Fuzzy Control of Mean Arterial Pressure During Anesthesia

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***Abstract*-- In this paper, a fuzzy logic controller is designed and developed for the control of Mean Arterial Pressure (MAP) of a patient during the anesthesia. The main purpose of the paper is to constitute a basis for the further real applications. In the simulation studies, the patient is represented with a linear mathematical model that includes time delay elements. The parameters of the fuzzy controller are tuned in order to obtain a robust control performance against the plant parameter variations. Since the modern control is usually realized by using a microprocessor, the discrete time analysis and design of the control system are given in the paper. The simulation results showing the robust control performance of the proposed fuzzy controller under the plant parameter variations are presented.**

***Index terms*-- Fuzzy logic, control, mean arterial pressure (MAP), dept of anesthesia.**

I. INTRODUCTION

The main task of an anesthetist is to control the depth of anesthesia during a surgery in order to provide a painless and comfortable operation for patients [1-3]. However, the depth of anesthesia cannot be easily measured or estimated. The measurement of the Mean Arterial Pressure (MAP), heart rate and pupil size provide some information about the depth of anesthesia. The MAP is used as the most reliable guide for dosing anesthetics by anesthesiologists. The patient's MAP should lie within a predefined range for a good control of depth of anesthesia. The main reason for automating the control of the depth of anesthesia is to release the anesthetist so that he/she can devote his/her attention to other tasks such as controlling the fluid balance, ventilation and drug applications which cannot be adequately automated, thus increasing the patient's safety. Thus, the control of MAP appears as one of the most effective way of controlling depth of anesthesia.

In recent years, fuzzy logic has been used for the control of MAP since the experts (anesthesiologists) knowledge can be embedded into the controller using fuzzy logic [1, 3]. However, the effects of parameter variations to the control performance of the system are not explicitly considered in most studies. In this paper, the design of a fuzzy logic controller providing a robust control under parameter variations is explained. Simulation results showing the effectiveness of the proposed fuzzy controller under parameter variations are presented.

II. MATHEMATICAL MODEL OF A PATIENT UNDER ANESTHESIA

In this study, the mathematical model of a patient developed in [1] is used for simulation and controller design. The relation between inflow concentration of isoflurane u(t) (input variable) and the resulting MAP y(t) (output variable) is modeled as the sum of two first order terms each with a pure time delay. The model includes the patient and also a semi-closed circuit that is used to deliver the anesthetic agent to the patient. The unit step response is determined as [1]

$$y(t) = K_1[1-e^{-a_1(t-t_1)}]u(t-t_1) + K_2[1-e^{-a_2(t-t_2)}]u(t-t_2) \qquad (1)$$

where K_1= -3, K_2= -7.3, τ_1= 23s, τ_2= 101s, a_1= 0.01, and a_2= 0.006. The Laplace transform of (1) is

$$Y(s) = K_1[\frac{1}{s} - \frac{1}{s+a_1}]e^{-t_1 s} + K_2[\frac{1}{s} - \frac{1}{s+a_2}]e^{-t_2 s} \qquad (2)$$

and since the input is unit step, i.e., U(s) = 1/s, the transfer function between output Y(s) and the input U(s) becomes

$$G_p(s) = \frac{Y(s)}{U(s)} = K_1[1 - \frac{s}{s+a_1}]e^{-t_1 s} + K_2[1 - \frac{s}{s+a_2}]e^{-t_2 s} \qquad (3)$$

or it can be written as

$$G_p(s) = G_{p_1}(s) + G_{p_2}(s) \qquad (4)$$

where

$$G_{p_1}(s) = \frac{K_1 a_1 e^{-t_1 s}}{s+a_1} \text{ and } G_{p_2}(s) = \frac{K_2 a_2 e^{-t_2 s}}{s+a_2}.$$

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In order to obtain a polynomial transfer function and get rid of the time delay elements, we may use Pade approximations [4]. However, since the design and simulations will be in discrete time, it is reasonable to use the modified Z-transform that is applied to systems containing time delay elements [5]. Note that in the closed loop digital control system, there is a zoh (zero order hold) preceding the $G_p(s)$. Therefore the modified Z-transform of (4) becomes

$$G_p(z) = \frac{z-1}{z} Z\left\{ \frac{z^{-\ell_1} a_1 K_1 e^{m_1 Ts}}{s(s+a_1)} \right\} + \frac{z-1}{z} Z\left\{ \frac{z^{-\ell_2} a_2 K_2 e^{m_2 Ts}}{s(s+a_2)} \right\} \qquad (5)$$

which can be written as

$$G_p(z) = \frac{zK_1 - zK_1 e^{-a_1 m_1 T} + K_1 e^{-a_1 m_1 T} - K_1 e^{-a_1 T}}{z^{\ell_1}(z - e^{-a_1 T})} + \frac{zK_2 - zK_2 e^{-a_2 m_2 T} + K_2 e^{-a_2 m_2 T} - K_2 e^{-a_2 T}}{z^{\ell_2}(z - e^{-a_2 T})} \qquad (6)$$

where

ℓ : an integer,
m : a positive real number less than 1,
T : sampling period,
$t_1 = \ell_1 T - m_1 T$ and $t_2 = \ell_2 T - m_2 T$.

If the nominal values of the parameters are used (K_1 =-3, K_2 =-7.3, a_1 =0.01, a_2 =0.006, τ_1 =23s, τ_2 =101s and T=10 s) then the modified Z-transform of $G_p(s)$ becomes

$$G_p(z) = \frac{-0.20282 z^{10} + 0.10834 z^9 + 0.077855 z^8 - 0.38375 z^2 + 0.30585 z + 0.037436}{z^{13} - 1.8466 z^{12} + 0.85214 z^{11}} \qquad (7)$$

Hence, the discrete time transfer function given by equation (7) is the mathematical model representing the patient (including the semi-closed circuit) in the simulation studies.

III. Design Of The Fuzzy Controller

Fuzzy theory was first introduced by Zadeh in 1965 [6]. During the last two decades, Fuzzy Logic Control (FLC) has emerged as one of the most attractive and fruitful areas for research in the application of the fuzzy theory to the real engineering problems. FLC is actually a practical alternative to the conventional control methods for a variety of control applications since it provides a convenient method for implementing linear and non-linear controllers via the use of both heuristic and mathematical information [7].

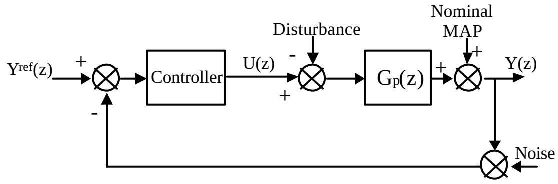

Fig. 1. The block diagram of the discrete time closed loop control system

Fig.1 shows the block diagram of the discrete time closed loop control system used in simulation studies. Note that the controller should mimic the control actions of the anesthetist. In other words, the knowledge and experience of the anesthetist should be embedded into the controller. This fact suggests the use of a rule-based controller like fuzzy controllers. The nominal MAP value used in the simulation is 100. The controller output is limited with an anti-wind up integrator in order to avoid any overdose and improve the steady state performance. The upper and lower limits for the isoflurane concentration are chosen as 4% and 0%, i.e., the controller output u(t) saturates at the values of 4 and 0.

In this study, a Sugeno type fuzzy controller [7] is designed and its parameters are tuned such that the good output responses are obtained under the variations of parameters K1, K2, a1, a2, τ_1 and τ_2. The reason behind the test under parameter variations is the fact that these parameters change from one patient to another in the real applications. Therefore, in order to provide a robust control performance, the controller should be designed by taking account the parameter variations. The trail and error method is used in tuning of the controller parameters.

Fig.2 shows the input membership functions of the fuzzy controller. The first input is the error between the reference (or desired) MAP and the actual measured MAP. The second input is the change of this error.

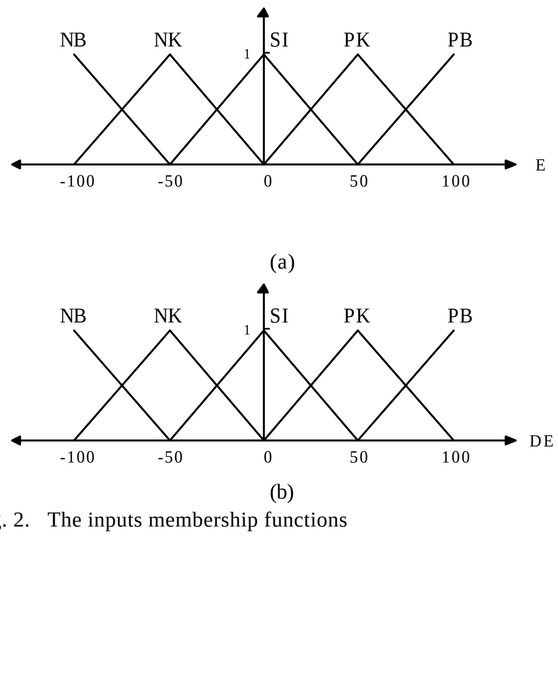

Fig. 2. The inputs membership functions

The rule base of the fuzzy controller is given in Table I. The numeric values of the output variables are given in Table II.

TABLE I
THE RULE BASE OF THE FUZZY CONTROLLER

U E / DE	NB	NK	SI	PK	PI
NB	D_1	D_2	D_3	D_4	D_5
NK	D_6	D_7	D_8	D_9	D_{10}
SI	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}
PK	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}
PB	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}

TABLE II
THE NUMERIC VALUES OF THE OUTPUT VARIABLES

D_1=4.4	D_2=4.175	D_3=3.95	D_4=3.725	D_5=3.5
D_6=2.375	D_7=2.15	D_8=1.925	D_9=1.7	D_{10}=1.475
D_{11}=0.35 D_{16}=-1.475	D_{12}=0.125 D_{17}=-1.7	D_{13}=0 D_{18}=-1.925	D_{14}=-0.125 D_{19}=-2.15	D_{15}=-0.35 D_{20}=-2.375
D_{21}=-3.5	D_{22}=-3.725	D_{23}=-3.95	D_{24}=-4.175	D_{25}=-4.4

The values in Table II are determined by trial and error method as stated before. In the following section, simulation results showing the robust performance of the fuzzy controller under parameter variations are presented.

IV. SIMULATION RESULTS

The closed loop system shown in Fig.1 is simulated and the robustness of the control performance is tested by changing the plant parameters.

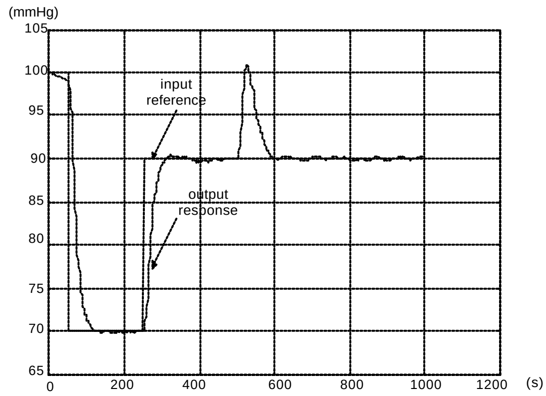

Fig. 3. Output response for the nominal parameters

Fig.3 shows the output response to a step input profile for the nominal system parameters K_1 =-3, K_2 =-7.3, a_1 =0.01, a_2 =0.006, τ_1 =23s, τ_2 =101s. A noise signal and disturbance applied at t= 500s are added as indicated in Fig.1. The output response to the same input step profile for t_2 = 151.5s (50% increased) is shown in Fig.4.

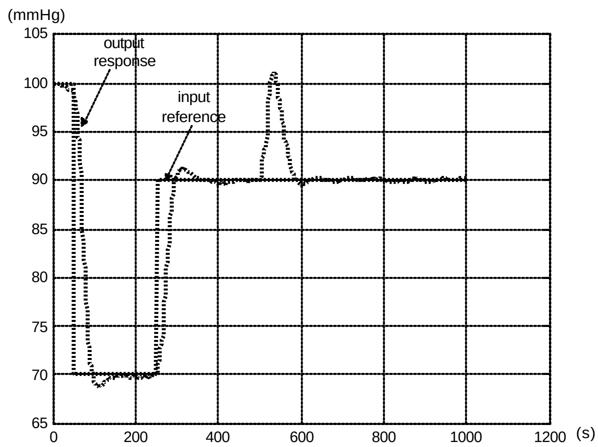

Fig. 4. Output response for t_2=151.5s (50% increased)

The output response for K2 = -10.95 (50% decreased) is shown in Fig.5.

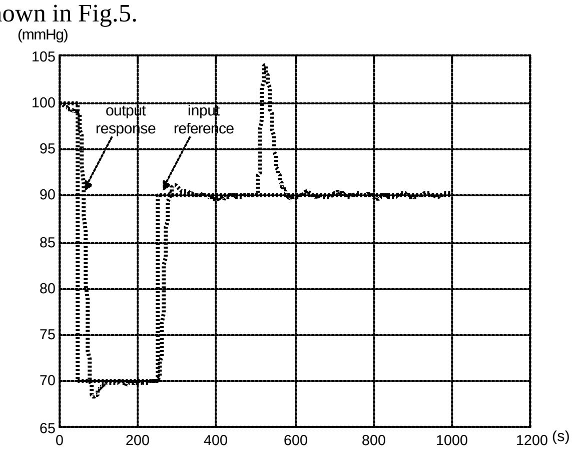

Fig. 5. Output response for K_2 = -10.95(50% increased)

Fig.6 shows the output response for a_2 = 0.018 (200% increased).

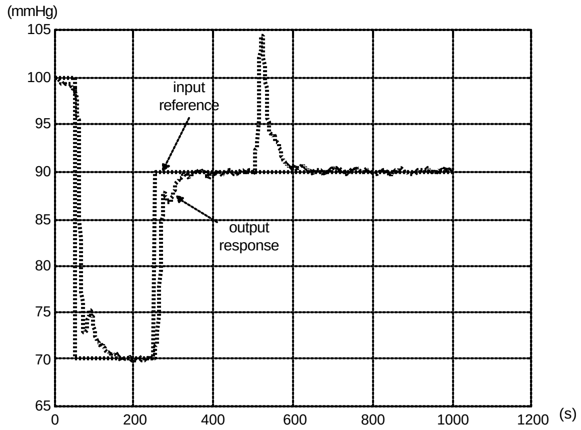

Fig. 6. Output response for a_2 = 0.018 (200% increased)

TABLE III
THE MAXIMUM AND MINIMUM LIMITS OF THE PARAMETERS WHICH CAN BE TOLERATED BY THE FUZZY CONTROLLER

Parameters	**Minimum values (decreasing)**		**Maximum values (increasing)**	
	Percentage (%)	**Values**	**Percentage (%)**	**Values**
τ_1	100	0 s	200	69 s
τ_2	50	50.5 s	50	151.5 s
K_1	600	-21	50	-1.5
K_2	50	-10.95	30	-5.11
a_1	75	0.0025	200	0.03
a_2	30	0.0042	200	0.018

Table III shows the maximum and minimum limits of the parameters that can be compensated or tolerated by the fuzzy controller, i.e., even if the parameters change within the limits given in Table III, the proposed fuzzy controller provides an acceptable (robust) control performance. Beyond this limits, the control performance is getting worse.

V. CONCLUSIONS

In this study, the control of the MAP of a patient under anesthesia is considered in order to prepare a basis for the real applications. A mathematical model representing the patient under anesthesia is derived and used in the simulation studies. A Sugeno type fuzzy controller is designed for the control of the MAP. The parameters of the controllers are determined by trial and error so that the robust control performances are obtained under plant parameter variations. The control performance of the proposed fuzzy controller is tested by changing the parameters of the mathematical model of the patient (including the semi-closed circuit). The simulation results showing the robust control performance of the fuzzy controller are presented in the paper. A table is given to show the maximum and minimum limits of the plant parameters that can be compensated by the fuzzy controller.

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