The helicopter rack control

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***Abstract*--The paper compares two approaches to the control of the helicopter rack model. The 1st one, based on the feedback linearization, uses systems equations modified with the aim to obey the naturally unstable zero dynamics. The approach requires a rigorous identification, but, in comparing with traditional PID-control, or with the traditional linear pole-assignment control based on the linearization around a fixed operating point, it yields much higher control quality. The 2nd approach is based on the design of two independent saturating minimum time pole assignment P(I)D controllers for each channel. The controller tuning is based on simplified double-integrator + time delay approximations specified by step or relay experiments.**

***Index Terms*--Exact linearization, unstable zero-dynamics, pole assignment control, constrained control, minimum-time pole assignment control, relay and step-based identification.**

I. INTRODUCTION

The helicopter rack model represents one of the physical systems frequently used in education for testing and demonstrating different control design approaches. One of the main streams in the development of the contemporary control theory is obviously represented by the nonlinear control. In order to be able to "touch" and evaluate the newest control techniques, students can experiment on several physical plant models with typically nonlinear behavior, whereby they can use a database of different already verified approaches.

II. DESCRIPTION OF THE LABORATORY HELICOPTER MODEL

The helicopter model consists of a body situated on a base support (Fig.1). The range of body rotation (measured by incremental sensors) is +-48 degree in elevation and +-175degree in azimuth. The angular velocity of both propellers is measured by integrated tachometers.

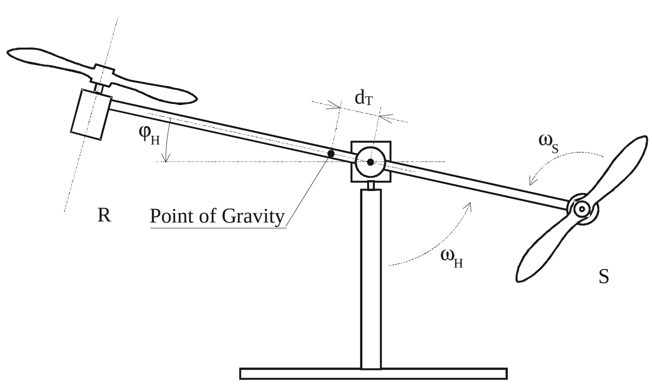

Fig. 1 Helicopter rack model

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The above given parameters correspond to the used model.

Rotor R

c_{uR}=0.0374 NmA^{-1}
$c_{\mu R}$=2.3956e-6 Nms
d_R =0.311 m
J_R=3.1064e-5 kgm^2
k_{MR}=6.9682e-8 kgm^2
k_{FR}=4.3056e-6 Ns^2

Rotor S

c_{uS}=0.0267 NmA^{-1}
$c_{\mu S}$=3.6016e-6 Nms
d_S =0.279 m
J_S=9.5369e-6 kgm^2
k_{MS}=1.7523e-8 kgm^2
k_{FS}=7.4472e-7 Ns^2

Horizontal axe

$c_{\mu H}$=0.0333 Nms
J_H=0.0912 kgm^2

Vertical axe

$c_{\mu V}$=0.0987 Nms
J_V=0.0747 kgm^2

m_G=0.96 kg
d_T=0.016 m
g=9.81 ms^{-1}
i_{Rmax}=0.4 A
i_{Rmin}=0 A
i_{Smax}=0.4 A
i_{Smin}=-0.4 A

The helicopter model is described by following differential equations:

$$\frac{d\omega_R}{dt}=\frac{1}{J_R}\left(c_{uR}i_R-k_{MR}sign\left(\omega_R\right)\omega_R^2-c_{\mu R}\omega_R\right)$$

$$\frac{d\varphi_H}{dt}=\omega_H$$

$$\frac{d\omega_H}{dt}=\frac{1}{J_H}(-k_{FR}sign\left(\omega_R\right)\omega_R^2 d_R+\frac{1}{2}c_{uS}i_S-$$
$$-c_{\mu H}\omega_H+m_G\cos\varphi_H(gd_T-d_S^2\omega_V^2\sin\varphi_H))$$

$$\frac{d\omega_S}{dt}=\frac{1}{J_S}\left(c_{uS}i_S-k_{MS}sign\left(\omega_S\right)\omega_S^2-c_{\mu S}\omega_S\right)$$

$$\frac{d\varphi_V}{dt}=\omega_V$$

$$\frac{d\omega_V}{dt}=\frac{1}{J_V+m_G d_{SL}^2\cos^2\varphi_H}$$
$$(k_{FS}sign\left(\omega_S\right)\omega_S^2 d_S\cos\varphi_H-\frac{1}{2}c_{MR}i_R\cos\varphi_H+$$
$$+2m_G d_S^2\omega_V\omega_H\sin\varphi_H\cos\varphi_H-c_{\mu V}\omega_V)$$

$$y=\left(\varphi_H,\varphi_V\right)^T \tag{1}$$

In the following, the system will be represented by the vector-matrix description:

$$\dot{x}=f(x)+g_1(x)u_1+g_2(x)u_2 \tag{2}$$

whereby

$$f(x)=\begin{bmatrix} \frac{1}{J_R}\left(-k_{MR}sign(x_1)x_1^2-c_{\mu R}x_1\right) \\ x_3 \\ \frac{1}{J_H}(-k_{FR}sign(x_1)x_1^2 d_R - c_{\mu H}x_3 + \\ +m_G\cos x_2(gd_{SP}-d_{SL}^2 x_6^2\sin x_2)) \\ \frac{1}{J_S}\left(-k_{MS}sign(x_4)x_4^2-c_{\mu S}x_4\right) \\ x_6 \\ \frac{1}{J_V+m_G d_S^2\cos^2 x_2} \\ (k_{FS}sign(x_4)x_4^2 d_S\cos x_2 + \\ +2m_G d_S^2 x_6 x_3\sin x_2\cos x_2 - c_{\mu V}x_6) \end{bmatrix}$$

$$g_1(x)=\begin{bmatrix} \frac{c_{uR}}{J_R} \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{c_{uR}\cos x_2}{2(J_V+m_G d_S^2\cos^2 x_2)} \end{bmatrix};\ g_2(x)=\begin{bmatrix} 0 \\ 0 \\ \frac{c_{uS}}{2J_H} \\ \frac{c_{uS}}{J_S} \\ 0 \\ 0 \end{bmatrix}$$

$$y=(x_2,x_5)^T \tag{3}$$

III. NONLINEAR REGULATION BASED ON THE BYRNES/ISIDORI NORMAL FORM.

Using the standard approaches of the feedback linearization [1,2] it is possible to show that the system has two subsystems with the relative degrees $r=[2,2]$. The associated zero dynamics is unstable. So, it is not directly possible to apply the input-output linearization.

A. Modification of systems equations

The problem of the unstable zero dynamics can be obeyed by a simple modification of systems equations (see e.g. [1,2]). After neglecting the relatively small terms in the control matrix, system description becomes

$$\dot{x}=f(x)+\overline{g}_1(x)u_1+\overline{g}_2(x)u_2$$

whereby

$$\overline{g}_1(x)=\begin{bmatrix} \frac{c_{uR}}{J_R} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \overline{g}_2(x)=\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{c_{uS}}{J_S} \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

B. Controller design for modified system

The modified system has relative degree $r=[3,3]$. Therefore, there exist a local diffeomorphism, $\Phi: X\to R^6$, with $\Phi(0)=0$

$$z=\Phi(x)=\begin{bmatrix} x_2 \\ x_3 \\ f_3(x_1,x_2,x_3,x_6) \\ x_5 \\ x_6 \\ f_6(x_2,x_3,x_4,x_6) \end{bmatrix} \tag{5}$$

such that the system can now be transformed to the Byrnes/Isidori normal form

$$\dot{z}=\begin{bmatrix} z_2 \\ z_3 \\ L_f f_3(\varphi^{-1}(z)) \\ z_5 \\ z_6 \\ L_f f_6(\varphi^{-1}(z)) \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ L_{\overline{g}_1} f_3(\varphi^{-1}(z)) \\ 0 \\ 0 \\ 0 \end{bmatrix}u_1+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ L_{\overline{g}_2} f_6(\varphi^{-1}(z)) \end{bmatrix}u_2 \tag{6}$$

For an appropriate choice of the controller parameters vectors k_1 and k_2, it will be transformed via nonlinear feedback

$$u_1=\frac{-L_f f_3(\varphi^{-1}(z))-z^T k_1}{L_{\overline{g}_1} f_3(\varphi^{-1}(z))}$$
$$u_2=\frac{-L_f f_6(\varphi^{-1}(z))-z^T k_2}{L_{\overline{g}_2} f_6(\varphi^{-1}(z))} \tag{7}$$

to a linear decoupled control subsystems. The controller parameters in vectors k_1 and k_2 have been designed by the pole assignment technique. The eigenvalues of the closed loop modified system have been set experimentally. By the simplification process, the real closed loop poles of the linearized system will be slightly different [2].

Even the propellers are driven by one type of motors, significant differences appear between the two channels! They are caused by different working regimes: while the horizontal systems works just with one voltage polarity, the vertical system is working in a commutation regime. This requires use of different amplifiers, what significantly influences the resulting dynamics.

It should also be stressed that the dynamical terms neglected in the design cause a steady state control error. Inverse nonlinear prefilters are used for its elimination.

C. Realization of controllers

Realization of regulators is connected with two problems. Firstly, we have not possibility to measure all state vector coordinates. So, the angular velocities of the vertical and horizontal axis are computed by differentiating the measured angle positions.

Secondly, the relative degree is not globally defined in states, where the velocities of rotors ω_R or ω_S are changing the sign. Constraining the computed control signals can simply solve the 2nd problem.

D. *Plant identification*

The crucial point in designing the feedback linearization controller lies in the plant identification. The estimation of a relatively high number of the nonlinear plant parameters can be simplified by carrying out several particular experiments. There are, however, problems in matching values determined by such experiments – especially the steady state values with those identified during transient responses.

Comparison of the simulation model) and real transients shows Fig.3. While the approximation of the horizontal axis seems to be quite precise, obvious differences can be observed in the movement around the vertical axis.

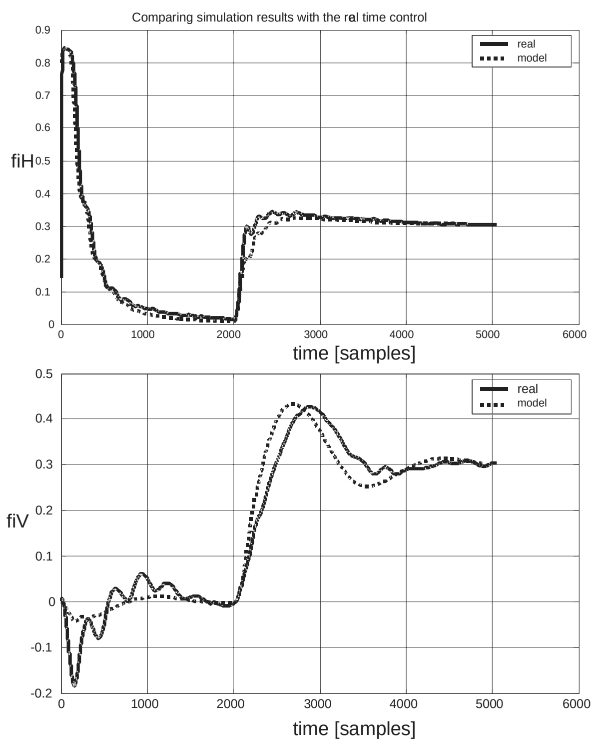

Fig.2 Simulation model and real plant behavior

IV. THE NONLINEAR CONTROLLER

The application of the feedback linearization approach requires rigorous plant identification. However, this is still not enough - the well-elaborated methodology of the feedback linearization represents just one half of the real problem: It is still necessary to find appropriate closed loop poles. For an unstable system of the 6^{th} order with a highly nonlinear dependence on the initial conditions it is far from to be a simple problem (Fig.3)! Choosing relatively fast poles, the closed loop system goes into oscillations. On the other side, due to the unstable plant character, the closed loop poles cannot be too slow!

The "optimal" tuning depends on the measurement (quantization) noise, neglected time lags, but also on the control signal saturation (or the initial deviation). So, the design of a well behaving dynamics is a really challenging and time-demanding task. Only after an experiential tuning it is possible to say that the approach gives excellent results and guarantees stability over the entire working range.

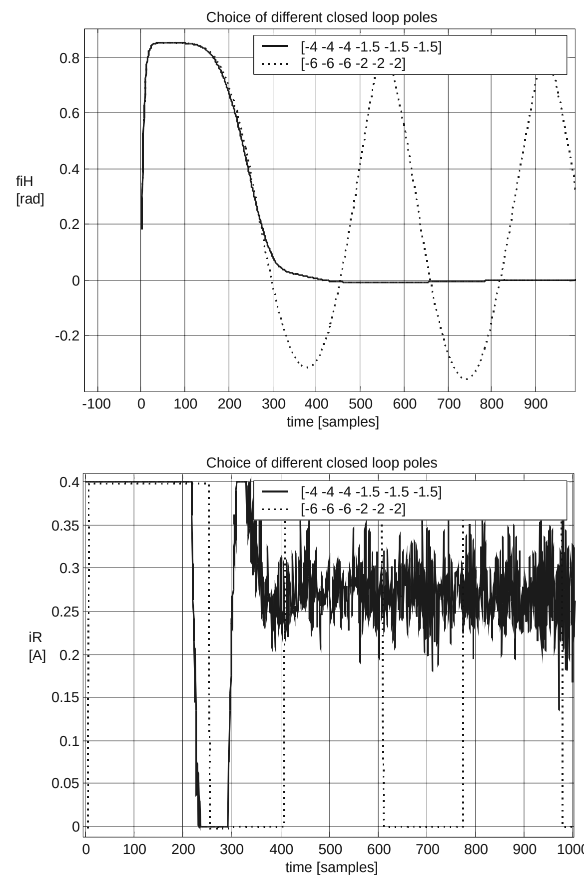

Fig. 3 Feedback linearization: Choice of different poles

V. MTPA CONTROLLER

It is well known that in systems with the control signal saturation, the linear (e.g. PD) controller cannot be sufficiently tuned both for the small and large disturbances. This motivated derivation of the Minimum Time Pole Assignment (MTPA), which involves both the minimum time control and the linear pole assignment control as limit cases of a more general approach. For a 2nd order system, its role is to decrease the representative point distance from a reference braking curve (Fig.4) by a quotient specified by one of the closed loop poles. The 2nd closed loop poles is used in specification of the reference braking curve (RBC). MTPA PD controller has been described e.g. in [3-6]. It enables to calculate optimal control sequences of the control signal u constrained to the interval

$$u \in [U_1, U_2]\,. \qquad (8)$$

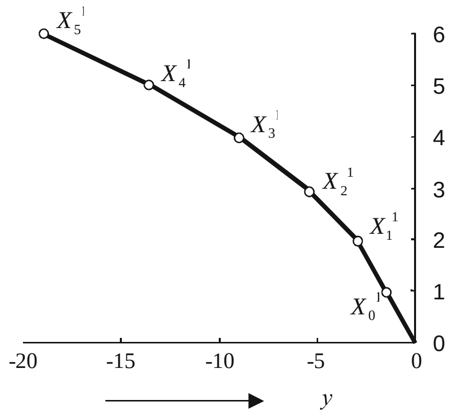

Fig. 4 Reference braking curve (RBC)

In the control algorithm, the limit control signal value U_j for breaking is determined with respect to the sign of the relative position

$$y = x - w \tag{9}$$

with x being the real output and w the reference signal. For $y > 0,\ U_j = U_2$, for $y \le 0,\ U_j = U_1$.

For chosen real closed loop poles λ, the minimum time pole assignment controller takes form of a piecewise linear PD-controller

$$u_f = -\frac{r_o(N)y + r_1(N)\dot{y} + r_C(N)}{K_s} \tag{10}$$

with a variable (integer) parameter N computed according to

$$N = FIX\left(\frac{1-3\lambda}{2(1-\lambda)} + \sqrt{\frac{1-6\lambda+\lambda^2}{4(1-\lambda)^2}} + \frac{p(x)}{K_s U_j T^2}\right) \tag{11}$$

$$for \quad \frac{p(x)}{K\ U_j T^2} > \frac{2\lambda}{(1-\lambda)^2} \quad else \quad N = 1$$

whereby

$$p(x) = 2y + \dot{y}T\,\frac{(1+\lambda)}{(1-\lambda)} \tag{12}$$

$$r_o = \frac{(1-\lambda)^2}{[1-(N-1)(1-\lambda)]T^2}$$
$$r_1 = \frac{(1-\lambda)[3+\lambda+2(N-1)(1-\lambda)]}{[1-(N-1)(1-\lambda)]T^2} \tag{13}$$
$$r_C = \frac{(1-\lambda)[(N-1)(1+\lambda)+(N-1)^2(1-\lambda)]}{2[1-(N-1)(1-\lambda)]}U_j$$

The output signal of the controller is finally constrained according to

$$u = sat(u_f) = \begin{cases} U_1\ ; u_f < U_1 \\ u_f \\ U_2\ ; u_f > U_2 \end{cases} . \tag{14}$$

VI. Equivalent Poles For The Double Integrator With time delays

In order to express the dependence of the closed loop poles on the system's parasitic time delays, a notion of the equivalent poles can be introduced [4,7].

Equivalent closed loop pole can be defined as number, which after a substitution into the algorithm derived for the delay-free system, gives the same controller gains as can be derived for the system with a specified parasitic delay. Here, we will consider equivalent poles for the double integrator system with the two types of time delays. The particular model will be denoted as

$$I_2T_d: \quad S(s) = \frac{K_s}{s^2}e^{-T_d s} \tag{15}$$

$$I_2T_1: \quad S(s) = \frac{K_s}{s^2(T_1 s + 1)} \tag{16}$$

Both approximations gives relatively closed solutions given graphically in Fig.5.

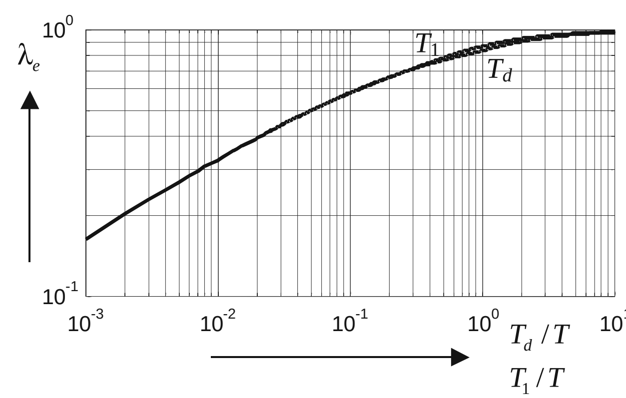

Fig. 5. Equivalent poles λ_e versus normalized dead time T_d/T, or the normalized time constant T_1/T, T being the sampling period.

VII. Plant Approximation

The methods most frequently used for the plant approximation in tuning the industrial controllers are usually based on different modifications of the method originally proposed by Ziegler and Nichols [8].

A. Step response method

The step response based methods are usually interpreted as an approximation of the measured process reaction curve by the single integrator with dead time [9]. In this paper, this idea has been generalized for higher order approximations by the two-parameter models (15) and (16) [5,7].

In applying this method for the helicopter rack model, the 1st problem is caused by the unstable plant behavior in combination with the highly nonlinear plant dynamics. In order to be able to approximate the plant in the vicinity of the most frequently used operating point, it is necessary to use a stabilizing controller. To eliminate the influence of noise and oscillations caused by non-perfect stabilizing controller, it is recommended to work on averaged process reaction curves.

Parameters of the approximative model (15) can be determined by appropriate software (based e.g. on the least-square method), or manually by means of a simple scheme (Fig.6). It is required to approximate the initial phase of the step response as good as possible (Fig.7).

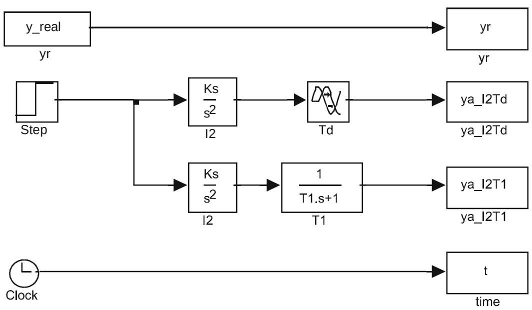

Fig.6. Simple Simulink scheme for "manual" determination of the model parameters

The 2nd problem occurs in approximation of the movement above the horizontal axis, when the unsymmetrical shape of propeller results in unsymmetrical responses (Fig.7).

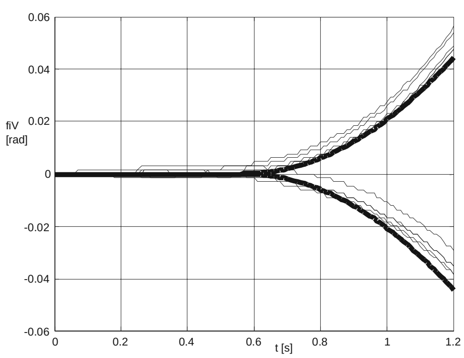

Fig.7. Approximation of the measured step responses – rotation around the horizontal axis

B. *Relay method*

Also in using the relay identification method proposed originally in [10] it is necessary to work with a stabilizing PD-controller expressed e.g. in the form

$$PD(s) = K_R + \frac{sT_D}{1 + s\alpha T_D} \tag{17}$$

From the basic control scheme in Fig.8, using conditions of the harmonic balance, it is possible to derive formulas for determination of the model parameters.

For the parameters of the I_2T_d model one derives

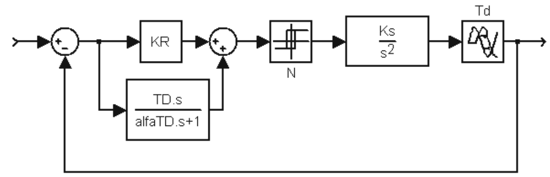

Fig.8. Basic scheme for derivation of formulas for model parameters estimation

$$K_S = \frac{\pi A}{4M} \frac{\omega^2 \sqrt{1 + \omega^2 \alpha^2 T_D^{\,2}}}{\sqrt{K_R^{\,2} + \omega^2 K_R^{\,2} \alpha^2 T_D^{\,2} + 2\omega^2 K_R \alpha T_D^{\,2} + \omega^2 T_{D2}}} \tag{18}$$

$$T_d = \frac{arctg\left(\omega \dfrac{K_R \alpha T_D + T_D}{K_R}\right) - arctg\left(\omega \alpha T_D\right) - 2k\pi}{\omega}$$

$$k = 0, \pm 1, \pm 2, \ldots \tag{19}$$

whereby:
- A – amplitude of sustained oscillations
- ω – angular frequency
- M – relay magnitude
- K_R, T_D, α –PD controller parameters

Similarly, for the K_S I_2T_1 model one gets:

$$K_S = \frac{\pi A}{4M} \frac{\omega^2 \sqrt{1 + \omega^2 \alpha^2 T_D^{\,2}} \sqrt{1 + \omega^2 T_1^2}}{\sqrt{K_R^{\,2} + \omega^2 K_R^{\,2} \alpha^2 T_D^{\,2} + 2\omega^2 K_R \alpha T_D^{\,2} + \omega^2 T_{D2}}} \tag{20}$$

$$T_d = \frac{arctg\left(\omega \dfrac{K_R \alpha T_D + T_D}{K_R}\right) - arctg\left(\omega \alpha T_D\right) - 2k\pi}{\omega} \quad ,$$

$$k = 0, \pm 1, \pm 2, \ldots \tag{21}$$

Combination of all basic approximation yields results shown in Tab.1.

	Relay	Step Response
I_2T_d	$F_H(s) = \frac{2.0321}{s^2} e^{-0.3925s}$ $F_V(s) = \frac{0.3545}{s^2} e^{-1.1195s}$	$F_H(s) = \frac{4.8369}{s^2} e^{-0.1832s}$ $F_V(s) = \frac{1.1105}{s^2} e^{-0.5674s}$
I_2T_1	$F_H(s) = \frac{2.4240}{s^2(0.4427s + 1)}$ $F_V(s) = \frac{0.5177}{s^2(1.4593s + 1)}$	$F_H(s) = \frac{10.249}{s^2(0.7688s + 1)}$ $F_V(s) = \frac{2.8717}{s^2(4.2845s + 1)}$

Tab.1. Results of the plant approximation

VIII. PLANT CONTROL

In controlling unstable system it is necessary to work with as small sampling periods as possible. On the other hand, due to the quantization of the position signals and their differentiation, it is preferable to work with longer periods.

As a result of these two contradictory requirements, the sampling period T=0.1s has been chosen. Then, by means of the normalized time delay values, the equivalent poles have been determined from Fig.5 (or by the corresponding analytical formulas). The 2nd tuning parameter of the controller described in chapter III was the gain K_s.

Due to unstable plant behavior, both the step response and the relay controller tuning require use of stabilizing controller. So, the controller tuning can be done just in an iterative way.

An example of the achieved closed loop responses is in Fig.9. The worst results correspond to the step based approximation using I_2T_1 model. In that case, usable results have been achieved just after a reasonable modification of the identified parameters. So, the next attention will be given to possible improvements of achieved results.

Results of both verified ways of tuning could be further improved experimentally. This was expected and so further research will be oriented in two directions:
- The results of the relay method can be improved by using the saturation with a finite slope [11].
- The results can also be improved by using identification formulas and control algorithms derived for more complicated approximations (e.g. some 3-parameter models).

IX. CONCLUSIONS

Two different approaches to the control of nonlinear unstable system have been verified. The 1st approach of feedback linearization is focused on identification and compensation of the plant nonlinearities. The always-present saturation limits and parasitic time delays can be taken into account just experimentally, by a choice of the closed loop poles.

The 2nd approach considers the saturation limits already in the controller design. The parasitic time delays are included in the process of the plant approximation by the double integrator + time delay model. This method neglects a detail description of the internal nonlinear couplings and so is much simpler to use. Despite of this, the results are comparable. Possibilities of further improvement of the 2nd approach will be investigated.

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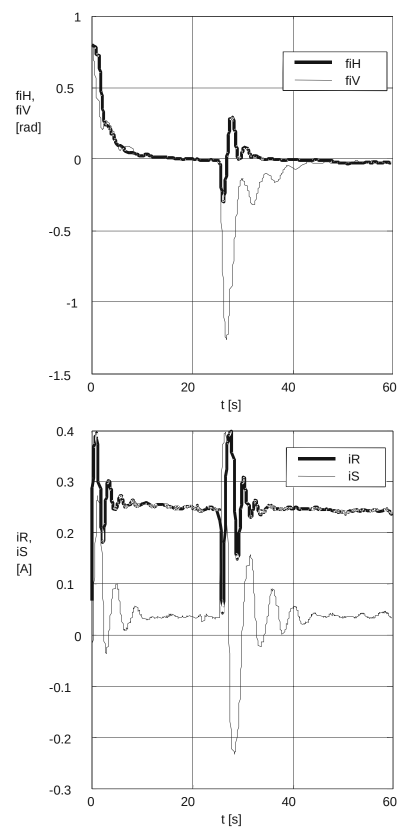

Fig.9. Controller based on I_2T_d-step approximation