ω and ω^I are then related by :

$$\omega^I = G(u^I) = G \circ P \circ F(\omega) = H^I(\omega) \tag{4}$$

Where H^I is a combination[1] of the mappings G, P and F.

3 Invariant line features from corresponding projective bases

Our purpose in this section is computing invariant encoding way (computation of the transformation P) such that any pair of corresponding lines will have the same features in an invariant parameter space. More specifically, let Image 1 and Image 2 be a pair of timely consecutive images obtained by moving a camera in a scene; and let $l : x\cos\theta + y\sin\theta - r = 0$ and $l' : x\cos\theta' + y\sin\theta' - r = 0$ be a pair of corresponding lines belonging in these images. l and l' have the respective feature vectors $\omega = (\theta, r)$ and $\omega' = (\theta', r')$. The computed encoding must transform ω and ω' to the same invariant vector, namely, ω^I. From analytic geometry, we know that an usual representation of a geometric entity (line in our case); is obtained by the coordinates in a given reference frame. On the other hand, one of the natural way of encoding lines in invariant manner is to find an unique transformation which maps a set of lines as a distinguished reference frame into a common canonical basis. The computed transformation is then applied to the remaining lines to obtain their coordinates in this canonical basis. Such coordinates are invariants for corresponding lines [8]. If we consider that the set of lines b_L is the chosen projective basis in Image 1, b_R is the set formed by corresponding lines to b_L . Le H^s be a projective relating lines in Image 1 and Image 2, we can ttthen write :

$$b_R = H_s(b_L)$$

Now, if we consider that P is an unique linear application which transforms the projective basis b_L into a canonical basis b_C :

$$b_C = P(b_L)$$

[1]The symbol $\circ$ means the combination of mappings.

If in Image 2, it exists an unique application P' which transforms the lines forming the set b_R into the canonical basis b_C :

$$b_C = P'(b_R)$$

we can deduce that :

$$b_C = P'(H_s(b_L)) = P(b_L)$$

and that :

$$P = P' \circ H_s$$

P associates to each line in Image 1 an homogeneous vector $P(l)$. In Image 2, P' also associates to each line l' a vector $P'(l')$. As l' corresponds to l, we can write :

$$l' = H_s(l)$$

We can then deduce :

$$P'(l') = P'(H_s(l)) = P(l) \tag{5}$$

The expression (5) proves that the representations obtained by applying respectively P and P' to a pair of corresponding lines l and l' are identical; so these representations are invariants under the projective transformation H_s. In the invariant parameter space, the feature vector ω^I is computed by applying the mapping G, and because of the equality between $P(l)$ and $P'(l')$, they have the same invariant parameter vector ω^I. So the lines l and l' are represented by the same invariant feature vector ω^I :

$$\omega^I = \begin{cases} G(P(F(\theta, r))) \\ G(P'(F(\theta', r'))) \end{cases} \tag{6}$$

3.1 Projective basis correspondence

In the Image 1, we have chosen a feasible set of four coplanar lines as a basis b_L. As we have seen before, to compute line features in the Image 2, we must identify the set of lines belonging to the Image 2 and corresponding to b_L. The key idea to identify such set, is indexing the set b_L by invariant parameters. In Image 2, the identification scheme will consist to the search of combination of lines which have the same index parameters as the basis b_L.

From projective geometry, it is known that there exists only two independents invariants resulting from a set of five lines which are directly related to the order of lines which enter in the computation. These scalars, I_1 and I_2, are largely used in numerous recognition algorithms. Let $F_L = \{u_1^b, u_2^b, u_3^b, u_4^b, u_5^b\}$ be a set of five homogeneous lines; the invariant scalars I_1 and I_2 computed from this set are :

$$\begin{cases} I_1 = \frac{|D_{431}||D_{521}|}{|D_{421}||D_{531}|} \\ I_2 = \frac{|D_{421}||D_{532}|}{|D_{432}||D_{521}|} \end{cases} \tag{7}$$

Where the matrix D_{ijk} is computed from homogeneous coordinates of lines u_i^b, u_j^b, and u_k^b :

$$D_{ijk} = \begin{pmatrix} \cos\theta_i^b & \cos\theta_j^b & \cos\theta_k^b \\ \sin\theta_i^b & \sin\theta_j^b & \sin\theta_k^b \\ -r_i^b & -r_j^b & -r_k^b \end{pmatrix}$$

$|D_{ijk}|$ is the determinant of the matrix D_{ijk}. In Image 2 we form a set of five lines $F_R = \{u_1'^b, u_2'^b, u_3'^b, u_4'^b, u_5'^b\}$, where :

$$u_i'^b = (\cos\theta_i^{b\prime}, \sin\theta_i^{b\prime}, -r_i^{b\prime}), \quad i = 1, \ldots, 5$$

Let I_1' and I_2' be the invariant scalars associated to the set F_R computed from D'_{ijk}, the matrix of lines $u_i'^b$, $u_j'^b$, and $u_k'^b$, $i, j, k = 1, \ldots, 5$.
If :

$$\begin{cases} I_1 = I_1' \\ I_2 = I_2' \end{cases} \tag{8}$$

We can deduce that :

$$u_i'^b = H_s u_i^b, \quad i = 1, \ldots, 5 \tag{9}$$

In Image 2, the corresponding projective basis, b_R, is then formed by the lines represented by the homogeneous vectors $u_i'^b$, $i = 1, \ldots 4$:

$$b_R = \{u_1'^b, u_2'^b, u_3'^b, u_4'^b\}.$$

The projective basis b_R is then identified.

3.2 Invariant encoding way computation

The computation of P and P' depends on the lines forming the sets b_L, b_R and b_C. P and P' are homogeneous matrices, with eight unknown coefficients, and so they are included in the projective transformation group. Usually, a pair of quadruples of corresponding lines are sufficient for the determination of projective transformation. Let $b_L = \{u_1^b, u_2^b, u_3^b, u_4^b\}$, be the projective basis. b_L line's are represented by the followings homogeneous vectors :

$$u_i^b = (\cos\theta_i, \sin\theta_i, -r_i^b), \quad i = 1, \ldots, 4$$

The set $b_C = \{e_1, e_2, e_3, e_4\}$ is the canonical basis whose the homogeneous vectors e_i, $i = 1, \ldots, 4$ are chosen such that :

$$\begin{cases} e_1 = (\cos 0, \sin 0, 0) = (1, 0, 0) \\ e_2 = (\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, 0) = (0, 1, 0) \\ e_3 = (\cos 0, \sin 0, -1) = (1, 0, -1) \\ e_4 = (\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, -1) = (0, 1, -1) \end{cases}$$

Now, if we consider that the inverse of P is given by the matrix :

$$P^{-1} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

We are dealing with homogeneous coordinates, vectors u_i^b and $\lambda_i u_i^b$, $i = 1, \ldots, 4$, represent the same line and so :

$$\lambda_i P u_i^b = e_i, \quad i = 1, \ldots, 4$$

We can then deduce the followings equations :

$$\lambda_1 = \frac{a_{11}}{\cos\theta_1^b} = \frac{a_{12}}{\sin\theta_1^b} = \frac{a_{13}}{-r_1^b} \tag{10}$$

$$\lambda_2 = \frac{a_{21}}{\cos\theta_2^b} = \frac{a_{22}}{\sin\theta_2^b} = \frac{a_{23}}{-r_2^b} \tag{11}$$

$$\lambda_3 = \frac{a_{11} - a_{31}}{\cos\theta_3^b} = \frac{a_{12} - a_{31}}{\sin\theta_3^b} = \frac{a_{13} - a_{33}}{-r_3^b} \tag{12}$$

$$\lambda_4 = \frac{a_{21} - a_{31}}{\cos\theta_4^b} = \frac{a_{22} - a_{32}}{\sin\theta_4^b} = \frac{a_{23} - a_{33}}{-r_4^b} \tag{13}$$

The resolution of equations (10), (11), (12) and (13), yields the coefficients $a_{ij}, i, j = 1, \ldots, 3$. The matrix P can be then written :

$$P = \frac{-1}{c_1 c_2 c_3 \lambda_4} \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \tag{14}$$

The coefficients p_{ij} and c_i, $i, j = 1 \ldots 3$ are given in the Annexe A.

Finally, in Image 1, for each line l represented by the homogeneous vector $u = (\cos\theta, \sin\theta, -r)$, we compute a pair of the invariant parameters (θ^I, r^I) which are given by ::

$$(\theta^I, r^I) = H^I(\theta, r) = G(P(F(\theta, r))) = G(P.u)$$

By Substitution of P defined in (14) and application of the mapping G described in the relation (1), the invariants parameters θ^I et r^I represented the line l can be written :

$$\begin{cases} \theta^I = \tan^{-1}(def/abc) \\ r^I = \frac{-gcf}{\sqrt{((abc)^2 + (def)^2)}} \end{cases} \tag{15}$$

The coefficients a, b, c, d, e, f, and g are explicited in the Annexe A.
The respective substitution of θ_i^b and r_i^b with $\theta_i'^b$ and $r_i'^b$, $i = 1, \ldots 4$ in the formulas (14), we can easily compute the mapping P' which transforms the projective basis b_R to the canonical basis b_C, and then calculate the invariant parameters for each line in Image 2.

4 Experimental results

The proposed approach was implemented on Station SUN using the C programming language. The used images are timely consecutive and obtained by an unknown displacement of camera in a polyhedral scene. The approach has been verified on both computer generated and real data :

4.1 Synthetic images

We have developed a software which generates synthetic scenes. The images taken with a virtual camera can be formed.The following figures present the result of line matching between images of the same planar object observed at different viewpoints.

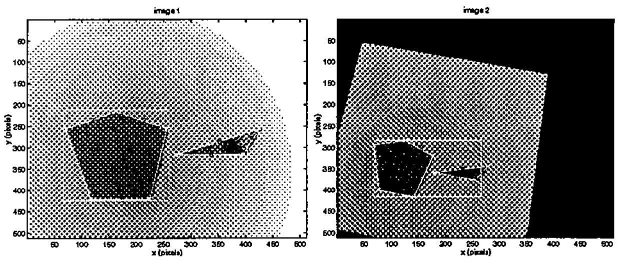

Figure 1: Polygon 1 —— Polygon 12

After the estimation of the projective transformation relating Polygon 1 and Polygon 12, we predicted the positions of segments in Polygon 12. The corresponding line segments in the pair Polygon 1 and Polygon 12 are presented in the figure 3.

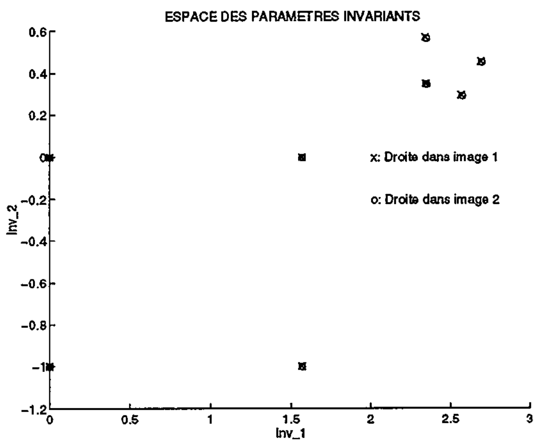

Figure 2: Representation of lines in Polygon 1 and Polygon 12 in the invariant parameter space (θ^I, r^I)

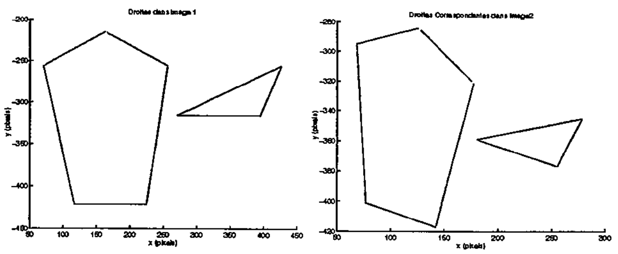

Figure 3: Matched line segments in Polygon 1 and Polygon 12

4.2 Real images

By the conducted tests on real images, we prove that the invariant line matching technique can be applied for line matching in image sequences which are different by their nature (indoor, outdoor image) and the aim of the line matching tasks (modeling, recognition, ...).
In the image Panel 4 of the couple in figure 4, we choose the set $F_L = \{20, 21, 22, 23, 28\}$, the associated invariants are I_1 =1.0928, $I_2 = 0.2529$. The projective basis b_L is composed by the following line segments $\{20, 21, 22, 23\}$. In Panel 7, F_R found by the searching process is formed by the lines $\{40, 46, 49, 50, 39\}$, the computed invariants are I_1' =1.07 and $I_2' = 0.2496$. The basis b_R is then identified.

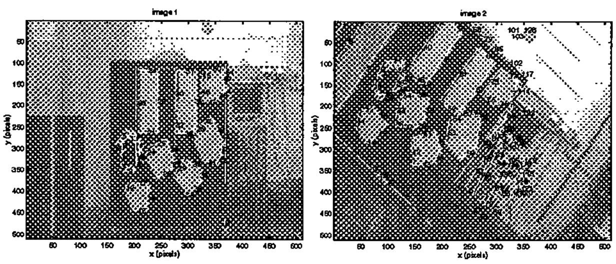

Figure 4: Panel 4 —— Panel 7

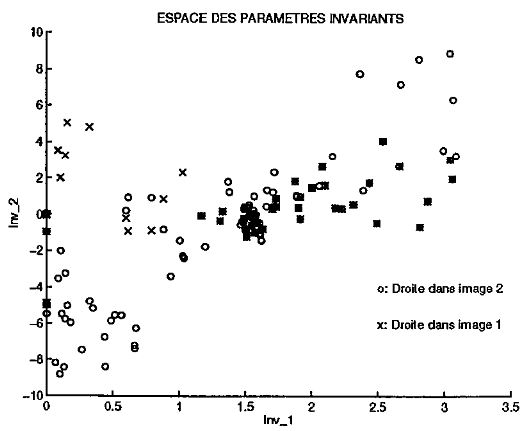

Figure 5: Representation of lines in Panel 4 and Panel 7 in the invariant parameter space (θ^I, r^I)

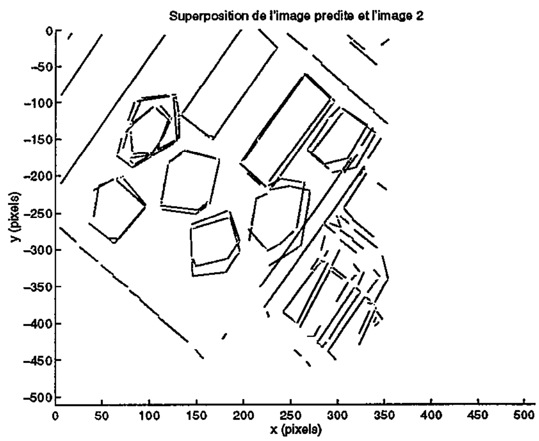

Figure 6: Real and predicted positions in Panel 7

The line segments in Panel 4 which have a corresponding line segment in Panel 7 are shown in the figure 7.

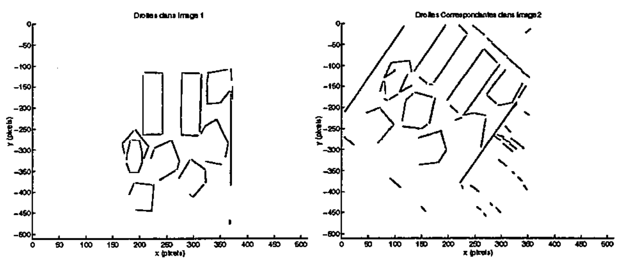

Figure 7: Matched line segments in Panel 4 and Panel 7

The figure 8 presents aerial images of an industrial city taken at two different viewpoints. Because the altitude of the camera position is bigger than the size of the observed objects size, the primitives in the images Aerial 1 and Aerial 2 are related by a projective transformation; and so, the proposed line matching technique can be applied to this type of images.In the image Aerial 1, the set F_L is selected. In Aerial 2, we idenify the set F_R and the projective basis b_R, as is fetched in the figure 8.

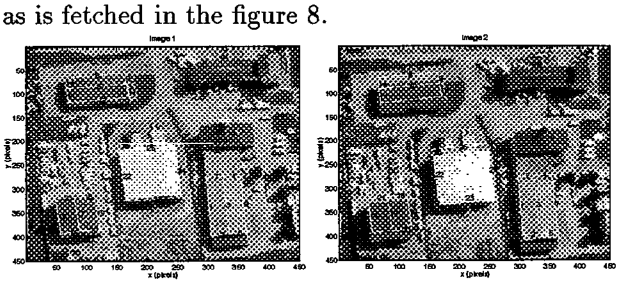

Figure 8: Aerial 1 —— Aerial 2

The invariants line features are then computed and represented in the same parameter space.

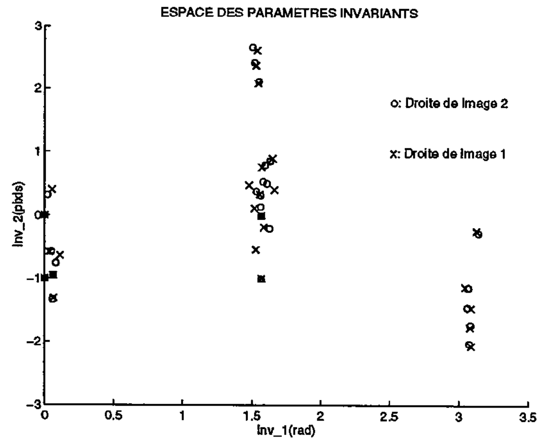

Figure 9: Representation of lines in Aerial 1 and Aerial 2 in the invariant parameter space (θ^I, r^I)

The predicted positions of line segments in Aerial 2 are presented in the figure 10.

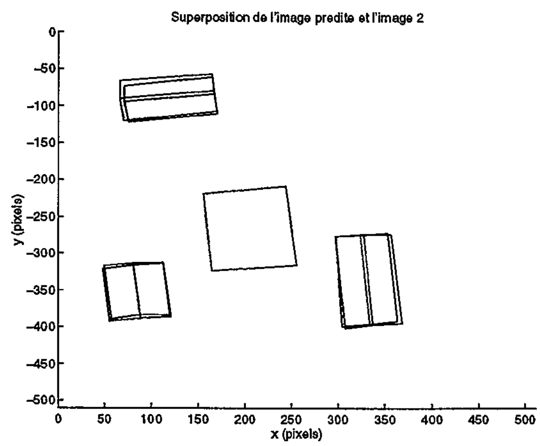

Figure 10: Real and predicted line segments of Aerial 2

After the verification scheme, the corresponding line segments in Aerial 1 and Aerial 2 are shown in the figure 11.

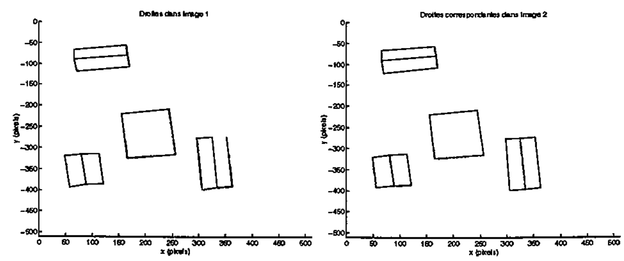

Figure 11: Matched line segments in Aerial 1 and Aerial 2

5 Conclusion

This report was dealing with the problem of analyzing of the timely consecutive images using feature-based approach. In the past, this problem has been discussed under various geometrical and physical settings. In the present work, a new approach was formulated; it is based on the transformation invariant indexing, and largely inspired by the classical geometric hashing. The lines in the images are represented by an invariant feature vector from the sets of corresponding lines chosen as frames. The line matching is then reduced to the clustering of two sets of primitives in a common parameter space. The verification scheme is included in the matching stage to ensure that correspondence of line features in the derived parameter space implies the correspondence of lines in the image plane. As the local properties of the invariants make this method efficient against to occlusions, the hypotheses verification by geometrical constraints provides a robust technique to scene clutter. We show that the approach can be used in a noisy environment where only line features can be rather reliably detected. Future research may include considering the sensitivity of the geometric matching with line features, analytically determining how the uncertainty of line parameters affects the computed invariants and resolve the problem of the false matchings due to the bad primitives extraction in the images.

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Annexe A

$$
\begin{aligned}
c_1 &= -r_4^b \sin(\theta_2^b - \theta_3^b) + r_3^b \sin(\theta_2^b - \theta_4^b) - r_2^b \sin(\theta_3^b - \theta_4^b)\\
c_2 &= -r_4^b \sin(\theta_1^b - \theta_3^b) + r_3^b \sin(\theta_1^b - \theta_4^b) - r_1^b \sin(\theta_3^b - \theta_4^b)\\
c_3 &= r_4^b \sin(\theta_1^b - \theta_2^b) - r_2^b \sin(\theta_1^b - \theta_4^b) + r_1^b \sin(\theta_2^b - \theta_4^b)\\
p_{11} &= c_2(c_1 r_2^b \sin\theta_1^b - c_1 r_1^b \sin\theta_2^b + c_3 r_3^b \sin\theta_2^b - c_3 r_2^b \sin\theta_3^b)\\
p_{12} &= -c_2(c_1 r_2^b \cos\theta_1^b - c_1 r_1^b \cos\theta_2^b + c_3 r_3^b \cos\theta_2^b - c_3 r_2^b \cos\theta_3^b)\\
p_{13} &= c_2(c_1 \sin(\theta_1^b - \theta_2^b) + c_3 \sin(\theta_2^b - \theta_3^b))\\
p_{21} &= c_1 c_3(r_1^b \sin\theta_3^b - r_3^b \sin\theta_1^b)\\
p_{22} &= -c_1 c_3(r_1^b \cos\theta_3^b - r_3^b \cos\theta_1^b)\\
p_{23} &= -c_1 c_3 \sin(\theta_1^b - \theta_3^b)\\
p_{31} &= c_1 c_2(r_1^b \sin\theta_2^b - r_2^b \sin\theta_1^b)\\
p_{32} &= c_1 c_2(r_1^b \sin\theta_2^b - r_2^b \sin\theta_1^b)\\
p_{33} &= -c_1 c_2 \sin(\theta_1^b - \theta_2^b).\\
a &= r_3^b \sin(\theta_1^b - \theta_2^b) - r_2^b \sin(\theta_1^b - \theta_3^b) + r_1^b \sin(\theta_2^b - \theta_3^b)\\
b &= r_4^b \sin(\theta - \theta_2^b) - r_2^b \sin(\theta - \theta_4^b) + r_1^b \sin(\theta_2^b - \theta_4^b)\\
c &= -r_4^b \sin(\theta_1^b - \theta_3^b) - r_3^b \sin(\theta_1^b - \theta_4^b) + r_1^b \sin(\theta_3^b - \theta_4^b)\\
d &= -r_3^b \sin(\theta - \theta_1^b) + r_1^b \sin(\theta - \theta_3^b) - r_1^b \sin(\theta_1^b - \theta_3^b)\\
e &= r_4^b \sin(\theta_1^b - \theta_2^b) - r_2^b \sin(\theta_1^b - \theta_4^b) + r_1^b \sin(\theta_2^b - \theta_4^b)\\
f &= r_4^b \sin(\theta_2^b - \theta_3^b) - r_3^b \sin(\theta_2^b - \theta_4^b) + r_2^b \sin(\theta_3^b - \theta_4^b)\\
g &= r_2^b \sin(\theta - \theta_1^b) - r_1^b \sin(\theta - \theta_2^b) + r \sin(\theta_1^b - \theta_2^b)
\end{aligned}
$$