Fault Accommodation of Output-Induced Actuator Failures for a Flexible Beam with Collocated Input and Output

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Abstract

In this note, we propose a nonlinear on-line parameter estimation method that utilizes neural network-based approximators for detecting changes due to actuator faults in a class of structural dynamical systems. The plant considered here is a cantilevered beam actuated via a pair of piezoceramic patches. We examine changes in the control input term, which provide a simple and practical model of actuator failures. Using Lyapunov redesign methods, a stable learning scheme for fault diagnosis is proposed. The resulting fault diagnosis scheme is utilized in a control reconfiguration in order to accommodate the system's actuator failure. A numerical algorithm is provided for the implementation of the detection and accommodation scheme and simulation studies are used to illustrate the applicability of the theoretical results.

1 Introduction

The detection and diagnosis of failures of dynamical systems is attracting the attention of many researchers working on diverse engineering problems, [2, 9, 15, 17]. As was noted in [17] and the references therein, many fault detection schemes deal with either linear or nonlinear finite dimensional systems (lumped parameter systems). Many physical systems though, are described by partial differential, integrodifferential and functional differential equations. These systems are infinite dimensional and their parameters are distributed in nature. For a class of distributed parameter systems (mainly flexible structures such as beams and trusses), damage detection based on the analysis of natural modes and frequencies was studied by several researchers. As was pointed out in [4] and argued in [3] and their references, methods based on natural modes (frequencies) are highly unreliable when dealing with estimation of (possibly) variable material parameters such as mass, stiffness and damping. The analysis of damages in parameterized partial differential equations with Galerkin approximation techniques validates non-destructive damage detection since it incorporates information on location/geometry of damages in structures. This motivates our current research efforts.

During the last two decades, a number of fault diagnosis schemes have been developed using the analytical redundancy approach [10, 16, 17]. According to this approach, input/output measurements are processed analytically to estimate the values of certain key system variables. The estimates are then compared with measured signals to generate a residual vector, which can be utilized to detect and isolate system failures.

The process of system failure characterization can be broken up into three steps: (i) *detection* deals with determining if a malfunction has occured in the system; (ii) *diagnosis* considers the problem of isolating and identifying a failure; and (iii) *accommodation* attempts to self-correct a particular failure through reconfiguration of the control system. Depending on the application, a diagnostic system may include some or all of the above tasks.

In this paper we discuss a theoretical investigation and present a numerical scheme for a *model-based* fault diagnosis and accommodation algorithm applied to a class of distributed parameter system. An estimated model of the plant is used to monitor the plant for any changes due to faults. The estimated model incorporates an on-line approximator [17], which estimates and monitors the parameters on-line via a learning algorithm. The output of the on-line approximator is used as an indicator of the occurrence of a fault and also as a method for identifying the location (fault isolation) and shape (fault identification) of system failures. A reconfiguration of the standard control is presented in order to accommodate the system failure.

The structure considered in the ensuing example is taken to be a cantilevered beam with two piezoceramic patches attached on the opposite sides of the beam. As was already mentioned in many works, see for example [5] and the references therein, a general structural (and even structural-acoustic) control problem has dynamics described by the second order evolution equation

$$M\ddot{w}(t) + D\dot{w}(t) + Kw(t) = Bu(t) \qquad (1)$$

with output

$$y(t) = C\dot{w}(t) \qquad (2)$$

where the state $w(t)$ belongs to a Hilbert space H and M, D, K and B, C are operators in the appropriate spaces.

The paper is organized as follows. In Section 2 we set up the abstract equations that govern the dynamics of the plant (which are assumed infinite dimensional) and in Section 3 we propose a model for the fault, which in this case is simply taken to be a change in actuator gain that is a function of the measurable output signals. The abstract formulation of the plant's estimator is presented in § 4 along with a discussion summary of well posedness. A standard controller for the nominal plant is proposed in § 5 and a modification to the standard control is presented to account for the plant changes due to failures. Convergence results follow and the numerical implementation scheme is proposed in § 6. Simulations studies with discussion follow in § 7 and conclusions with future directions are summarized in § 8.

2 Plant Dynamics

It is assumed that the beam satisfies the Euler-Bernoulli displacement hypothesis with Kelvin-Voigt damping (damping proportional to strain rate) and air damping (damping proportional to velocity). Two piezoceramic patches are bonded to the beam at the location $x_l \leq x \leq x_r$ and are excited out-phase, which results in pure bending of the beam [5]. The moment due to patches is localized to the region covered by the patches. When the structure is subject to moments generated by the patches, it leads to the equation

$$\rho(x)\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 \mathcal{M}}{\partial x^2} = -\frac{\partial^2 \mathcal{M}_p}{\partial x^2}, \quad 0 < x < l, \qquad (3)$$

where $w = w(t,x)$ is the transverse displacement, $\mathcal{M}$ is the internal moment given by

$$\mathcal{M} = EI(x)\frac{\partial^2 w}{\partial x^2} + c_D I(x)\frac{\partial^3 w}{\partial x^2 \partial t}, \qquad (4)$$

and $\mathcal{M}_p$ is the external moment due to the piezoceramic patches. This piezoceramic moment is given by

$$\mathcal{M}_p = -\mathcal{K}_A \chi_p(x) u(t), \qquad (5)$$

where $u(t)$ is the voltage applied to the patches, $\mathcal{K}_A$ is a constant that depends on the piezoceramic material properties [5] and $\chi_p(x)$ is the characteristic function, which is equal to 1 for $x_l \leq x \leq x_r$ and zero elsewhere. The above (spatially varying) parameters $\rho(x)$, $EI(x)$, and $c_D I(x)$ above are the *mass density*, *stiffness* coefficient and *damping* coefficient, respectively. Using the above equations for the moments (i.e. equations (4), (5)), we arrive at the following partial differential equation (PDE) for the transverse displacement of the beam

$$\rho w_{tt} + [EIw_{xx} + c_D I w_{txx}]_{xx} = [\mathcal{K}_A \chi_p(x) u]_{xx}, \qquad (6)$$

for $x \in \Omega = [0,1]$, with collocated output

$$y(t) = \int_0^l \mathcal{K}_S \chi_p(x) w_{txx}(t,x)\, dx, \qquad (7)$$

where $\mathcal{K}_S$ is a sensor constant which is a piezoceramic material and geometry related quantity, [5, 8]. Associated with the above beam equation are the appropriate boundary (cantilevered beam) and initial conditions given by

$$w(t,0) = w_x(t,0) = 0 = w_{xx}(t,l) = w_{xxx}(t,l), \qquad (8)$$

$$\text{and} \quad w(0,x) = w_0(x), \quad w_t(0,x) = w_1(x). \qquad (9)$$

In order to analyze the above system and propose its state estimator, we consider the problem in an abstract setting. We consider the above PDE as a second order differential equation in a Hilbert space. Let the Hilbert space $H = L^2(0,l)$ be the state space and consider the space of *test functions* $V = H_L^2(0,l)$. The Sobolev space $H_L^2(0,l)$ is

$$H_L^2(0,l) = \left\{\varphi \in H^2(0,l) \,:\, \varphi(0) = \varphi'(0) = 0\right\}.$$

In addition, we define the *negative* Sobolev space $V^* = H^{-2}(0,l)$ as the continuous dual of $H_L^2(0,l)$, see [1].

When the plant (6) with output (7) is written as a second order evolution equation in the larger space V^*, it becomes

$$M\ddot{w}(t) + D\dot{w}(t) + Kw(t) = Bu(t) \qquad (10)$$

with velocity output

$$y(t) = C\dot{w}(t). \qquad (11)$$

See, for example, [5] for details on how to write (6) in an abstract setting. The operators M, D, K B and C expressed in weak form are given as follows

$$\begin{aligned}
\langle Mw, \phi\rangle &= \textstyle\int_0^l \rho(x) w(t,x)\phi(x) dx \\
\langle D\dot{w}, \phi\rangle &= \textstyle\int_0^l c_D I(x) w_{txx}(t,x)\phi_{xx}(x) dx \\
\langle Kw, \phi\rangle &= \textstyle\int_0^l EI(x) w_{xx}(t,x)\phi_{xx}(x) dx \\
\langle Bu(t), \phi\rangle &= \textstyle\int_0^l \mathcal{K}_A \chi_p(x) u(t)\phi_{xx}(x)\, dx, \\
C\dot{w}(t) &= \textstyle\int_0^l \mathcal{K}_S \chi_p(x) w_{txx}(t,x)\, dx,
\end{aligned}$$

for all $\phi \in H_L^2(0,l)$, see also the companion paper [7] for a complete description of these operators. It is easily seen that the output operator C is a constant multiple of the adjoint of the input operator B, given by $C = \alpha_p B^*$ with $\alpha_p = \frac{\mathcal{K}_S}{\mathcal{K}_A}$.

3 Modeling of Failure

The failure is modeled as a *time varying additive perturbation* (incipient additive perturbation) of an actuator fault due to a nonlinear gain depending on the measured output signal and is given via $Bf(y)u$ with $f : \mathbb{R}^1 \rightarrow \mathbb{R}^1$ being a smooth vector field. Thus, we have

$$\begin{aligned} M\ddot{w} + D\dot{w} + Kw &= Bu + \beta(t-T)Bf(y)u \\ y &= C\dot{w} \end{aligned} \tag{12}$$

where the term $\beta(t-T)$ denotes the time profile of the failure and is given by

$$\beta(\tau) = \begin{cases} 0 & \text{if } \tau < 0 \\ 1 - e^{-\lambda\tau} & \text{if } \tau \geq 0. \end{cases} \tag{13}$$

with $\lambda > 0$ an unknown constant, see also [7].

We will now make the assumption that for the class of systems under study, we have *admissible plants*.

Assumption 3.1 (Admissible plant) *We assume that the (perturbed) system*

$$\begin{aligned} M\ddot{w}(t) + D\dot{w}(t) + Kw(t) &= Bu(t) + \beta Bf(y)u \\ y &= C\dot{w}(t) \end{aligned}$$

for $t \geq T$, is well posed in the sense that a weak solution $w \in L^2(0,\infty; H_0^2(\Omega))$ with $w_t \in L^2(0,\infty; L^2(\Omega))$, $w_{tt} \in L^2(0,\infty; H^{-2}(\Omega))$ exists that satisfies (13) with (8) (see [5, 19]) and that has $y \in L^\infty(0,\infty; \mathbb{R}^1)$.

4 Model Estimator and Convergence

In this section we propose a state estimator to monitor the plant for possible changes in dynamics and hence to detect possible failures in the system. These changes will be utilized by the controller to accommodate the failure. The same state estimator presented in [7] is used here and is given by

$$\begin{aligned} &M\ddot{\widehat{w}}(t) + D\dot{\widehat{w}}(t) + K\widehat{w}(t) = Bu(t) + B\widehat{f}(y;\widehat{\theta})u(t), \\ &\widehat{w}(0) = w(0), \quad \widehat{w}_t(0) = w_t(0) \\ &\widehat{y}(t) = C\dot{\widehat{w}}(t). \end{aligned} \tag{14}$$

where $\widehat{w}(t,x)$ is the estimate of the state $w(t,x)$ and $\widehat{f}(y;\widehat{\theta}) : \mathbb{R} \times \mathbb{R}^q \rightarrow \mathbb{R}$ is the estimate of the time varying failure term $\beta(t-T)f(y)$.

Let the state error be given by $e(t,x) = w(t,x) - \widehat{w}(t,x)$. In order to extract the adaptation rule for the estimate $\widehat{f}(y;\widehat{\theta})$, we use the state error equation in abstract form

$$\begin{aligned} M\ddot{e} &= -D\dot{e} - Ke + B\left[\beta f(y) - \widehat{f}(y;\widehat{\theta})\right]u \\ &= -D\dot{e} - Ke + B\nu u \\ &\quad + B\left[\beta\widehat{f}(y;\theta^*) - \widehat{f}(y;\widehat{\theta})\right]u, \end{aligned} \tag{15}$$

having zero initial conditions $e(0,x) = e_t(0,x) = 0$, where $\nu(t)$ is the *approximation error* given by

$$\nu(t) = \beta(t-T)\left[f(y) - \widehat{f}(y;\theta^*)\right].$$

The "optimal " parameter θ^* is chosen as the value of $\widehat{\theta}$ that minimizes the L_2-norm distance between $f(y)$ and $\widehat{f}(y;\widehat{\theta})$. The *output error* $\varepsilon(t)$ given by

$$\varepsilon(t) = \frac{\mathcal{K}_S}{\mathcal{K}_A}B^*\dot{e}(t) = \alpha_p B^*\dot{e}(t) = C\dot{e}(t). \tag{16}$$

Using Lyapunov redesign methods [12], the adaptation law for the adjustment of parameter estimates is given by

$$\begin{aligned} \dot{\widehat{\theta}}(t) &= \mathcal{P}\left\{\varepsilon(t)Z(t)u(t)\right\} \\ \widehat{\theta}(0) &= 0, \end{aligned} \tag{17}$$

where $Z \in \mathbb{R}^q$ is $Z(t) = \frac{\partial \widehat{f}(y;\widehat{\theta})}{\partial\widehat{\theta}}$ and $\mathcal{P}$ is the projection operator that constrains the parameter $\widehat{\theta}$ to some selected compact, convex region of the parameter space, [17]. As was mentioned in [7, 17, 18], in the case of the compact region being a hypersphere, the adaptive law can then be expressed as

$$\dot{\widehat{\theta}}(t) = \varepsilon(t)Z(t)u(t) - \chi^*\frac{\widehat{\theta}\widehat{\theta}^T}{|\widehat{\theta}|^2}\varepsilon(t)Z(t)u(t), \tag{18}$$

where the *indicator function* χ^* is given by

$$\chi^* = \begin{cases} 0 & \text{if } \quad (|\hat{\theta}| < M) \text{ or } (|\hat{\theta}| = M \text{ and } \hat{\theta}^T Z\varepsilon u \leq 0) \\ 1 & \text{if } \quad (|\hat{\theta}| = M \text{ and } \hat{\theta}^T Z\varepsilon u > 0). \end{cases}$$

Using the smoothness assumption on $\widehat{f}$, it then follows from (15) that

$$\begin{aligned} M\ddot{e} = &-D\dot{e} - Ke - [1-\beta]\,Bu\widehat{f}(y;\theta^*) \\ &-BZ^T\left(\widehat{\theta} - \theta^*\right)u - \Delta(y;\widehat{\theta})Bu + Bu\nu \end{aligned}$$

where $\Delta(y;\widehat{\theta})$ is given by

$$\Delta(y;\widehat{\theta}) = \widehat{f}(y;\theta^*) - \widehat{f}(y;\widehat{\theta}) - \frac{\partial\widehat{f}(y;\widehat{\theta})}{\partial\widehat{\theta}}.$$

If we let $\tilde{\theta}(t) = \widehat{\theta}(t) - \theta^*$, $\omega(t) = -\Delta(y(t);\widehat{\theta}(t)) + \nu(t)$, we have

$$M\ddot{e} = -D\dot{e} - Ke - Z^T\tilde{\theta}Bu - \Phi Bu\widehat{f}(y;\theta^*) + Bu\omega \tag{19}$$

where $\Phi(t) = 1 - \beta(t-T)$.

We use the following Lyapunov functional in order to analyze the stability and performance properties of the proposed diagnosis scheme,

$$V(t) = \langle e(t), Ke(t)\rangle + \langle \dot{e}(t), M\dot{e}(t)\rangle + |\tilde{\theta}(t)|^2 + |\Phi(t)|^2.$$

When the derivative of $V(t)$ is evaluated along the trajectories of the state error equation (19), it yields

$$\begin{aligned}
\dot{V}(t) &= 2\langle e, K\dot{e}\rangle - 2\langle \dot{e}, D\dot{e}\rangle - 2\langle \dot{e}, Ke\rangle - 2\langle \dot{e}, BuZ^T\tilde{\theta}\rangle \\
&\quad -2\langle \dot{e}, \Phi Bu\widehat{f}\rangle + 2\langle \dot{e}, Bu\omega\rangle + 2\dot{\tilde{\theta}}^T\tilde{\theta} + 2\dot{\Phi}^T\Phi \\
&= -2\langle \dot{e}, D\dot{e}\rangle - 2\langle \dot{e}, \Phi Bu\widehat{f}\rangle + 2\langle \dot{e}, B\omega u\rangle \\
&\quad -2\chi^*\tilde{\theta}^T\frac{\widehat{\theta}\widehat{\theta}^T}{|\widehat{\theta}|^2}Z^T\varepsilon u - 2\lambda\Phi^T\Phi, \qquad (20)
\end{aligned}$$

where we used the fact that $\dot{\Phi} = -\lambda\Phi$. Using established results in the theory of robust adaptive control [11, 13] and in automated fault detection [17, 18], we have that the projection term can only make the derivative of $V(t)$ more negative, i.e.

$$\chi^*\tilde{\theta}^T\frac{\widehat{\theta}\widehat{\theta}^T}{|\widehat{\theta}|^2}Z^T\varepsilon u \geq 0.$$

Using the coercivity of the operator D (implicitly assumed in Assumption 3.1), the smoothness of the input $u(t)$ and output $y(t)$ and the smoothness assumption on $\widehat{f}$, we have that

$$\begin{aligned}
\dot{V}(t) &\leq -2\langle \dot{e}, D\dot{e}\rangle - 2\langle B^*\dot{e}, \Phi\widehat{f}u\rangle + 2\langle B^*\dot{e}, \omega u\rangle \\
&\quad -2\lambda\Phi^T\Phi \\
&\leq -c_1|\dot{e}|^2 - c_2|\Phi|^2 + c_3|\omega|^2.
\end{aligned}$$

When $c_1|\dot{e}|^2 + c_2|\Phi|^2 \geq c_3|\omega|^2$ we have that $\dot{V} \leq 0$, which yields the uniform boundedness of V and $\tilde{\theta}$. Thus by integrating over a finite interval $[T, T+\tau]$ we have that

$$\begin{aligned}
V(T+\tau) + c_1\int_T^{T+\tau} |\dot{e}(t)|^2\,dt + c_2\int_T^{T+\tau} |\Phi(t)|^2\,dt \\
\leq V(T) + c_3\int_T^{T+\tau} |\omega(t)|^2\,dt.
\end{aligned}$$

The above yields the uniform boundedness of $\widehat{\theta}$ and $V(t)$ for $t \geq T$. It is easily observed that $V(t) = 0$ for $t < T$. Using the observability condition, we have that both $\widehat{f}(y, \widehat{\theta}(t))$ and $\varepsilon(t)$ are zero prior to the failure time T and become nonzero for $t \geq T$. Hence, by monitoring either the output error $\varepsilon(t)$ or the on-line approximator output $\widehat{f}(y, \widehat{\theta}(t))$ we can detect the time of failure T. Furthermore, we have that the extended L^2 norm of the state estimation velocity error (and by observability, the output error) over any finite time interval is at most of the same order as the extended L^2 norm of $\omega(t)$.

5 Accommodation

The standard control law for the nominal plant (10) without failure terms can be chosen as $u(t) = u_0(t)$ with

$$u_0(t) = -G_1\widehat{w}(t) - G_2\dot{\widehat{w}}(t) + G_3r(t), \qquad (21)$$

where the gains G_1, G_2 are, for example, chosen as the LQE feedback gains obtained by solving an Algebraic Riccati equation for the nominal plant (10), and the signal r is a reference signal with G_3 a reference gain, that is used if the control objective is model reference.

In the presence of a failure the nominal control law (21) needs to be modified to account for the additive failure term $Bu(t)f(y)$. This takes the form

$$u(t) = \frac{1}{1+\widehat{f}(y;\widehat{\theta})}u_0(t). \qquad (22)$$

The closed loop state estimator is now given by

$$M\ddot{\widehat{w}} + [D + BG_2]\dot{\widehat{w}} + [K + BG_1]\widehat{w} = BG_3r.$$

A modification to the control law (22) must be made in order to ensure that $1 + \widehat{f}(y;\widehat{\theta}) \neq 0$. Implicitly it was assumed that the fault term $f(y) \neq -1$ (hence no loss of controllability in (12)) and thus the output of the on-line approximator must not cancel out the control signal in (14). In practice, the control reconfiguration must ensure that $|1 + \widehat{f}(y;\widehat{\theta})| > c$ with $0 < c \ll 1$ in order to avoid large voltages in the patch. This leads to the reconfigured controller

$$u(t) = \begin{cases} \dfrac{u_0(t)}{1+\widehat{f}(y;\widehat{\theta})} & \text{if } |1+\widehat{f}(y;\widehat{\theta})| > c \\[2ex] \dfrac{u_0(t)}{c} & \text{if } |1+\widehat{f}(y;\widehat{\theta})| \leq c. \end{cases}$$

Alternatively, one can switch the adaptation off for $\widehat{\theta}$ when $1 + \widehat{f}(y;\widehat{\theta})$ is near 0 or impose additional constraints on the adaptation rule (17) such that $1 + \widehat{f}(y;\widehat{\theta}) \neq 0$.

6 Numerical Implementation

In this section we summarize the numerical approximation scheme. Assume that the beam displacement is approximated by

$$w^n(t,x) = \sum_{i=1}^{n+1}\alpha_i(t)\phi_i^n(x), \quad i = 1, 2, \ldots, n+1 \quad (23)$$

where $\phi_i^n(x)$, $i = 1, \ldots, n+1$ are modified cubic splines on $[0, l]$. Then using results in [5, 14] the beam equation can be written in a matrix form as

$$\begin{aligned}
M^n\ddot{\alpha}(t) + D^n\dot{\alpha}(t) + K^n\alpha(t) &= B^nu(t) + \beta B^nf(y)u \\
y(t) &= C^n\dot{\alpha}(t)
\end{aligned}$$

where the above matrices are given explicitly in [5, 6]. In this simulation study, Radial basis function networks are used as the on-line approximator model given by

$$\widehat{f}(y,\widehat{\theta}) = \sum_{k=1}^{m} \widehat{\theta}_i(t)\exp\left(-\frac{(y-c_i)^2}{\sigma^2}\right) = Z^T(t)\widehat{\theta}(t).$$

The finite dimensional estimator is given by

$$M^n\ddot{\hat{\alpha}}(t)+D^n\dot{\hat{\alpha}}(t)+K^n\hat{\alpha}(t) = B^n u(t)+B^n\widehat{f}(t,\widehat{\theta})u(t).$$

The adaptation laws are given by

$$\dot{\widehat{\theta}}(t) = \gamma Z(t)\varepsilon(t)u(t),$$

and γ is the *adaptive gain*, [13].

7 Numerical Results

For the specific set of simulations, we assumed that the beam length is $l = 0.4573$m, with the patches placed at $x_l = 0.15$m and $x_r = 0.25$m. The beam stiffness coefficient is $EI_b = 0.491Nm^2$ and the beam damping coefficient is $c_D I_b = 0.649 \times 10^{-3} sNm^2$. In the damping component, we assumed air damping with damping parameter $0.013 sN/m^2$. The corresponding values for the patch are $EI_p = 0.793Nm^2$, $c_D I_p = 1.255 \times 10^{-3} sNm^2$ with patch linear mass density $\rho_p = 0.433kg/m$ and thickness $h_p = 0.000254m$. The beam had a mass density $\rho_b = 0.093kg/m$, thickness $h_b = 0.0016m$, and width $b = 0.0203m$. The piezoceramic constant $\mathcal{K}_A = 1.746\times10^{-2}Nm/V$ with the one used for sensing $\mathcal{K}_S = 1 \times 10^{-3}\mathcal{K}_A$.

The failure term is given by

$$\beta(t-1)f(y) = 100\left(1-e^{-0.5(t-1)}\right)\frac{y}{1+y^2}$$

which models an incipient fault commencing at $T = 1$ seconds. The adaptive gain is $\gamma = 10^6$. The feedback gains G_1, G_2 were found by solving the Riccati equation $\Pi\mathcal{A} + \mathcal{A}^T\Pi - \Pi\mathcal{B}R^{-1}\mathcal{B}\Pi + Q = 0$ for the nominal system (10) written as a first order system with $\dot{x} = \mathcal{A}x + \mathcal{B}u$ and Q given by $Q = \text{diag}(500K, 10^4M)$. Finally, the reference term $G_3r(t) = 10(\sin(150\pi t) + \cos(250\pi t) + \sin(225\pi t) + \cos(175\pi t) + \cos(20\pi t))$.

The evolution of the on-line approximator output $\widehat{f}(y;\widehat{\theta})$ is presented in Figure 1b. In the same figure we plot the actual failure term $\beta(t-1)f(y)$ (Figure 1a). It is observed that the on-line approximator (OLA) is able not only to detect but to diagnose the failure as well.

When the output error $\varepsilon(t)$ is plotted vs time in Figure 2 we notice that it has a value of zero prior to $t = 1$ seconds, attains a nonzero value and then converges to zero. As a result we can conclude that both the on-line approximator output $\widehat{f}$ and the output error $\varepsilon(t)$ can be used for failure detection and furthermore the OLA output can be used for failure diagnosis as well.

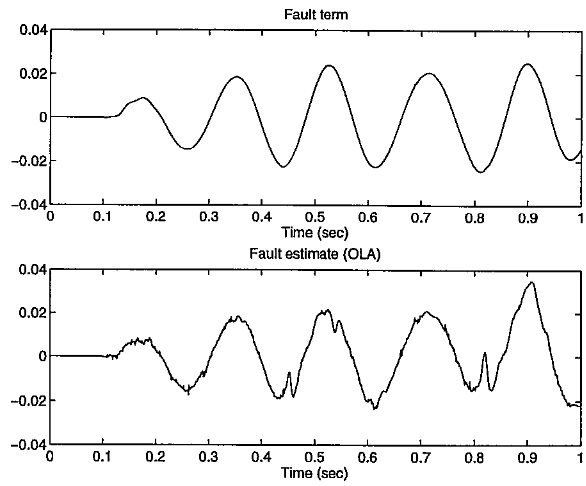

Figure 1: Evolution of OLA (solid) and failure terms (dashed): incipient failure time profile.

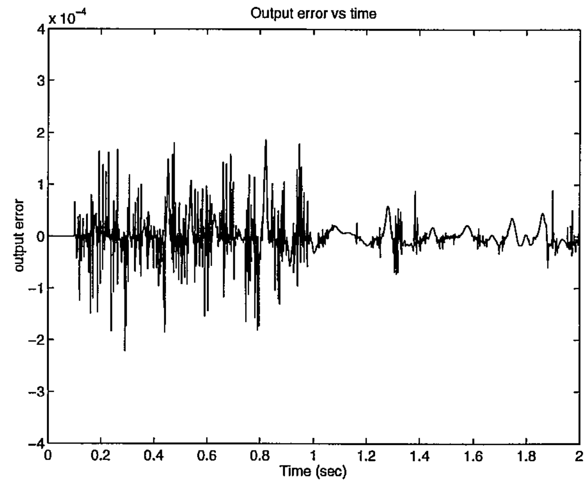

Figure 2: Evolution of output error $\epsilon(t)$.

8 Conclusion

In this note an on-line approximation scheme was proposed for the detection, diagnosis and accommodation of actuator failures in a plant whose dynamics are governed by a partial differential equation. The plant describes the transverse vibration of a flexible cantilevered beam actuated with a pair of piezoceramic patches that are also used as sensors. The failure was modeled as a time varying output-induced additive perturbation of an actuator failure. The proposed scheme, through both theoretical and numerical results, was shown to actually detect, diagnose and accommodate the actuator failure with incipient time profiles.

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