# Modular Design for the Computation of Vehicle Dynamic behaviour

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**Abstract**: In the last years many efforts have been done to develop simulation models. These models need a lot of computational time. This paper presents a new modular simulation to avoid these problems. This approach requires a good tire model to take the nonlinearities into account which have a significant influence on the dynamic behavior of the car. The process of composing the model is modular and consists of several sub-models (tires, chassis, suspension, steering, wind, road). Each of them can be developed independently. The advantage of the simulation is the reduction of time and costs for designing and engineering different controller concepts or vehicle safety systems. It will be proved with different examples that this simulation gives good results for the realistic behavior of the vehicle. The time of simulation is reduced by minimizing the coordinate transformations.

**Key Words**: Modeling, Vehicle model, Simulation, Nonlinear model.

## 1. Introduction

The manufacturers are always concerned with improving the dynamics of vehicles in order to have a more efficient behavior. Knowledge of the vehicle state [1] is essential to determine the dynamic behavior of a vehicle and to design automotive control systems for example Bosch FDR [2] which increase safety and improve handling characteristics.

The simulation of a car dynamic behavior could be of necessity for the conception of a new car model and essential for structural engineering. This simulation of a realistic car can also be used to understand the dynamic behavior in normal and critical situations.

The advantages of simulation in opposite to driving tests are the reproducibility of the maneuvers, the availability of different environmental conditions and the variability of the vehicle properties. This simulation will be used to reduce the time and costs for designing and engineering different controller concepts (ABS) or vehicle safety systems seen in [3], [4]. Additionally, comparison can be made under the same conditions independently of seasonal environmental and without time consuming preparations of experimental vehicles.

Moreover, this simulation can be used to develop new control systems such as electrical steering, control of lateral acceleration of the car, detection of tire deflation and to develop algorithms for the estimation of several dynamic parameters as for example the stiffness and the damping coefficients of the suspension.

The goal of this paper is to explain the dynamics of a vehicle as a connection of subsystems based on theoretical and experimental studies.

In a first step the entire system "automobile" has been separated into several autonomous sub-models (tires, wheel suspension, car body, steering system), so that each sub-model can be developed independently. The submodel itself is based on physical relations. All relevant nonlinearities are comprised.

It is necessary to define the interfaces between the different sub-models, i.e. which signals are required as inputs and outputs. This concept allows independent development of each subsystem.

For a realistic nonlinear simulation model, it is very important to have a complex nonlinear tire model, because the influence of the dynamic tire on the chassis is dominant. The detailed description of the tire model will be followed by the suspension car model. The body of the car was modeled with 6 degrees of freedom and the dynamic effect of the steering was also taken into account. Finally, different examples will be given to demonstrate the realistic behavior of the simulation in different driving situations.

## 2. Modular implementation

Figure 1 represents the different sub-models which are implemented on the software Matlab/Simulink.

The input variables for this simulation are the steering angle, acceleration or braking torque at the wheels and the initial speed of the car.

The model was adapted to a test car, a BMW520i. Some geometrical parameters have been found in technical magazines. However, there are still a lot of unknown parameters which must be estimated. But the simulation can also be easily adapted to other test cars.

The resulting simulation program is easy to use. It allows to develop each submodel independently and to connect them afterwards.

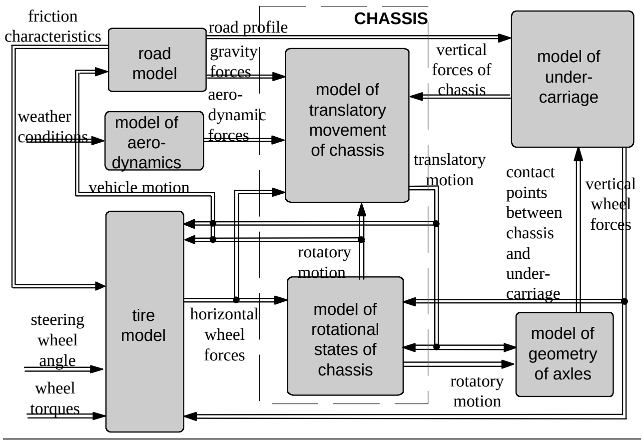



**Figure 1**: Modular implementation

The great power of the simulation is on the one hand the reproducibility of simulation runs. Therefore the simulation can be used during the process of development to investigate the performance of controller algorithms. On the other hand, environmental parameters as well as vehicle parameters can be modified.

## 3. Chassis model

The lateral dynamics of the vehicle are described by three force equations

$$m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = T_{FaIn}\begin{bmatrix}F_{XVL}+F_{XVR}+F_{XHL}+F_{XHR}+F_{WX}+F_{GX}+F_{R}\\ F_{YVL}+F_{YVR}+F_{YHL}+F_{YHR}+F_{WY}+F_{GY}\\ F_{ZCVL}+F_{ZCVR}+F_{ZCHL}+F_{ZCHR}+F_{WZ}+F_{GZ}\end{bmatrix}$$

whereas $T_{FaIn}$ is a matrix that transforms the coordinate system of the vehicle into the inertial system.

The indices V and H stand for front and rear wheels, respectively. The calculation of the wheel forces $F_X$ and $F_Y$ is explained later on chapter 4.2. And the vertical tire forces are calculated on chapter 4.3.

The wind forces $F_{WX}$, $F_{WY}$, and $F_{WZ}$ are calculated by transforming the wind speed into the coordinate system of the vehicle, then the velocity of the vehicle is added, and finally the wind forces can be determined. The lift force of the vehicle was neglected.

$$\begin{bmatrix}F_{WX}\\ F_{WY}\\ F_{WZ}\end{bmatrix} = \begin{bmatrix} c_{WX}A_L\frac{\rho}{2}\left(v_{ChassisX}-v_{WX}\cos\psi-v_{WY}\sin\psi\right)^2 \\ \left(c_{WY}A_S\frac{\rho}{2}\left(v_{ChassisY}+v_{WX}\sin\psi-v_{WY}\cos\psi\right)^2\right). \\ \text{..........}\operatorname{sgn}\left(-v_{WX}\sin\psi+v_{WY}\cos\psi\right) \\ 0\end{bmatrix}$$

Having a road pitch $\chi_{Road}$ and a road slope $\varphi_{Road}$, the gravity forces can be obtained by a multiplication by a transformation matrix:

$$\begin{bmatrix}F_{GX}\\ F_{GY}\\ F_{GZ}\end{bmatrix} = \begin{bmatrix}\cos\chi_{Road} & \sin\chi_{Road}\sin\varphi_{Road} & \sin\chi_{Road}\cos\varphi_{Road}\\ 0 & \cos\varphi_{Road} & -\sin\varphi_{Road}\\ -\sin\chi_{Road} & \cos\chi_{Road}\sin\varphi_{Road} & \cos\chi_{Road}\cos\varphi_{Road}\end{bmatrix} * \begin{bmatrix}0\\0\\-mg\end{bmatrix}$$

A positive road pitch implies a rising road and a positive slope an elevation on the right hand side.

The force $F_R$ describes the rolling resistance which is approximated in [7] by:

$$F_R = -f_{R0}F_Z - f_{R1}F_Z\frac{v}{30} - f_{R2}F_Z\frac{v^4}{30^4}$$

The rotatory motion of the vehicle can by described by three torque equations in the coordinate system of the vehicle.

Torque equation for the yaw rate:

$$\Theta_Z\ddot{\psi} = \left(F_{XVR}-F_{XVL}\right)\cdot\frac{1}{2}b_V + \left(F_{XHR}-F_{XHL}\right)\cdot\frac{1}{2}b_H$$
$$+\left(F_{YVL}+F_{YVR}\right)\cdot l_V - \left(F_{YHL}+F_{YHR}\right)\cdot l_H$$

Torque equation round a longitudinal axis:

$$\Theta_X\ddot{\chi} = \left(F_{ZVL}+F_{ZHL}\right)\cdot\frac{b}{2} - \left(F_{ZVR}+F_{ZHL}\right)\cdot\frac{b}{2} + ma_Y h$$

Torque equation round a lateral axis:

$$\Theta_Y\ddot{\varphi} = \left(F_{ZVL}+F_{ZVR}\right)\cdot l_V - \left(F_{ZHL}+F_{ZHR}\right)\cdot l_H + ma_X h$$

## 4. Tire model

### *4.1. Slip calculation*

The calculation of the slip is important, because the gradient of the adhesion curve is extremely significant and an error of 0.1 per cent produces a big variation of the tire forces.

Without a tire side slip angle a, the determination of the slip can't be exact. In the literature there can be found several definitions of the transversal and lateral slip. Burckhardt [5] defines the longitudinal slip in the direction of the movement of the tire, and the same for the tire forces, but Reimpell [6] defines it in the direction of the plane of the tire. In both cases an absolute slip can be defined:

$$s_A = \frac{v_S - v_R}{v_S} \qquad \text{with} \quad v_R = r_R\cdot\omega$$

$v_R$ : speed of the tire circumference
$v_S$: speed of the contact area between tire and road
$r_R$ : dynamic radius of the tire
$w$ : angular speed of the tire rotation

From the definition of Burckhardt, longitudinal slip is calculated by:

$$s_L = \frac{v_S - v_R\cdot\cos\alpha}{v_S} \qquad \text{for braking} \qquad (4.1)$$

$$s_L = \frac{v_S - v_R\cdot\cos\alpha}{v_S} \qquad \text{for acceleration (4.2)}$$

And the lateral slip is defined by:

$$s_S = \frac{v_R \cdot \sin\alpha}{v_S} \qquad \text{for braking}$$

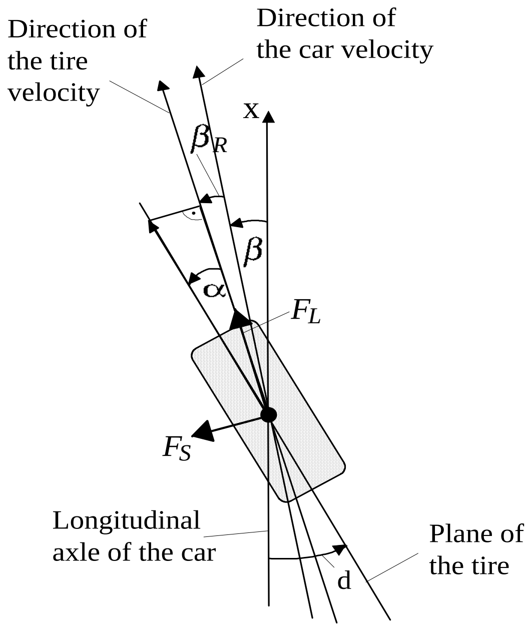
$$s_S = \sin\alpha \qquad \text{for acceleration}$$

We can define the resulting slip as a function of the lateral slip and the longitudinal slip:

$$s_{RES} = \frac{\sqrt{v_S^2 + v_R^2 - 2 \cdot v_S \cdot v_R \cdot \cos\alpha}}{v_S} = \sqrt{s_L^2 + s_S^2}$$

The braking slip is negative and the driving slip is positive.

In figure 2, it' s shown that $s_L$ is in the direction of the tire velocity and $s_S$ is perpendicular to it. We note that the tire velocity forms an angle a with the plane of the tire.




Front tire

**Figure 2**: Forces and angles of the tire model with side slip angle

### *4.2. Adhesion curve of the tire*

One of the most important parts of a car concerning dynamic behavior and safety is the tire, because all forces will act in the small area between tire and road. If we want an exact tire model the best method to describe it is to use the finite element method (FEM.). However, such a method requires too much computation time. To overcome this problem, we introduce simplifications, considering the tire to be isotropic and the parameters of the friction coefficient of the curve to be independent of the direction of the tire. Then we define by Burckhardt the resulting friction coefficient :

$$\mu_{RES} = \left[ c_1 \cdot \left( 1 - e^{\left(-c_2 \cdot s_{RES}\right)} \right) - c_3 \cdot s_{RES} \right] \cdot e^{-c_4 \cdot s_{RES} v} \cdot K_N$$

with $K_N = 1 - c_5 F_Z^2$, a coefficient as a function of the load, and the parameter's $c_1$, $c_2$, $c_3$, $c_4$, give us the characteristics of the road surfaces (dry asphalt, snow, ice, wet asphalt) as it can be seen in figure 3.

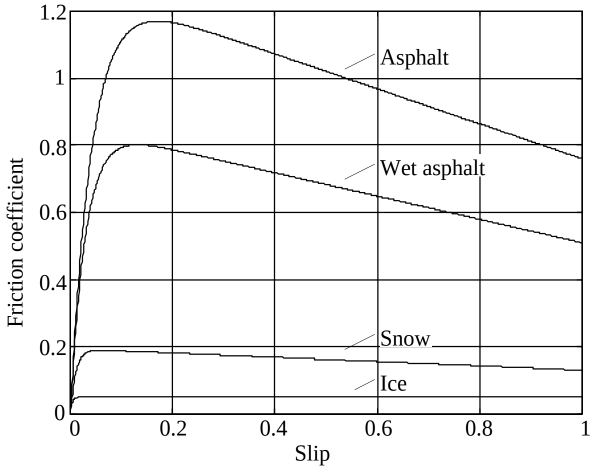



**Figure 3** : Friction characteristics for some typical surfaces

Then the lateral and transversal components are :

$$\mu_L = \mu_{RES} \cdot \frac{s_L}{s_{RES}} \qquad \mu_S = \mu_{RES} \cdot \frac{s_S}{s_{RES}}$$

After that, it is defined that the tire is subject to a longitudinal force and a lateral force as a function of the load $F_Z$ and the friction coefficient :

$$F_L = F_Z \cdot \mu_L$$

$$F_S = F_Z \cdot \mu_S$$

These forces are defined in figure 2 in the tire coordinate system.

By this figure 2, a convention is defined : when a is positive then the forces are on the left side, and a positive longitudinal slip yields forces in the front direction.

a is mathematically negative in the trigonometric direction, and is defined as the angle between the direction of the tire and the direction of the tire velocity.

### *4.3. Model for the normal forces*

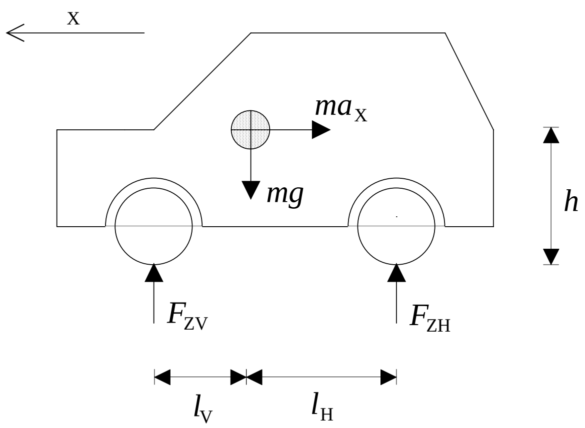



**Figure 4** : Model of the car for shifting of the axle load

For the calculation of the tire forces $F_L$ and $F_S$ it' s important to determine the dynamic load of the wheels corresponding to the normal forces $F_Z$. The normal forces change opposite to their static values due to different

driving situations. The load of the axle changes due to braking or accelerating or driving at a hill.
The wheel load shifts from the inner to the outer wheel during cornering.

Figure 4 shows the forces applying to the chassis of an accelerated car. The car is simplified to one mass model. The change of the wheel loads is only dependable of the position of the center gravity and the actual longitudinal acceleration. This model can be used because the body frequency of the car's spring-damper-system is higher than possible frequencies of the acceleration. The accelerating force (m $a_x$) causes a pitching moment with a lever arm of the height h of the center of gravity. This results in a dynamic load shifting from the front to the rear axis. Calculating the balance of moments about the contact area of the rear axis the dynamic wheel load for the front axle using the abbreviations of the literature [6] results in the following equations:

for front wheels

$$F_{ZV} = m \cdot \left[ \frac{l_H}{l_V + l_H} \cdot g - \frac{h}{l_V + l_H} \cdot a_X \right]$$

for rear wheels

$$F_{ZH} = m \cdot \left[ \frac{l_V}{l_V + l_H} \cdot g + \frac{h}{l_V + l_H} \cdot a_X \right]$$

These considerations can be also made for a decelerated car.

### *4.4. Equation of the torque for the tire model*

The model consists of one wheel with the radius $r$ and the moment of inertia $\Theta_R$. The load of the wheel is the normal force $F_Z$. Based on the wheel load and the adhesion coefficient m, an accelerating force $F_L$ is applied according to Coulomb's law $F_L = F_Z \cdot \mu_L$ to the contact area between the tire and road. This force has an accelerating effect on a decelerated wheel. Furthermore, the wheel is decelerated by the braking torque $M_x$ through the brake.

Resulting out of a balance of moments the following equation can be written :

$$\Theta_R \, \dot{\omega} = M_X - r F_L$$

The same equation can be written for the three other wheels, and gives us a state space vector:

$$\mathbf{x}_{Rad}^T = [\omega_{VL} \quad \omega_{VR} \quad \omega_{HL} \quad \omega_{HR}]$$

the inputs of this model are the longitudinal tire forces and the moment of braking or acceleration.

$$\mathbf{u}_{Rad}^T = [F_{LVL} \quad F_{LVR} \quad F_{LHL} \quad F_{LHR} \quad M_{VL} \quad M_{VR} \quad M_{HL} \quad M_{HR}]$$

For this calculation the parameter $\Theta_R$, $r$, and $M_x$ must be measured.

## 5. Model of suspension

In the literature[1], [8], a large variety of tire models are proposed ranging from a simple mechanical spring up to complex finite element models. It is useful to evaluate wheel axis vibrations as it is shown in [9]. In order to study the influence of alterations of tire stiffness $C_{RH}$ due to variation of inflation pressure, it is useful to consider a simple point model of vehicle suspension.

Figure 5 is an example of ¼ rear left side of the car. All the four suspensions of the vehicle show the same model structure, with different parameters.

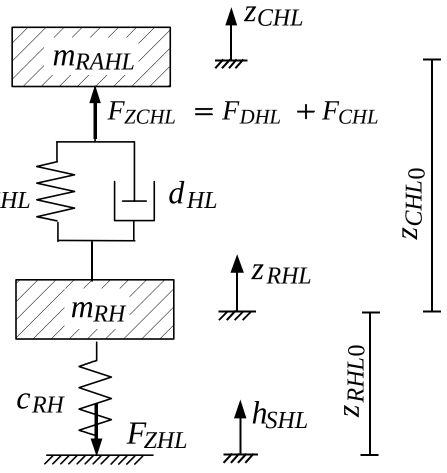



**Figure 5** : Suspension model

It consists of the suspension spring with stiffness $C_{FH}$ and the shock absorber with damping coefficient $d_H$ which are both mounted between axle mass $m_{RH}$ and vehicle body mass $m_{RAHL}$. Moreover, the suspension spring and the shock absorber are treated as linear elements.

The static spring distances $Z_{RHL0}$ and $Z_{CHL0}$ are chosen in a way to compensate the static forces when the car is in horizontal position. We calculate the relative distance and the forces in comparison with the balance position of the car.

Relative distance : $\Delta z_{RHL} = z_{RHL} - z_{RHL0}$ and

$$\Delta z_{CHL} = z_{CHL} - z_{CHL0}$$

Forces :

$$F_{ZHL} = c_{RH}\,(h_{SHL} - z_{RHL} + z_{RHL0})$$
$$F_{CHL} = c_{FH}\,(z_{RHL} - z_{CHL} + z_{CHL0})$$
$$F_{DHL} = d_{FH}\,(\dot{z}_{RHL} - \dot{z}_{CHL})$$

Equation of the forces :

$$m_{RH}\,\ddot{z}_{RHL} = F_{ZHL} - F_{CHL} - F_{DHL}$$

Finally:

$$m_{RH}\,\ddot{z}_{RHL} = c_{RH}\,(h_{SHL} - \Delta z_{RHL}) + c_{FH}\,(-\Delta z_{RHL} + \Delta z_{CHL}) + d_{FH}\,(-\Delta \dot{z}_{RHL} + \Delta \dot{z}_{CHL})$$

But, to have a good model of the suspension we must take into account that the spring of the vehicle is non-linear. With the help of measurements, we can estimate the force on the spring by a polynomial function with order 4. We give here the example for the force applied on the rear axes. The procedure for the front axis is similar :

with $xd = \dot{z}_R - \dot{z}_C$ then

$$F_{HZ} = d_{HZ1}\, xd + d_{HZ2}\, xd^2 + d_{HZ3}\, xd^3 + d_{HZ4}\, xd^4 \quad \textit{for} \quad xd < 0$$

$$F_{HD} = d_{HD1}\, xd + d_{HD2}\, xd^2 + d_{HD3}\, xd^3 + d_{HD4}\, xd^4 \quad \textit{for} \quad xd \geq 0$$

With the use of the non-linear curve of the damper, on each step of the integration the parameters have to change, then :

$$m_R \ddot{z}_R = c_F\,(z_C - z_R) + c_R\,(h - z_R) + a_1\,(\dot{z}_C - \dot{z}_R) + a_2\,(\dot{z}_C - \dot{z}_R)^2 + a_3\,(\dot{z}_C - \dot{z}_R)^3 + a_4\,(\dot{z}_C - \dot{z}_R)^4$$

After that, this model is represented with a nonlinear state space equation as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{c_F}{m_R}(u_2 - x_1) + \frac{c_R}{m_R}(u_1 - x_1) + \frac{a_1}{m_R}(x_2 - u_3) + \frac{a_2}{m_R}(x_2 - u_3)^2 + \frac{a_3}{m_R}(x_2 - u_3)^3 + \frac{a_4}{m_R}(x_2 - u_3)^4$$

$$y_1 = c_R\,(x_1 - u_1)$$

$$y_2 = c_F\,(u_2 - x_1) + a_1\,(x_2 - u_3) + a_2\,(x_2 - u_3)^2 + a_3\,(x_2 - u_3)^3 + a_4\,(x_2 - u_3)^4$$

Finally for the four wheels it becomes so :

$$\mathbf{x}_V^T = [\Delta z_{RVL} \quad \Delta\dot{z}_{RVL} \quad \Delta z_{RVR} \quad \Delta\dot{z}_{RVR} \quad \Delta z_{RHL} \quad \Delta\dot{z}_{RHL} \quad \Delta z_{RHR} \quad \Delta\dot{z}_{RHR}]$$

$$\mathbf{u}_V^T = \begin{bmatrix} h_{SVL} \ \Delta z_{CVL} \ \Delta\dot{z}_{CVL} \ h_{SVR} \ \Delta z_{CVR} \ \Delta\dot{z}_{CVR} \cdots\cdots\cdots \\ \cdots\cdots\cdots h_{SHL} \ \Delta z_{CHL} \ \Delta\dot{z}_{CHL} \ h_{SHR} \ \Delta z_{CHR} \ \Delta\dot{z}_{CHR} \end{bmatrix}$$

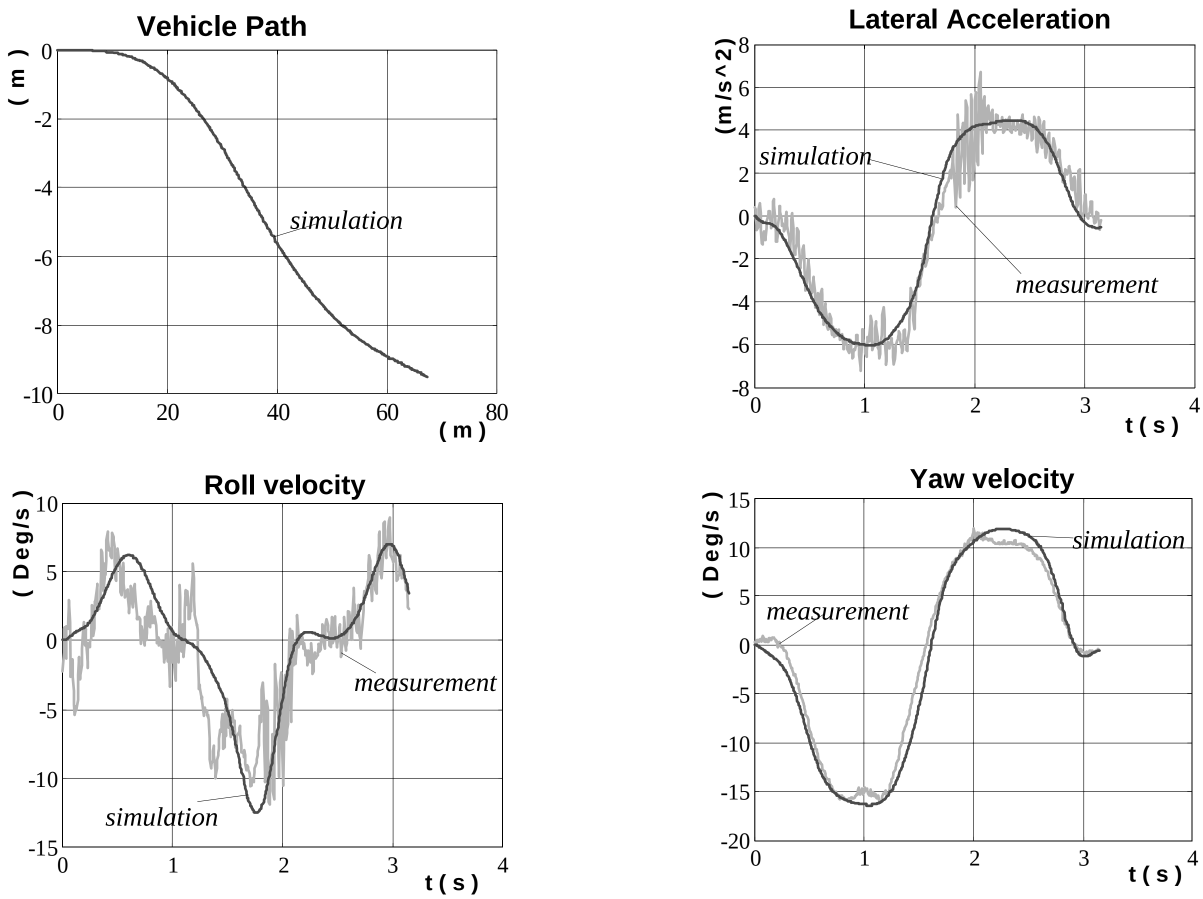
$$\mathbf{y}_V^T = [\mathbf{x}_V^T \quad \Delta F_{ZVL} \quad \Delta F_{ZCVL} \quad \Delta F_{ZVR} \quad \Delta F_{ZCVR} \quad \Delta F_{ZHL} \quad \Delta F_{ZCHL} \quad \Delta F_{ZHR} \quad \Delta F_{ZCHR}]$$

## 6. Results

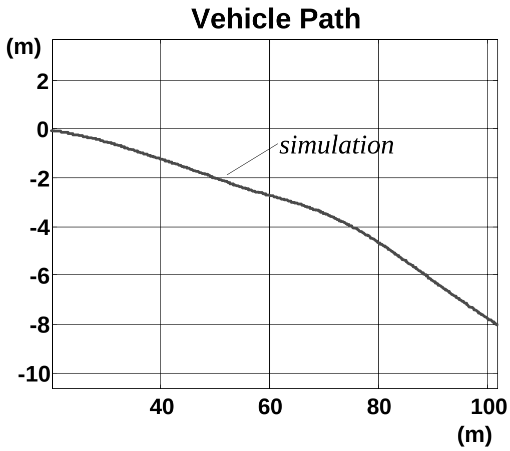
Two application examples have been selected in order to demonstrate the realistic behavior of the simulation. For performance evaluation the signals measured on a test car and the outputs of the simulation are compared under the same driving condition. Different driving situations are regarded in order to have a large representation of the vehicle behavior.

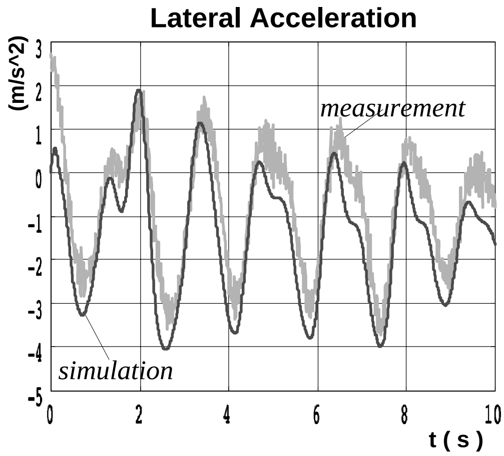
In these two examples, lateral acceleration, yaw and roll velocity show a good correspondence between measurements and simulation results. This comparison verifies that the above simulation model can be used for the development of control systems.

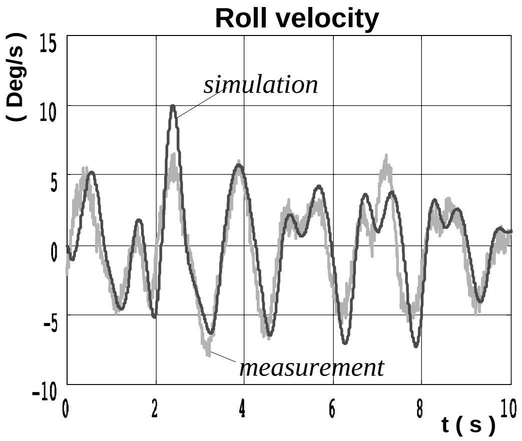
*__Example 1:__* The first example is characterized by a rapid step of the steering angle, and the condition of a constant vehicle speed about 20 m/s and wet road.

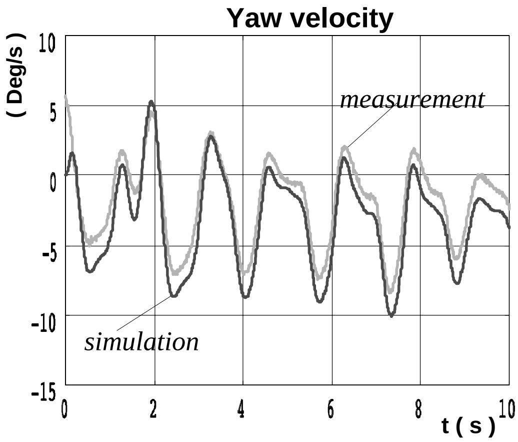


*Example 2 :* The second example shows fast oscillations of steering angle.
The speed is also about 20 m/s and the road is dry.




## 7. Conclusions

This modeling of nonlinear vehicle dynamics is a modular concept with different sub-models, beginning with an exact nonlinear tire model, which is the most important submodel to describe the dynamic behavior of cars more exactly. The suspension model is also described with nonlinearities of the spring and its link with the chassis. The body car has 6 degrees of freedom. To take the environmental parameters into account, also the influence of the wind and the slope angle are included in this simulation.

The computation time of this simulation is fast because a simulation of 10 seconds last in real-time 30 seconds.

This modular simulation gives good results and demonstrates the realistic car behavior during different driving situations, in spite of difficulties to simulate the roll velocity due to the fact that big forces result in very little angles. Finally, it can be outlined, that the simulation has sufficient exactness to be used instead of driving tests which consume a lot of time for the preparations of a experimental vehicle.

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