Controller design for stable processes using user specified gain and phase margin specifications and two degree-of-freedom IMC structure

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Abstract-- In real industrial practice, controller designs are usually performed based on an approximate model. Furthermore, the parameters of the physical systems can vary with operating conditions and time. Therefore, it is essential to design a control system which will show a robust performance in the case of aforementioned situations. Gain and phase margins are well known measures for maintaining the robustness of a control system. This paper presents a new two degree-of-freedom Internal Model Control (IMC) structure and simple tuning rules to tune/design PI controllers for stable processes with a small dead time to meet specified gain and phase margins. Simulation examples are given to illustrate that the proposed design method can give better closed loop system performances than existing design methods which is also designed based on user-specified gain and phase margins.

***Index Terms*-- IMC Design, PI controller, Gain margin, Phase margin, Time delay**

I. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are still widely used in industrial systems despite the significant developments of recent years in control theory and technology. This is because they perform well for a wide class of processes. Also, they give robust performance for a wide range of operating conditions. Furthermore, they are easy to implement using analogue or digital hardware and familiar to engineers.

In the practice, the model used to analyze or design control systems is only an approximation of the actual plant transfer function. The most common models used for stable plant transfer functions are a first order plus dead time (FOPDT) or second order plus dead time (SOPDT) model. Also, the parameters of the physical systems, usually, vary with operating conditions and time. Hence, robustness of a control system has always been an important issue. Gain and phase margins are two well-known measures for maintaining the robustness of a control system. Recently, there has been a renewed interest in designing a control system to satisfy the specified gain and phase margins (Ho *et al.* 1995; Fung *et al.* 1998; Wang *et al.* 1999; Wang and Shao, 1999).

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In this paper, a two-degree of freedom Internal Model Control (IMC) (Rivera *et al.,* 1986; Morari *et al.*, 1989) structure is proposed for controlling stable processes with small time delays. Since, in the IMC procedure pole-zero cancellations are used, a controller designed based on the IMC principles results in good set point tracking. However, the disturbance rejection of the IMC structure may be sluggish, in some cases, again due to pole zero cancellation used in the design procedure. To eliminate this shortcoming, in this paper a second controller acting on the signal difference between the plant output and the model output is introduced so that a better disturbance rejection can be achieved. The main controller used for set point tracing is designed based on user-specified gain and phase margins. For this, the classical single input single output (SISO) feedback control system is represented as its equivalent IMC. This provides the parameters of PID type controllers used in the SISO system to be defined in terms of the desired closed-loop time constant, which can be adjusted by the operator, and the parameters of the process model. This means that only one parameter, namely the desired closed-loop time constant, is left for tuning, assuming that the model parameters have been obtained from a relay autotuning (Kaya, 1999; Kaya and Atherton, 2001). The details of the identification method are not given here and interested readers can refer to the cited references. The second controller, which is introduced for a better disturbance rejection, is designed using the Nyquist stability criteria. The proposed design method is compared with some existing ones, which are also based on the specified gain and phase margins, and it is shown by examples that the proposed design method gives better closed loop performances for both the set-point response and disturbance rejection.

The paper organised as follows: Since, the tuning rules to tune/design the main controller (which is a PI controller) are derived based on IMC principles, the next section gives a brief review of the IMC design. In section three, the new two degree-of-freedom IMC structure is introduced. Tuning rules for the main and the second controller are derived in section four. Section five gives simulation examples which show that with the design method given in the paper better closed loop system performances can be obtained when compared to existing design methods which are also designed with specifications on gain and phase margins. The paper ends with conclusions given in section 6.

II. INTERNAL MODEL CONTROL (IMC)

The IMC configuration suggested Rivera *et al* (1986) is given in Fig. 1.

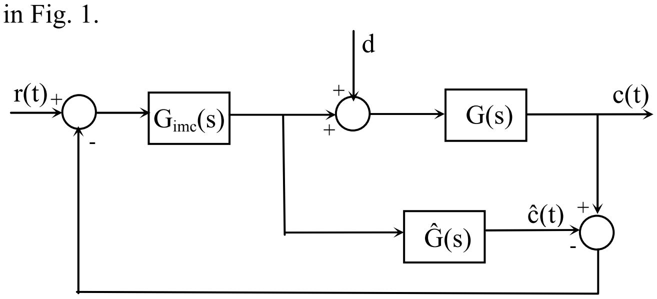

Fig.1: IMC Control Strategy

This control structure is referred to as Internal Model Control (IMC) since the plant model, $\hat{G}(s)$, appears in the control structure. Here, $G(s)$ and $\hat{G}(s)$ are the actual process and process model transfer functions, respectively. When $G(s)=\hat{G}(s)$, that is perfect modelling, and $d=0$, the system is basically open loop. This provides the open loop advantages. When $G(s)\neq\hat{G}(s)$ or $d\neq 0$ the system is a closed loop system. Thus, the IMC control strategy has the advantages of both the open loop and closed loop structures.

The first step in the IMC controller design is to factor the process model

$$\hat{G}(s) = \hat{G}_{+}(s)\hat{G}_{-}(s) \tag{1}$$

where $\hat{G}_{+}(s)$ contains all the time delays and right-half plane zeros.

The second step is to define the IMC controller as

$$G_{imc}(s) = \hat{G}_{-}^{-1}(s)F(s) \tag{2}$$

where $F(s)$ is a low pass filter with a steady state gain of one. The filter is introduced for physical realizability of the IMC controller, $G_{imc}(s)$. The simplest filter has the following form (Rivera, *et al*, 1986; Morari *et al*, 1989)

$$F(s) = \frac{1}{(\lambda s+1)^{n}} \tag{3}$$

III. TWO DEGREE OF FREEDOM IMC STRUCTURE

Since the IMC design approach is based on pole zero cancellations, while the response for the set-point change is quite satisfactory, the response to disturbance rejections may be sluggish. Therefore, a two degree-of-freedom IMC structure (TDF IMC), shown in Fig. 2, is proposed to eliminate aforementioned shortcoming of the original IMC structure.

The closed loop transfer function and disturbance transfer function of the improved IMC structure are now given by

$$T_r(s) = \frac{G(s)G_{imc}(s)[1+\hat{G}(s)G_d(s)]}{1+G_{imc}(s)[G(s)-\hat{G}(s)]+G(s)G_d(s)} \tag{4}$$

and

$$T_d(s) = \frac{G(s)\{1-\hat{G}(s)G_{imc}(s)\}}{1+[G(s)-\hat{G}(s)]G_{imc}(s)+G_d(s)G(s)} \tag{5}$$

respectively, where $G_d(s)$ is introduced for a better disturbance rejection. Substituting eqn. (2) into eqns. (4) and (5) and assuming perfect modelling and non-minimum phase systems, the closed loop and disturbance transfer functions reduce to

$$T_r(s) = G_{imc}(s)G(s) \tag{6}$$

and

$$T_d(s) = \frac{G(s)\{1-F(s)\}}{1+G_d(s)G(s)} \tag{7}$$

Clearly, the set-point response given by eqn. (6) and disturbance response of the improved IMC structure given by eqn. (7) are decoupled from each other and can be designed independently. Therefore, this structure corresponds to a two degree-of-freedom IMC structure.

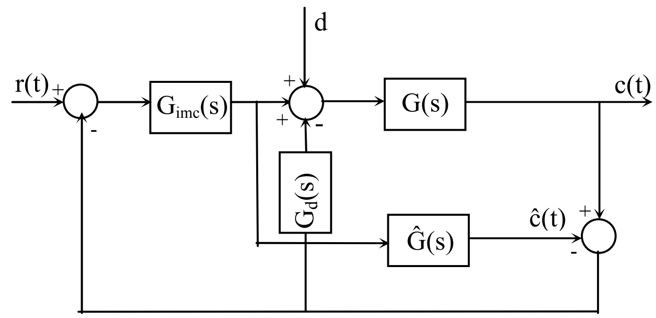

Fig. 2: Two degree-of-freedom IMC structure

Note that if $G_d(s)=0$, the proposed two degree-of-freedom IMC structure reduces to the original IMC structure for non-minimum phase systems. Therefore, the controller $G_d(s)$ can be used for improving the disturbance rejection capability of the original IMC structure.

IV. CONTROLLER DESIGN

The design of the two controllers $G_{imc}(s)$ and $G_d(s)$ are done separately since the closed loop and disturbance transfer functions are decoupled from each other. First, the main controller $G_{imc}(s)$, is considered. The closed loop transfer function of a classical SISO feedback system and the two degree-of-freedom IMC structure, for perfect matching, are respectively given by

$$T_{siso}(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \tag{8}$$

and

$$T_{imc}(s) = G_{imc}(s)G(s) \tag{9}$$

In order to have the same output for the both configurations, it is straightforward to illustrate, by comparing eqns. (8) and (9), that the IMC controller, $G_{imc}(s)$, is related to the classic controller, $G_c(s)$, through the transformation

$$G_{imc}(s)=\frac{G_c(s)}{1+G_c(s)G(s)} \tag{10}$$

or

$$G_c(s)=\frac{G_{imc}(s)}{1-G_{imc}(s)G(s)} \tag{11}$$

Therefore, from eqn. (10) a classical SISO feedback system can be put into the two degree-of-freedom IMC structure, assuming a perfect modelling, that is, $G(s)=\hat{G}(s)$, as shown in Fig. 3.

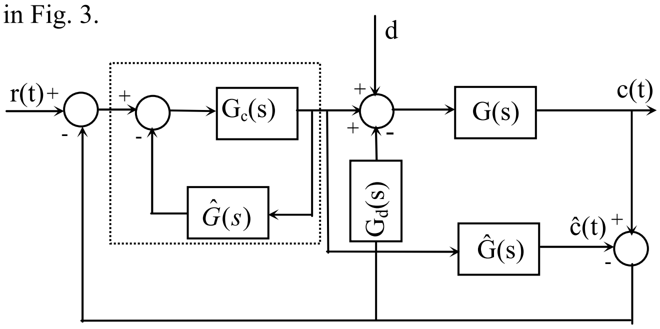

Fig. 3: Two degree-of-freedom IMC representation of a SISO control system

To find the tuning parameters of the controller $G_c(s)$ in Fig. 3, a stable first order plus dead time (FOPDT) plant transfer function is considered. Note that the FOPDT model is only used for simplifying calculations and that the actual process may be a higher order process, a process with complex poles, etc. In order to obtain the IMC controller, the process model, $\hat{G}(s)=Ke^{-\theta s}/(Ts+1)$, must be factored as in eqn. (1):

$$\hat{G}_+(s)=e^{-\theta s} \tag{12}$$

$$\hat{G}_-(s)=\frac{K}{(Ts+1)} \tag{13}$$

The IMC controller can be obtained from eqn. (2), assuming a filter with $n=1$, as

$$G_{imc}(s)=\frac{(Ts+1)}{K(\lambda s+1)} \tag{14}$$

Using a first order Taylor series expansion for the time-delay approximation, the classic controller, $G_c(s)$, can be obtained from eqn. (11)

$$G_c(s)=\frac{Ts+1}{K(\lambda+\theta)s} \tag{15}$$

Eqn. (15) can be rearranged as an ideal PI controller, which has the following controller parameters

$$K_p=\frac{T}{K(\lambda+\theta)} \tag{16}$$

$$T_i=T \tag{17}$$

The only unknown in the last two equations is the filter time constant, λ, since it is assumed that the plant transfer function model is obtained from the exact relay feedback identification method given in (Kaya, 1999; Kaya and Atherton, 2001). Thus, if a proper value of λ is obtained, then the design procedure for the controller $G_c(s)$ in Fig. 3, will be completed. In this paper, gain and phase margin specifications are used to find a proper value for λ.

The characteristic equation of a SISO control system is given by $1+G_c(s)G(s)$. Hence, the open loop transfer function of the SISO control system, with $G_c(s)$ given by eqn. (15), is

$$G_c(s)G(s)=\frac{e^{-\theta s}}{(\lambda+\theta)s} \tag{18}$$

Therefore, from the basic definitions of the gain and phase margins the following equations can be obtained:

$$\arg\{G_c(j\omega_p)G(j\omega_p)\}=-\pi \tag{19}$$

$$A_m\left|G_c(j\omega_p)G(j\omega_p)\right|=1 \tag{20}$$

$$\left|G_c(j\omega_g)G(j\omega_g)\right|=1 \tag{21}$$

$$\phi_m=\pi+\arg\{G_c(j\omega_p)G(j\omega_p)\} \tag{22}$$

where the gain margin is given by eqns. (19) and (20), and the phase margin by eqns. (21) and (22). The frequency ω_p is known as phase crossover frequency, where the Nyquist curve has a phase lag of $-\pi$, and the frequency ω_g is known as the gain crossover frequency, where the Nyquist curve has an amplitude of 1.

Substituting eqn. (18) into eqns. (19)-(22), results in the following set of equations:

$$\omega_p\theta=\frac{\pi}{2} \tag{23}$$

$$A_m=\omega_p(\lambda+\theta) \tag{24}$$

$$\omega_g=\frac{1}{\lambda+\theta} \tag{25}$$

$$\phi_m=\frac{\pi}{2}-\omega_g\theta \tag{26}$$

From eqns. (24) and (25) one can obtain

$$A_m\omega_g=\omega_p \tag{27}$$

Multiplying both sides of eqn. (27) with θ and then substituting values of $\omega_g\theta$ and $\omega_p\theta$ from eqns. (24) and (26), the relation between gain and phase margin can be obtained as

$$\phi_m = \frac{\pi}{2}(1 - \frac{1}{A_m}) \tag{28}$$

The recommended ranges for the gain and phase margins are between 2 to 5 and 30° to 60°, respectively (Åström and H gglund, 1995). Choosing $A_m=3$, then $\phi_m=60^o$. Therefore, the closed loop time constant λ is obtained, by rearranging eqns. (23) and (24), as

$$\lambda = \theta(\frac{2A_m}{\pi} - 1) = 0.91\theta \tag{29}$$

Hence, the PI controller parameters are given by eqn. (16) and (17) with λ given by eqn. (29). So, the settings of controller $G_c(s)$, which is a PI controller, have been identified. For a satisfactory load disturbance rejection, the tuning parameters for disturbance rejection controller $G_d(s)$ have to be found as well. For this, the Nyquist stability criteria is applied to the characteristic equation of eqn. (7):

$$1 + G_d(s)G(s) = 0 \tag{30}$$

The controller $G_d(s)$ is assumed to be a gain only controller, $G_d(s)=K_d$. Thus,

$$1 + \frac{KK_d e^{-\theta s}}{(Ts+1)} = 0 \tag{31}$$

Choosing K_d to give a gain margin of G_m results in

$$K_d = \frac{\sqrt{(T+\theta)^2 + (\pi T)^2}}{K(T+\theta)G_m} \tag{32}$$

So, the settings of disturbance rejection controller has also been determined. Extensive simulation examples illustrate that a gain margin of 3, usually, results in a good load disturbance rejection. Hence, through out the paper this gain margin value is used, unless otherwise has been stated. It should be pointed out that, by no means, G_m is the gain margin of the system given by Fig. 3. It just appears as a procedure to analyze the roots of eqn. (31).

V. SIMULATION EXAMPLES

Two examples are considered to illustrate the use of the proposed method. The identification method given in (Kaya, 1999; Kaya and Atherton, 2001) has been used for all transfer functions in the examples but since it gives essentially exact results on simulation data the estimated plant transfer functions are only given for original plants of higher order. In all the examples, the main controller of the proposed design method is designed for a gain and phase margin of 3 and 60°, respectively. The design methods used for comparison are also designed for the same gain and phase margins. The second controller used for a better disturbance rejection is designed for the gain margin of $G_m=3$. Each example is taken from different publications, which also consider controller design based on gain and phase margin specifications. Designed controllers by the procedure given in this paper are compared with controller design methods where they are taken from.

Example 1:
Consider a second order plus dead time plant transfer function of $G(s) = e^{-0.5s}/(s+1)(0.5s+1)$, which was used in Ho et al. (1995). The identification method given in (Kaya, 1999; Kaya and Atherton, 2001) was used to find the FOPDT model as $\hat{G}(s) = e^{-0.796s}/(1.741s+1)$. Hence, the main PI controller parameters were obtained from eqns. (16) and (17), in conjunction with eqn. (29), to be $K_p=1.145$ and $T_i=1.741$. The tuning parameters of the disturbance rejection controller, which is a gain only controller, was found to be $K_d=0.792$, from eqn. (32). The controller parameters for the design method proposed by Ho *et al*. (1995) are $K_p=1.050$, $T_i=1.000$ and $T_d=0.500$. The closed loop responses for both design methods are given in Fig. 4 for a unity step set-point change and load disturbance, introduced at time 20s. The figure illustrates that the proposed design method gives superior performance for set point tracking when compared to the design method of Ho *et al*. (1995). The disturbance rejection of TDF IMC structure is also very satisfactory, if not better than that of Ho *et al*. (1995).

Example 2:
A high order oscillating plant transfer function of $G(s) = e^{-s}/(s^2+s+1)(s+3)$, which was also given in (Wang and Shao, 1999), is considered. Again, the parameter estimation method given in (Kaya, 1999; Kaya and Atherton, 2001) was used to obtain the FOPDT model as $\hat{G}(s) = 0.333e^{-3.1s}/(0.075s+1)$. Therefore, once a proper model is found, the main PI controller tuning parameters were calculated to be $K_p=0.038$ and $T_i=0.075$, using eqns (16) and (17) in conjunction with eqn. (29). The disturbance rejection controller gain was found to be $K_d=1.003$. Wang and Shao (1999) suggested a PID controller with settings of $K_p=1.298$, $T_i=1.034$ and $T_d=1.017$. With these calculated controller settings, the step response of the closed loop system to a unity step set-point change and a disturbance of magnitude of -1 introduced at time 40s is shown in Fig. 5. Again, the proposed design method results in a better performance, especially for set point response.

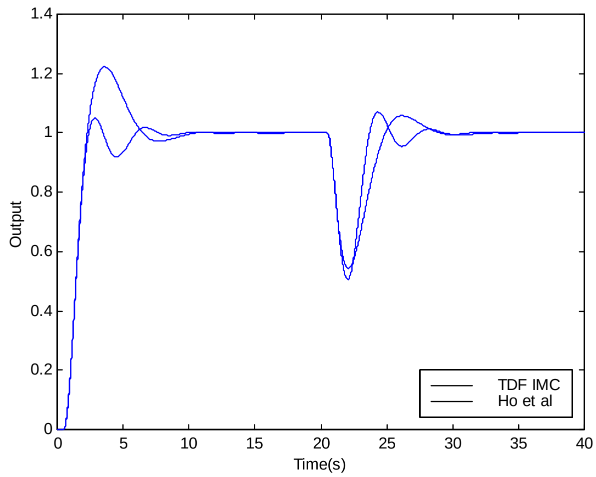

Fig 4. Step responses of Example 1.

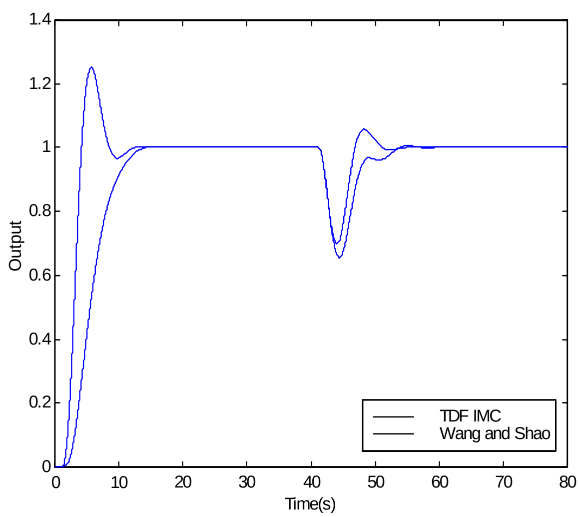

Fig. 5: Step responses for example 2

VI. CONCLUSIONS

A two degree-of-freedom IMC structure for controlling stable processes with small time delays based on specified gain and phase margin specifications is introduced. The original IMC structure can lead to sluggish load disturbance rejections as the IMC design approach is based on pole zero cancellations. With the introduced two degree-of-freedom IMC structure this shortcoming has been eliminated. Since, the design method given in the paper is model based, first an FOPDT plant transfer function model was obtained from a single relay feedback test with exact limit cycle analysis. Once the model is found, simple tuning rules provided in the paper were used to control the process. Simulation examples have shown that the proposed two degree-of-freedom IMC structure and design method gives very satisfactory results for controlling stable processes with small time delays.

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