Two-Layer Control Of An Ion Exchange Process For Desalination Of Water

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Abstract--The problem for optimal control design of a counter current ion exchange process subjected to the influence of slowly varying disturbances is considered. The approach of repetitive optimization in a two-layer control structure with layers of steady-state optimization and optimal stabilization is applied. The problem for optimal control is decomposed in the corresponding to the layers problems. The first one is solved by a decomposition method of prediction of the dual variables in the Lagrange's function. The second one is solved in two phases - design of a nonlinear controller to linearize the closed loop system and design of an optimal controller for the linearized system. The common control is sum from the controls of the two layers. The developed algorithms and programmes are applied for the model of an ion exchange lab scale plant.

Index Terms—ion exchange, optimal control, two-layer control, steady state, nonlinear controller design

I. Introduction

Counter current ion exchange processes used for removal of salts from water employs a method that involves passing the water through columns of cation exchange and anion-exchange beads in the H^+ and OH^- forms respectively. The mechanism for desalination is to convert the salt into acid by strong cation exchange (in the cation load column with strong cation resin) and subsequently to remove the acid by absorption on a weak anion resin (in the anion load column). The functioning of the columns is counter current- the salt water is coming from the bottom and the resin is coming from the top of the column.

Counter current ion exchange plant for desalination (NaCl) of water was built in Peninsula Technikon, Fig. 1. The basic ion exchange configuration consists of four columns, two columns for cation resin loading and regeneration and two columns for anion resin loading and regeneration. Every column is divided into eight stages by means of multi-orifice stage separator plates. The functioning of the columns is a cyclic one as the control action is the time of the up-flow cycle of wastewater. Sensors, for pH and conductivity measurements in order to determine the water salinity in the input and output column flows are used.

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The problem for design of an optimal control is to find optimal value of the up-flow period such that the concentration of salt in the output flow is less or equal to a previously specified concentration in spite of the influence of slowly varying salt concentration of the input flow of salt water. The slowly varying disturbance leads to indefinite optimal control execution – it causes a drift to set points for the process output. Therefore it becomes necessary first to find a control that will determine the new, depending on the disturbances value of the set points and second to lead the control system to these new steady state values. The hierarchical approach of multi-layer control in the framework of optimization and direct control layers is most suitable for this purpose.

The aim is to develop a method for design of optimal control of the ion exchange process subjected to the influence of slowly varying disturbances, which ensures action integrity of the separate layers for optimization and direct control in order to achieve the specifications of a global criterion function. The method is based on the approach of the repetitive optimization [Celikovsky, 1993; Kozietulski, 1981], realized in the frameworks of a two-layer control structure. The problem is decomposed into two sub problems for: steady state optimization and optimal stabilization. The steady state optimization deals with the determination of new optimal steady state values of process variables. These values directly depend on the values of the disturbances, considered constant for some interval of time i.e. until the next disturbance occurs. The solution of this sub-problem is based on the function of Lagrange The solution of the second sub problem for optimal stabilization is a state feedback in order to reach and maintain the obtained optimal steady state of the process. This sub problem is nonlinear one as the model equation is bilinear. The approach that is accepted is first to linearize the closed loop system by nonlinear control [Isidori, 1995], and then to design linear controller for the linearized closed loop system, which will make the whole system optimal [Tzoneva et al., 1996]. The system is linearized in an input-output sense. In order to overcome the effect of the process parameter variation [Henson and Seborg, 1991, Frecman and Kokotovic, 1995] over the linearising effect of the nonlinear controller its qualities are improved by design of linear control of the linearized system based on quadratic criterion for optimality. Based on the solution of the two sub problems, the global aim is reached by layering of the controls from the two interconnected sub layers, Fig 3.

II. THE PROCESS MODEL

A model of the ion exchange column is developed using mass-balance equations written for every stage [Dodds *et al,* 1973, Xin and Guihua, 1991]. The equations connect all input and output flows of the stages and the column. They express the rates of changing of the concentration of salt in to the water. The following assumptions are made [Tzoneva, 2002]:

1. Both the volumes of every stage and the amount (hold-ups) of resin h and liquid H in each stage are equal just before the transfer. Equal volumes of resin are transferred between the stages.
2. The resin particles are uniform both in size and density at all conversion levels, so that segregation does not occur The transfer of resin between two stages is instantaneous and neither ion exchange reactions, nor adsorption action take place during this period. Hydrodynamic delays in liquid and resin streams are neglected.
3. The fluidized phase is perfectly mixed on each stage and the expanded fluidized bed fills the entire stage volume, this means that the concentration is the same everywhere. There is not back mixing.
4. The operation of the column is at a steady state. The electrical neutrality is maintained. There is a linear equilibrium relationship between the liquid and resin.

The model is obtained on the basis of *Na* component mass balance on the stage n, where the number of stages is *N*. The component mass balance equation for the n-th stage is

$$d(H_n y_n)/dt + d(h_n y_n)/dt = F_{L,n-1} y_{n-1} - F_{L,n} y_n +$$
$$+ F_{R,n+1} x_{n+1} - F_{R,n} x_n, \quad n = \overline{1,N}, \qquad (1)$$

where $H_n, h_n[l], F_L, F_R[mol/h], y_n, x_n[mol]$ are the liquid hold up, the resin hold up, the liquid flow rate, the resin flow rate, the fraction of *Na* in the liquid phase and the fraction of *Na* in the resin phase for the n-th stage. After fulfillment of the assumptions (1-4) $H_n = const = H, h_n = const = h, F_{L.n} = const = F_{L,}$ where a_n, b_n are the slope and intercept of the pseudo equilibrium curve, the model of the n-th stage can be written in the form

$$dy_n/dt = [F_L(y_{n-1} - y_n) + a_{n+1} F_R y_{n+1} - a_n F_R y_n]/$$
$$/(H + a_n h) + F_R(b_{n+1} - b_n), \; n = \overline{1,N}, \qquad (2)$$

where $y_o = y_f, x_{N+1} = 0$, are the initial concentrations of *Na* in the liquid and resin respectively. After selecting as a vector of state space $y = [y_1, y_2, ..., y_n, ...y_N]^T$, the state space model of the ion exchange process can be written in the following form:

$$\dot{y}(t) = Ay(t) + By(t)F_R(t) + B1F_R(t) + Wy_f(t), \quad y(0) = y_0,$$
$$z(t) = Cy(t) \qquad (3)$$

The coefficients of the models are calculated on the basis of theoretical considerations and of the experiments with the plant. The method of the least squares is used. For the purposes of the control calculation and implementation the model is discretized using representation of the first derivative as first difference. The model matrices are

$$A = \begin{bmatrix} \frac{-F_L}{H+a_1h} & 0 & 0 & ... & 0 \\ \frac{F_L}{H+a_2h} & \frac{-F_L}{H+a_2h} & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \frac{-F_L}{H+a_{N-1}h} & 0 \\ 0 & 0 & ... & \frac{F_L}{H+a_Nh} & \frac{-F_L}{H+a_Nh} \end{bmatrix}, B1 = \begin{bmatrix} b_2 - b_1 \\ b_3 - b2 \\ ... \\ b_N - b_{N-1} \\ -b_N \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{-a_1}{H+a_1h} & \frac{a_2}{H+a_1h} & 0 & ... & 0 \\ 0 & \frac{-a_2}{H+a_2h} & ... & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & \frac{a_N}{H+a_{N-1}h} & 0 \\ 0 & 0 & ... & 0 & \frac{-a_N}{H+a_Nh} \end{bmatrix}, W = \begin{bmatrix} \frac{F_L}{H+a_1h} \\ 0 \\ ... \\ 0 \\ 0 \end{bmatrix}$$

$C = [0 0 .....1], C \in R^{1xN}, A \in R^{NxN}, B \in R^{NxN}, \mathbf{B1} \in R^{Nx1}, W \in R^{Nx1},$

$z(t)$ is the process output. Control action for the process is the up-flow time. It is in connection with the value of the resin flow rate on the basis of mass balance of resin in every stage $F_R^s T^s = hd, \quad T^s = hd / F_R^s$, (4)

where the upper index s is for the steady state balance d is the part of the resin hold-up which is transferred from one stage to the next during the pull down period.

III. THE PROBLEM FOR OPTIMAL CONTROL IN THE PRESENCE OF SLOWLY VARYING DISTURBANCES

A. Formulation of the Problem for Optimal Control in the Presence of Slowly Varying Disturbances

Find the control, $F_R(k), \; k = \overline{q, q+K-1}, \; i = \overline{1,N}$, which minimizes the functional

$$J = \frac{1}{2}\left\|y(q+K) - y^{sp}\right\|_S^2 + \qquad (5)$$
$$+ \sum_{k=q}^{q+K-1} \frac{1}{2}\left[\left\|y(k) - y^{sp}\right\|_Q^2 + \left\|F_R(k) - F_R^{sp}\right\|_R^2\right], K \to \infty, K \neq \infty$$

and satisfies the constraints

$$y(k+1) = [1 + \Delta t A] y(k) + \Delta t B \, y(k) \, F_R(k) +$$
$$+ \Delta t B_1 \, F_R(k) + \Delta t \, W y_f(k), \; y(q) = y_q, \qquad (6)$$
$$z(k) = y_N(k) = C \, y(k), \qquad (7)$$

where $k = \overline{q, q+K-1}$, and is determined at the current moment $q = 1,2,3,...., S \in R^{NxN}, Q \in R^{NxN}, R \in R^{1x1}$ are the symmetrical, positively definite weighing matrices, q is the moments at which the slowly varying disturbance changes its value, K – is the number of steps in the optimization problem, y_q – is the initial condition of the state vector at the moments $q = 1,2,3...$, $y_f(k) = y_f = const$,

$k = \overline{q, q+K-1}$ is the slowly varying disturbance, y^{sp}, F_R^{sp} are the state and control set points, determined as nominal values for the ion exchange process. In the formulation of the problem it is assumed that: 1) the control plant (6), (7) is controllable and observable and works in real time, 2) the disturbance is determined - $y_f(k) = y_f$ = constant, can be measured and remains constant during the whole period $[q, q+K-1]$, 3)the initial condition y_q is determined by the actual measured state at the moment $q \geq 0$, $q = 0,1,2...$, 4) in each sequential moment $q \geq 0$, the optimal control can be determined for the period $[q, q+K-1]$. The above assumptions allow the multiplayer strategy of control to be applied to the problem (5)-(7).

B. Decomposition of the Optimal Control Problem into Two Layer Structure

When the determined disturbance, $y_f(k)$ is slowly varying: $y_f(k) = const,\ \ k = \overline{q, q+K-1}$, the optimal solution of the problem can be represented as a sum of steady state and dynamical components:

$$y(k) = y^s + e_y(k), \tag{8}$$

$$F_R(k) = F_R{}^s + e_{F_R}(k), \tag{9}$$

$$z(k) = z^s + e_z(k), \tag{10}$$

where, $y^s, F_R^s,\ z^s$ - are the components of the optimal solution of the initial problem (6) – (7) which depend on the slowly varying disturbance, $e_y(k),\ e_{F_R}(k),\ e_z(k)$ – are the components expressing the transitional behavior of the process moving from the steady state values of the variables at the moment (*q*-1) to the values of the steady state variable at the moment *q*. The components $e_y(k)$, $e_{F_R}(k)$, $e_z(k)$ do not depend on the action of the slowly varying disturbances.

The model can now be decomposed into steady state and dynamic sub-models that can be used in solving the optimal control problem in the two-layer structure [Popchev. & Tsoneva, 1992]. After substitution of the state, control and output (8) into the dynamical equations, the model (6) – (7) can be represented as a sum of two models:

- steady state model

$$y^s = [I + \Delta t A] y^s + \Delta t B y^s F_R{}^s + \Delta t B_1 F_R{}^s + \Delta t\, W y_f,$$

$$z^s = C\, y^s, \tag{11}$$

- and dynamic model

$$e_y(k+1) = [I + \Delta t A + \Delta t B F_R{}^s]\, e_y(k) +$$

$$+ \left[\Delta t B y^s + \Delta t B_1\right] e_{F_R}(k) + \Delta t B e_y(k)\, e_{F_R}(k),$$

$$e_y(q) = y_q - y^s = e_{yq}, \tag{12}$$

$$e_z(k) = C\, e_y(k). \tag{13}$$

As the disturbance y_f can be considered piecewise constant, the steady state optimization problem can be solved where the solution will depend on the value of the disturbance, as y^s, and $F_R{}^s$ in equation (11) directly depend on the value of y_f. The criterion is also decomposed into two parts - for steady state optimization and for the dynamic stabilization. $J = J_s + J_d$, (14)

$$J_s = \frac{1}{2}[\left\|y^s - y^{sp}\right\|^2_{S+KQ} + \left\|F_R^s - F_R^{sp}\right\|^2_{KR}], \tag{14a}$$

$$J_d = \tfrac{1}{2}\{2(y^s - y^{sp})^T S e_y(q+K) + \sum_{k=q}^{q+K-1}[\left\|e_y(k)\right\|^2_Q + \left\|e_{F_R}(k)\right\|^2_R +$$

$$+ 2(y^s - y^{sp})^T Q e_y(k) + 2(F_R^s - F_R^{sp})^T \operatorname{Re}_{F_R}(k)]\} \tag{14b}$$

Then using both models and criteria, separate problems for a steady state optimization and dynamic stabilization can be solved. The errors $y^s - y^{sp}$, $F_R^s - F_R^{sp}$ in (14b) are considered very small because their minimization through the solution of the steady state optimization problem. Their values are accepted to be approximately zero in the solution of the dynamic sub problem.

C. The Problem for Steady State Optimization

The steady state optimization problem is formulated as follows: Find the control $F_R{}^s$ in such a way that, (14a) is minimized under the model constraints (11). The solution is based on the function of Lagrange [Tzoneva, 2002]

$$L_s = 1/2\left[\left\|y^s - y^{sp}\right\|^2_{S+KQ} + \left\|F_R{}^s - F_R{}^{sp}\right\|^2_{KR}\right] + \\ + \lambda_s{}^T\left[A y^s + B y^s F_R{}^s + B_1 F_R{}^s + W y_f\right], \tag{16}$$

where, $\lambda_s \in R^N$ - is the vector of the Lagrange multipliers.

The solution of the problem can be found from the necessary conditions for optimality of the Lagrange function. The necessary conditions for optimality are:

$$\partial L_s / \partial y^s = (S + KQ)(y^s - y^{sp}) + A^T \lambda_s + B^T \lambda_s F_R{}^s = e_y^s,$$

$$\partial L_s / \partial F_R{}^s = KR(F_R^s - F_R^{sp}) + (B y^s)^T \lambda_s + B_1{}^T \lambda_s = e_{F_R^s}, \tag{17}$$

$$\partial L_s / \partial \lambda_s = A y^s + B y^s F_R{}^s + B_1 F_R{}^s + W y_f = e_{\lambda_s}. \tag{18}$$

The second derivatives according to the state and control

$$\partial L_s{}^2 / \partial^2 y^s = S + KQ > 0, \quad \partial L_s{}^2 / \partial^2 F_R{}^s = KR > 0, \tag{19}$$

are positive. In the considered case the necessary conditions for optimality are also sufficient and the obtained solution is unique. This problem is a convex one. The solution of the obtained nonlinear system of equations will determine the optimal solution of the steady state problem. The system (17),(18) has three equations with three unknown variables, y^s, F_R^s and λ_s. y^s and F_R^s are the solutions of the primal (initial) problem and λ_s is the solution of the dual problem. Because the solution of the primal problem y^s and F_R^s depends on the values of the Lagrange multipliers the calculation procedure could be built using the hierarchical principle of coordination of the aims of the subsystems, where λ_s has some previously given value. This means that if the optimal solution for λ_s is obtained, the optimal solutions for y^s and F_R^s can be obtained. For this purpose it becomes necessary to introduce some hierarchical computing structure, which will realize this strategy of the solution of the system of equations. This structure is shown in the Fig.2. The calculation is decomposed into two levels: Second level – coordinating sub-problem and First level – calculating for every value of the coordinating variable λ_s the values of the state and the control. The solution of the coordinating sub-problem is based on the necessary condition for optimality (18). As analytical solution for the dual variable is not possible to be obtained, the gradient method is used in the form:

$$\lambda_s^{(j+1)} = \lambda_s^{(j)} + \alpha^{(j)} e_{\lambda_s}^{(j)}, \qquad (20)$$

The process of calculation of λ_s is gradient one where $\alpha^{(j)}$ is the step of calculation procedure, $e_{\lambda s}^{(j)}$ is the value of the gradient function of Lagrange at the j^{th} iteration and j is the index of iterations. The value of the gradient $e_{\lambda s}^{(j)}$ (18) gives the direction of the search of maximum of Lagrange function towards λ_s. Its j^{th} value is obtained using the values of the calculated state y^s and control F_R^s at each j^{th} iteration. The optimal solution will be obtained when $e_{\lambda_s}^{(j)}$ is very close to zero, or $\left|e_{\lambda_s}^{(j)}\right| \le \varepsilon$, (21)

where, $\varepsilon > 0$ - is a small (positive) value of the error. When the error is smaller than ε, then the necessary condition for optimality of λ_s is fulfilled. The obtained values of $\lambda_s^{(j)}$ are substituted in the equations of the necessary conditions for y_s and F_R^s in order to solve the sub-problems for the state and the control calculation (First Level). Then y^s and F_R^s are obtained analytically from equations (17) – (18) after some mathematical transformations, as

$$\begin{aligned} y^{s(j)} &= y^{sp} - Q^{-1}A^T\lambda_s^{(j)} - \\ &- Q^{-1}B^T\lambda_s^{(j)}\{(R - \lambda_s^{T(j)}Q^{-1}B^T\lambda_s^{(j)})^{-1}RF_R^{sp} + \\ &+(R-\lambda_s^{T(j)}Q^{-1}B^T\lambda_s^{(j)})^{-1}. \\ &.[(y^{sp})^T B^T\lambda_s^{(j)} + \lambda_s^{T(j)}AQ^{-1}B^T\lambda_s^{(j)}] - B_1^T\lambda_s^{(j)}\}. \end{aligned} \qquad (22)$$

$$\begin{aligned} F_R^{s(j)} &= (R - \lambda_s^{T(j)}Q^{-1}B^T\lambda_s^{(j)})^{-1}RF_R^{sp} + \\ &+(R-\lambda_s^{T(j)}Q^{-1}B^T\lambda_s^{(j)})^{-1}. \\ &.[(y^{sp})^T B^T\lambda_s^{(j)} + \lambda_s^{T(j)}AQ^{-1}B^T\lambda_s^{(j)}] - B_1^T\lambda_s^{(j)}. \end{aligned} \qquad (23)$$

The equations (22),(23) use only the values of the weighting matrices R and Q, the matrices of the model and the value of the dual variable, λ_s. The obtained values are used for calculation of the gradient of $e_{\lambda s}$ and so on. The optimal solution is obtained when the condition (21) is fulfilled.

D. *Problem for Design of the Closed Loop Control*

The problem for design of the closed loop control is formulated for the dynamic part of the model, the part describing the deviation of states from the steady states. The criterion is the dynamic part of the common criterion. The problem is: Find the control $e_{F_R}(k)$, $k = \overline{0, K-1}$ such that the criterion (14b) is minimized under the model equations (12). The problem (14b), (12) is a nonlinear one. It is not possible to design a linear state space controller. The approach that can be accepted is first to linearize the closed loop system by nonlinear control, and then to design linear controller for the linear closed loop system, which will make the whole system optimal.

1) *Nonlinear Control Design*

The input/output linearizing control design consists of calculating a state feedback that transforms the nonlinear initial system into a closed loop linear system. The problem is to find a static state feedback law of the form:

$$e_{F_R}(k) = \varphi[\cdot] + \beta[\cdot]\nu(k), \qquad (24)$$

$$[\cdot] = [y(k), A, B, B_1, W, \Delta t, y^s, F_R^s]. \qquad (25)$$

such that:1)the system (12), (24) is locally stable around the steady state point y^s, F_R^s; 2) the tracking error $y^s - y(k)$ is governed by a pre-specified stable linear model, called reference model; 3) the closed loop system (12), (24) is robust in some limits towards, parameter variation, where $\nu(k)$ – is an external reference signal for the nonlinear controller, φ and β – are smooth vector functions in a neighborhood of the set-point and, $\beta[\cdot] \ne 0$.

The applicability conditions for the linearization of multi-variable systems by nonlinear control [Isidori, 1995]: the system must have a well defined vector relative degree and the system must be minimum phase, are fulfilled. It is assumed that the process is completely controllable and all states are accessible to feedback control.

The aim is the map between the input $\nu(k)$ and the state error vector $e_y(k)$ to be linear, equal to the reference model. The problem is to synthesize the control in such a way that the process dynamics exactly tracks desired ones, given by the reference model.

$$e_y(k+1) = Le_y(k) + \nu(k), \qquad (26)$$

$$e_z(k) = e_y(k), \qquad (27)$$

where L – is the diag$\{li\}$, li = const, $i = \overline{1, N}$, $L \in R^{NxN}$ – is the constant matrix, given by the desired dynamics.

2) *Nonlinear Control Problem Solution*

The right part of the equations (12) and (26) are equalized

$$\begin{aligned} &[I + \Delta tA + \Delta tBF_R^s]e_y(k) + \Delta tBe_y(k)e_{F_R}(k) + \\ &+[\Delta tBy^s + \Delta tB_1]e_{F_R}(k) = Le_y(k) + \nu(k). \end{aligned} \qquad (28)$$

The nonlinear control can be expressed as a function of the state space error after some algebraic transformations of equation (28). Then the control can be obtained as,

$$e_{F_R}(k) = M[L - I + \Delta tA + \Delta tBF_R^s]e_y(k) + M\nu(k), \qquad (29)$$

$$\begin{aligned} M = &\{[\Delta tBe_y(k) + \Delta tBy^s + \Delta tB_1]^T. \\ &.[\Delta tBe_y(k) + \Delta tBy^s + \Delta tB_1]\}^{-1}.[\Delta tBe_y(k) + \Delta tBy^s + \Delta tB_1], \end{aligned} \qquad (30)$$

The matrix $[\Delta tBe_y(k) + \Delta tBy^s + \Delta tB_1]^T[\Delta tBe_y(k) + \Delta tBy^s + \Delta tB_1]$ is a square matrix and it is supposed that its inverse exists.

Equations (29), (30) represent the nonlinear control. It depends on the values of desired linear dynamics *L*, the reference input ν and the current error $e_y(k)$. This control can be realized in real time after measurement of y(*k*). It can be seen that the control e_{F_R} is very sensitive to variations in the model parameters, and also to the values of previously calculated y^s and F_R^s. The closed loop system (12), (29), (30) is linearized. It has linear dynamics and it is possible to use optimal control theory to find the external reference ν such that the error $e_y(k)$ is minimized and the control reference signal is also minimized.

3) *Synthesis of Linear Stabilizing Control*

In equation (29) the value of the reference signal is not known. It can be obtained as linear optimal control according to the criterion (14b) and the reference model. The problem can be formulated in the following way: Find the control $\nu(k)$, $k=\overline{0,K-1}$, which minimizes the functional (14b), where only the optimizing part of the control is considered, under the constraints (26), (27). The solution of the optimal control problem is based on the functional of Hamilton: (31)

$$H=\sum_{k=q}^{q+K-1}\{\|e_y(k)\|_Q^2+\|\nu(k)\|_R^2+\,p^T(k+1)\left[Le_y(k)+\nu(k)\right]\},$$

where $p(k)\in R^{\,n}$ – is the conjugate variables' vector. The optimal control is found on the basis of the necessary condition for optimality of H . The control is obtained as

$$\nu(k)=-R^{-1}(L^T)^{-1}\left[G(k)-Q\right]e_y(k)=He_y(k),$$

$$H=-R^{-1}(L^T)^{-1}\left[G(k)-Q\right], \qquad (32)$$

$$G(k)=Q+L^TG(k+1)\left[I+R^{-1}G(k+1)\right]^{-1}L\,,G(K)=S\,,(33)$$

where the obtained equation (33) is a Riccatti equation. It is solved from the last moment $k=q+K$ till the beginning $k=q$. Control depends on the real value of the error $e_y(k)$.

4) *Common Dynamic Control*

The dynamic nonlinear control according to (29) and (32) is

$$e_{F_R}(k)=M[L-1+\Delta tA+\Delta tBF_R^s]e_y(k)-$$

$$-MR^{-1}(L^T)^{-1}[G(k)-Q]e_y(k)=M[V-H]e_y(k). \qquad (34)$$

$$V=[L-I+\Delta tA+\Delta tBF_R^s]\,. \qquad (35)$$

When the optimization interval is sufficiently long K → ∞, K ≠ ∞ then the solution of the Ricatti equations tend to be constant $G(k)\to G(0)=G=const$, and this constant value can be used in (34). The control consists of two parts: nonlinear one, which linearizes the nonlinear model, and linear one, which minimizes the error and control effort and reduces the influences of parameter variations. The common optimal control in the two layer structure is given by (29) and (34). This control can be realized in a real time, as it is a function only of the current value of the state error and a measured disturbance, Fig. 3.

IV. OPTIMIZATION METHOD APPLICATION

The calculations are carried out for the model of the ion exchange process plant for desalination of water built at the Department of Chemical Engineering, Peninsula Technikon. The process is for removing of *NaCl* from wastewater. The values of the process parameters are *a*=[5.67 4.58 4.32 4.18 4.05 3.4 2.7 2.92], *b*=[0.0009 0.016 0.016 0.016 0.016 0.01 0.014 0.005], *h*=28.96 $[dm^3]$, *H*=37.62$[dm^3]$, $F_L=2000[dm^3/h]$, $y_f(i)=1.2[g/l], i=\overline{1,N}$, $y_0(i)=2.2[g/l],\ i=\overline{1,N}$,

$y^{sp}=[0.05\ 0.1\ 0.4\ 0.5\ 0.6\ 0.8\ 1.2\ 1.4][g/l]$. The calculated matrices of the model are $C=[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1]^T$,

$$B1=|0.01\ \ 0.01\ \ 0.01\ \ 0.01\ \ 0.01\ \ 0.01\ \ 0.01\ \ -0.15|^T$$

$$W=[28.787\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$$

$$A=\begin{bmatrix}-28787 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 27.635 & -27.635 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 24673 & -24673 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 2424 & -2424 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 23822 & -23822 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 23339 & -23339 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 23182 & -23182 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 22285 & -22285\end{bmatrix}$$

$$B=\begin{bmatrix}-0.0158 & 0.0173 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -0.0166 & 0.0207 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -0.0185 & 0.0191 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -0.0188 & 0.0194 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -0.0191 & 0.0198 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -0.0194 & 0.0196 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -0.0195 & 0.0209\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0201\end{bmatrix}$$

The result from the program for simulation of the process with the proposed control is shown on Fig 4.

V. CONCLUSION

The aim is to develop a unified method for synthesis of optimal control of ion exchange process which ensures action integrity of the separate layers for optimization and direct control to achieve a global criterion function, in the presence of slowly varying disturbances. This methodology is usually based on the approach of the repetitive optimization. The problem is decomposed into two sub-problems for steady state optimization and for steady state stabilization. The steady state and dynamic sub-problems are solved repetitively after long periods of time, at moments when the value of the main slowly varying disturbance is changed. The designed control structure has characteristics of an optimal feed-forward and feedback control. The feed-forward control attempts to eliminate the effects of measurable disturbance – the input flow concentration. The feedback control corrects for immeasurable disturbances and modeling errors. It performs two actions : 1)linearizing the closed loop system according to desired dynamics of a linear stable system, and 2)optimally stabilizing the state of the linearized system according to the optimal steady state value. In this way the total behavior of closed loop system is linearized and robust according to unmeasured disturbances and model parameter variations.

The control structure requires measurement of the input flow concentration as main disturbance, which can be done easily with the existing pH and conductivity meters and measurements of the concentrations of salt in every stage, which at the moment cannot be realized because the stages are not accessible. These measurements will be developed at later phases of the building of the ion exchange plant.

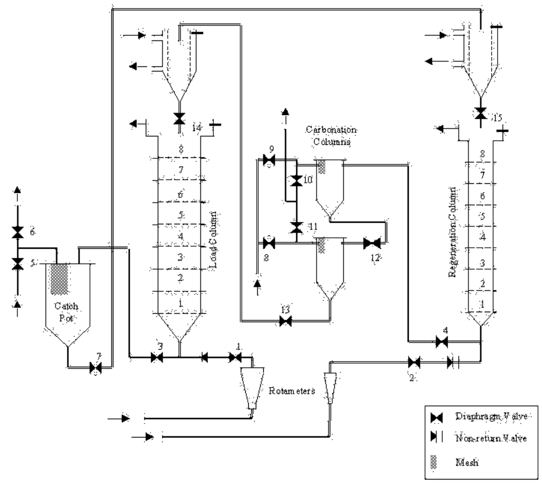

Fig. 1 Ion exchange pilot plant

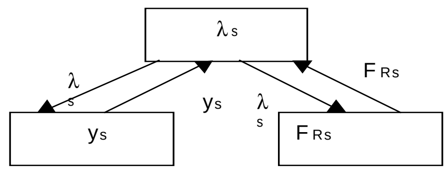

Fig. 2. Two level calculating structure

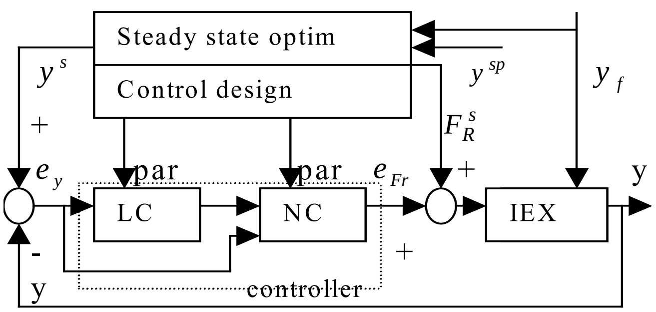

Fig. 3. Two layer control of the process

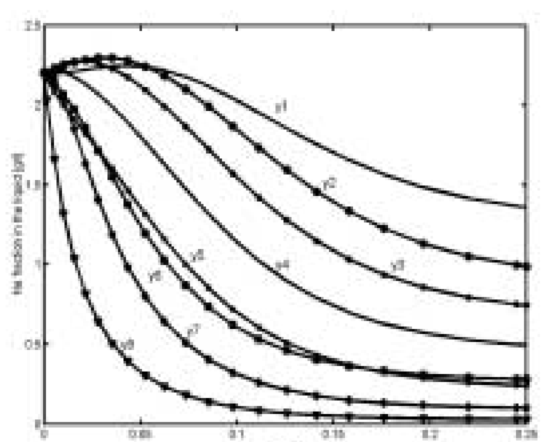

Fig.4. State trajectories under two-layer control

VI. ACKNOWLEDGEMENTS

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