# Visualized Simulation of Off-tracking Elimination in the *n*-trailer Problem

Vassilios Deligiannis, Gregory Davrazos, Stamatis Manesis, Nick Koussoulas

**Abstract**--The motion of a multi-articulated either robotic or transportation vehicle is characterized by the deviation of the path of each intermediate vehicle from that of the leading one (off-tracking phenomenon). The advanced visualization of systems response is frequently used for verification of a system's behavior. In this paper, we present the visualization of the motion of a *n*-trailer system with *n=3,5* and its off-tracking elimination using an innovative movable junction and a nonlinear controller in both cases.

**I*n*dex Te m** *n*-trailer systems, off-tracking, nonlinear controller

## I. Introduction

Multi-articulated (or train-like or multi-body) vehicles or *n*-trailer systems can be found in two different research fields: autonomous robotics and transportation systems (referring to vehicles). In autonomous robotics, the goal is to build mobile multi-body robots that accomplish useful tasks without human intervention while operating in unknown environments. On the other hand, in intelligent transportation systems the goal is similarly to construct transportation vehicles intelligent enough to be driven with as less as possible human intervention. In both of the above areas, one major common problem is the undesired excess in motion due to off-tracking. This term refers to the deviation of the path of each articulated vehicle from the paths of preceding vehicles, especially that of the tractor's. The reduction or elimination of off-tracking will result in much improved performance in terms of safety during turns, cornering, overtaking other especially small cars, and backtracking.

The motion of the *n*-trailer system is subject to nonholonomic constraints (rolling without slipping) so it has been studied as a class of nonholonomic systems by many researchers and has both theoretical and practical interest. The research work [4] constitutes an excellent survey of the recent advances in control of nonholonomic systems. The main problem that has attracted most of the attention is path following while only few works consider the off-tracking problem. A closed form expression for the off-tracking of the rear pivot point of a simple tractor-trailer vehicle can be found in [1] while off-tracking bounds for a car pulling trailers have been derived in [3].

In [2] the path following problem with reduced off-tracking is addressed for the *n*-trailer system. This is achieved by keeping track of the error distance of each of the middle points of the axles of the vehicle from the path using different moving frames.

In [9] different passive steering mechanisms as well as control laws are presented for nonholonomic trailer systems. The main focus of these mechanisms is on reducing the passive tracking error with respect to the tractor's trajectory and little attention was paid to active motion control.

The rest of the paper is organized as follows. Section 2 presents the *n*-trailer vehicle model, the off-tracking problem and its consequences in the motion of such vehicles. In section 3 a short description of the innovative junction technique called "sliding kingpin mechanism" is given. In section 4, a nonlinear controller is analytically designed based on the compensation for the steady-state off-tracking deviation. In section 5, visualization of simulation results is presented comparing the performance with and without sliding the analytically derived nonlinear controller. Last section ends the paper with concluding remarks about the results and some open research problems.

## II. The *N*-trailer system

In this section, we describe the kinematic model of the multi-articulated vehicle, which is suitable for both transportation and robotic vehicles. Furthermore, the notion of off-tracking and its consequences in the motion of an *n*-trailer system is explained briefly for each domain of application. The *n*-trailer system is defined as a long and complex vehicle system consisting of a suitable power tractor (leading vehicle) pulling a number of passive robot bodies or semi-trailers as shown in figure 1.

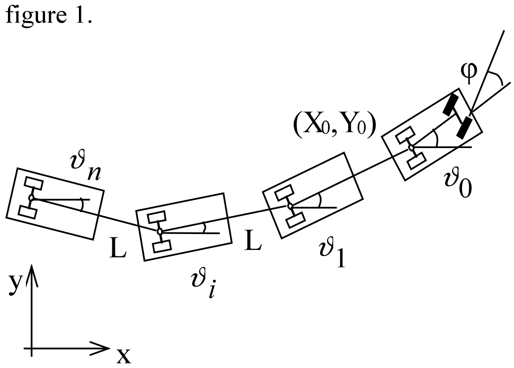


Fig. 1. Illustration of the multi-articulated vehicle coordinates.

The kinematic equations of the *n*-trailer system with a two-axle tractor (of which the front one provides driving and steering) are:

The authors are with Division of Systems and Automatic Control, Department of Electrical & Computer Engineering, University of Patras Rio, Patras 2600 e-mail: stam.manesis@ee.upatras.gr

$$
\begin{aligned}
\dot{x}_0 &= U_1 \cos\theta_0 \\
\dot{y}_0 &= U_1 \sin\theta_0 \\
\dot{\varphi} &= U_2 \\
\dot{\theta}_0 &= \frac{U_1}{L}\tan\varphi \\
\dot{\theta}_1 &= \frac{U_1}{L}\sin(\theta_0-\theta_1) \\
\dot{\theta}_2 &= \frac{U_1}{L}\cos(\theta_0-\theta_1)\sin(\theta_1-\theta_2) \\
\dot{\theta}_i &= \frac{U_1}{L}\left[\prod_{j=1}^{i-1}\cos(\theta_{j-1}-\theta_j)\right]\sin(\theta_{i-1}-\theta_i)
\end{aligned}
\tag{1}
$$

where $x_0, y_0$ are the Cartesian coordinates of the leading vehicle and $U_1, U_2$ are the two control inputs, the linear velocity and the steering angle rate, respectively [8]. The other state variables represent the orientation angles $\theta_i$ for each vehicle *i*, as shown in figure 1. The above state equations are derived from the algebraic manipulation of the *2n* holonomic constraints and the *n+1* nonholonomic constraints. Off-tracking relates to the question of how much road is needed for the rear wheels of a vehicle during a turn and can be defined for cars as well as for multi-articulated vehicles, where the interest focuses on the last vehicle where the phenomenon achieves its strongest demonstration.

In the research field of mobile robotics, the major problems are to find an obstacle free path and a path following control law. When finding an obstacle-free path for multi-articulated robotic vehicles, we must take into consideration the presence of off-tracking. The reason is that the last trailer may collide with obstacles if the vehicle attempts to follow the designed path for the leading vehicle with off-tracking neglected. One efficient way to solve this problem is to find an obstacle-free path for the leading vehicle, add a controller for path following and use another kingpin controller for off-tracking elimination.

In the case of truck-trains it is imperative that the last semi-trailer follows exactly the path of the (lead) tractor during a turn for lane change or a turn due to the curvature of the highway. Otherwise it will be possible for at least the last semi-trailer to violate the outer boundary of the highway or to crash with an adjacent car during a lane change even when both tractor and the car keep their relative velocity within safe limits.

Due to the significance of the off-tracking phenomenon for safety reasons mainly in both research fields a novel movable junction technique is described briefly in the next section.

## III. The Sliding Kingpin Technique

The off-tracking can be eliminated by sliding each trailer with respect to the previous one, a technique firstly described in [5]. According to this technique the kingpin hitch in each semi-trailer slides in a direction perpendicular to the longitudinal axle (i.e. along the rear axle) of the trailer by a distance $S_i$. In this section we present briefly the sliding kingpin system and the state-equations of the multi-articulated vehicle when sliding is used, together with the assumptions that are made during the derivation of the equations. Consider two intermediate semi-trailers of a truck train as shown in figure 2. The position of each semi-trailer $P_i$, is taken to be the middle point of the *ith* semi-trailer's rear axle.

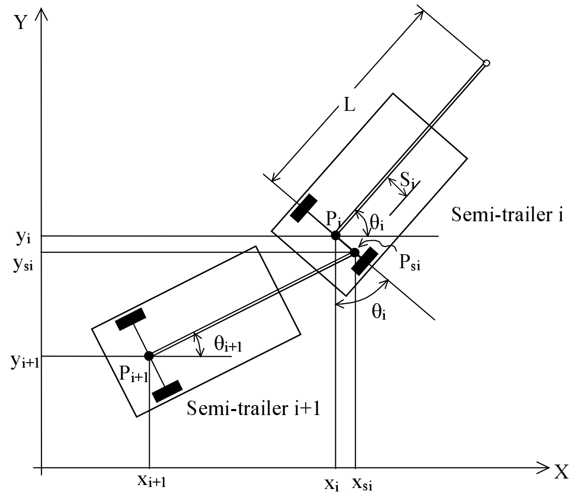


Fig.2. The kingpin slides along the axle when the semi-trailer turns.

Position $P_i$ is defined by the pair $(x_i, y_i)$ in the Cartesian coordinates system while $\theta_i$ is the orientation of the *ith* semi-trailer with respect to the horizontal axis. To simplify derivation of the truck train model we will not consider initially a steering angle for the tractor, since the extension of the model to cover this case is simple. It has been pointed out [3] that when the lead car of a single trailer system is traveling along a circle of radius $R_l$, then the trailer is traveling along a circle of radius $R_t$ with the same center, where $R_t < R_l$. In order to compensate for this path deviation of the trailer, we suppose that the kingpin hitching point slides from the point $P_i$ to the point $P_{si}$ by a distance $S$. In order to be able to derive a rather simple mathematical model for the kinematic equations without taking into consideration the complex effects of this technique in the dynamic behavior of the vehicle we make some assumptions that can be found in detail in [7].

In the general case of a *n*-trailer truck train, we have the classical *n+1* nonholonomic constraints imposed by the rolling and non-slipping condition

$$
\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0 \tag{2}
$$

and *2n* holonomic equations introduced by the corresponding links, which, because of the sliding $S = \overline{P_i P_{si}}$ (see figure 2), are of the form

$$
\begin{aligned}
x_{i+1} &= x_i - L\cos\theta_{i+1} + S_i \sin\theta_i \\
y_{i+1} &= y_i - L\sin\theta_{i+1} - S_i \cos\theta_i
\end{aligned}
\tag{3}
$$

Taking the derivatives of the holonomic Eq. (3), combining them with Eq. (2) and eliminating $\dot{x}_i, \dot{y}_i$ leads to a system of

$n+1$ equations.
The complete solution of this system combined with the equations of motion of the tractor with steering angle for *n=3,5* can be found using Mathematica v4.01. A more detailed version of this work is presented in [7] while here we include the kinematic equations under the assumption $\dot{S}_i = 0$.

$$\dot{x}_0 = U_1 \cos\theta_0$$
$$\dot{y}_0 = U_1 \sin\theta_0$$
$$\dot{\phi} = U_2$$
$$\dot{\theta}_1 = \frac{U_1}{L^2}\sin(\theta_0 - \theta_1)\left[L + S_1(t)\tan\phi\right] \quad (4)$$
$$\dot{\theta}_2 = \frac{1}{L^3}U_1\left[L + S_1(t)\tan\phi\right]\sin(\theta_1 - \theta_2)\ \left[L\cos(\theta_0 - \theta_1) + S_2(t)\ \sin(\theta_0 - \theta_1)\right]$$
....
$$\dot{\theta}_n = \frac{1}{L^{n+1}}U_1\left[L + S_1(t)\tan\phi\right]\sin(\theta_{n-1} - \theta_n)\ \prod_{i=1}^{n-1}\left[L\cos(\theta_{i-1} - \theta_i) + S_{i+1}(t)\sin(\theta_{i-1} - \theta_i)\right]$$

The complex nature of these equations makes very difficult and time consuming but not impossible in principle the derivation of kinematic equations for more semi-trailers but only after making the assumption above we were able to obtain a recursive form for the kinematic equations.

## IV. NONLINEAR CONTROLLER DESIGN

Equations (1) and (4) describe the kinematic behavior of an n-trailer system without and with sliding, respectively. In a multi-articulated vehicle two different controllers are used the one for path following (human driver in transportation vehicles) and the other for off-tracking elimination regulating the sliding distance in the sliding kingpin system. For path following issues the linear velocity and the steering angle rate of the leading vehicle are the control inputs, which are regulated by the "driver".
In the sequel we apply the results in [3] in which, was proven that if the leading vehicle travels along a circular trajectory with radius $r$ (where $r > L$) then the trailer converges to a circular trajectory with radius $R = \sqrt{r^2 - L^2}$ . It is known that the curve radius for a vehicle is given by $r = \frac{U}{\omega} = \frac{U}{\dot{\theta}}$. So in general and for the *ith* trailer this radius will be given by $r = \frac{U_i}{\dot{\theta}_i}$ .By combining this equation and the last equation of (1) and taking into consideration the relation $U_n = U_1 \prod_{j=0}^{n-1}\cos(\theta_j - \theta_{j+1})$ and after some algebraic manipulation this yields that:

$$r_i = L\cot(\theta_{i-1} - \theta_i) \quad (5)$$

We claim that using as sliding distances for the different trailers $S_i = \sqrt{r_i^2 + L^2} - r_i$ the *ith* and the *i-1th* trailers move in the same circular trajectory.

Using the result in [3] that we previously referred and the equations above and after some algebraic manipulation that can be found in detail in [6,7] we conclude with the nonlinear controller for regulating the sliding distance in the *ith* trailer is given by:

$$S_i = L\frac{1 - \cos(\theta_{i-1} - \theta_i)}{\sin(\theta_{i-1} - \theta_i)} \quad (6)$$

## V. VISUALIZATION OF SIMULATION RESULTS

The animated visualization of a system response has become a very competitive alternative to costly experimental studies. In order to test the nonlinear controller, which has been analytically derived, we used the Matlab/Simulink simulation environment. In order to verify the correctness of the previous simulation results Mathematica also have been used. Visualization of simulation results is shown in the following figures with two different ways the first is to show the successive frames of the motion of the n-trailer system with or without the sliding kingpin mechanism and the other one show in one figure the different frames.
Analytically in figure 3 four different frames of the motion of a multi-articulated vehicle with three trailers without the sliding kingpin mechanism are shown while in figure 4 the same frames are shown when the sliding kingpin mechanism is in use. In figure 7 the same frames as in the previous figures are shown in one figure for each case. Visualization of the same simulation results in the case of a multi-articulated vehicle with five trailers is shown in figures 5,6,8. Finally in figure 9 we show in detail the motion of a multi-articulated vehicle with three trailers in order to pinpoint the sliding action of the kingpin mechanism during a sharp turn.

## VI. CONCUSIONS

Off-tracking is one of the most significant problems with potentially dangerous consequences occurring in multi-articulated vehicles. This problem creates difficulties or even renders prohibitive the wide application of multi-articulated vehicles in both robotic and transportation domains. The sliding kingpin mechanism is a mechanism whose principle of operation allows correcting such deviations. In this paper, the main contribution consist primarily of the idea of sliding kingpin and particularly of visualization-verification of the method. We present the visualization of motion of multi-articulated vehicles using the complete kinematic equations for multi-articulated vehicle equipped with a novel movable junction technique the so called "sliding kingpin mechanism". We concentrate on the three and five semi-trailer cases due to the complex nature of these equations. Based on the analysis, we derive a nonlinear controller for adjusting the sliding distance of the sliding kingpin mechanism based on the theoretical steady-state off-tracking when the leading vehicle moves along a circular trajectory. The analytically derived nonlinear controller performed exceptionally eliminating off-tracking in all phases of the vehicle motion.

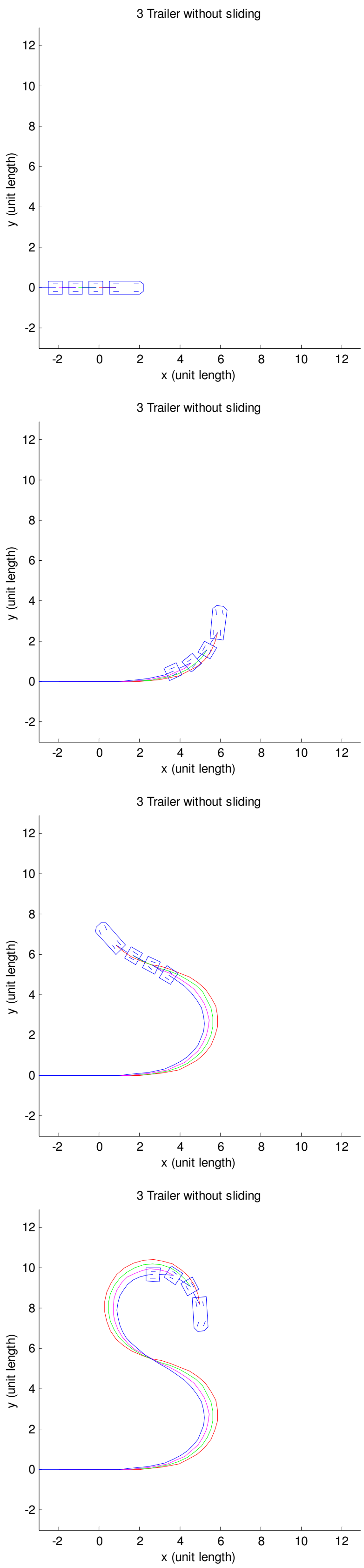


Fig. 3. Different frames of the motion of a multi-articulated vehicle with three trailers mechanism.

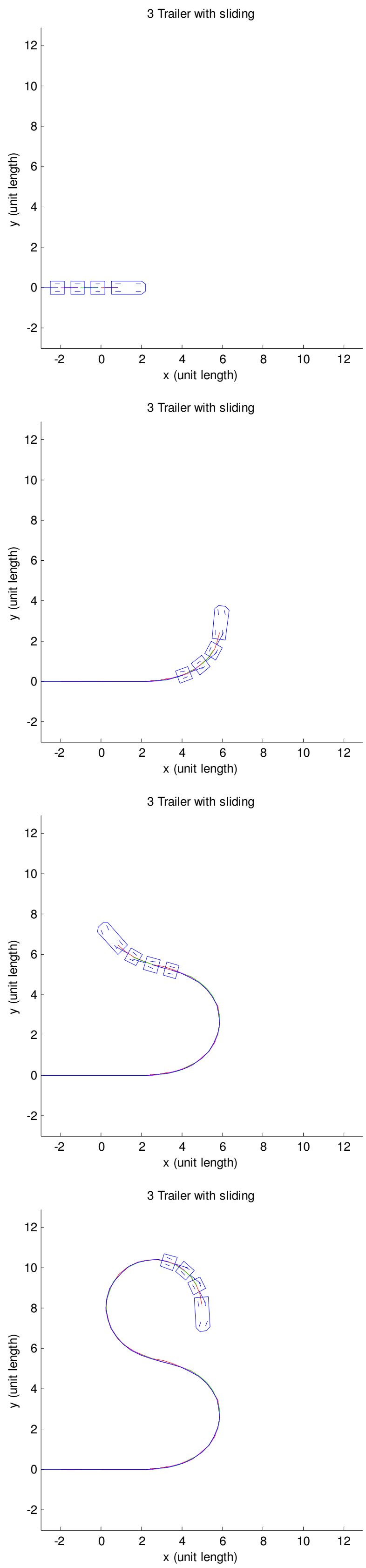


Fig. 4. Different frames of the motion of a multi-articulated vehicle with three trailers with sliding kingpin mechanism.

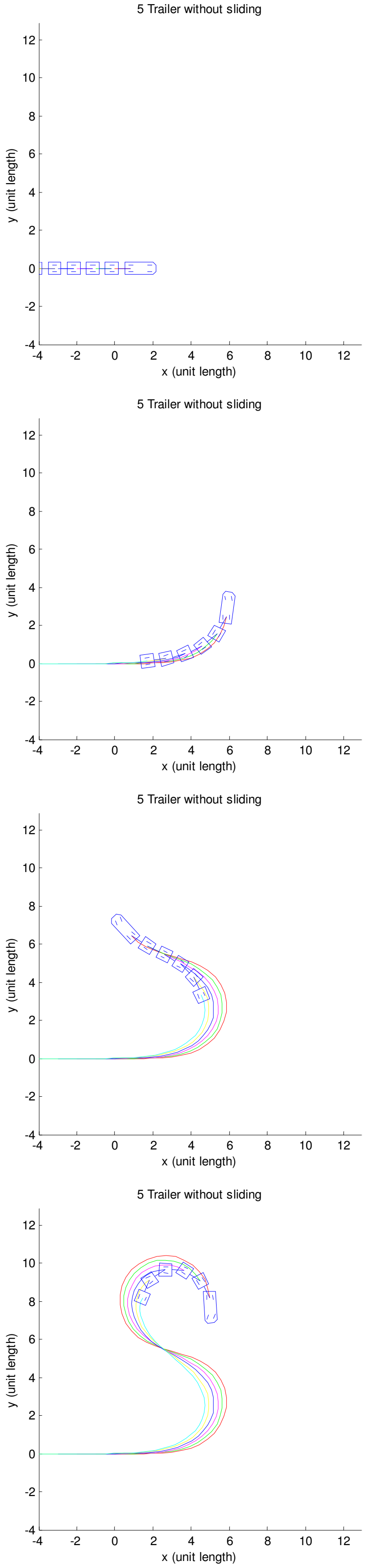


Fig. 5. Different frames of the motion of a multi-articulated vehicle with three trailers mechanism.

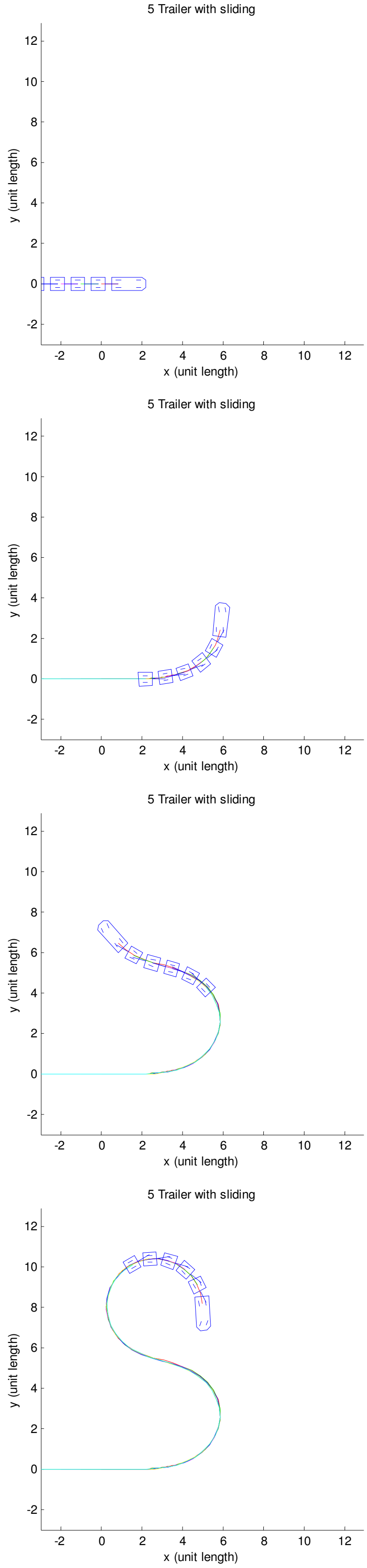


Fig. 6. Different frames of the motion of a multi-articulated vehicle with three trailers with sliding kingpin mechanism.

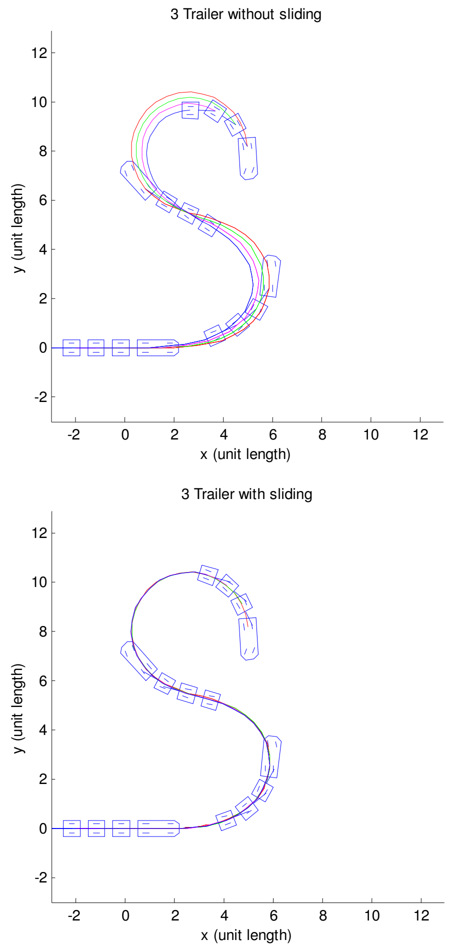


Fig. 7. Different frames of the motion of a multi-articulated vehicle with three trailers without/with sliding kingpin mechanism in one figure

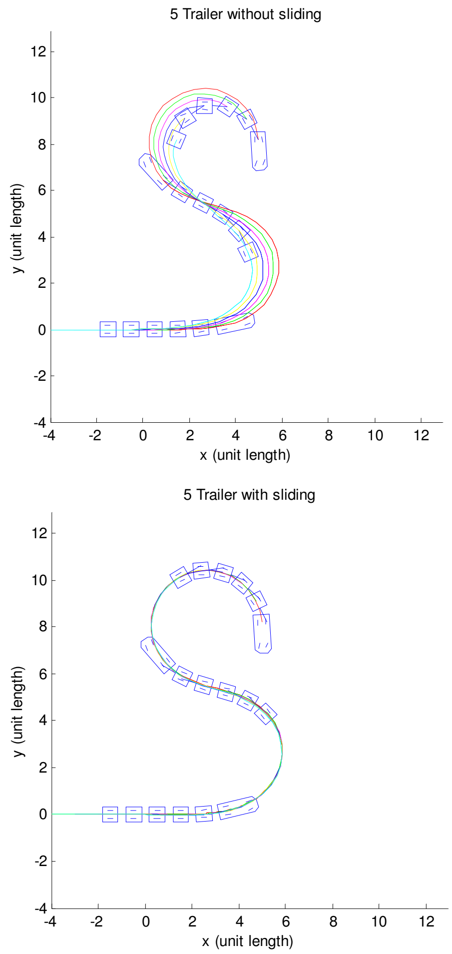


Fig. 8. Different frames of the motion of a multi-articulated vehicle with five trailers without/with sliding kingpin mechanism in one figure.

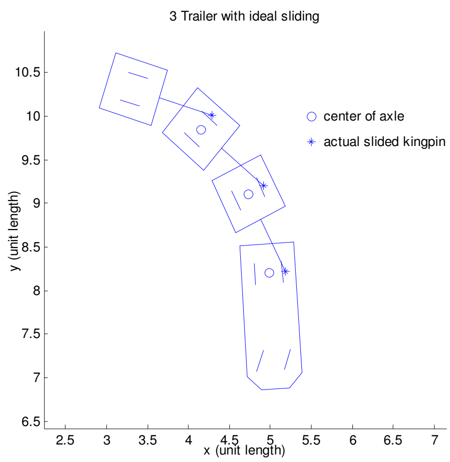


Fig. 9. Sliding action on kingpin in the case of a sharp turn.


## ACKNOWLEDGEMENTS

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