THE USE OF FUZZY COGNITIVE MAPS IN MODELING SYSTEMS

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Abstract

This paper investigates a new theory, Fuzzy Cognitive Map (FCM) Theory, and its implementation in modeling systems. First the description and the methodology that this theory suggests is examined and then the application of FCM in a process control problem is described. The presentation indicates how effective FCMs are and some interesting points for further research are included. In the recent years, a wide discussion has started about the autonomy and intelligence of systems, so the application of FCM in the field of control and systems may contribute in the development of more intelligent and autonomous control systems.

1. Introduction

It is widely recognized that conventional methods in modeling and control systems have contributed a lot in the research and the solution of many control problems, but their contribution on the solution of the increasingly complex dynamical systems will be limited. It has became quite clear that the requirements in the control and modeling systems can not be met with the existing conventional control theory and it is necessary to use new methods that will exploit past experience, will have learning capabilities and will be supplied with failure detection and identification qualities. One new theory for modeling systems which is proposed in this paper and which will contribute to the effort for more intelligent control methods, is Fuzzy Cognitive Map (FCM) Theory.

Fuzzy Cognitive Map (FCM) Theory uses a symbolic representation for the description and modeling of the system. It utilizes concepts to illustrate different aspects in the behavior of the system and these concepts interact each other showing the dynamics of the system. A Fuzzy Cognitive Map (FCM) integrates the accumulated experience and knowledge on the operation of the system, as a result of the method by which it is constructed, i.e., using human experts that know the operation of system and its behavior in different circumstances.

At first, a political scientist R. Axelrod [1] introduced cognitive maps for representing social scientific knowledge and describing the methods that are used for decision making in social and political systems. Then B. Kosko[2],[3] enhanced the power of cognitive maps considering fuzzy values for the concepts of the cognitive map and fuzzy degrees of interrelationships between concepts. After this pioneering work, Fuzzy Cognitive Maps attracted the attention of scientists in many fields and have been used in a variety of different scientific problems. Fuzzy Cognitive Maps have been used for planning and making decisions in the field of international relations and political developments [4] and for analyzing graph theoretic behavior [5], been proposed as a generic system for decision analysis [6] and for distributed cooperative agents [7]. Fuzzy Cognitive Maps also have been used to analyze electrical circuits [8], to structure Virtual worlds[9]. In the control related themes FCMs have been used to model and support plant control [10], to represent Failure Models and Effects Analysis for a system model [11]-[13] and to model the supervisor of control systems [14]. It is obvious that there is high interest in the use of FCM in a wide range of different fields. In this paper the objective is to define and construct Fuzzy Cognitive Maps for modeling systems.

2. Fuzzy Cognitive Maps

In fact, Fuzzy Cognitive Maps (FCM) could be regarded as a combination of Fuzzy Logic and Neural Networks. In a graphical illustration FCM seems to be a signed directed graph with feedback, consisting of nodes and weighted arcs. Nodes of the graph stand for the concepts that are used to describe the behavior of the system and they are connected by signed and weighted arcs representing the causal relationships that exist between the concepts (Figure 1). It must be mentioned that all the values in the graph are fuzzy, so concepts take values in the range between [0,1] and the weights of the arcs are in the interval [-1,1]. Observing this graphical representation, it becomes clear which concept influences other concepts showing the interconnections between concepts and it permits updating in the construction of the graph, such as the adding or deleting of an interconnection or a concept.

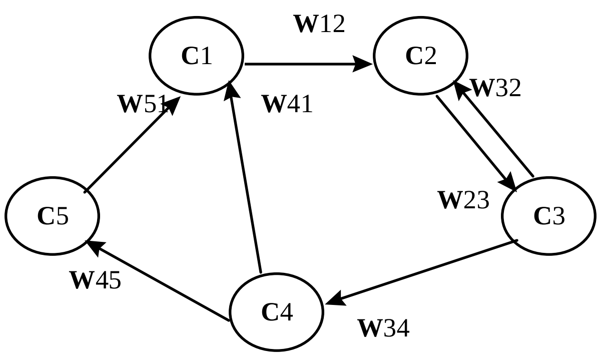

Figure 1. A simple Fuzzy Cognitive Map

A Fuzzy Cognitive Map consists of nodes-concepts and arcs between concepts. Each concept represents a characteristic of the system; in general it stands for events, actions, goals, values, trends of the system that is modeled as an FCM. Each concept is characterized by a number A_i that represents its value and it results from the transformation of the real value of the system's variable, for which this concept stands, in the interval [-1,1].

Between concepts, there are three possible types of causal relationships, that express the type of influence from one concept to the others. The weights of the arcs between concept C_i and concept C_j could be positive $(W_{ij} \succ 0)$ which means that an increase in the value of concept C_i leads to the increase of the value of concept C_j, and a decrease in the value of concept C_i leads to the decrease of the value of concept C_j. Or there is negative causality $(W_{ij} \prec 0)$ which means that an increase in the value of concept C_i leads to the decrease of the value of concept C_j and vice versa.

Beyond the graphical representation of the FCM there is its mathematical model. It consists of an $1 \times n$ state vector $\mathbf{A}$ which includes the values of the n concepts and an $n \times n$ weight matrix $\mathbf{W}$ which gathers the weights W_{ij} of the interconnections between the n concepts of the FCM. The matrix $\mathbf{W}$ has n rows and n columns where n equals the total number of distinct concepts of the FCM and the matrix diagonal is zero since it is assumed that no concept causes itself.

The value of each one concept is influenced by the values of the connected concepts with the appropriate weights and by its previous value. So the value A_i for each concept C_i is calculated by the following rule :

$$A_i = f\Big(\sum_{\substack{j=1 \\ j \neq i}}^{n} A_j W_{ji}\Big) + A_i^{old} \tag{1}$$

where A_i is the activation level of concept C_i at time t+1, A_j is the activation level of concept C_j at time t, A_i^{old} is the activation level of concept C_i at time t, and W_{ji} is the weight of the interconnection between C_j and C_i, and f is a threshold function.

$$\mathbf{A}_{new} = f(\mathbf{A}_{old} \circ \mathbf{W}) + \mathbf{A}_{old} \tag{2}$$

So the new state vector $\mathbf{A}_{new}$ is computed by multiplying the previous state vector $\mathbf{A}_{old}$ by the weight matrix $\mathbf{W}$. The new vector shows the effect of the change in the value of one concept in the whole Fuzzy Cognitive Map. But, equation (2) includes also, the old value of each concept, and so the FCM possesses memory capabilities and there is a smooth change after each new cycling of the FCM.

3. Constructing Fuzzy Cognitive Maps

From the presentation of FCMs, discussed in the previous paragraph, it is clear that the most critical part is the drawing of the FCM. In order to build a FCM, the knowledge and experience of one expert on the system's operation must be used. At first the expert determines the concepts that best describe the system; a concept can be a characteristic of the system, a state or a variable or an input or an output of the system; he has known which factors are crucial for the modeling of the system and he represents a concept for each one. Moreover he has observed which elements of the system influence others elements; and for the corresponding concepts he determines the negative or positive effect of one concept on the others, with a fuzzy value for each interconnection, since it has been considered that there is a fuzzy of degree of causation between concepts.

It is possible to have better results in the drawing of the FCM, if more than one experts are used. In that case, all experts are polled together and they determine the relevant factors and thus the concepts that should be presented in the map. Then, experts are individually asked to express the relationship among concepts, during the assigning of weights three parameters must be considered: how strongly concept C_i influences concept C_j, what is the sign of the weight and whether concept C_i causes concept C_j, or vice versa. Therefore there will be a collection of individual FCMs that must be combined into a collective map. But if there are experts of different credibility, for them, their proposed maps must be multiplied with a nonnegative 'credibility' weight b_i before combining them with other's experts FCMs. So the combination of these different FCMs will produce an augmented FCM and its weight matrix $\mathbf{W}$ is created by adding the matrices $\mathbf{W}_i$ of the FCMs, that each one of the N experts have drawn :

$$\mathbf{W} = \sum_{1}^{N} b_i \mathbf{W}_i$$

(3)

where $\mathbf{W}$ is the whole FCM, b_i is the weight for i_{th} expert and $\mathbf{W}_i$ is the connection matrix of i_{th} expert's fuzzy cognitive map and N is the number of the experts.

Sometimes it is necessary to construct more than one FCMs, each one describing different parts of the same system. And then, using a similar method, these different FCMs can be integrated into one augmented FCM. This is another quality of FCM which allow knowledge bases to grow by connecting different Fuzzy Cognitive Maps.

When a FCM has been constructed, it can be used to model and simulate the behavior of the system. Firstly, the FCM should be initialized, the activation level of each of the nodes of the map takes a value between -1 and +1 based on expert's opinion for the current state and then the concepts are free to interact. In each step of the cycling the values of concepts change according to the equation (1). This interaction between concepts continues until:

i) A fixed equilibrium is reached
ii) A limited cycle is reached
iii) Chaotic behavior is exhibited

A Fuzzy Cognitive Map is a powerful tool that can be used for modeling systems exploiting the knowledge on the operation of the system. It can avoid many of the knowledge-extraction problems which are usually present in by rule based systems and moreover it must be mentioned that cycles are allowed in the graph.

Usually, concepts and causality are determined by experts involved in the construction of FCM, but this methodology would lead to a distorted model of the system, since it is possible that experts have not consider the appropriate factors of the system. In order to minimize this likelihood learning methods can be utilized to train the FCM. If there is data of the real system about the changes in the values of concepts, the Differential Hebbian learning method can be used as it has proposed in [2], in order to train the FCM, which means adjusting the weights of the interconnections between concepts.

4. Implementation of FCM in a Process Control Problem

In this section the modeling of a practical problem will be examined. As it has become clear, the most important component in drawing a FCM is the determination of the concepts that best describe the system and the direction and grade of causality between concepts. These aspects will be represented through this example.

A system is considered, which is part of a large plant that is widely known and used in chemical process. The system consists of two identical tanks depicted in figure 2. Each tank has an inlet valve and a outlet valve. The outlet valve of the first tank is the inlet valve of the second.

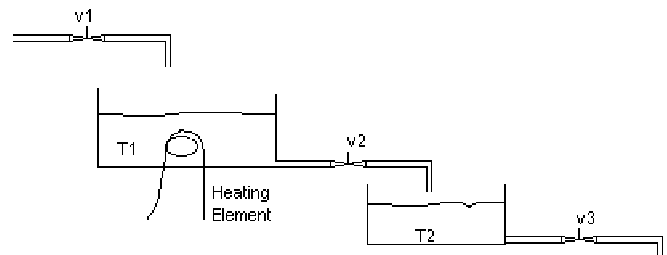

Figure 2. Example of a process system to be controlled

The control objective of the system is to keep the amount of liquid, in both tanks, between some limits, an upper $H_{\max}$ and a low limit $H_{\min}$, and furthermore, the temperature of the liquid in both tanks must be kept between maximum value $T_{\max}$ and a minimum value $T_{\min}$. The target is keeping these variables in the middle of their range of values:

$$H^1_{\min} \le H^1 \le H^1_{\max}$$
$$H^2_{\min} \le H^2 \le H^2_{\max}$$
$$T^1_{\min} \le T^1 \le T^1_{\max}$$
$$T^2_{\min} \le T^2 \le T^2_{\max}$$

(4)

The temperature of the liquid in tank1 is increased through a heating element. A thermostat continuously senses the temperature of the liquid in tank1 and turns the heater on and off. The temperature of the liquid in the tank2 is measured through a thermometer; when the temperature of the liquid2 decreases, this causes the valve2 to open, so hot liquid comes into tank2.

A FCM will be constructed which will model and control the whole system. In order to determine the concepts of the FCM that describe the system, the variables of the system must be taken into account, such as the height of the liquid in each tank or the temperature. Then concepts are assigned for the system's elements that affect the variables such as the state of the valves.

For this simple plant FCM, eight concepts are supposed and they give a good model of the system, later on, any other concept can be added, which will help the view and control of the system:

Concept1	The amount of the liquid which tank1 contains. This amount is dependent on valve1 and valve2.
Concept2	The amount of the liquid in the tank2. This amount is related to valve1 and valve2.

Concept3	The state of the valve1. The valve is open, closed or partially open.
Concept4	The state of the valve2.
Concept5	The state of the valve3.
Concept6	The temperature of the liquid in tank1.
Concept7	The temperature of the liquid in tank2.
Concept8	Describes the operation of the heating element which increases the temperature of the liquid in tank1.

These concepts must be connected with each other. First for each concept it must be decided with which other concepts it will be connected. Then the sign of the connection is decided, and then the weight of each connection is determined. For this procedure the experience on the system's operation is used.

The connections between concepts are:

Event1	It connects concept1 with concept3. It relates the amount of the liquid in tank1 with the operation of the valve1.When the height of the liquid in the tank is low we want to increase the amount of incoming liquid and we open valve1.
Event2	It relates concept1 with concept4; when the height of the liquid in tank1 is high we open valve2 (concept4) and reduce the amount of liquid in tank1.
Event3	It connects concept2 with concept4; when the height of the liquid in tank2 is low we open valve2 (concept4) so that liquid comes into tank2.
Event4	It relates concept2 with concept5; when the height of the liquid in tank2 is high, we open valve3 (concept5) and keep the amount of the liquid below a limit.
Event5	It connects concept3 (valve1) with concept1 (tank1); any change in valve1 influences the amount of liquid in tank1.
Event6	The value of concept4 (valve2) causes the decrease or not of the value of concept1(tank1)
Event7	The value of concept4 (valve2) causes the increase or not of the amount of liquid in tank2 (concept2).
Event8	It relates concept5 (valve3) with concept2 (tank2), the value of concept5 causes the decrease or not of the amount of the liquid in tank2.
Event9	It connects concept6 (temperature in tank1) with the concept8 (the operation of the heating element).When the temperature in tank1 is low, it causes the opening of the heating element.
Event10	It connects concept8 with concept6; the value of concept8 (operation of the heating element) increases the value of concept6 (temperature in tank1).
Event11	It connects concept6 with concept3 (valve1); when the temperature in tank1 reaches an upper limit, we open valve so that liquid of low temperature will enter tank1.
Event12	It relates concept7 (temperature in tank2) with concept4 (valve2); when the temperature in tank2 is below a limit, we open the valve2 and so new hot liquid enters tank2 from the tank1.
Event13	It shows the effect of concept4 (valve2) on concept7 (the temperature in tank2); when the valve2 (concept4) is open then hot liquid comes into tank2 and the temperature in tank2 (concept7) is increased.

It is obvious that connections can easily be added or removed between the concepts that describe the system. Moreover, a concept can be added or removed if this improves the system's description. For example, another concept, that could be added later, is a concept which will include the desirable output of the valve3.

Each event (connection between concepts) has a weight which ranges between [-1,1] and in this case it was determined arbitrarily and then has changed during the training period of the FCM. Each concept has a value which ranges in the interval [0,1] and it is obtained after thresholding the real value of the concept. It is apparent that an interface is needed which will transform the real measures of the system to their representative values in the FCM and vice versa. It should be mentioned that the transformation from the real values of the physical measurements to the values of the concepts, needs investigation and must take into consideration the actual mechanisms depicted in the FCM.

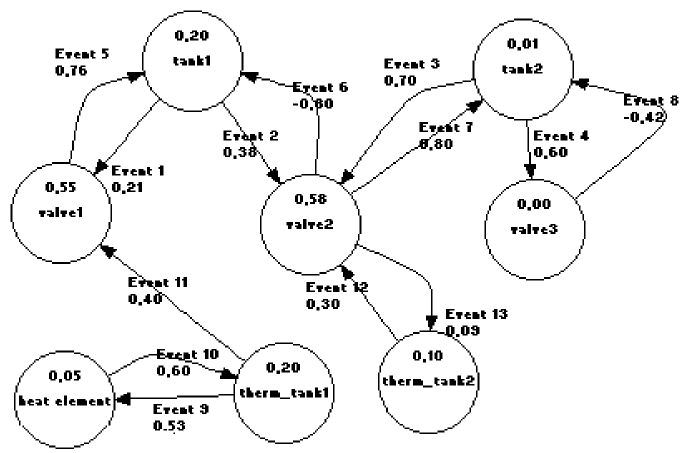

Figure 3. The initial FCM, with the first values for the concepts

Figure 3 shows the FCM that is used to describe and control the system, with the initial value of each concept and the interconnections between concepts. The values of concepts correspond to the real measurements of the physical magnitude. The values of the events have been determined after observation of the changes in the real experimental system and then training the FCM according to the Differential Hebbian learning method [2].

At each running step of the FCM, the value of each concept is defined by the result of taking all the causal event weights pointing into this concept and multiplying each weight by the value of the concept that causes the event, according to equation(1). Then the sigmoid function is applied on the result of calculation and it is transformed in the interval between 0.00 and 1.00. As running step or running cycle of the FCM is defined the time unit during which the values of the concepts are calculated and change.

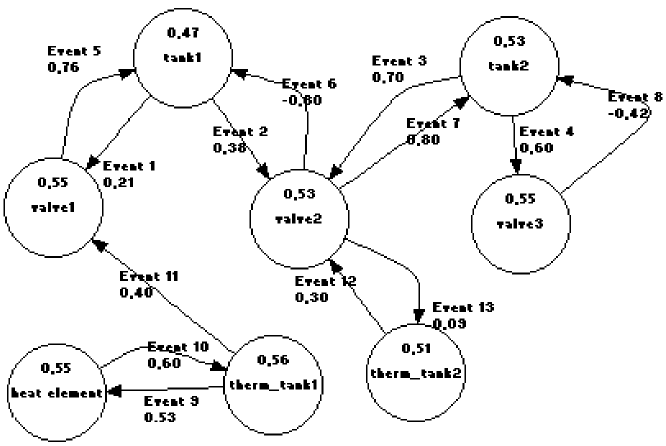

Figure 4. The FCM after 50 running cycles

Figure 4 shows the FCM after 50 running cycles; it must be mentioned that each running cycle holds for a time unit. Table I represents the value of each concept for the first eighteen (18) cycles. It can be seeing that after some cycles the FCM reaches a limit cycle and its values have a slight variation.

Table I. The values of FCM concepts for the first 18 running cycles.

	tank1	tank2	valve1	valve2	valve3	heat	therm_tank1	therm_tank2
1	0,20	0,01	0,55	0,58	0,00	0,05	0,20	0,10
2	0,41	0,57	0,53	0,52	0,50	0,50	0,50	0,50
3	0,48	0,51	0,52	0,63	0,50	0,53	0,57	0,51
4	0,44	0,50	0,56	0,62	0,54	0,53	0,52	0,51
5	0,46	0,48	0,57	0,57	0,52	0,56	0,53	0,51
6	0,56	0,53	0,57	0,54	0,54	0,52	0,50	0,50
7	0,48	0,47	0,58	0,57	0,52	0,54	0,53	0,51
8	0,42	0,48	0,54	0,59	0,53	0,52	0,52	0,51
9	0,47	0,56	0,53	0,59	0,55	0,54	0,53	0,51
10	0,44	0,49	0,51	0,60	0,53	0,53	0,52	0,51
11	0,53	0,55	0,52	0,64	0,54	0,52	0,58	0,50
12	0,45	0,52	0,52	0,58	0,54	0,56	0,53	0,51
13	0,47	0,52	0,54	0,64	0,53	0,55	0,55	0,51
14	0,46	0,50	0,53	0,53	0,51	0,55	0,53	0,51
15	0,45	0,49	0,54	0,58	0,52	0,51	0,58	0,50
16	0,47	0,47	0,52	0,55	0,56	0,55	0,57	0,50
17	0,55	0,58	0,54	0,56	0,55	0,57	0,52	0,50
18	0,40	0,51	0,54	0,53	0,57	0,50	0,52	0,51

The FCM never reaches a fixed point, because random noise has been considered which influences the value of the interconnections (events) between concepts. In this way, the disturbances that influence the real system and our uncertainty about the FCM's weights, pass into the model of the system.

If the weights of the interconnections are considered fixed, without any noise, and the FCM runs for the same initial values, as in the previous example, in Table II it can be seen that after only 5 running steps, the FCM reaches a fixed point. After this fixed point, if a disturbance occurs in the real system, which will cause the change in the value of one or more concepts, the FCM in a limited number of cycles will reach again another fixed point.

Table II. The values of FCM concepts for fixed event values

	tank1	tank2	valve1	valve2	valve3	heat	therm_tank	therm_tank
1	0,20	0,01	0,55	0,58	0,00	0,05	0,20	0,10
2	0,49	0,61	0,53	0,53	0,50	0,53	0,51	0,51
3	0,50	0,55	0,58	0,68	0,59	0,57	0,58	0,51
4	0,47	0,57	0,58	0,67	0,58	0,58	0,58	0,52
5	0,48	0,57	0,58	0,68	0,59	0,58	0,59	0,52
6	0,48	0,57	0,58	0,68	0,59	0,58	0,59	0,52

In this problem it has been assumed that there is no time relationship in the changes of the concepts values, when the value of one concept changes, in the same time unit the values of the rest concepts change according to their influence of the first. This is referred to as a running cycle. But in a realistic system effects take place in different unit times. For example, in Figure3 a change in the concept6 (the temperature of the liquid in tank1) will lead almost immediately to a change on the state of the heat element(concept8) but a change in the state of the valve1 take some time to have full effect in the amount of liquid in the tank1.

Thus, time tags would be introduced corresponding to each effect, but then there would appear problems on estimating the time tags for each effect but it could be followed the methodology that has been proposed [15].

5. Supervisor Control for the process control problem

In the previous section a model for a process control problem has been proposed, this model could be enhanced if a two-level structure model is considered (Figure 5). In the lower level of the structure will lie the FCM that it has just constructed and it will reflect the model of the process during normal operation conditions. In the upper another FCM will be constructed and will be used for failure modes, effects analysis and decision analysis.

The FCM on the upper level will consist of concepts that may represent the irregular operation of some elements of the system, failure mode variables, failure effects variables, failure cause variables, severity of the effect or design variables. In this example, the FCM will describe the failure states of the valves, possible malfunction in the heating element, leaks in the tanks or other alarm schemes. Moreover, this FCM will include concepts for determination of a specific operation of the system. As an example, in a similar chemical process, as the one represented in the previous section, it could need different amounts of liquid in the output at different times, according to the requisite density of the liquid.

The two FCMs will interact with each other and there will be an amount of information that must pass from the one FCM to the other. So two interfaces are needed, one will pass information from the FCM in the lower level to the FCM in the upper level and the other interface in the opposite direction. The two interfaces are necessary because changes on two or more concepts in the FCM on the lower level could mean change in one concept in the upper level and the corresponding procedure, when information descends from the FCM on the upper level towards the lower level. As it can be seen from figure 5, two or more concepts of the FCM in the lower level pass through the interface and influence one concept in the FCM on the upper level, an analogous interface exists for the inverse transmission of information.

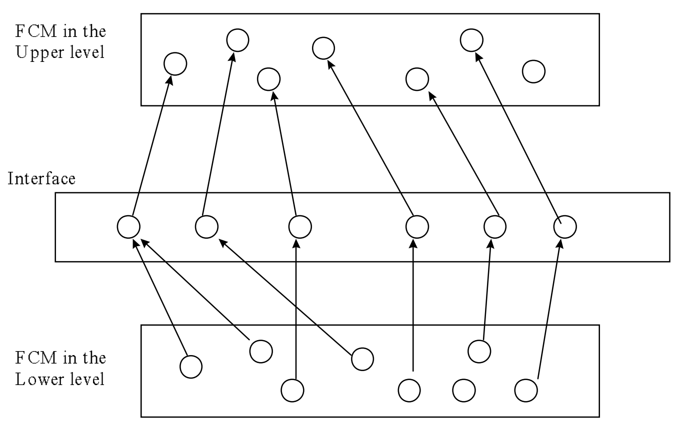

Figure 5. The path from the lower level to the upper.

The cooperation of two-level FCM seems to be alluring and could lend itself to more sophisticated systems. Moreover, it gives the stimulus to investigate another approach, where in the lower level there is a more conventional controller, like a Neural Network, and the supervisor in the upper level is a FCM.

6. Summary

Fuzzy Cognitive Map Theory, a new theory used to model the behavior of complex systems, which best utilizes existing experience in the operation of the system, has been examined. For such systems it is extremely difficult to describe the entire system by a precise mathematical model. Thus, it is more attractive and useful to represent it, in a graphical way showing the causal relationships between states-concepts. Since this symbolic method of modeling and control of a system is easily adaptable and relies on human expert experience and knowledge it can be considered intelligent.

The implementation of this method in a process control problem has been presented and it has been shown how simply it describes the system's operation. The prospect for it to be expanded in more advanced control schemes has been discussed, by adding a second FCM in a higher level which will be used for failure analysis, prediction and planning.

Fuzzy Cognitive Maps seem to be a useful method in modeling and control of complex systems which will help the designer of a system in decision analysis and strategic planning. Fuzzy Cognitive Maps appear to be an appealing tool in the description of the supervisor of complex control systems, which can be complemented with other techniques and will lead to more sophisticated control systems.

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