State estimation in presence of disturbance using neural networks

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Abstract

A state reconstruction method of non-linear systems is proposed. It concerns the systems which can be broken up into linear and non-linear parts and requires the implementation of two observers. The first one is used for determining approximately the target for a neural network providing the estimation of the non-linearity. This later can be the consequence of modeling errors of the system parameters. The second observer allows to reconstruct the state of the non-linear system.

1. Introduction

The state reconstruction of non-linear systems has received many attention specially as part of continuous cases [10]. The non-linearities can be due to uncertainties on the model of the system. To reconstruct the system state, assumptions have to be verified. Some papers require that the non-linearities are lipschitz [9] [3] [4] [2]. Others need the transformation of the model of the system into appropriate observable canonical forms [1] [6], where the development of the non-linear time-variable form requires regularity of the non-linear time-variable observability matrix of the system. Others techniques use algorithms based on neural networks, for control [7] or diagnosis [5] goals. However, methods allowing the estimation of the uncertainties are seldom treated. [8] has proposed a method to estimate the non-linearities and to reconstruct the state. The model of the non-linearities is identified by a neural network and the state is reconstructed using a sliding mode observer. The technique requires the measurement of all state components.
A method is proposed here, for discrete cases, to reconstruct the state and to estimate the non-linear part, which can depend on the system input and output when only few state components are measured.

2. Problem Statement

A non-linear system can be presented in state form by:

$$x(k+1) = f(x, u)$$

where x and u are the state and the input of the system, respectively. In practice, this representation is often avoided by operating around an operating point; the system is then linearized and can be written as:

$$x(k+1) = A\, x(k) + B\, u(k)$$

If the system is observable, its state estimation is "easily" obtained by using a Luenberger's observer. Actually, the system operates away from this operating point and consequently, its structure moves.

This kind of system is considered in order to propose a method able to reconstruct the state and to estimate the unmodeled non-linearities. It is assumed that the system remains stable in spite of disturbance.

3. Non-linearities Estimation

Let us consider a non-linear system defined by:

$$x(k+1) = A\, x(k) + B\, u(k) + F\, \Psi(y, u) \tag{1.a}$$
$$y(k) = C\, x(k) \tag{1.b}$$
$$C = [I \quad 0] \tag{1.c}$$

where $x \in R^n$, $y \in R^m$, $u \in R^p$ are the state, the output and the input of the system, respectively; $\Psi \in R^q$ and A, B, C and F are matrices with appropriate dimensions. The variable Ψ represents the disturbances due, for example, to modeling errors. It depends on the input and on the state or the output of the system. Note that only the first m components of the state are directly measured.

***Assumption 1**: the pair (A, C) is detectable*

Observer 1
A Luenberger's observer is used to generate the non-linearities estimation. The system output can be reconstructed by:

$$\hat{x}(k+1) = A_0\, \hat{x}(k) + B\, u(k) + L\, y(k) \tag{2.a}$$
$$\hat{y}(k) = C\, \hat{x}(k) \tag{2.b}$$

where $\hat{x}(k) \in R^n$, $A_0 = A - L\,C$, with L, the observer gain.

It is clear that the output estimation error does not converge to zero when k tends to infinite because the observer (2) cannot provide an acceptable estimation on account of non-linearities.

By substituting the system output (1.b) and its estimate (2.b) by their expressions, the output error $\varepsilon(k) = y(k) - \hat{y}(k)$ becomes:

$$\varepsilon(k) = C\,(qI - A_0)^{-1} F\,\Psi(y,u) \tag{3}$$

where q is the forward shift operator. This equation is also equivalent to:

$$v(k+1) = A_0\, v(k) + F\,\Psi(y,u) \tag{4.a}$$

$$\varepsilon(k) = C\, v(k) \tag{4.b}$$

Partitioning the variable v into $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$ and taking into account the expression (1.c) of C, the system (4) is written under the form:

$$\begin{bmatrix} v_1(k+1) \\ v_2(k+1) \\ v_3(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \Psi(y,u) \quad \begin{pmatrix} 5.a \\ 5.b \\ 5.c \end{pmatrix}$$

$$\begin{bmatrix} \varepsilon_1(k) \\ \varepsilon_2(k) \end{bmatrix} = \begin{bmatrix} I_{q.q} & 0_{q.(m-q)} & 0_{q.(n-m)} \\ 0_{(m-q).q} & I_{(m-q).(m-q)} & 0_{(m-q).(n-m)} \end{bmatrix} \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \end{bmatrix} \tag{5.d}$$

$v_1 \in R^q$, $v_2 \in R^{m-q}$, $v_3 \in R^{n-m}$, $\varepsilon_1 \in R^q$, $\varepsilon_2 \in R^{m-q}$, A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23}, A_{31}, A_{32}, A_{33} and F_1, F_2 and F_3 correspond to the partitioning of A_o and F.

Assumption 2: *the matrix* F_1 *is regular*

The substitution of the equation (5.d) in (5.a) leads to:

$$\Psi(y,u) = F_1^{-1}\left(\varepsilon_1(k+1) - A_{11}\varepsilon_1(k) - A_{12}\varepsilon_2(k) - A_{13}v_3(k)\right) \tag{6}$$

The non-linear variable $\Psi(y,u)$ is expressed in terms of an unknown parameter v_3. Its estimation will be performed after generating, firstly, an equation with a state variable v_3, and secondly its corresponding measurement equation.

Replacing the non-linearity $\Psi(y,u)$ by its expression (6) in equation (5.c); leads then to:

$$v_3(k+1) = \bar{A}\, v_3(k) + r(k) \tag{7.a}$$

where:

$$\bar{A} = A_{33} - \gamma\, A_{13}$$

$$r(k) = (A_{31} - \gamma A_{11})\varepsilon_1(k) + (A_{32} - \gamma A_{12})\varepsilon_2(k) + \gamma\varepsilon_1(k+1)$$

with $\gamma = F_3\, F_1^{-1}$

Expression (7.a) represents an equation with a state variable v_3. Its corresponding measurement equation is obtained by substituting the non-linearity (6) in (5.b):

$$f(k) = D\, v_3(k) \tag{7.b}$$

where $D = A_{23} - F_2\, F_1^{-1} A_{13}$ (8.a)

and

$$f(k) = \varepsilon_2(k+1) - D_1\varepsilon_1(k+1) + D_2\varepsilon_1(k) + D_3\varepsilon_2(k) \tag{8.b}$$

with $D_1 = F_2\, F_1^{-1}$

$D_2 = D_1\, A_{11} - A_{21}$

$D_3 = D_1\, A_{12} - A_{22}$

Assumption 3: *the pair* $(\bar{A}, D)$ *is detectable*

If this assumption holds, the unmeasured variable v_3 can be reconstructed by:

$$\hat{v}_3(k+1) = (\bar{A} - M\,D)\,\hat{v}_3(k) + r(k) + M\, f(k) \tag{9}$$

where M is the observer gain and $\hat{v}_3$ the estimation of v_3.

It can be noticed that the proposed estimation (9) depends on unavailable variables defined at the discrete time k+1. This problem can be overcome by defining an intermediate variable w:

$$w(k) = \hat{v}_3(k) - (\gamma - M\, D_1)\,\varepsilon_1(k) - M\,\varepsilon_2(k) \tag{10}$$

Taking into account the expression (9), the dynamic of w is written under the form:

$$w(k+1) = G\, w(k) + H_1\,\varepsilon_1(k) + H_2\,\varepsilon_2(k) \tag{11}$$

After determination of w, the estimation of v_3 is deduced. The expression of the non-linearity contains unavailable parameters. At infinite, this non-linearity becomes:

$$\tilde{\Psi}(y,u) = F_1^{-1}\left((I - A_{11})\varepsilon_1(k) - A_{12}\varepsilon_2(k) + A_{13}\hat{v}_3(k)\right) \tag{12}$$

The structure of the non-linear variable $\Psi(y,u)$ can be approximated and defined using a neural network (multilayer perceptron, for example). The neural network is used to learn $\tilde{\Psi}(y,u)$ as a function of the system output and input. As shown fig.1, the network input is composed of y and u and its output gives the estimation of $\Psi(y,u)$.

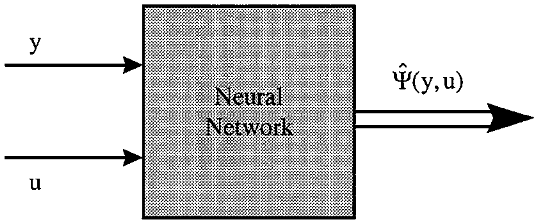


Fig. 1. Neural Network inputs and output.

4. State Reconstruction

Once the neural network has been trained, its output is used as feedforward signal to compensate the modeling errors. The accurate estimation of the system state is then:

Observer 2

$$\hat{\hat{x}}(k+1) = (A-KC)\,\hat{\hat{x}}(k) + B\,u(k) + K\,y(k) + F\,\hat{\Psi}(y,u) \quad (13.a)$$

$$\hat{\hat{y}}(k) = C\,\hat{\hat{x}}(k) \quad (13.b)$$

It is clear that the output error

$$e(k) = y(k) - \hat{\hat{y}}(k) \quad (14)$$

is asymptotically null but presents picks because of the approximation (12).

<u>*Proof*</u>

Let us compute the state reconstruction error defined by:

$$\beta(k) = x(k) - \hat{\hat{x}}(k)$$

The dynamic of this error is obtained by substituting the state x and its estimation by their respective expressions (1) and (13):

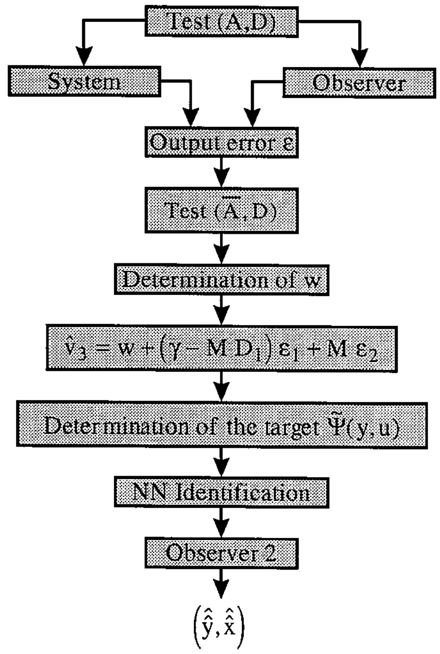
$$\beta(k+1) = (A-KC)\,\beta(k) + F\left(\Psi(y,u) - \hat{\Psi}(y,u)\right)$$

As the eigenvalues of the matrix $(A-KC)$ *have to be chosen in the unit circle, the state reconstruction error* $\beta \to 0$ *if* $k \to \infty$ *and* $\hat{\Psi}(y,u) \to \Psi(y,u)$*; consequently, the output estimation error* $\to 0$ *when* $k \to \infty$.

End of proof.

5. Algorithm

The proposed method can be summarized by the following algorithm.



For a diagnosis goal, the properties of the residuals 'e' can be used to detect and identify the faulty system components (actuators, sensors, ...). Then, a threshold λ has to be defined:

* if $|e| < \lambda$: no faults,

* if $|e| > \lambda$: presence of faults.

6. Simulation Results

A non-linear system defined by:

$$x(k+1) = A\,x(k) + B\,u(k) + 0.01\,F\,x(1{:}2)(k)\;x(3)(k)\cos(100\pi\,u(k))$$
$$y(k) = C\,x(k)$$

is considered, where the system matrices are:

$$A = \begin{bmatrix} 0.5 & 0.03 & 0 & 0.1 \\ 0 & 0.9 & 0.2 & 0.3 \\ 0.01 & 0.02 & 0.6 & 0.2 \\ 0.2 & 0.01 & 0.1 & 0.1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.1 \\ 1.3 \\ 1 \\ 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The system input and output are shown on figures 2 and 3 respectively.

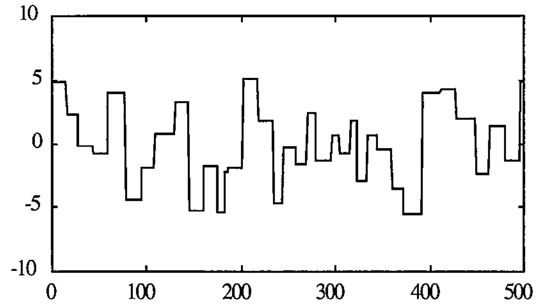


Fig. 2. System input.

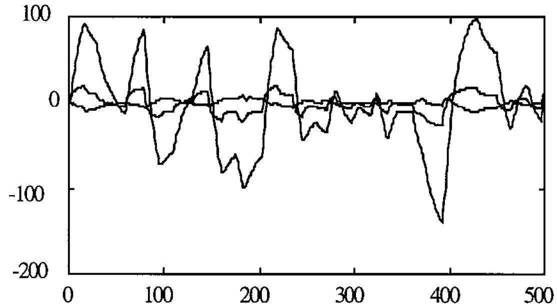


Fig. 3. System output.

After using the pole assignment method, the observer 1 is designed in order to generate the first output estimation error 'ε' (fig. 4) which allows the estimation of the variable v_3.

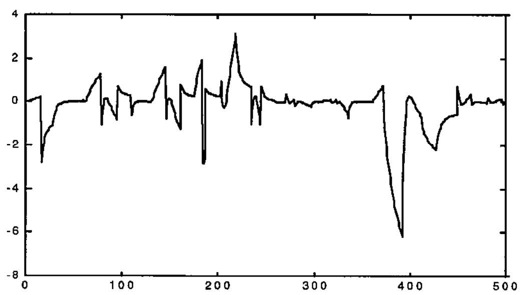


Fig. 4a. First component of ε.

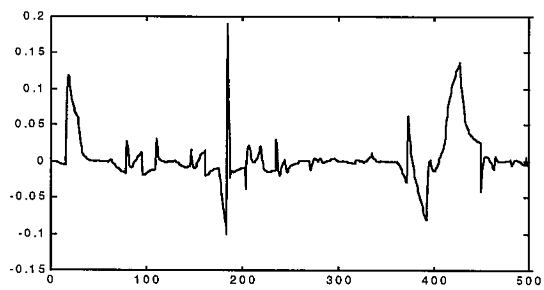


Fig. 4b. Second component of ε.

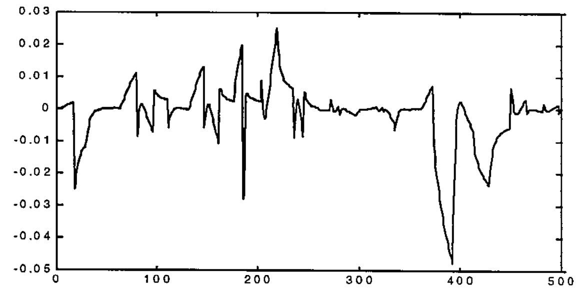


Fig. 4c. Third component of ε.

Consequently, the target of the Neural Network is obtained. It represents an approximation of the actual value of the non-linearities $\Psi(y,u)$. Their estimations are obtained using a multilayer Neural Network 4x8x2 which is trained off-line and are presented on the figures 5 and 6.

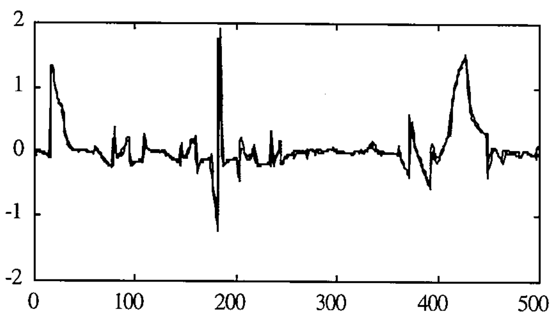


Fig. 5a. First non-linearity and its estimation.

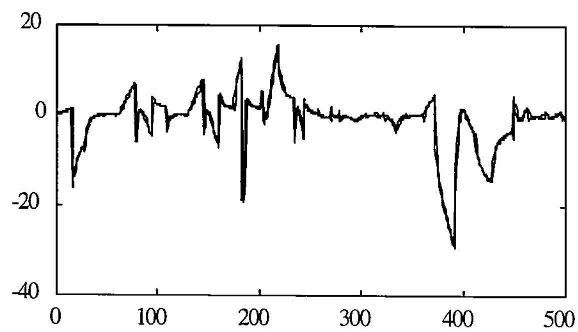


Fig. 5b. Second non-linearity and its estimation.

The non-linearity estimation is used as feedforward signal to compensate the modeling errors. The observer 2 provides the accurate reconstructed output (figures 6).

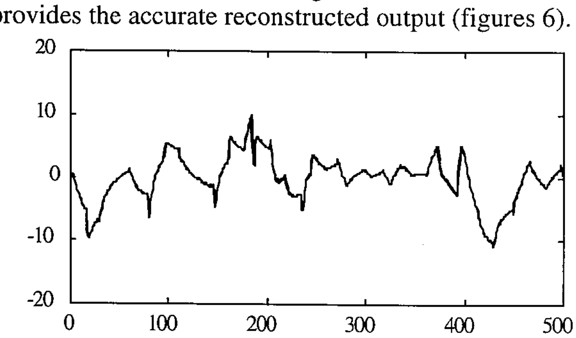


Fig. 6.a. First output and its reconstruction.

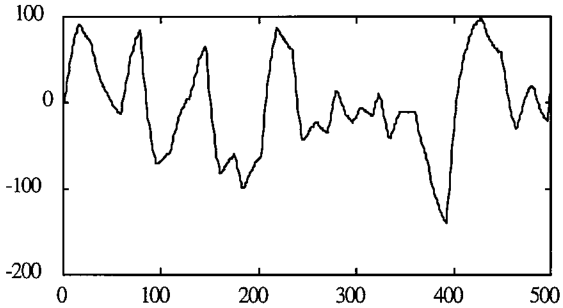


Fig. 6.b. Second output and its reconstruction.

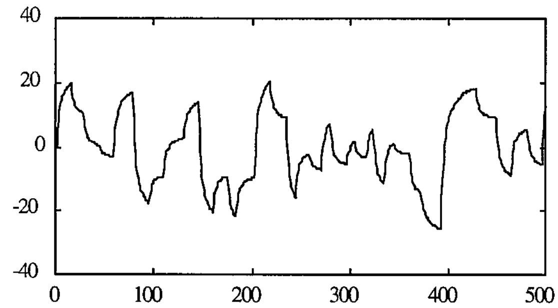


Fig. 6.c. Third output and its reconstruction.

7. Conclusion

By respecting some assumptions, we have proposed a method able to estimate system non-linearities. Their structures can be found using Neural Network and expressed in terms of the known variables y and u. The observer used for reconstructing the actual state takes into account the estimated non-linearities. The method requires the transformation of the measurement matrix C into [I 0] which is always possible if material redundancies are avoided.

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