Stabilization Independent Of Delay For Saturated Linear Systems

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Abstract

This paper deals with the problem of global stabilization independent of delay for a class of delayed linear systems subject to bounded controls.

A new sufficient condition addressing the global asymptotic stabilization (G.A.S.), via saturated (static or dynamic) feedback, of such class of systems is proposed. It concerns the class of systems for which the open-loop system without time-delay term is Hurwitz.

Keywords

Linear systems, saturating controls, global stability, time-delay, observer.

1 Introduction

Several study upon independent time-delay stabilization of delayed linear systems, by means of linear controls, have been reported in the literature [2], [7], [9], [10], [11], [16], [17]. Some of these studies have been extended to the class of delayed linear systems with saturating controls [3], [12], [15]. In these papers, neither the form of a state feedback required to obtain G.A.S. nor the estimated region of local stability is specified. Furthermore, their results were obtained for a nonlinearity contained in a conic sector where saturation was included as in [8]. By addressing directly the saturation issue, one should expect to obtain less restrictive stability conditions [14].

In this paper, we focus attention on the global stabilization independent of delay for a class of internally delayed systems with saturating controls. The resulting closed-loop system is then of a nonlinear type. A sufficient condition independent of delay, improving that one in [5] is proposed, when the matrix of the open-loop system without time-delay term is Hurwitz. Firstly a saturating static state feedback is used, afterwards a saturated dynamic controller, built from delayless observer, is considered.

The following notations and terminology are used. The inner product of two vectors $x, y \in R^n$, is denoted by $\prec x, y \succ$, the symbol "$grad\ V$" denotes the gradient vector of the function V, "$Re(.)$" denotes the real part of (.), and I_n represents the identity matrix of dimension $(n \times n)$. Finally we denote by $||x(t)||$ the euclidean norm of vector $x(t)$, $||A||$ the following matrix norm $\lambda_{max}^{1/2}(A^t A)$, and $\mu(A)$ the matrix measure defined by $\frac{1}{2}\lambda_{\max}(A^t + A)$.

2 Preliminaries

Consider the following linear time-delay system described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + \bar{A}x(t-h(t)) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where matrices $A, \bar{A} \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, and $C \in \Re^{l\times n}$ with $rank(C) = l$. The varying-time-delay $h(t)$ is nonegative, bounded and continuous function satisfying $0 \le h(t) \le \tau$, with $\dot{h}(t) \le \delta < 1$, where $\tau < \infty$ is any constant. Further, for system (1) the following properties hold:

(P.1) $x(t) = \Psi(t),\ \forall\ t \in [t_0 - \tau,\ t_0]\ ,\ t_0 \ge 0$, where $\Psi(t)$ is a continuous vector-valued initial function.

(P.2) The control vector $u(t)$ is assumed to belong to a compact set $\Omega \subset \Re^m, \forall t \ge 0$, defined by:

$$\begin{aligned} \Omega = \ & \{u(t) \in \Re^m | -u_m^i \le u^i(t) \le u_M^i\ ; \\ & u_m^i, u_M^i > 0; \forall i = 1, 2, ..., m\} \end{aligned} \tag{2}$$

where u^i represents the i^{th}component of the vector $u(t)$.

(P.3) The pairs (A, B) and (A, C) are assumed to be stabilizable (or controllable) and detectable (or observable) respectively.

(P.4) Matrix A is assumed to be Hurwitz.

Assume that all states are available to a measure. Then by implementing a saturated static controller:

$$u(t) = sat(Fx(t)), F \in \Re^{m\times n} \tag{3}$$

system (1) becomes:

$$\dot{x}(t) = Ax(t) + \bar{A}x(t-h(t)) + Bsat(Fx(t)) \tag{4}$$

where the saturation term

$$sat(Fx(t)) = [(sat(Fx(t))^1, ..., (sat(Fx(t))^m]^t$$

is defined, for $i = 1, 2, ..., m$, by:

$$sat(Fx(t))^i = \begin{cases} u_M^i & \text{if} \quad (Fx(t))^i > u_M^i \\ (Fx(t))^i & \text{if} \quad -u_m^i \le (Fx(t))^i \le u_M^i \\ -u_m^i & \text{if} \quad (Fx(t))^i < -u_m^i \end{cases} \tag{5}$$

The saturation term can be written as

$$sat(Fx(t)) = \Phi(\alpha(x))Fx(t) \tag{6}$$

whose entries $\alpha^i(x(t))$ of the diagonal matrix Φ are defined for $i = 1, 2, ..., m$, by:

$$\alpha^i(x(t)) = \begin{cases} \frac{(u_M)^i}{(Fx(t))^i} & \text{if} \quad (Fx(t))^i > u_M^i \\ 1 & \text{if} \quad -u_m^i \le (Fx(t))^i \le u_M^i \\ -\frac{(u_m)^i}{(Fx(t))^i} & \text{if} \quad (Fx(t))^i < -u_m^i \end{cases} \tag{7}$$

and satisfy

$$0 < \alpha^i(x(t)) \le 1, \ \forall \ i = 1, ..., m. \tag{8}$$

Thus, from (6)-(8), system (4) can be rewritten equivalently as:

$$\dot{x}(t) = (A + B\Phi(\alpha(x))F)x(t) + \bar{A}x(t - h(t)) \tag{9}$$

Note that system (4) is of a nonlinear type.

Further, it is well-known that if matrix A in (1) is Hurwitz, then there always exists a symmetric positive definite matrix P ($P > 0$) solution of:

$$A^tP + PA = -N^tN \tag{10}$$

where N is any nonsingular matrix.

We study, in this paper, the global asymptotic stabilization, independent of varying-time-delay, by using a saturating (static or dynamic) feedback law. The case of constant time-delay systems follows as a particular case. Then, we give a feedback law type and a new sufficient condition for which G.A.S of system (4) is guaranteed.

The following Lemma concerning the G.A.S of system (4), gives us a way to choose a suitable matrix F.

Lemma 2.1 : *If for system (4), there exists a Lyapunov function $V(x(t))$ for which:*

$$\begin{aligned} \dot{V}(x(t)) = \ & \prec grad \ V(x(t)), Ax(t) + \bar{A}x(t - h(t)) + \\ & Bsat(Fx(t)) \succ < 0, \ \forall h(t) \le \tau \end{aligned} \tag{11}$$

for any $x(t) \in \Re^n \backslash \{0\}$, then $V(x(t))$ must necessarily be a Lyapunov function of the following open-loop system

$$\dot{x}(t) = Ax(t) + \bar{A}x(t - h(t)) \tag{12}$$

The proof of this lemma follows by using the same approach as in [5].

3 Main results

Following Lemma 2.1, system (12) must be stable. On the basis of the previous result, the determination of the feedback matrix F can be made from a suitable Lyapunov function of system (12). Hence, we introduce the next theorem which provides a new sufficient condition for global asymptotic stability of system (4).

Theorem 3.1 : *Assume that matrix A is Hurwitz ; the state feedback control law (3) with the following matrix*

$$F = -D(\gamma)B^tP \tag{13}$$

where $D(\gamma)$ is a diagonal matrix of positive elements, and matrix $P > 0$ is solution of (10), globally stabilizes (G.A.S) system (4), independently of time-delay, provided that:

$$\|(N^{-1})^tP\bar{A}N^{-1}\| < \sqrt{\beta(1-\beta)(1-\delta)}, \ \beta \in]0 \ 1[\tag{14}$$

Proof : Consider the coordinate transformation $x = N^{-1} \ z$. Hence the system (4) may be transformed into:

$$\dot{z}(t) = A_0z(t) + \bar{A}_0z(t - h(t)) + B_0sat(F_0z(t)) \tag{15}$$

where $A_0 = NAN^{-1}$, $\bar{A}_0 = N\bar{A}N^{-1}$, $B_0 = NB$ and $F_0 = FN^{-1}$. Substituting matrix $A = N^{-1}A_0N$ into (10), one gets:

$$A_0^t(N^{-1})^tPN^{-1} + (N^{-1})^tPN^{-1}A_0 = -I_n \tag{16}$$

To examine the global stability of system (4), we define a Lyapunov function candidate $V(z(t))$ as:

$$V(z) = z^t(N^{-1})^tPN^{-1}z + \beta \int_{t-h(t)}^{t} z^t(\theta) \, z(\theta) \, d\theta \tag{17}$$

where $\beta > 0$ and $(N^{-1})^tPN^{-1} > 0$ is a solution of (16). The time-derivative of $V(z)$ along the trajectories of system (15) is evaluated by:

$$\begin{aligned} \dot{V}(z) = \ & -z^tz + 2z^t(N^{-1})^tPBsat(FN^{-1}z) + \\ & 2z^t(N^{-1})^tP\bar{A}N^{-1}z(t - h(t)) + \beta z^tz - \\ & \beta(1 - \dot{h}(t)) \ z^t(t - h(t))z(t - h(t)) \end{aligned} \tag{18}$$

From the equivalent form of the saturation term, given by (6), and using matrix F defined in (13), equation (18) becomes:

$$\begin{aligned} \dot{V}(z) = \ & -(1-\beta)z^tz - 2z^t(N^{-1})^tPB\Phi D(\gamma)B^tPN^{-1}z \\ & +2z^t(N^{-1})^tP\bar{A}N^{-1}z(t - h(t)) - \beta(1 - \dot{h}(t)). \\ & z^t(t - h(t))z(t - h(t)) \end{aligned} \tag{19}$$

and can be majorized by:

$$\dot{V}(z) \leq \; -(1-\beta)\|z(t)\|^2 + 2\|(N^{-1})^t P\bar{A}N^{-1}\| \; \|z(t)\|. \\ \|z(t-h(t))\| - \beta\,(1-\delta)\,\|z(t-h(t))\|^2 \tag{20}$$

since

$$z^t(t)(N^{-1})^t PB\Phi(\alpha)D(\gamma)B^t PN^{-1}z(t) \geq 0, \; \forall t \geq 0.$$

In terms of $\eta(t) = [\|z(t)\| \;\; \|z(t-h(t))\|]^t$ yields:

$$\dot{V}(z(t)) \leq \; -\eta^t(t)\; R\,\eta(t),$$

where

$$R = \begin{bmatrix} (1-\beta) & -\|(N^{-1})^t P\bar{A}N^{-1}\| \\ -\|(N^{-1})^t P\bar{A}N^{-1}\| & \beta(1-\delta) \end{bmatrix} \tag{21}$$

If the condition (14) is satisfied, then matrix R is positive-definite and we get:

$$\dot{V}(z(t)) < 0, \forall z(t) \in \Re^n \backslash \{0\}$$

This implies the global asymptotic stability of system (15) and therefore that one of system (4). ■

Remark 3.1 : *Note that the maximized value of the term $\sqrt{\beta(1-\beta)}$ with $\beta \in]0\;1[$ is obtained for $\beta = \frac{1}{2}$. Thus, the condition (14) can be replaced by:*

$$\|(N^{-1})^t P\bar{A}N^{-1}\| \; < \frac{1}{2}\sqrt{(1-\delta)} \tag{22}$$

Dynamic output feedback

Suppose that the state vector is not completely available for measurement. In order to reconstruct needful states for feedback, we can use dynamic feedback, built from minimal-order observer. Let us consider a reduced-order observer, realized as follows:

$$\begin{cases} \dot{w}(t) = & D\,w(t) + E\;w(t-h(t)) + Gsat(F\hat{x}(t)) + \\ & H\,y(t) + J\;y(t-h(t)) \\ \hat{x}(t) = & M\,w(t) + K\;y(t) \end{cases} \tag{23}$$

where $w(t) \in \Re^{(n-l)}$ and $D,\ E,\ G,\ H,\ J,\ M,\ K$, are constant matrices of appropriate dimensions, which can be determined as shown in Appendix. The existence conditions of such class of observers are given below.

Theorem 3.2 : *[4] If $rank[C^t \quad \bar{A}^t C^t] = n$, then the necessary and sufficient condition for the existence of a delayless observer with E null is that all transmission zeros for $(A,\ \bar{A}\Pi^t,\ C)$ be stable, where Π is any matrix satisfying $range[\Pi^t] = null[C]$. Note that s_0 is said to be a transmission zero of $(A,\ \bar{A}\Pi^t,\ C)$ if*

$$rank \begin{bmatrix} s_0 I_n - A & \bar{A}\Pi^t \\ C & 0 \end{bmatrix} < 2n - l$$

The conditions under which the state $w(t)$ is an estimate of $Tx(t)$, for some $T \in \Re^{(n-l)\times n}$, i.e.,

$$\lim_{t\to\infty} \epsilon(t) = 0,\ \forall w(0),\ x(0),\ u(t) \tag{24}$$

where

$$\epsilon(t) = w(t) - Tx(t) \tag{25}$$

are:

$i)$ $G = TB$,

$ii)$ $\begin{bmatrix} DT & 0 \\ 0 & ET \end{bmatrix} - \begin{bmatrix} TA & 0 \\ 0 & T\bar{A} \end{bmatrix} = -\begin{bmatrix} HC & 0 \\ 0 & JC \end{bmatrix}$,

$iii)$ $MT + KC = I_n$,

$iv)$ $\mu(D) < -\|E\|$.

(26)

Specify that condition (iv), in (26), is only sufficient for the observer's convergence. Nevertheless, conditions (i), (ii), (iii) are necessary, for the existence of delayless observer.

Followig (25) and conditions (26), the reconstruction error vector, defined by:

$$e(t) = \hat{x}(t) - x(t)$$

can be expressed as:

$$e(t) = M\epsilon(t)$$

with

$$\dot{\epsilon}(t) = D\,\epsilon(t) + E\,\epsilon(t-h(t)) \tag{27}$$

Hence, it appears clearly that if matrices D and E can be chosen such that the condition (iv) holds, the observer (23) converges, i.e., $\epsilon(t) \to 0$ as $t \to \infty$.

The objective is then, to give a sufficient condition for which the feedback matrix F, given in (13), globally stabilizes the following composite system:

$$\begin{cases} \dot{x}(t) = Ax(t) + \bar{A}x(t-h(t)) + B\;sat(F(x(t) + M\epsilon(t))) \\ \dot{\epsilon}(t) = D\,\epsilon(t) + E\,\epsilon(t-h(t)) \end{cases} \tag{28}$$

Theorem 3.3 : *Assume that matrix A is Hurwitz and that conditions (26) hold. The composite system (28) is globally asymptotically stabilizable, independent of time-delay, by means of the feedback matrix F given in (13) if the condition (22) is satisfied.*

Proof: Let the coordinate transformation $x = N^{-1}z$. Hence, the system (28) may be transformed into:

$$\begin{cases} \dot{z}(t) = & A_0 z(t) + \bar{A}_0 z(t-h(t)) + \\ & B_0 sat(F_0(z(t) + NM\epsilon(t))) \\ \dot{\epsilon}(t) = & D\,\epsilon(t) + E\,\epsilon(t-h(t)) \end{cases} \tag{29}$$

where $A_0 = NAN^{-1}$, $\bar{A}_0 = N\bar{A}N^{-1}$, $B_0 = NB$ and $F_0 = FN^{-1}$.

Now, consider the candidate Lyapunov function described by (17), for which equation (16) holds. Its time-derivative along trajectories of system (29) is given by:

$$\begin{aligned}\dot{V}(z) \; &= -(1-\beta)z^t z + 2z^t (N^{-1})^t P\bar{A}N^{-1}z(t-h(t)).\\ &+2z^t(N^{-1})^t PBsat(FN^{-1}z) - \beta z^t(t-\dot{h}(t))\\ &z(t-h(t)) + 2z^t(N^{-1})^t PB\ f(z,\epsilon)\end{aligned} \tag{30}$$

where $f(z,\epsilon) = [sat(FN^{-1}(z+NM\epsilon)) - sat(FN^{-1}z)]$ is globally Lipschitz function [13], i.e., which satisfies:

$$\|f(z,\epsilon)\| \; \leq \rho\ \|\epsilon(t)\|,\ \rho = k\,\|F_0\|\ \|NM\| > 0 \tag{31}$$

Substituting the equivalent form of the saturation term, given in (6), and matrix F in (13), into (30) we have:

$$\begin{aligned}\dot{V}(z) = \; & -(1-\beta)z^t z + 2z^t(N^{-1})^t P\bar{A}N^{-1}z(t-h(t))\\ & -2z^t(N^{-1})^t PB\Phi(\alpha)D(\gamma)B^t PN^{-1}z\\ & -\beta(1-\dot{h}(t))z^t(t-h(t))z(t-h(t))\\ & +2z^t(N^{-1})^t PB\ f(z,\epsilon)\end{aligned} \tag{32}$$

By using (31), equation (32) can be majorized by:

$$\begin{aligned}\dot{V}(z) \leq \; & -(1-\beta)\|z\|^2 + 2\|(N^{-1})^t P\bar{A}N^{-1}\|\|z\|.\\ & \|z(t-h(t))\| - \beta(1-\delta)\,\|z(t-h(t))\|^2\\ & +2\rho\|(N^{-1})^t PB\|\|z\|\ \|\epsilon\|\end{aligned} \tag{33}$$

In terms of $\eta(t) = [\|z(t)\|\ \ \|z(t-h(t))\|]^t$ yields:

$$\dot{V}(z(t)) \leq \; -\eta^t(t)\ R\ \eta(t) + 2\sigma\,\|z(t)\|\ \|\epsilon(t)\| \tag{34}$$

where R is given by (21) and $\sigma = \rho\,\|(N^{-1})^t PB\,\| > 0$.

If condition (14) is satisfied, one gets $R > 0$ and we can write:

$$\dot{V}(z(t)) \leq \; -\lambda_{\min}(R)\,\|\eta(t)\|^2 + 2\sigma\,\|z(t)\|\ \|\epsilon(t)\| \tag{35}$$

Taking into acount that:

$$\lambda_1\ \|z(t)\|^2 \leq V(z(t)) \leq \lambda_2\|z(t)\|^2 + \tau\|z(\xi)\|^2 \tag{36}$$

where $\lambda_1 = \lambda_{\min}((N^{-1})^t PN^{-1})$, $\lambda_2 = \lambda_{\max}((N^{-1})^t PN^{-1})$, $\xi \in [t-h(t),\, t]$ and that $\|z(\xi)\| \leq \|\eta(t)\|$, we have $\|\eta(t)\| \geq \sqrt{\frac{V(z(t))}{\lambda_2+\tau}}$. This allows to obtain an upper bound on $\dot{V}(z(t))$. Therefore

$$\dot{V}(z(t)) \leq -a\,V(z(t)) + 2b\,\sqrt{V(z(t))}\ \|\epsilon(t)\| \tag{37}$$

where $a = \frac{\lambda_{\min}(R)}{\lambda_2+\tau}$ and $b = \frac{\sigma}{\sqrt{\lambda_1}}$.

Letting $W(t) = \sqrt{V(z(t))}$, from (37) yields:

$$\dot{W}(t) \leq -\frac{a}{2}\ W(t) + b\,\|\epsilon(t)\| \tag{38}$$

By integrating the both sides of (38) we obtain:

$$W(t) \leq W(t_0)\ e^{-\ \frac{a}{2}(t-t_0)} + b\int_{t_0}^{t} e^{-\frac{a}{2}(t-\theta)}\ \|\epsilon(\theta)\|\ d\theta \tag{39}$$

From the left side of (36) and (39), it follows:

$$\|z(t)\| \leq \frac{W(t_0)}{\sqrt{\lambda_1}}\ e^{-\ \frac{a}{2}(t-t_0)} + \frac{b}{\sqrt{\lambda_1}}\int_{t_0}^{t} e^{-\frac{a}{2}(t-\theta)}\ \|\epsilon(\theta)\|\ d\theta \tag{40}$$

Hence, if the condition (22) is satisfied, so $a > 0$, then by taking into account that conditions (26) and those of Theorem 3.2 hold, by assumption, one gets:

$$\lim_{t\to\infty} z(t) = 0$$

since $\epsilon(t) \to 0$ as $t \to \infty$. This means that system (28) is globally asymptotically stable. ■

Suppose $h(t) = \tau < \infty$, $\forall t \geq 0$, so $\delta = 0$, then from Theorem 3.1 and Theorem 3.3, derived the following Corollary.

Corollary 3.1 *Under assumptions* **(P.1)**–**(P.4)**, *the feedback matrix F in (13) globally stabilizes system (4) and the composite system (28), satisfying conditions (26), with constant time-delay, if*

$$\|(N^{-1})^t P\bar{A}N^{-1}\| \; < \frac{1}{2} \tag{41}$$

Proof : Follows from Theorem 3.1, Theorem 3.3 and Remark 3.1, by taking $\delta = 0$. ■

Remark 3.2 : *The case of full-order observer can be obtained, as particular case, by taking $w(t) = \hat{x}(t)$, $D = A - HC$, $E = \bar{A}$, $G = B$, $J = 0_{n\times l}$, $M = I_n$, $K = 0_{n\times l}$ and $T = I_n$.*

Remark 3.3 : *Notice that condition (22) is less conservative than that one given in [2], [5], [12], that is,*

$$\|P\bar{A}\| \; < \frac{\lambda_{\min}(Q)}{2}\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} \tag{42}$$

and also less restrictive than the well-known condition:

$$\mu(A) < -\|\bar{A}\|$$

Remark 3.4 : *An interesting open problem consists in finding the pair of matrices (N, P) which minimizes the term $\|(N^{-1})^t P\bar{A}N^{-1}\|$. This defines the following constrained optimization problem:*

$$\min_{N}\ \|(N^{-1})^t P\bar{A}N^{-1}\|$$
$$\textit{Subject to: } A^t P + PA = -N^t N$$

for which condition (41) must holds.

4 Conclusion

Design of linear saturated controller, built from delayless observer, to globally stabilize continuous time-delay systems with constrained controls has been developed in this paper. It has been established that if there exists a Lyapunov function of the open-loop delayed system, for which matrix A is Hurwitz and the condition (14) is fulfilled, then the state feedback matrix, built from this function, globally stabilizes the composite system (system+observer) independently of time-delay.

When matrix A is not Hurwitz, the local stabilization independent of delay, can be envisaged to determine some positively invariant and asymptotically stable domains, in which the behavior is of a nonlinear type. This case is studied in [6].

5 Appendix

To determine all matrices of system (23), we can consider the method proposed in [1], for undelayed linear systems. It's well known that there exists an arbitrary choice of real constant matrix $\Delta \in \Re^{(n-l)\times n}$ such that matrix $S^t = [C^t \ \ \Delta]^t$ is nonsingular. Hence, using the similarity transformation $\chi(t) = Sx(t)$, system (1) can be described by:

$$\begin{cases} \dot{\chi}(t) = SAS^{-1}\chi(t) + S\bar{A}S^{-1}\chi(t-h(t)) + SBu(t) \\ y(t) = CS^{-1}\chi(t) = [I_l \ \ 0]\chi(t) \end{cases} \tag{43}$$

and partitioned as:

$$\begin{aligned} \begin{bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \chi_1(t) \\ \chi_2(t) \end{bmatrix} + \\ & \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \chi_1(t-h(t)) \\ \chi_2(t-h(t)) \end{bmatrix} + \\ & \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \\ y(t) &= \chi_1(t) \end{aligned} \tag{44}$$

where $\chi_1(t) \in \Re^l, \ \chi_2(t) \in \Re^{(n-l)}$.

If (A, C) is observable, then (A_{22}, A_{12}) is also observable [1]. Only the last $(n-l)$ components of $\chi(t)$ have to be estimated.

By setting $w(t) = \hat{\chi}_2(t) - Ly(t) = \hat{\chi}_2(t) - L\chi_1(t)$, where

$$\begin{aligned} \dot{\hat{\chi}}_2(t) = \ & A_{22}\ \hat{\chi}_2(t) + \bar{A}_{22}\ \hat{\chi}_2(t-h(t)) + A_{21}\chi_1(t) \\ & + \bar{A}_{21}\chi_1(t-h(t)) + B_2u(t) \end{aligned} \tag{45}$$

we obtain:

$$\begin{aligned} \dot{w}(t) = \ & (A_{22} - LA_{12})\,w + (\bar{A}_{22} - L\bar{A}_{12})w(t-h(t)) \\ & + [(A_{22} - LA_{12})L + (A_{21} - LA_{11})]y(t) \\ & + [(\bar{A}_{22} - L\bar{A}_{12})L + (\bar{A}_{21} - L\bar{A}_{11})]y(t-h(t)) \\ & + (B_2 - LB_1)\ u(t) \end{aligned} \tag{46}$$

where L is a suitable matrix, chosen to satisfy:

$$\mu(A_{22} - LA_{12}) < -\|\bar{A}_{22} - L\bar{A}_{12}\|$$

Thus, if we define:

$$\epsilon(t) = \hat{\chi}_2(t) - \chi_2(t) = w(t) - Tx(t)$$

then, from (23), (27) and (46) one gets:

$$\begin{aligned} D &= (A_{22} - LA_{12}) \\ E &= (\bar{A}_{22} - L\bar{A}_{12}) \\ G &= (B_2 - LB_1) \\ H &= [(A_{22} - LA_{12})L + (A_{21} - LA_{11})] \\ J &= [(\bar{A}_{22} - L\bar{A}_{12})L + (\bar{A}_{21} - L\bar{A}_{11})] \\ T &= [-L \quad I_{(n-l)}]\ S \end{aligned} \tag{47}$$

According to [4], it is clear that if matrix E is chosen null, then matrices J and H can always be calculated once L is determined so that matrix D is Hurwitz and $\bar{A}_{22} - L\bar{A}_{12} = 0$.

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