High performance variable structure control of a magnetic levitation system

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Abstract- **In this paper the position-tracking problem of a voltage-controlled magnetic levitation system is considered. It is well known that the control problem is quite complicated and challenging duo to inherent nonlinearities associated with the electromechanical dynamics. A sliding mode control is employed for controlling the system. The proposed controller exhibits satisfactory robustness in response to parameter uncertainties. Simulation results reveal the effectiveness of the proposed robust controller.**

Index Terms- **Magnetic levitation system, voltage control, nonlinear control, sliding mode control.**

I. INTRODUCTION

Magnetic levitation systems are widely used in various fields, such as frictionless bearings, high-speed maglev passenger trains, levitation of wind tunnel models, etc., and it is an important task to construct a high performance feedback controller to control the position of the levitated object, since a magnetic levitation system is usually open-loop unstable. Duo to inherent nonlinearities associated with the electromechanically dynamics, the control problem is usually quite challenging to the control engineers, since a linear controller is valid only about a small region around a nominal operating point. In recent years a lot of controllers have been reported in the literature for controlling a magnetic levitation system by actively taking nonlinearities of the system model into account [1-5].

Sliding mode control [6] is a particular type of variable structure control systems that is designed to drive and then constrain the system to lie within a neighborhood of the switching function [7,8]. There are two main advantages of this approach. Firstly, the dynamic behavior of the system may be tailored by the particular choice of switching functions. Secondly, the closed-loop response becomes totally insensitive to a particular class of uncertainty [9]. In addition, the ability to specify performance directly makes sliding mode control attractive from the design perspective. This design approach consists of two components. The first, involves the design of a switching function so that the sliding motion satisfies design specifications. The second is conserved with the selection of a control law, which will make the switching function attractive to the system state.

In this paper we will design a sliding mode control for position control of magnetic levitation system via voltage control. In order to compare the proposed controller with the other methods the performance of the sliding mode controller will be compared with that of feedback linearization method designed in [3]. Simulation results show that the former have better performance than the other one. Using sliding mode the closed-loop systems gains more robustness in response to parameter uncertainty.

The organization of the paper is as follows: In Section II the model of the magnetic levitation system is presented. Section III reviews the sliding mode control design. Simulation results are provided in Section IV and section V concludes the paper.

II. MODEL OF THE MAGNETIC LEVITATION SYSTEM

Consider the magnetic levitation system shown in Fig. 1. this is a popular gravity-based one degree-of-freedom magnetic levitation system, in which an electromagnet exerts attractive force to levitate a steel ball (in some references a steel plate is levitated). The system dynamics can be described in the following equations

$$M\ddot{x} = Mg + \frac{1}{2}i^2\frac{dL}{dx}$$
$$u = Ri + \frac{d}{dt}(Li) \tag{1}$$

where the coil inductance is given as

$$L(x) = \frac{Q}{x_\infty + x} + L_\infty \tag{2}$$

Definition of the parameters in above equations is:

x : air gap (vertical position) of the steel ball, i : coil current
g : gravity acceleration, M : mass of the steel ball
R : electrical resistance, u :voltage control input applied to the system.
L_∞, x_∞ and Q are positive constants determined by the characteristics of the coil, magnetic core and steel ball.

Defining the state variable vector as $x = [x_1, x_2, x_3]^T = [x, \dot{x}, i]^T$ and rewriting equations (1) we have the following state space model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}x_2\\ \alpha(x)\\ \beta(x)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \gamma(x)\end{bmatrix}u \tag{3}$$

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where

$$\alpha(x) = g - \frac{Qx_3^2}{2M(x_\infty + x_1)^2}$$

$$\beta(x) = \frac{x_3[Qx_2 - R(x_\infty + x_1)^2]}{Q(x_\infty + x_1) + L_\infty(x_\infty + x_1)^2} \qquad (4)$$

$$\gamma(x) = \frac{x_\infty + x_1}{Q + L_\infty(x_\infty + x_1)}$$

Denote the nominal values of the physical parameters as $g_o, M_o, R_o, L_{\infty o}, Q_o$ and $x_{\infty o}$. It is assumed here these nominal parameters which are only rough estimates of their values are known a priori. Then the system model can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = f_o(x) + \Delta f(x) + g_o(x)u + \Delta g(x)u$$
$$= \begin{bmatrix} x_2 \\ \alpha_o(x) + \Delta_\alpha(x) \\ \beta_o(x) + \Delta_\beta(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \gamma_o(x) + \Delta_\gamma(x) \end{bmatrix} u \qquad (5)$$

where

$$\alpha_o(x) = g_o - \frac{Q_o x_3^2}{2M_o(x_{\infty o} + x_1)^2}$$

$$\beta_o(x) = \frac{x_3[Q_o x_2 - R_o(x_{\infty o} + x_1)^2]}{Q_o(x_{\infty o} + x_1) + L_{\infty o}(x_{\infty o} + x_1)^2} \qquad (6)$$

$$\gamma_o(x) = \frac{x_\infty + x_1}{Q_o + L_{\infty o}(x_{\infty o} + x_1)}$$

are the nominal nonlinear functions, and

$$\Delta_\alpha(x) = \alpha(x) - \alpha_o(x)$$
$$\Delta_\beta(x) = \beta(x) - \beta_o(x) \qquad (7)$$
$$\Delta_\gamma(x) = \gamma(x) - \gamma_o(x)$$

it is remarkable that an external constant mechanical disturbance can be viewed as a biased error of g equivalently, i.e. the gravity acceleration is biased equivalently. Therefore, we will not treat such a disturbance explicitly for the sake of simplicity.

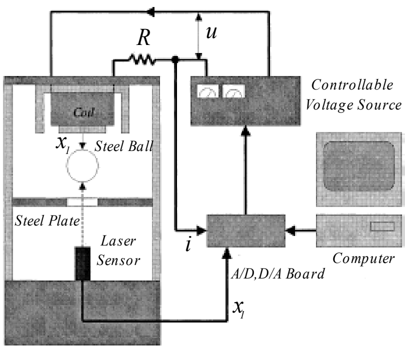

Fig. 1. Diagram of the magnetic levitation system.

III. Sliding Mode Control Design

A Sliding Mode Controller is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that map plant state to a control surface, and the switching among different functions is determined by plant state that is represented by a switching function.

Consider the design of a sliding mode controller for the following system

$$\dot{x}(t) = A(x(t) - x_d) + Bu(t) + f(x, u, t) \qquad (8)$$

where x_d is reference trajectory and $u(t)$ is the input to the system. The following is a possible choice of the structure of a sliding mode controller [10]

$$u = -k\,\mathrm{sgn}(s) + u_{eq} \qquad (9)$$

where u_{eq} is called equivalent control which is used when the system state is in the sliding mode [11]. k is a constant, representing the maximum controller output. s is called switching function because the control action switches its sign on the two sides of the switching surface $s = 0$. s is defined as [10]

$$s = \dot{e} + \lambda e \qquad (10)$$

where $e = x - x_d$ and x_d is the desired state. λ is a constant. The definition of e here requires that k in (9) be positive. $\mathrm{sgn}(s)$ is a sign function, which is defined as

$$\mathrm{sgn}(s) = \begin{cases} -1 & if \quad s < 0 \\ 1 & if \quad s > 0 \end{cases} \qquad (11)$$

The control strategy adopted here will guarantee a system trajectory move toward and stay on the sliding surface $s = 0$ from any initial condition if the following condition meets

$$s\dot{s} \le -\eta |s| \qquad (12)$$

where η is a positive constant that guarantees the system trajectories hit the sliding surface in finite time [10].

Using a sign function often causes chattering in practice. One solution is to introduce a boundary layer around the switch surface [11]

$$u = -k\,sat(\frac{s}{\phi}) + u_{eq} \qquad (13)$$

where constant factor ϕ defines the thickness of the boundary layer. $sat(\frac{s}{\phi})$ is a saturation function that is defined as

$$sat(\frac{s}{\phi}) = \begin{cases} \frac{s}{\phi} & if \quad \left|\frac{s}{\phi}\right| \le 1 \\ \mathrm{sgn}(\frac{s}{\phi}) & if \quad \left|\frac{s}{\phi}\right| > 1 \end{cases} \qquad (14)$$

This controller is actually a continuous approximation of the ideal relay control [10]. The consequence of this control scheme is that invariance property of sliding mode control is lost. The system robustness is a function of the width of the boundary layer.

A variation of the above controller structures is to use a hyperbolic tangent function instead of a saturation function

$$u = k\tanh\left(\frac{s}{\phi}\right) + u_{eq} \qquad (15)$$

It is proven that if k is large enough, the sliding model controllers of (9), (13) and (15) are guaranteed to be asymptotically stable [9,10].

For a 2-dimensional system, the structure of controllers (9), (13) and (15) and the corresponding control surface are illustrated in Fig 2.

In this work we will use the sliding control law in the form of (15).

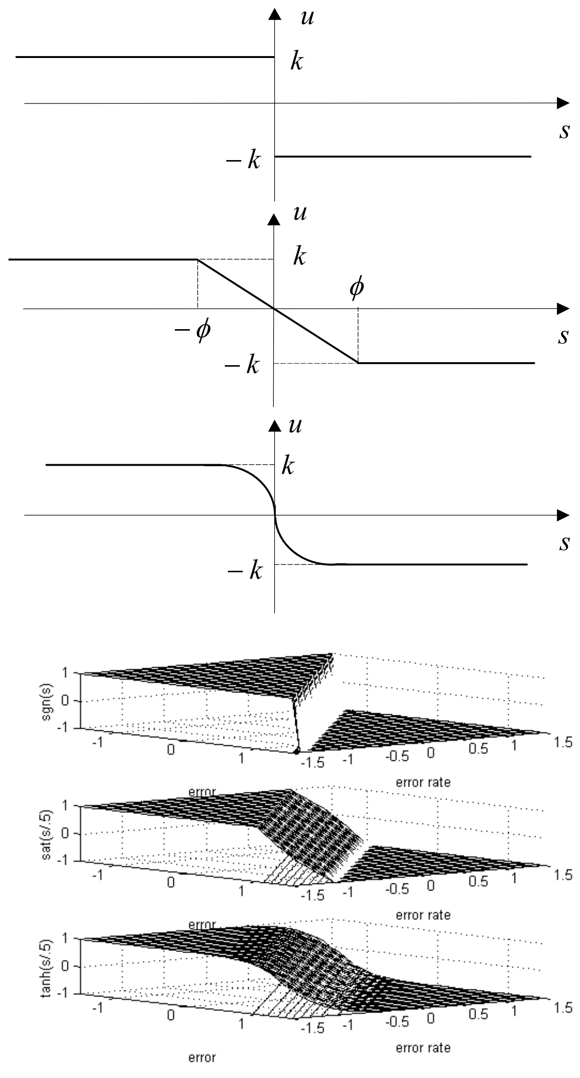

Fig. 2. Various Sliding Mode Controllers and control surfaces.

IV. SIMULATION RESULTS

To verify the performance of the proposed robust controller, the closed-loop system is simulated using MATLAB, whose physical parameters are given in the Table. The physical allowable operating region of the steel ball is limited to $0[mm] \le x_1 \le 13[mm]$, and the output of the controllable voltage source is limited to $-60[V] \le u \le 60[V]$. The velocity x_2 is measured by pseudo-differentiation of the measured position x_1 as $sx_1/(0.0004s+1)$.

Table. Physical parameters of the magnetic levitation system.

M	0.64	$[kg]$
g	9.8	$[m/s^2]$
x_∞	0.007	$[m]$
Q	0.001599	$[Hm]$
L_∞	0.8052	$[H]$
R	11.68	$[\Omega]$

The following nominal system parameters with considerable errors are used for the simulation, to verify the robust performance of the proposed sliding mode controller, in the presence of parameter uncertainties.

$$M_o = 0.8kg$$
$$x_{\infty o} = 0.005m$$
$$L_{\infty o} = 0.5H$$
$$g_o = 9m/s^2$$
$$Q_o = 0.001Hm$$
$$R_o = 10\Omega$$

To design with a suitable reference trajectory initialization, the reference trajectory is initialized based on the initial conditions for the steel ball ($x_1(0) = 0.013m$ and $x_2(0) = 0m/s^2$). Before the feedback controller's start, a step input $u = 15V$ is applied to the system during $-0.5\sec \le t \le 0\sec$ in order to produce an appropriate initial coil current $x_3(0)$.

In order to show the effectiveness of the sliding mode controller, its performance is compared with that of feedback linearization method designed in [3]. Fig. 3 shows the closed-loop response of the system using the feedback linearized control law. As it can be seen, the steel ball oscillates roughly that is not desirable. The responses using the proposed sliding mode controller is depicted in Fig. 4. Comparison of these figures conforms that the sliding mode control action is much better the other one. Using the sliding mode controller, the position, x_1, could track the set point better and the system is robust in response to the parameter uncertainties.

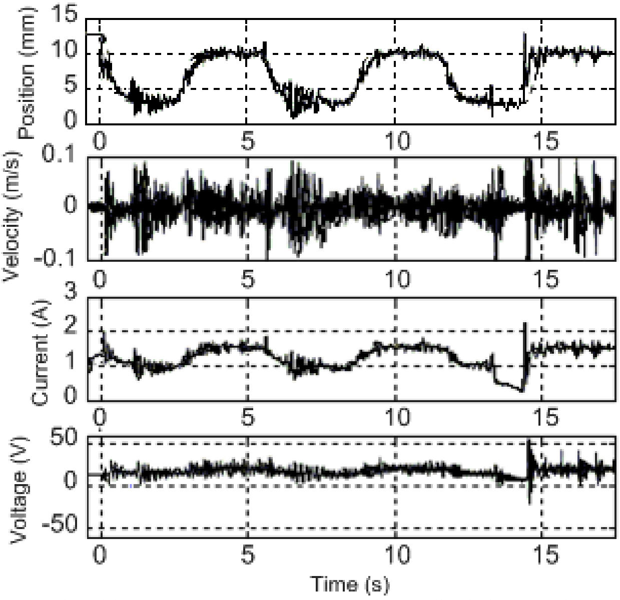

Fig. 3. Closed-loop system responses using the feedback linearized control law.

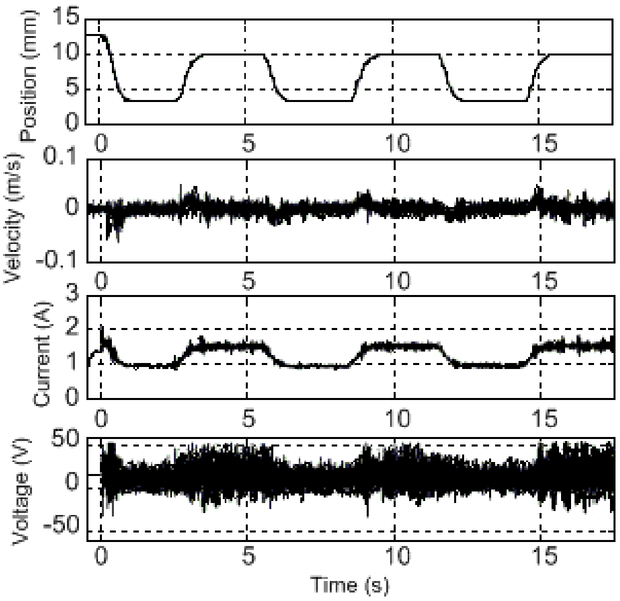

Fig. 4. Closed-loop system responses using the sliding mode control law.

IV. CONCLUSION

A sliding mode controller was proposed for position control of a voltage-controlled magnetic levitation system. For the design of the sliding controller a soft switching function was used. The proposed control action was a robust controller in response to parameter uncertainties and its performance was better than that of feedback linearized controller. Simulation results showed the effectiveness of the sliding mode controller.

REFERENCES

[1] Joo, A.J. and J.H. Seo, Design and analysis of the nonlinear feedback linearization control of an electromagnetic suspension system, IEEE Trans. Control system technology, vol. 5, pp.135-144, 1997.

[2] De Queroz, M.S. and D.M. Dawson, Nonlinear control of active bearings: a backstepping approach, IEEE Trans. Control system technology, vol. 4, pp.545-552, 1996.

[3] Yang, Z.J. and M. Minashima, Nonlinear control of a feedback linearizable voltage-controlled magnetic levitation system, Trans. IEEE Japan, vol. 127-3, No 7, pp.1203-1211, 2001.

[4] Barie, W. and J. Chasson, Linear and nonlinear state-space controllers for magnetic levitation, Int. J. System Science, vol. 27, pp.1153-1163, 1996.

[5] Lin L.C. and T.B. Gao, Feedback linearization and fuzzy control of conventional magnetic bearing, IEEE Trans. Control system technology, vol. 5, pp.913-920, 1997.

[6] Edwards, C. & Spurgeon, S. K., Sliding mode control. *Taylor & Francis Ltd*, 1998.

[7] Bartolini, G. & Pydynowski, P., An improved, chattering free, VSC: scheme for uncertain dynamics systems. *IEEE Transaction on Automatic Control* ,**41(8)**, August, 1996.

[8] Jezerink K., Rodic M., Sabanovic A., Sliding mode application in speed senseless torque control of an induction motor. *Proc.15th IFAC world congress. Barcelona, Spine,* July, 2002.

[9] Young, D. K., Utkin,V. I. & Özgüner, A control engineer's guide to sliding mode control. *IEEE Transaction on Control System Technology* **7(3)**, May, 1999.

[10] John Y. Hung, W. Gao and James C. Hung, "Variable Structure Control: A Survey," *IEEE Transactions on Industrial Electronics*, vol.40, no.1, pp.2-21, 1993.

[11] J.J. Slotine and W. Li, Applied Nonlinear Control, *Englewood Cliffs, NJ: Prentice-Hall*, 1991.