Variable Structure Control of a robotic assistance system

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Abstract

The development of an autonomous robotic system for the assistance to disabled people (composed of a 'car-like' mobile base and a robot arm) is the primary aim of the MOVAID Project, supported by the CEC in the framework of the TIDE Programme. In the context of the MOVAID Project, an automatic navigation system is currently being developed at the Robotics Lab at the Dipartimento di Elettronica ed Automatica. From the functional viewpoint, the navigation system can be decomposed into a motion planner and a collision avoidance module. Both blocks, however, rely on a suitable low-level control procedure, which has to drive the wheel actuators in order to ensure the tracking of the planned trajectory with a sufficient degree of precision, opposing eventual parameter variations and external disturbances. In this paper, a recently developed discrete-time Variable Structure Control (VSC) technique is applied to the control of the steering-wheel robot of the MOVAID Project. The algorithm has been designed in the discrete-time domain, therefore allowing to avoid the well known problems due to the discretization of continuous-time controllers. The simulation study reported in this note is preliminary to the experimental testing of the control algorithm, which is being carried out.

1 Introduction

The development of robotic systems with increasing degree of autonomy is, at present, a challenging issue in robotics research. A promising application field covering both technical and social motivations is the design of robotic systems for the assistance of disabled, handicapped or elderly people. The development of such systems is particularly demanding, due to the remarkable autonomy and the high level of friendliness required to the robotic system in order to compensate the reduced ability of the users.

The development of an autonomous robotic system for the assistance to disabled people (composed of a 'car-like' mobile base and a robot arm) is the primary aim of the MOVAID Project, supported by the CEC in the framework of the TIDE Programme [2] [6] [3]. In the context of the MOVAID Project, an automatic navigation system is currently being developed at the Robotics Lab at the Dipartimento di Elettronica ed Automatica [7]. Its testing is being performed on the LABMATE vehicle which, although strictly speaking belonging to the so called 'unicycle' category, can be considered equivalent to the MOVAID mobile base. From the functional viewpoint, the navigation system can be decomposed into a motion planner and a collision avoidance module. Both blocks, however, rely on a suitable low-level control procedure, which has to drive the wheel actuators in order to ensure the tracking of the planned trajectory with a sufficient degree of precision, opposing eventual parameter variations and external disturbances.

In this paper, a recently developed discrete-time VSC technique [1] is applied to the control of the steering-wheel robot of the MOVAID Project. The algorithm has been designed in the discrete-time domain, therefore allowing to avoid the well known problems due to the discretization of continuous-time controllers. There are two main reasons justifying the choice of the VSC technique [4] [8] [5]:

- the linearized model of the vehicle is not controllable: techniques based on the nonlinear model of the system are therefore needed;
- the widely recognised robustness of VSC can be exploited in case of parameter variations and/or presence of external disturbance in the environment.

The simulation study reported in this note is preliminary to the experimental testing of the control algorithm, which is being carried out. The LABMATE vehicle available at the Robotics Lab can be controlled by setting the steering angle of the vehicle and the incremental angular position of the left and right wheel: a dedicated DSP board then provides for mapping the control inputs into properly generated motor torques. The experimental availability of the above control variables induced to consider first the kinematic model of the vehicle. But the adoption of this model precludes any on-field testing of the VSC algorithm in presence of parameter variation, since the kinematic model cannot account for the vehicle and the wheels dimensions, the robot mass and its inertia. This is the reason why, in this paper, the vehicle dynamic model has been also considered, and simulation results obtained using this model have been reported. In this case, however, the testing on the

LABMATE vehicle cannot be early foreseen, since it would imply the designer intervention inside the inner control loops of the mobile robot in order to make the wheel torques directly available to control. Moreover, a state observer should be added to the scheme to provide the VSC algorithm with estimates of the velocity variables, being VSC a state feedback technique.

The paper is organised as follows. In Section 2 the kinematic and dynamic model of the vehicle are reported, and the adopted control law is briefly recalled. Section 3 contains some simulation results obtained with reference to the kinematic model, while Section 4 provides simulations relative to the dynamic model, including some tests with parameter variations. Finally, conclusions are drawn in Section 5.

2 Models of the robotic system.

In this section a description of the robotic system will be given. (Fig.1). It is equipped with two driving wheels mounted on the same axis, and additional free wheels not shown in the picture. The motion and orientation are achieved by independent actuators, i.e. DC motors providing the necessary torques to the wheels. The vehicle position is described by the coordinates (x, y) of the midpoint between the two driving wheels, and by the orientation angle θ with respect of a fixed frame, as shown in Fig.1.

2.1 Kinematic model of the wheeled robot.

The kinematic model of the steering-wheel robot has the following form:

$$\left\{\begin{array}{lcl} \dot{x} & = & v(t)cos(\theta) \\ \dot{y} & = & v(t)sin(\theta) \\ \dot{\theta} & = & \omega(t) \end{array}\right. \tag{2.1.1}$$

where $\omega(t)$ is the angular velocity, and $v(t)$ is the translational velocity. The discretization of the model (2.1.1) assuming a sampling interval T_c and a zeroth-order-hold provides:

$$\left\{\begin{array}{lcl} x_{k+1} & = & x_k + v_k cos(\theta_k)T_c \\ y_{k+1} & = & y_k + v_k sin(\theta_k)T_c \\ \theta_{k+1} & = & \theta_k + \omega_k T_c \end{array}\right. \tag{2.1.2}$$

the subscript k indicating the variable evaluated in kT_c.

2.2 Dynamic model of the wheeled robot.

The non-holonomic constraint has the following form:

$$-\dot{x}cos(\theta) + \dot{y}sin(\theta) = 0 \tag{2.2.1}$$

formalising the fact that the vehicle can move only in the direction normal to the drive-wheels axis, i.e. the mobile base satisfies the condition of pure rolling and non slipping. The dynamic equations of the robot resulting by the Newton's equations are:

$$\left\{\begin{array}{l} m\ddot{x} = \dfrac{cos(\theta)}{r}\tau_1 + \dfrac{cos(\theta)}{r}\tau_2 - Nsin(\theta) \\ m\ddot{y} = \dfrac{sin(\theta)}{r}\tau_1 + \dfrac{sin(\theta)}{r}\tau_2 - Ncos(\theta) \\ I\ddot{\theta} = \dfrac{R}{r}\tau_1 - \dfrac{R}{r}\tau_2 \end{array}\right. \tag{2.2.2}$$

being m and I the vehicle mass and the moment of inertia, respectively, R the robot length, r the wheels radius, N the centripetal force, and τ_1 e τ_2 the input torques. Using the following relation for N:

$$N = m(\dot{x}cos(\theta) + \dot{y}sin(\theta))\dot{\theta} \tag{2.2.3}$$

the model (2.2.2) becomes:

$$\left\{\begin{array}{lcl} m\ddot{x} & = & -msin(\theta)(\dot{x}cos(\theta) + \dot{y}sin(\theta))\dot{\theta} + \\ & & +\dfrac{cos(\theta)}{r}\tau_1 + \dfrac{cos(\theta)}{r}\tau_2 \\ m\ddot{y} & = & mcos(\theta)(\dot{x}cos(\theta) + \dot{y}sin(\theta))\dot{\theta} + \\ & & +\dfrac{sin(\theta)}{r}\tau_1 + \dfrac{sin(\theta)}{r}\tau_2 \\ I\ddot{\theta} & = & \dfrac{R}{r}\tau_1 - \dfrac{R}{r}\tau_2 \end{array}\right. \tag{2.2.4}$$

Equations (2.2.4) can be rewritten in a state-space form introducing the state vector $\mathbf{z} = [\mathbf{p}^T\ \mathbf{q}^T]^T$, with $\mathbf{p} = [p_1\ p_2\ p_3]^T = [x\ y\ \theta]^T$, and $\mathbf{q} = [q_1\ q_2\ q_3]^T = [\dot{x}\ \dot{y}\ \dot{\theta}]^T$. Denoting the input vector by $\mathbf{u} = [\tau_1\ \tau_2]^T$, the model (2.2.4) provides:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) + \mathbf{g}(\mathbf{z})\mathbf{u} \tag{2.2.5}$$

being:

$$\begin{array}{lcl} \mathbf{f}(\mathbf{z}) & = & \left[\begin{array}{c} \mathbf{q} \\ -sin(p_3)[q_1 cos(p_3) + q_2 sin(p_3)]q_3 \\ cos(p_3)[q_1 cos(p_3) + q_2 sin(p_3)]q_3 \end{array}\right] \\ & = & \left[\begin{array}{c} \mathbf{q} \\ \tilde{\mathbf{f}}(\mathbf{z}) \end{array}\right] \end{array} \tag{2.2.6}$$

$$
\mathbf{g}(\mathbf{z}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{cos(p_3)}{mr} & \frac{cos(p_3)}{mr} \\ \frac{sin(p_3)}{mr} & \frac{sin(p_3)}{mr} \\ \frac{\frac{R}{Ir}}{} & -\frac{R}{Ir} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{3\times 2} \\ \tilde{\mathbf{g}}(\mathbf{z}) \end{bmatrix} \tag{2.2.7}
$$

A discretization of equation (2.2.5) can be performed integrating it from kT_c to $(k+1)T_c$. Using the integration by parts for the equation $\dot{\mathbf{p}} = \mathbf{q}$, and assuming that the sampling frequency is high enough to make neglectable the variations of the term $\tilde{\mathbf{f}}(\mathbf{z}) + \tilde{\mathbf{g}}(\mathbf{z})\mathbf{u}$ within a sampling interval, one gets the following discrete-time model:

$$
\mathbf{z}_{k+1} = \mathbf{F}(\mathbf{z}_k) + \mathbf{G}(\mathbf{z}_k)\mathbf{u} \tag{2.2.8}
$$

where

$$
\mathbf{F}(\mathbf{z}_k) = \begin{bmatrix} \mathbf{p}_k + T_c\mathbf{q}_k + \frac{1}{2}T_c^2\tilde{\mathbf{f}}(z_k) \\ \mathbf{q}_k + T_c\tilde{\mathbf{f}}(z_k) \end{bmatrix} \tag{2.2.9}
$$

$$
\mathbf{G}(\mathbf{z}_k) = \begin{bmatrix} \frac{1}{2}T_c^2\tilde{\mathbf{g}}(z_k) \\ T_c\tilde{\mathbf{g}}(z_k) \end{bmatrix} \tag{2.2.8}
$$

2.3 Variable Structure Optimal Control

In this section, a recently presented control algorithm [1] designed for nonlinear uncertain discretized multivariable systems will be summarized. It is given the following nonlinear discrete-time model:

$$
\mathbf{z}_{k+1} = \mathbf{F}(\mathbf{z}_k) + \mathbf{G}(\mathbf{z}_k)[\mathbf{u}_k + \mathbf{D}(\mathbf{z}_k, \mathbf{u}_k)] \tag{2.3.1}
$$

where $\mathbf{z} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the input vector, $\mathbf{F}(\mathbf{z}_k)$ e $\mathbf{G}(\mathbf{z}_k)$ are properly dimensioned vectorial functions representing the nominal system, and the vectorial function $\mathbf{D}(\mathbf{z}_k, \mathbf{u}_k)$ accounts for external disturbances and parameter variations. It is assumed that a bound is available for each entry of the uncertain term $\mathbf{D}(\mathbf{z}_k, \mathbf{u}_k)$:

$$
|\mathbf{D}^i(\mathbf{z}_k, \mathbf{u}_k)| \le \rho^i(\mathbf{z}_k, \mathbf{u}_k) \qquad i = 1 \dots n \tag{2.3.2}
$$

The purpose of the control is to find the signal $\mathbf{u}_k$ minimizing the following loss function:

$$
I_{k+1} = \mathbf{z}_{k+1}^T\mathbf{R}\mathbf{z}_{k+1} + \mathbf{u}_k^T\mathbf{T}\mathbf{u}_k \tag{2.3.3}
$$

being $\mathbf{R}$ and $\mathbf{T}$ symmetric nonnegative definite weighting matrices of proper dimensions. The control objective can be achieved introducing the following sliding surface:

$$
\mathbf{s}_{k+1} = \mathbf{G}^T(\mathbf{z}_k)\mathbf{R}\mathbf{z}_{k+1} + \mathbf{T}\mathbf{u}_k = \mathbf{0} \tag{2.3.4}
$$

In fact, the minimization of index (2.3.3) provides equation (2.3.4) through differentiation with respect to $\mathbf{u}_k$. A quasi sliding mode on the surface (2.3.4) can be imposed setting:

$$
\mathbf{u}_k = \mathbf{u}_k^{eq} + \mathbf{u}_k^n \tag{2.3.5}
$$

where $\mathbf{u}_k^{eq}$ solves the equation (2.3.4) in the nominal case, and $\mathbf{u}_k^n$ will be defined below in order to compensate the system uncertainties. Let's define the quantities:

$$
\mathbf{A}(\mathbf{z}_k) = \mathbf{G}^T(\mathbf{z}_k)\mathbf{R}\mathbf{G}(\mathbf{z}_k) \tag{2.3.6a}
$$

$$
\mathbf{L}(\mathbf{z}_k) = \mathbf{A}(\mathbf{z}_k)\mathbf{A}^{-1}(\mathbf{z}_k) \tag{2.3.6b}
$$

$$
\mathbf{E}^i(\mathbf{z}_k) = \sum_{j=1}^{m} |A^{i,j}(\mathbf{z}_k)|\rho^i(\mathbf{z}_k, \mathbf{u}_k) \qquad (i = 1 \dots m) \tag{2.3.6c}
$$

$$
\bar{\mathbf{u}}_k^n = [\mathbf{A}(\mathbf{z}_k) + \mathbf{T}]\mathbf{u}_k^n \tag{2.3.6d}
$$

Theorem 2.1 *[1] It is given a system of the form (2.3.5). The control law (2.3.5) guarantees the achievement of a discrete sliding motion on the hyperplane $\mathbf{s}_{k+1} = 0$ if $\mathbf{u}_k^{eq}$ solves the equation (2.3.4) in the nominal case, and each entry $\bar{u}_k^n(i)$ $(i = 1 \dots m)$ of $\bar{\mathbf{u}}_k^n$ is chosen as:*

$$
\bar{u}_i^n(i) = \begin{cases} \theta^i(|s_k(i)| - \mathbf{E}^i(\mathbf{z}_k)) \\ \qquad if \quad |s_k(i)| > \mathbf{E}^i(\mathbf{z}_k) \quad (3.2.7a) \\ -\sum_{j=1}^{m} L^{i,j}(\mathbf{z}_k)[s^j(k) - \bar{u}_n^j(k-1)] \\ \qquad if \quad |s_k(i)| \le \mathbf{E}^i(\mathbf{z}_k) \quad (2.3.7b) \end{cases}
$$

where θ^i $(i = 1 \dots m)$ are design parameters within the set $\theta^i \in (-1, 1)$.

3 Results.

As discussed in the Introduction, a preliminary simulation study on both the kinematic and the dynamic model of the vehicle has been performed before the experimental testing. The planned trajectory in the x-y plane is depicted in Fig.2, while Fig.3 shows the desired trajectories versus time. The control law of Theorem 1 has been first applied to the kinematic model of the MOVAID mobile base. Considering the nominal system, the errors relative the variables x, y, θ reported in Fig.4 are obtained, while Fig.5-6 show the two control variables, i.e. the translational velocity and the angular velocity, respectively . When the dynamic model of the vehicle is used, results reported in Figs 7-10 are obtained. Figures 7-8 are relative to the nominal system, and show the error variables and the control torques applied to the vehicle wheels, respectively. When a parameter variation of 20% is applied to all the parameters appearing in the model (2.2.4)-(2.2.7), results reported in Figs 9-10 are obtained: Fig.9 depicts the error variables, while Fig.10 shows the control torques.

4 Conclusions.

In this note, the control problem of a wheeled mobile base, described by a nonlinear model with nonholonomic constraints, has been addressed. This issue, however, has to be set against a wider context, being part of a CEC Project for the development of a robotic assistance system. A VSC algorithm, recently developed for discrete-time systems, has been applied to both the kinematic and the dynamic model of the vehicle. Beyond the control of the nominal system, perturbed conditions have been also considered in the dynamic model, obtaining satisfactory results in both cases. The simulation study reported in this study is preliminary to the experimental testing of the control algorithm, which is currently being carried out. It is important to point out that, as discussed in the Introduction, experiments are at the moment limited to the only kinematic model, due to the unavailability of motor torques to direct control.

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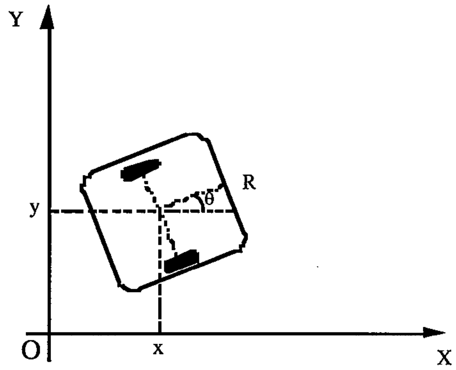

Figure 1: Schematic representation of the vehicle.

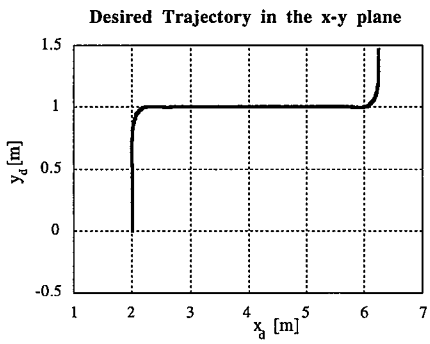

Figure 2: Desired trajectory in the x-y plane.

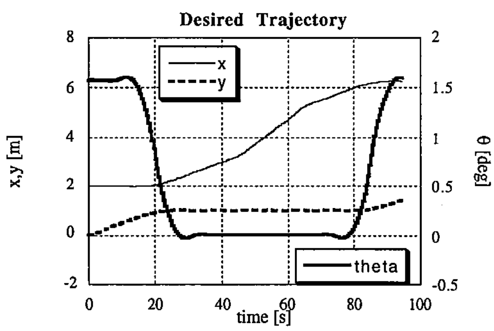

Figure 3: Desired trajectory.

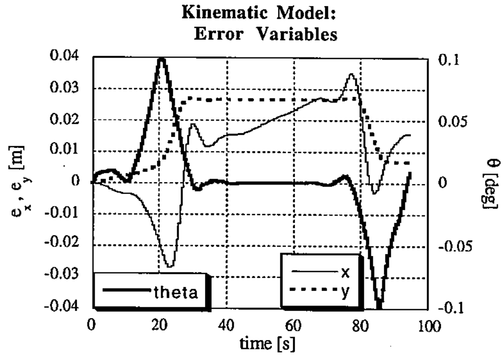

Figure 4: Kinematic model: error variables.

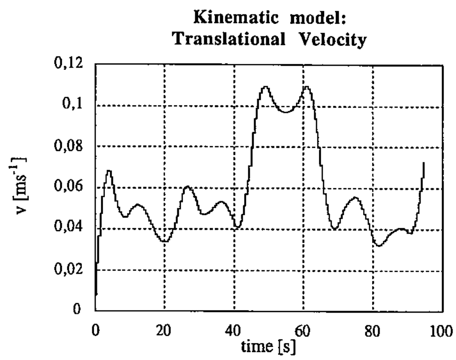

Figure 5: Kinematic model: translational velocity.

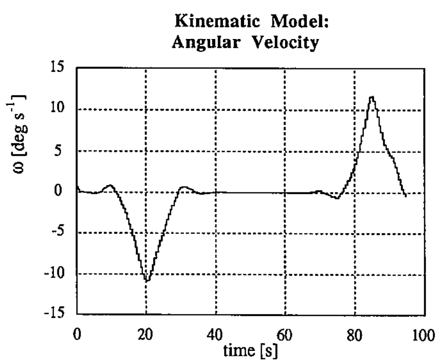

Figure 6: Kinematic model: angular velocity.

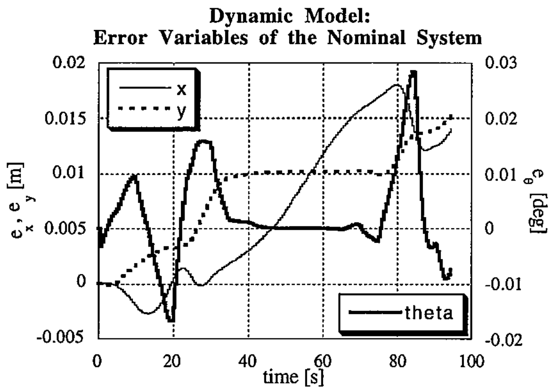

Figure 7: Dynamic model: error variables of the nominal system.

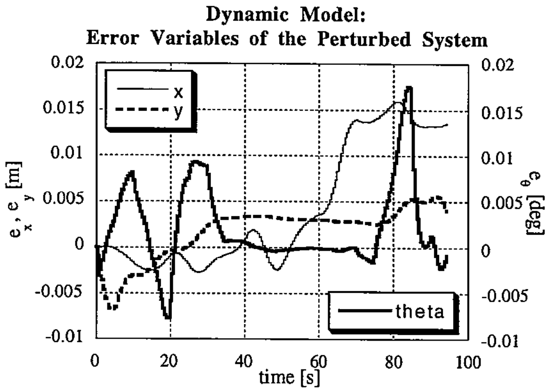

Figure 9: Dynamic model: error variables of the perturbed system.

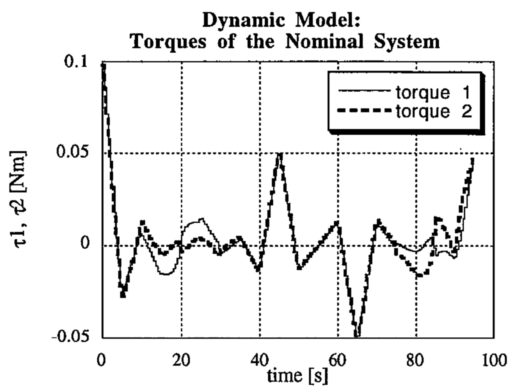

Figure 8: Dynamic model: torques of the nominal system.

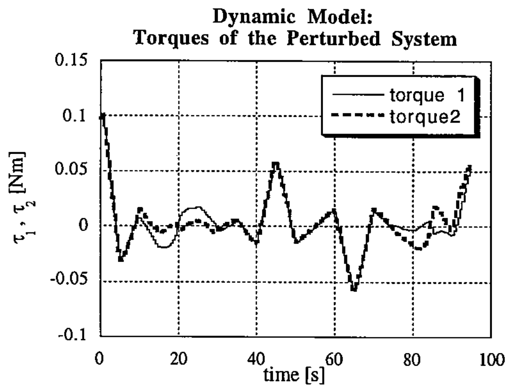

Figure 10: Dynamic model: torques of the perturbed system.