Control of the Planar Rotation in Human Head-Eye-Coordination

Magnus Egerstedt*
magnuse@math.kth.se
Division of Optimization and Systems Theory
Department of Mathematics
Royal Institute of Technology
S - 100 44 Stockholm, Sweden

Clyde F. Martin†
Department of Mathematics
Texas Tech University
Lubbock, 79409 Texas

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1 Introduction

In this article, two somewhat separate questions are being discussed, and the first one concerns finding a mathematical model of the combined, horizontally rotational movements of the human head and eye. Therefore we devote Section 2 to finding systems of differential equations for describing the movements, based on simplified physical models of the muscular configurations in the neck and the eye respectively. However, it must be stressed that even though we use simplified models, our aim is to come up with a physically feasible model for the human muscular actions. This model could be of some interest in robotics, but our primary goal is that it will help us understand the dynamics of the actual muscles.

The next task, when it comes to finding a mathematical model, is to link the two separate systems, constituted by the head and the eye respectively, together, so that we can move on to the next major problem investigated in this article; How do we combine the movements of the head and the eye in order to follow a moving object with a given, known trajectory, at a constant distance from the head? This question is discussed in Section 3, where control laws are developed for activating the neck and the eye muscles in such a way that the pupil follows the desired trajectory, at the same time as both the head and eye trajectories, viewed separately, are three times continuously differentiable.

The reason for investigating the known trajectory case is that even though we in practice do not know the trajectory, we believe that since the problem involves four actuators, one for each muscle, the control of the overall switching system is interesting for it's own sake. We also hope that we further on are going to be able to make predictions of the observed object's trajectory, and then base the tracking on these predictions. This can be done using the same strategies as those suggested in this article. But any control laws that accomplish this will not do and in this article, we try to use controls that make the energy produced in the movement as small as possible, since we believe that to be a reasonable, physical control-criterion.

We also investigate what role time delay plays when it comes to switching between active and inactive muscles. This phenomena could hopefully help explaining what happens in esotropia cases, were the eye is stuck on one side.

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2 Head-Eye-Dynamics

2.1 Head Rotation

What we want to do in this paper is to model the complex behavior of the more than 20 pairs of muscles that control the human head movements. We want to do the modeling in such a way that the rotation of the head is given account for in a simple way. Therefore we chose to model the muscles as just one pair of muscles, conducting the same actions as all of the actual muscles together. This is because we are more interested in the principles of the controls behind the muscular contractions, than in finding an exact muscular model at the price of clarity.

We chose to model these muscles as damped springs with a second order linear dynamics of the form [11]

$$\ddot{x} = -k(x - L) - g\dot{x} + v(t), \tag{1}$$

where L and x are the lengths of the unstretched and the stretched spring respectively, and k and g are frequency and damping parameters of the spring. A controller, $v(t)$, is added to the spring, and the control term is produced by an active and an inactive part, corresponding to the active and the inactive muscle in the rotational movement, respectively.

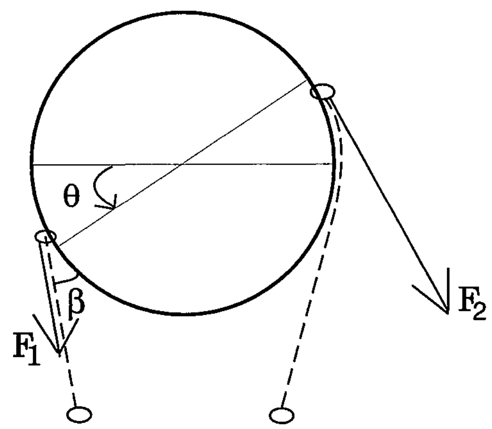

Figure 1: The two forces producing a rotation of the head.

If the angle θ is chosen to be the system state variable, as seen in Figure 1, we get

$$\ddot{\theta} = \frac{R}{I}(F_1 \cos\beta - F_2), \tag{2}$$

where I is the moment of inertia of the disc that is being rotated, since the angular acceleration is given by the torque, produced by the two tangential forces $F_1 \cos\beta$ and F_2. If we now let x_1 and x_2 be the lengths of the left and the right spring respectively and consider the fact that we now have two springs affecting the lengths simultaneously, we, after some calculations get a system on the form

$$\begin{aligned} \ddot{\theta} &= \operatorname{sign}(\theta) f(|\theta|, \operatorname{sign}(\theta)\dot{\theta}) \\ &+ u_1(\theta)v_1(t) + u_2(\theta)v_2(t), \end{aligned} \tag{3}$$

where

$$v_1(t) = \begin{cases} v_{\text{active}}(t) & \text{if } \dot{\theta} > 0 \\ v_{\text{inactive}} & \text{if } \dot{\theta} < 0, \end{cases} \tag{4}$$

and the opposite for $v_2(t)$. The inactive muscle control is only a constant term, which is due to the fact that a muscle is never completely at rest. This last condition (equation 4) gives us a physically inspired description of when we are to switch between actively controlling one muscle to the other.

2.2 Eye Rotation

The *external* and *internal recti*, the muscles behind the rotation of the eye, both attach on the so called Annulus of Zinn, behind the eye, and they also attach rather high up on the eye itself, which makes the modeling a bit easier than in the head case, since the geometry is simplified by the fact that the forces, produced by the two muscles, can be assumed to always be tangential to the eye itself. However, in almost the same way as in the neck case case [11], we get the system describing the eye rotation to be

$$\ddot{\phi} = -2(g\dot{\phi} + k\phi) - \frac{1}{r}(v_1(t) - v_2(t)), \tag{5}$$

with $v_i(t), (i = 1, 2)$ determined in the same way as in the neck case above.

2.3 The Combined Movement

So if we return to our initial problem; How do we combine the movements of the head and the eye in order to follow an object with a given trajectory, $\psi(t)$, at a constant distance from the head? Simple geometrical considerations, mainly involving the Law of Cosines, gives us that the acceleration of the head and of the eye, can be linked together by a function

$$\ddot{\phi} = F(\theta, \psi, \dot{\theta}, \dot{\psi}, \ddot{\theta}, \ddot{\psi}), \tag{6}$$

but we still have equation 5, which gives us a control law on the form

$$\begin{aligned} v_{\text{eye}}(t) &= -\text{sign}(\dot{\phi})(v_1(t) - v_2(t)) \\ &= -\text{sign}(\dot{\phi}) r[F(\theta, \psi, \dot{\theta}, \dot{\psi}, \ddot{\theta}, \ddot{\psi}) \\ &+ 2(g\dot{\phi} + k\phi)]. \end{aligned} \tag{7}$$

This way of letting the eye do most of the tracking is a product of the so called *occulocentric view*. This means that the main tracking is performed by the eye, while the head just moves in a general way. This approach is a rather reasonable one, since the fast *saccadic* movements of the eye make the eye better suited for following fast movements than the head [6].

3 Control Laws

Now that we have a model for the combined process of activating both the muscles of the neck and of the eye, the next task is to find the control laws. A reasonable approach is to try to minimize the energy produced in the movement, and since the mass of the head, M, is so much larger than the mass of the eye, m, one criterion for finding our control could be that it should make the angular acceleration of the head as small as possible. This would make the energy, given by the torque, small since

$$E = \int_{\theta_0}^{\theta_f} |N\, d\theta| = \int_{\theta_0}^{\theta_f} |\ddot{\theta} I\, d\theta|. \tag{8}$$

In order to accomplish this, we divide the trajectory of the head into subparts, where in some parts the head accelerates, and in others the angular acceleration is zero. This is because one obvious control that makes $|\ddot{\theta}|$ small is the one that makes $\ddot{\theta} = 0$. Therefore we want the major part of the trajectory to be of this type. If we recall equation 3 - 4, we directly see that to achieve this, we simply let

$$\begin{aligned} v_{1/2}(t) &= -\frac{\text{sign}(\theta) f(|\theta|, \text{sign}(\theta)\dot{\theta})}{u_{1/2}(\theta)} \\ &- \frac{u_{2/1}(\theta) v_{\text{inactive}}}{u_{1/2}(\theta)}. \end{aligned} \tag{9}$$

This linear approach is unfortunately not enough. First of all, we assume that we start following the object when the head and the eye both are at rest at some fixed angle, and therefore we need to find controls that can accelerate the systems up to some suitable velocity when the tracking is initiated. Secondly, when the followed trajectories are not well behaved, we have to take into account that that the eye may rotate out of bound if no modification of the head's zero acceleration trajectory is being made. These two cases show that we need to be able to accelerate the head in a controlled way in some situations, according to some desired trajectory.

Inspired by the feed forward control system concept in [13], the general idea behind the control laws we chose to use, can be illustrated by the block chart in Figure 2.

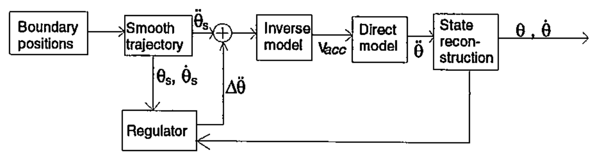

Figure 2: Block chart for the feed forward non-zero acceleration case.

If we want to find the control producing the desired trajectory, we simply use the inverse for $\ddot{\theta}$, and then use a feedback to take care of numerical and model errors,

$$\begin{aligned} \Delta\ddot{\theta}(t) &= C_1(\theta_{\text{calc}}(t) - \theta_{\text{actual}}(t)) \\ &+ C_2(\dot{\theta}_{\text{calc}}(t) - \dot{\theta}_{\text{actual}}(t)), \end{aligned} \tag{10}$$

where θ_{calc} is the desired, calculated trajectory. If we use a polynomial for describing the accelerations, which, for calculation reasons, is a good choice, we need a polynomial for describing $\theta(t)$ with a degree of at least seven. This is because we need eight coefficients, since the continuous differentiability conditions on each boundary for θ up to $\ddot{\theta}$ gives us eight continuity conditions that need to be fulfilled.

This approach would for instance give the head's starting trajectory as shown in Figure 3, and a total tracking scenario as seen in Figures 4-6.

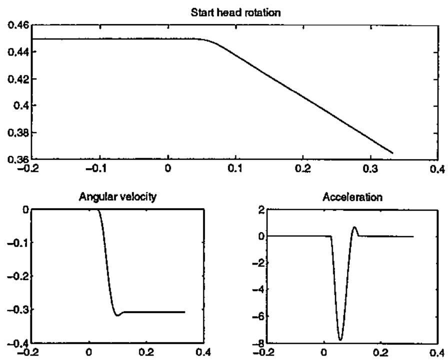

Figure 3: Head rotation when starting with zero velocity.

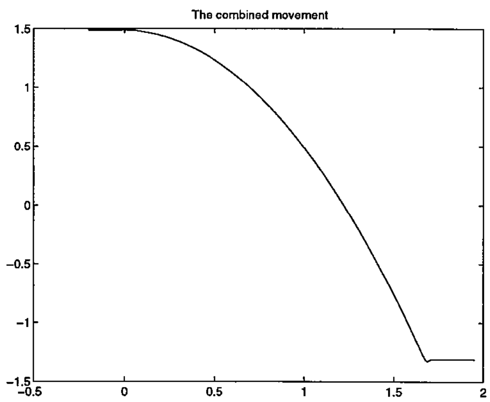

Figure 4: Head and eye rotation when for a quadratic tracked trajectory, $\psi(t) = \psi_0 - (t - t_0)^2$. One correction of the head acceleration was necessary, since the eye was rotating out of bound.

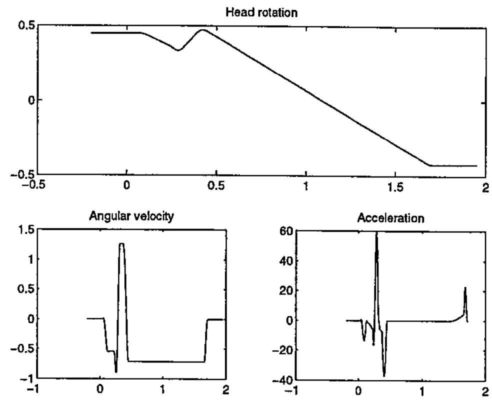

Figure 5: Head rotation for the quadratic tracking case.

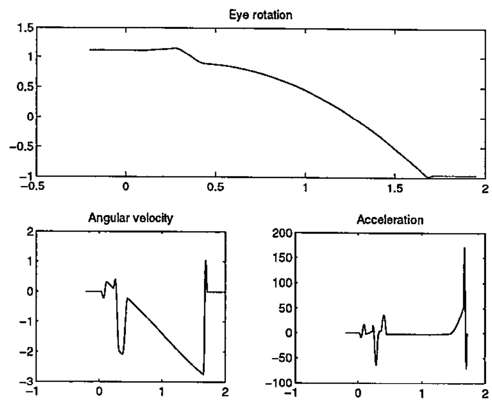

Figure 6: Eye rotation for the quadratic tracking case.

When it comes to physical adequacy, it can be worth comparing our results to the trajectories found in Guitton's *Eye-Head Coordination in Gaze Control* [6]. It turns out that our piecewise linear approach is not so bad after all, since an actual combined movements seem to have somewhat of the same piecewise linear characteristics as our trajectories, even though they are somewhat more complex. This should not, however, disqualify our model as not being an interesting step towards an understanding of the complex behavior of human head-eye-coordination.

4 Delayed Switchings

If we now impose a time delay on the switchings between the active and the inactive muscles in the eye, we get, at each switching occasion, a short period of time were both the muscles are inactive.

$$\begin{aligned} v^1_{\text{active}} &\rightarrow v^1_{\text{inactive}} \quad \text{when } \dot{\phi}(t) > 0 \\ &\rightarrow \dot{\phi}(t) < 0 \\ v^1_{\text{inactive}} &\rightarrow v^1_{\text{active}} \quad \text{when } \dot{\phi}(t - t_d) < 0 \\ &\rightarrow \dot{\phi}(t - t_d) > 0 \end{aligned} \tag{11}$$

This way of introducing delays into the switching system is probably a reasonable assumption, since we are trying to model real, physical systems.

If, for instance, the eye is following a sinusodial trajectory with high frequency, we get a scenario were the pupil is pulled far towards one side. In Figure 7 this can be seen, where the the flat line indicates that the eye is rotated more than what is actually physically possible, since we have that $\phi_{\max} = 1.17$rad. [11] This could maybe help explaining what happens in esotropia cases, were the eye is stuck on one side. Esotropia is believed to start by extreme muscular contractions, which later shortens the muscle and turns it into a mechanical problem instead of a control problem.

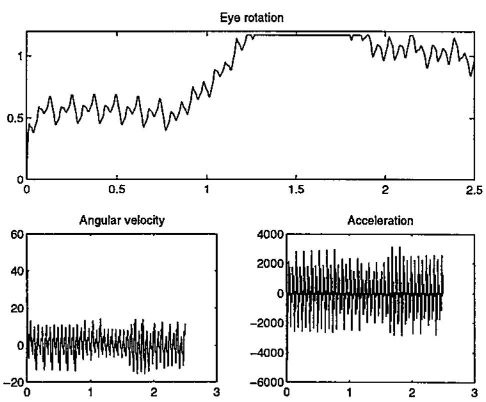

Figure 7: The eye is pulled towards one side when a delay of 0.01s is imposed on the system. The tracked sinusodial curve has a 100Hz frequency.

5 Conclusions

When it comes to the developed model, the weakest part is probably that of trying to model muscles as second order springs, since an actual muscle has a dynamics that is more complicated than that. However, this approach has the major advantage that it makes the mathematics reasonably simple. It is also sufficiently complete when it comes to actually start thinking about how to control the head and the eye muscles simultaneously. As we have seen, this is a non-trivial problem.

The control strategy we chose to use was based on a desire to keep the energy produced in the movement small, since we believed this to be a physically reasonable approach. We therefore let the angular acceleration of the head be zero most of the time, since this would make the energy small.

The introduction of a time delay in the control of the eye, gave rise to a phenomenon that resembled estropia, were the eye is pulled towards one side. It would thus be of great interest if it would turn out that our model could help explaining this phenomenon.

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