Adaptive Variable Structure Control of Robot Manipulators with Exponentially Stable Trajectories

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Abstract

An adaptive variable structure control approach for robot manipulators is proposed in this paper. The proposed approach avoids the requirement of the uncertainty parameter bounds which is adopted in the most of robust control methods, and makes the tracking errors converge to zero exponentially without requiring any persistent excitation. In addition, the fast parameter variations (e.g. payload variation) are allowed, unlike the conventional adaptive control method which assumed that the parameter variations are slower than the adaptive mechanism.

1 Introduction

The two main techniques almost universally adopted for controlling uncertain systems are adaptive control (Craig et al., 1986; Slotine and Li, 1988; Johansson, 1990), and robust control (Utkin, 1977; Slotine, 1985; Spong, 1992). Adaptive control uses on–line identification in which either the plant parameters are identified using the prediction errors (indirect adaptive control) or, the controller parameters are adjusted using tracking errors (direct adaptive control) in an attempt to 'learn' the uncertain parameters of the system. Adaptive control is applicable to a wide range of parameter variations, but is sensitive to the unstructured uncertainties (Yu et al., 1993b). On the other hand, a robust controller is designed to make the system insensitive to all uncertainties, and the final controller has a fixed structure. Robust control is suitable for dealing with small uncertainties (Slotine, 1985; Spong, 1992).

Adaptive control techniques are further divided into "model–based" methods and "performance–based" methods (Seraji, 1989). Model–based methods fully use the robot system structure properties and guarantees the stability of the closed–loop system (Craig et al., 1986; Slotine and Li, 1988; Johansson, 1990),. However, the transient response, the robustness to external disturbances and unmodelled dynamics and the computational burden are three major problems of stability–based adaptive control of robot manipulators. Performance–based methods, which are also named as decentralized methods (Seraji, 1989), are designed under the assumption that the dynamic terms (functions of the system states) are "slowly varying". The method is systematic and simple but for the robot manipulator the stability analysis is not strict since it requires that the inertia and the coupling terms are slowly–varying. An alternative approach is proposed by Stepanenko and Yuan (1992) based on a similar control design philosophy as in Seraji (1989). The adaptive parameters in Stepanenko and Yuan (1992) do not include any unbounded states ($\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$). These unbounded states are moved into the regressors. However, the identification parameters in Stepanenko and Yuan (1992) are still the complicated state functions ($\mathbf{D}(\mathbf{q})$, $\mathbf{V}_m(\mathbf{q})$, and $\mathbf{G}(\mathbf{q})$) although they are bounded from the robot properties, that is, the controlled plant parameters are time–varying (Narendra and Annaswarmy, 1989). In order to overcome the time–varying parameter effect, the constant feedback gain must be chosen sufficiently large. This results in a high gain control system and the closed–loop systems proposed in Stepanenko and Yuan (1992) is uniformly stable, not asymptotically stable (see Theorem 1 in Stepanenko and Yuan (1992)).

On the other hand, variable structure control is simple and robust to external disturbances, but there are two main effects of the conventional variable structure (VS) control: 1) control input chattering; 2) the assumption of the known uncertainty bounds. The former has received extensive attention. To reduce the control input chattering, a boundary layer is introduced in Slotine (1985) and Spong (1992), and a continuous function, $\frac{x}{|x|+\delta}$, is used in Yu et al., (1993a) to replace a switch function, $sgn(x)$. However, the closed loop system then loses its convergence, i.e. the tracking error no longer tends to zero (Yu et al., 1993b). The exponentially stable robust controller for robot manipulators proposed in Yu et al., 1994) shows a promising solution to the problem of control input chattering.

The uncertainty bounds used in Slotine (1985) are a function of the joint positions and velocities, so they are difficult to select. Recently, robust controllers for robot manipulators based on the adaptive control structures (Craig et al., 1986; Slotine and Li, 1988; Johansson, 1990), have been developed in Spong (1992) and Yu et al., (1993a, 1994). The uncertainty bounds required to derive the robust control laws in Spong (1992) and Yu et al., (1993a, 1994) depend only on the inertia parameters.

The shortcoming of the approach proposed in Spong (1992) and Yu et al., (1993a, 1994) is that the bounds of the uncertainty parameters are required to design the stable robust control law. In this paper, we use the estimated bounds to replace the lower bounds to avoid this problem. The proposed adaptive variable structure controller has a same control structure as recently proposed robust control method by Yu et al., (1994). Thus the controller proposed in this paper is called a adaptive variable structure (AVS) controller. Difference between AVS control and VS control is the estimated bounds used in the former and the fixed lower bounds used in the latter. This approach does not require the assumption that the variation of the parameters is slower than that of the adaptive mechanism, which is required by most of the adaptive control methods Craig et al., 1986; Slotine and Li, 1988; Johansson, 1990). It requires very general structure information of robot manipulators. In addition, the tracking errors converge to zero exponentially without requiring any persistent excitation. Recently, two papers (Su and Leung, 1993; Koo and Kim, 1994) studied robust control with the estimation bounds based on the robust control structure proposed in Spong (1992). However, neither of them have proved the exponential convergence of the tracking errors.

This paper is organized as follows. In **Section 2** we briefly discuss the manipulator mode and some fundamental properties used in this paper. An AVS control approach is developed in **Section 3.** Some implementing considerations are discussed in **Section 4**, and some conclusions are included in **Section 5.**

2 Manipulator Model and Main Properties

The dynamics equation of a general rigid link manipulator having n degree of freedom in free space can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + T_d = T \qquad (1)$$

where $q \in R^n$ denotes the vector of generalized displacements in robot coordinates, $T \in R^n$ denotes the vector of generalized control input forces in robot coordinates; $D(q) \in R^{n\times n}$ is the manipulator inertial matrix, $C(q,\dot{q})\dot{q} \in R^n$ is the vector of centripetal Coriolis torque, $G(q) \in R^n$ is the vector of gravitational torque, and T_d is the external disturbance which includes joint friction force, input disturbance, and the other unmodelled dynamics. It is assumed that only the joint positions and velocities, not accelerations, are available from measurements. Some fundamental properties of the equations of motion follow Craig et al., 1986; Slotine and Li, 1988; Johansson, 1990).

(i) The inertial matrix $D(q)$ is symmetric, uniformly positive definite, and bounded above and below, i.e.,

$$0 < \alpha_m(q)I_n \leq D(q) \leq \alpha_M(q)I_n < \infty,\ \forall q \in R^n$$

where I_n is the $n\times n$ identity matrix, α_m(q) and $\alpha_M(q)$ are scalar positive constants for a revolute arm and generally scalar functions of q for an arm containing prismatic joints.

(ii) Using a proper definition of the matrix $C(q,\dot{q})$, $D(q)$ and $C(q,\dot{q})$ satisfy

$$X^T[\dot{D} - 2C(q,\dot{q})]X = 0$$

where X^T is the transpose of $X \in R^{n\times 1}$, which is an arbitrary vector. That is $(\dot{D} - 2C)$ is a skew–symmetric matrix.

(iii) The structure of Eq (1) is linear in terms of a suitably selected set of robot and load parameters, i.e.,

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = W(t)\Theta + W_0(t)$$

where $W, W_0 \in R^{n\times p}$ are matrices composed of known functions of $q, \dot{q}$ and $\ddot{q}$, and $\Theta \in R^{p\times 1}$ is a vector containing the unknown manipulator and load parameters.

The control objective can be stated as follows: given desired trajectories $q_d, \dot{q}_d, \ddot{q}_d \in R^{n\times 1}$ which are bounded functions of time, determine a control law, T, in the presence of parameter and other uncertainties, such that $q \to q_d$ and $\dot{q} \to \dot{q}_d$ as $t \to \infty$. When the desired signals are not available, the desired trajectories of the robot can be generated from the reference model:

$$\ddot{q}_d + K_v\dot{q}_d + K_p q_d = Kr \qquad (2)$$

where $r, q_d \in R^{n\times 1}$ and $K, K_v, K_p \in R^{n\times n}$.

3 Adaptive Variable Structure Control of Robot Manipulators

In this section, we consider the case that $T_d = 0$ and define the state errors as,

$$\tilde{x}(t) = \begin{bmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{bmatrix} = \begin{bmatrix} \dot{q} - \dot{q}_d \\ q - q_d \end{bmatrix} \qquad (3)$$

The proposed control laws are:

$$T(t) = T_f(t) + T_l(t) \qquad (4)$$

$$\begin{aligned} T_l(t) &= -(P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s + P_{cc}\tilde{q} \\ &= -(P_{ll} + P_{cc}\Gamma^{-1}P_{cc})\dot{\tilde{q}} - P_{ll}P_{cc}^{-1}\Gamma\tilde{q} \end{aligned} \qquad (5)$$

$$\begin{aligned} T_f(t) &= \hat{D}(q)[\dot{q}_r - \mu s] + \hat{C}(q,\dot{q})q_r + \hat{G}(q) \\ &= \bar{W}(t)\hat{\Theta}(t) + \bar{W}_0(t) \end{aligned} \qquad (6)$$

where q_r and s are reference velocity and reference error vectors [4], respectively,

$$q_r = \dot{q}_d - P_{12}\tilde{q} \qquad (7)$$

$$s = \dot{q} - q_r = \dot{\tilde{q}} + P_{12}\tilde{q} \qquad (8)$$

$T_f(t)$ is a non-linear adaptive feedforward vector which complements the robot non-linear part, $T_l(t)$ is a linear feedback vector which guarantees the overall system stability in Lyapunov's sense, and $P_{cc}, \Gamma, P_{ll} \in R^{n\times n}$ are symmetric positive definite matrices, $P_{12} = P_{cc}^{-1}\Gamma$, $\hat{\Theta}$ is the estimated values of the uncertainty parameters, and $\bar{W}(t)$ and $\bar{W}_0(t)$ are the relevant regressors, $\bar{W}_0(t)$ is "nominal" control vector (Spong, 1992). Note the extra control term, $-\mu D(q)s$, introduced for proof of exponential stability. It is noted that $\hat{\Theta}(t)$ is implemented using estimation algorithms (e.g. gradient, least–square algorithms) in adaptive control Craig et al., 1986; Slotine and Li, 1988; Johansson, 1990), and is implemented using the lower bounds of the uncertainty parameters, $\bar{\Theta}$, plus a switching-function (or approximated switching-function for a continuous input) in robust control (Spong, 1992; Yu et al., 1993a). Both methods have advantages and disadvantages. Robust control proposed in Yu et al., (1994) can guarantee an exponentially stable tracking of the joint positions and velocities, but requires the knowledge of $\bar{\Theta}$. On the other hand, adaptive control can make the tracking errors converge to zero asymptotically without using $\bar{\Theta}$. However, the exponential stability of the system requires the persistent excitation condition (Narendra and Annaswarmy, 1989) which is hardly true in the robot control problems. With these considerations in mind, we combine the above control philosophies and use both advantages of them. This control approach is in the spirit of the combined adaptive and variable structure control approaches proposed in Yu et al., (1993b). The proposed adaptive control law is

$$\hat{\Theta}(t) = -\frac{\bar{W}^T(t)s(t)}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)}\theta_s(t) \qquad (9)$$

$$\dot{\tilde{\theta}}_s(t) = \dot{\theta}_s(t) = c_1\frac{\|\bar{W}^T(t)s(t)\|^2}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)} \qquad (10)$$

where δ_1 and δ_2 are positive constant which are selected by the designer, $\tilde{\theta}_s = \theta_s - \bar{\theta}$, $\bar{\theta} \geq \|\Theta\|$, and $c_1 > 0$ is the gain of adaptation law.

Considering Eqs (1), (3–6), the error equation can be obtained in the form

$$\begin{aligned} D(q)(\dot{s}+\mu s) &+ C(q,\dot{q})s + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})s - P_{cc}\tilde{q} \\ &= \bar{W}(t)(\hat{\Theta} - \Theta) \end{aligned} \quad (11)$$

Theorem. 1 *Consider the closed loop system, Eq (11). If the adaptive variable structure control law, Eqs (9) and (10), is used, then all signals in the system are bounded and $\tilde{x}(t)$ tends to zero with at least an exponential rate which is independent of the excitation.*

The proof of this theorem is divided into two parts. First we will prove that all signals in the system are bounded. Then we will prove that $\tilde{x}(t)$ converges to zero exponentially.

Proof: Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}\tilde{x}^T P_q(q)\tilde{x} + \frac{1}{2}z^T z + \frac{1}{2c_1}\tilde{\theta}_s^2 \quad (12)$$

where

$$\begin{aligned} P_q(q) &= P_q^T(q) \\ &= \begin{bmatrix} I & 0 \\ P_{12}^T & I \end{bmatrix} \begin{bmatrix} D(q) & 0 \\ 0 & P_{cc} \end{bmatrix} \begin{bmatrix} I & P_{12} \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} D(q) & D(q)P_{12} \\ P_{12}^T D(q) & P_{cc} + P_{12}^T D(q) P_{12} \end{bmatrix} > 0 \end{aligned} \quad (13)$$

$z = \sqrt{2(\nu+1)\frac{\delta_1\bar{\theta}}{\delta_2}exp(\delta_2 t)}$ is exponentially convergent, $\nu > 0$, and definitions of other notations are as before. The pseudo-state z is first introduced in Yu et al., (1994) for the convenience of stability analysis.

The time derivative of the Lyapunov function Eq (12) is

$$\dot{V}_1 = \tilde{x}^T P_q(q)\dot{\tilde{x}} + \frac{1}{2}\tilde{x}^T \dot{P}_q \tilde{x} + z\dot{z} + \tilde{\theta}_s \dot{\tilde{\theta}}_s \quad (14)$$

By using the error equation (11) and considering Eq (9), the first term is

$$\begin{aligned} \tilde{x}^T P_q(q)\dot{\tilde{x}} &= \tilde{x}^T \begin{bmatrix} D(q)\ddot{\tilde{q}} + D(q)P_{12}\dot{\tilde{q}} \\ P_{12}^T D(q)\ddot{\tilde{q}} + (P_{cc} + P_{12}^T D(q)P_{12})\dot{\tilde{q}} \end{bmatrix} \\ &= -\tilde{x}^T\{\mu \begin{bmatrix} D(q)(\dot{\tilde{q}} + P_{12}\tilde{q}) \\ P_{12}^T D(q)(\dot{\tilde{q}} + P_{12}\tilde{q}) \end{bmatrix} + \\ &\quad \begin{bmatrix} C(q,\dot{q})(\dot{\tilde{q}} + P_{12}\tilde{q}) + (P_{ll} + P_{cc}\Gamma^{-1}P_{cc})\dot{\tilde{q}} + P_{ll}P_{12}\tilde{q} \\ C(q,\dot{q})(\dot{\tilde{q}} + P_{12}\tilde{q}) + P_{12}^T P_{ll}(\dot{\tilde{q}} + P_{12}\tilde{q}) \end{bmatrix}\} \\ &\quad + s^T(t)\bar{W}(t)(\hat{\Theta} - \Theta) \\ &= -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q \tilde{x} \\ &\quad - \frac{\|\bar{W}^T(t)s(t)\|^2}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)}\theta_s(t) \\ &\quad - s^T(t)\bar{W}(t)\Theta \end{aligned} \quad (15)$$

where

$$Q = Q^T = \begin{bmatrix} P_{ll} + P_{cc}\Gamma^{-1}P_{cc} & P_{ll}P_{cc}^{-1}\Gamma \\ \Gamma P_{cc}^{-1}P_{ll} & \Gamma P_{cc}^{-1}P_{ll}P_{cc}^{-1}\Gamma - \mu P_{cc} \end{bmatrix} \quad (16)$$

and it can be proved that Q is a positive definite matrix (Yu et al., 1994). Considering Eq (13), the second term of Eq (14) is

$$\frac{1}{2}\tilde{x}^T \dot{P}_q \tilde{x} = \tilde{x}^T \begin{bmatrix} \frac{1}{2}(\dot{D}\dot{\tilde{q}} + \dot{D}P_{12}\tilde{q}) \\ \frac{1}{2}(P_{12}^T \dot{D}\dot{\tilde{q}} + P_{12}^T D(q)P_{12}\tilde{q}) \end{bmatrix} \quad (17)$$

The third term is

$$z\dot{z} = -(\nu+1)\delta_1\bar{\theta}exp(-\delta_2 t) \quad (18)$$

The fourth term is

$$\tilde{\theta}_s\dot{\tilde{\theta}}_s = \frac{\tilde{\theta}_s\|\bar{W}^T(t)s(t)\|^2}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)} \quad (19)$$

Collecting the above terms, using **Property (ii)**, and considering $\bar{\theta} \geq \|\Theta\|$, we have

$$\begin{aligned} \dot{V}_1 &= -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q\tilde{x} \\ &- \frac{\|\bar{W}^T(t)s(t)\|^2}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)}\theta_s(t) - s^T(t)\bar{W}(t)\Theta \\ &- (\nu+1)\delta_1\bar{\theta}exp(-\delta_2 t) + \frac{(\theta_s - \bar{\theta})\|\bar{W}^T(t)s(t)\|^2}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)} \\ &\leq -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q\tilde{x} - \nu\delta_1\bar{\theta}exp(-\delta_2 t) \\ &- \frac{(\bar{\theta} - \|\Theta\|)\|\bar{W}^T(t)s(t)\|^2}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)} \\ &- [\delta_1\bar{\theta}exp(-\delta_2 t) - \frac{\|\Theta\|\delta_1 exp(-\delta_2 t)\|\bar{W}^T(t)s(t)\|}{\|\bar{W}^T(t)s(t)\| + \delta_1 exp(-\delta_2 t)}] \\ &\leq -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q\tilde{x} - \nu\delta_1\bar{\theta}exp(-\delta_2 t) \leq 0 \end{aligned} \quad (20)$$

Thus we have proved that $\tilde{x}$ and $\tilde{\theta}$ are bounded. From Eqs (7)–(9), it is easy to prove that q_r, s, and $\hat{\Theta}$ are also bounded. To prove that $\tilde{x}$ converge to zero exponentially, we choose another Lypunov function as

$$V_2 = \frac{1}{2}\tilde{x}^T P_q(q)\tilde{x} + \frac{1}{2}z^T z \quad (21)$$

If $s(t) \neq 0$, $\exists t_1 \geq 0$, such that $\theta_s(t) \geq ||\Theta|| \; \forall t \geq t_1$ from Eq (10). Thus, the derivative of V_2 is

$$\begin{aligned}
\dot{V}_2 &= -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q\tilde{x} \\
&- \frac{||\bar{W}^T(t)s(t)||^2}{||\bar{W}^T(t)s(t)|| + \delta_1 exp(-\delta_2 t)}\theta_s(t) - s^T(t)\bar{W}(t)\Theta \\
&- (\nu+1)\delta_1\bar{\theta}exp(-\delta_2 t) \\
&\leq -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q\tilde{x} - \nu\delta_1\bar{\theta}exp(-\delta_2 t) \\
&- \frac{(\theta_s - ||\Theta||)||\bar{W}^T(t)s(t)||^2}{||\bar{W}^T(t)s(t)|| + \delta_1 exp(-\delta_2 t)} \\
&- [\delta_1\bar{\theta}exp(-\delta_2 t) - \frac{||\Theta||\delta_1 exp(-\delta_2 t)||\bar{W}^T(t)s(t)||}{||\bar{W}^T(t)s(t)|| + \delta_1 exp(-\delta_2 t)}] \\
&\leq -\mu\tilde{x}^T P_q(q)\tilde{x} - \tilde{x}^T Q\tilde{x} - \nu\delta_1\bar{\theta}exp(-\delta_2 t) \quad \forall t \geq t_1 \\
&\leq -\eta V_2 \qquad \forall t \geq t_1 \qquad (22)
\end{aligned}$$

where $\eta = min\{2\mu, \nu\delta_2/(\nu+1)\}$. This implies the exponential convergence of V_2, i.e.

$$V_2(t) \leq V_2(0)e^{-\eta t}, \; \forall t \geq t_1 \qquad (23)$$

Putting Eq (24) into Eq (21) gives

$$||\tilde{x}||^2 \leq \frac{2}{\alpha_q}[V_2(0)e^{-\eta t} - (\nu+1)\frac{\bar{\theta}\delta_1}{\delta_2}exp(-\delta_2 t)] \; \forall t \geq t_1 \qquad (24)$$

where α_p is the smallest eigenvalue of $P_q(q)$. This completes the proof of the theorem.

Remark. 1 *It is noted that the lower bounds $\bar{\theta}$ is only used in the proof of the theorem, and is not used in the implementation of the control law, Eqs (5), (6), (9), and (10). The exponential convergent operator $\delta_1 exp(-\delta_2 t)$ is introduced to avoid the discontinuous control law (Yu et al., 1994). This control approach only requires very general robot structure information and avoids the assumption that the uncertainty parameters Θ are 'constant' or 'slowly changing'. This assumption is required by the adaptive control methods. More importantly, the exponential convergence of the tracking errors is obtained.*

4 Practical Considerations

To reduce the parameter drift (Narendra and Annaswarmy, 1989), the following adaptation law may be used to replace Eq (10),

$$\begin{aligned}
\dot{\tilde{\theta}}_s(t) &= \dot{\theta}_s(t) = -\lambda\theta_s \\
&+ c_1\frac{||\bar{W}^T(t)s(t)||^2}{||\bar{W}^T(t)s(t)|| + \delta_1 exp(-\delta_2 t)} \qquad (25)
\end{aligned}$$

where $\lambda \geq 0$.

Corollary. 1 *In Theorem 1, if the adaptation law, Eq (10), is replaced by Eq (25), and the other conditions are same, then all signals in the system are bounded.*

The unmodelled dynamics (such as friction, actuator dynamics) can not be avoided in a practical system. This kind of disturbances must be reduced in order to satisfy the control objective. To overcomes the input disturbances, an external control term is required. we assume that

$$||\tau_d|| \leq d_3 + d_2||q|| + d_1||\dot{q}|| \qquad (26)$$

$$\begin{aligned}
-s^T T_d &= -s^T(d_3 + d_2||q|| + d_1||\dot{q}||) \\
&\leq ||s||(d_3 + d_2||\tilde{q}|| + d_2||q_d|| + d_1||\dot{\tilde{q}}|| \\
&+ d_1||dotq_d||) \\
&\leq (d_3 + d_2||q_d|| + d_1||\dot{q}_d)||s|| \\
&+ [d_2 + d_1\lambda(P_{12})]||\tilde{q}||||s|| + d_1||s||^2 \\
&= d_0||s|| + d_4||\tilde{q}||||s|| + d_1||s||^2 \qquad (27)
\end{aligned}$$

where $d_0 = (d_3 + d_2||q_d|| + d_1||\dot{q}_d$ and $d_4 = d_2 + d_1\lambda(P_{12})$.

The non-linear feedforward adaptive control law is modified as

$$T_f = \bar{W}(t)\hat{\Theta}(t) + \bar{W}_0(t) + T_c(t) \qquad (28)$$

$$T_c(t) = -\hat{d}_0 s - \hat{d}_1\frac{s}{||s|| + \delta_1 exp(-\delta_2)} \qquad (29)$$

$$\begin{aligned}
\dot{\tilde{d}}_0 &= \dot{\hat{d}}_0 = c_{d0}||s|| \qquad (30) \\
\dot{\tilde{d}}_1 &= \dot{\hat{d}}_1 = c_{d1}||s||^2 \qquad (31)
\end{aligned}$$

Corollary. 2 *If the disturbances, which satisfies the assumption Eq (26), are considered, and the adaptive control law Eqs (28)-(31) is used to replace Eq (6), and the other conditions are the same as in Theorem 1, then the conclusions in Theorem 1 are also true.*

Having a single number θ_s to estimate the parameter uncertainty may lead to an overly conservative design and result in an excessive input effort. To reduce this problem, we propose the following adaptation law.

$$\hat{\theta}_i = -\theta_{si}\frac{f_i}{|f_i| + \delta_{1i}exp(-\delta_{2i}t)} \quad for \; i = 1, 2, \ldots, r \qquad (32)$$

$$\theta_{si} = c_i \frac{f_i^2}{|f_i| + \delta_{1i} exp(-\delta_{2i} t)} \quad for\ i = 1, 2, \ldots, r \quad (33)$$

where $f_i = \sum_{j=1}^{n} \bar{W}_{ji} s_j$, for $i = 1, 2, \ldots, r$.

Corollary. 3 *In Theorem 1, if the adaptation law Eqs (9) and (10) is replaced by Eqs (32) and (33), then the conclusions of Theorem 1 are also true.*

5 Conclusion

An adaptive variable structure (AVS) control approach, which combines variable structure control with adaptive control, is proposed in this paper. The proposed AVS controller has the same control structure as in the variable structure controller proposed in Yu et al., (1994), but the estimated bounds are used to replace the lower bounds used in Yu et al., (1994). The exponential convergence of the tracking errors are warranted using the same priori knowledge of the system as in Su and Leung (1993) and Koo and Kim (1994) without requiring the persistent excitation. This is main difference between this paper and the work in Su and Leung (1993) and Koo and Kim (1994). In addition, the assumption that the variation of the uncertainty parameters is slower than that of the adaptive mechanism is avoided.

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