A Fuzzy Controller based on Fuzzy Lyapunov Synthesis for a Single-Link Flexible Manipulator

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Abstract

In this paper tracking control for a single-link flexible manipulator is considered. Then the controller is designed based on the fuzzy Lyapunov synthesis (FLS). The control strategy assumes no knowledge about the system dynamics, except for some structural properties of the system model which are proved first. Thereby, some important problems regarding the application of the FLS particularly to nonminimum-phase systems are considered too which have not been discussed in the literature yet. Finally, the simulation results compare the application of this methodology with that of a joint PD controller to prove the effectiveness of the proposed algorithm.
Keywords: Lyapunov function, stability, tracking, fuzzy control, internal dynamics, flexible manipulator

1. Introduction

Lightweight manipulators offer many challenges in comparison with rigid and bulky robot manipulators. Smaller energy consumption, larger payload-to-arm weight ratio and faster movements are some advantages. Because of their characteristics, this class of manipulators are specially suitable for a number of non-conventional robotic applications, including space missions as well as applications in heavy, large-scale and macro-micro manipulators [1] where it is no longer possible to assume negligible link deformations. All of these factors make the study of flexible-link manipulators, interesting.

The control of flexible robot arm suffer from the lack of an independent control input for each degree of freedom and the non-collocated nature of sensors and actuators resulting in the nonminimum phase behavior of the tip. Practical considerations like joint friction add to these, too. In the literature, several different kinds of controllers have been used for a single-link flexible manipulator. The joint PD controller while ensuring the closed-loop stability, can not damp the link vibrations [2]. Many controllers such as LQG, H_∞ and input shaping as well as singular perturbation, feedback linearization, manifolds and output redefinition techniques have been used [1,2].

All of the model-based classic or modern controllers suffer from the lack of an exact simple-enough model of the system and this calls for the use of the intelligent controllers. Neural network controllers assuming a priori knowledge of the system have been used as an identifier [3, 4] or an observer [5] or together with a singular perturbation controller to model and control the unknown dynamics of the system. In [2], four controllers are suggested for tracking of a suitable defined output with the first two of them requiring the linear model of the system and the others release this need and the results have been tested both theoretically and experimentally. Fuzzy controllers in the supervisory form to tune the PID coefficients [6] or selecting the lower-order controllers [7] have been used too. In [8], a fuzzy weight is used to combine the output of two SMC controllers for joint tracking and vibration damping. In [9], the linguistic model of the system has been used to derive fuzzy relations suitable for quantitative analysis and inverse-model control of the system. In [10], a three-stage method has been suggested to formulate the expert knowledge for tracking and vibration damping of a single-link flexible manipulator.

Most of these works however, suffer from the lack of a systematic design and suitable framework for stability and performance analysis. Model-based fuzzy controllers are one of the solutions to this problem [11]. The scheme is based on Takagi-Sugeno (TS) models, which employ linear models of the system. Stability and performance indices form several LMIs to be solved. Practical system modelings are prone to error and uncertainties always exist in system models. In that case, model-free control methods are the best solutions. This however, will complicate the stability analysis of the closed-loop system. The authors of [12] use the phase-diagram reasoning to derive the stabilizing fuzzy rules for the error dynamics together with a fuzzy weight. In [9], fuzzy relations and their converted matrices are used for modeling and control of a flexible-link manipulator. In [12], a rule base was derived using phase portrait of error with an adjustable parameter to control a flexible-link manipulator. In [13], a fuzzy controller was designed based on the sliding mode control with the idea of dividing the trajectory to several non-isoclinal segments in the phase plane and regard them as piecewise sliding surfaces. Then, two fuzzy controllers were designed, one for achieving hitting motion and the other for preserving the sliding condition for each region. In [14], a fuzzy hyperbolic state space model has been suggested and a discussion about the stability and optimal performance was given. The authors also introduced a fuzzy Lyapunov synthesis and applied it to some simple minimum-phase SISO problems. The idea is to choose a Lyapunov function candidate and derive the fuzzy rules to make its derivative negative. The only knowledge about the model was the output relative degree.

This paper inspects this idea more through showing the applicability of the FLS to the tracking problem of a single-link flexible manipulator, which is a nonmimimum phase system. The control strategy assumes no knowledge about the system dynamics, except for some structural properties of the system model which are proved first. The control decision rules are essentially the same as those suggested in [14] with suitable modifications to improve the performance and address the problem of internal dynamics.

The outline of the paper is as follows: in Section 2, the problem formulation is given. Section 3 proves some structural properties of the model. In Section 4 the FLS is introduced and applied, and in Section 5, simulation results are presented. Finally conclusions are given in Section 6.

2. Manipulator model

The dynamics of a macro-micro manipulator with locked macro joints can be given by [15]:

$$M(\theta,q)\begin{pmatrix}\ddot{\theta}\\ \ddot{q}\end{pmatrix}+\begin{pmatrix}h_1(X)+f_1(\dot{\theta},\theta)+g_1(X)\\ h_2(X)+f_2\dot{q}+g_2(X)+Kq\end{pmatrix}=\begin{pmatrix}u\\ 0\end{pmatrix} \qquad (1)$$

where $X^T=[\theta^T,q^T,\dot{\theta}^T,\dot{q}^T]$, $\theta\in\Re$ is the joint angle, $q\in\Re^m$ is the vector of flexible modes, h_1 and h_2 are the coriolis/centripetal forces, g_1 and g_2 represent gravity forces, f_1 and f_2 are the joint friction and structural damping terms, and M and K are the positive-definite mass and stiffness matrices, respectively. Define $N=M^{-1}=\begin{pmatrix}N_1 & N_2\\ N_2^T & N_3\end{pmatrix}$. Then, (1) can be written as:

$$\begin{pmatrix}\ddot{\theta}\\ \ddot{q}\end{pmatrix}=\begin{pmatrix}N_1 & N_2\\ N_2^T & N_3\end{pmatrix}\left[-\begin{pmatrix}h_1+f_1+g_1\\ h_2+f_2\dot{q}+g_2+Kq\end{pmatrix}+\begin{pmatrix}u\\ 0\end{pmatrix}\right] \qquad (2)$$

The end-effector of the micro can be expressed as:

$$y=\theta+\phi q \qquad (3)$$

where ϕ is a constant matrix. The goal of the control is that y tracks a reference trajectory while the closed-loop and internal dynamics are stable. Then, by successive differentiation of the output, we have

$$\ddot{y}=A+Bu$$
$$B=(N_1+\phi N_2^T)$$
$$A=-B(h_1+f_1+g_1)-(N_2+\phi N_3)(h_2+Kq+f_2\dot{q}+g_2) \qquad (4)$$

Since M is positive definite, N and hence N_1 are positive definite. By continuity, B is positive definite in a neighborhood of $\phi=0$ and the output relative degree is two.

3. Structural properties of the system model

Most practical systems have complicated nonlinear and uncertain models. However, the dynamics of such systems are based on physical laws, which result in some structural mathematical properties. Although the exact model is difficult to derive, those properties are known and held in the presence of uncertainties. Here two properties of the system model are stated and proved.

Claim 1: The coefficient B in (4) is independent of the joint angle.

Claim 2: The sign of B in (4) is independent of the vibration modes.

Proofs: see the appendix.

As a result of claim 2, the sign of B can be seen as for a rigid link. Moreover, it can be seen that this sign depends on the vector ϕ (i.e. output definition), being positive for hub angle. Therefor, once the sign of B is determined, it remains unchanged versus hub angles and link deflections. Expert knowledge can be used to finding it positive as in case of a single rigid link like inverted pendulum.

4. The fuzzy lyapunov synthesis (FLS)

The FLS as introduced in [14] is a fuzzy model-free approach based on first selecting a Lyapunov function candidate and then make its derivative negative by designing control rules. In [14], the FLS was applied to some minimum phase SISO plants where only the output relative degree and some fuzzy rules defining the linguistic relation between the input and the output were assumed known. No direct insight to the system structural dynamic properties was given there and hence the applicability of the FLS and the stability analysis were less tractable. Here, the formulation of [14] is reviewed to show the essentials of the theory and its control rules are modified to illustrate their applicability to nonminimum phase systems and improve the performance.

4.1. The basic idea

Consider the nonlinear affine system

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad x \in R^n, \quad u \in R^p, \quad y \in R \qquad (5)$$

The control objective is that the error $e = y - y_r$ goes to zero asymptotically where y_r is the reference trajectory. To this end. one way is to choose a positive-definite function V of the error (and its related derivatives/integrals) as a Lyapunov function candidate and design u to make its time-derivative negative along the system trajectory, i.e.

$$V > 0, \quad \dot{V} < 0 \qquad (6)$$

If the knowledge about (5) is limited to some fuzzy descriptions of the system, (6) may be used again, but this time as a linguistic inequality yielding u in terms of mamdani or Takagi-Sugeno IF-THEN conditions. This methodology is called fuzzy Lyapunov synthesis. Like its classical counterparts, choosing V except in simple cases is not an easy task. Other than positive-definiteness, several other measures like simpler operations and more physical insight may be important as well. As explained, the control strategy of FLS is based on designing u to make the $\dot{V}$ negative, therefor one key point is how u appears in $\dot{V}$ To investigate this, let the output relative degree in (5) be r i.e.

$$y^{(r)} = A(x) + B(x)u, \quad \forall x \in \Omega \subset R^n, \quad B(x) \neq 0 \qquad (7)$$

where A and B are functions of the state vector x Some immediate considerations are of main concern.

First, it can be seen that to apply the FLS to this system, V should be chosen such that $\dot{V}$ includes u.

Second, A and B depend on internal as well as external dynamics of the system and therefor the stability of the zero dynamics should be ensured too. Since V is chosen based on only the output error and its derivatives/integrals, it does not include all the system states and as a result $\dot{V} < 0$ does not guarantee the stability of the zero dynamics. For minimum phase systems, the zero dynamics is always stable and there is no problem with this point. In case of nonminimum phase systems, different approaches like the output redefinition or adding a state feedback of the unobservable states to locally stabilize them may be adopted appropriately.

Third, the condition $B(x) \neq 0$ is a local controllability condition which is necessary for all control strategies including FLS to perform efficiently.

As a result, it is seen that although A and B are not known and this makes the Lyapunov reasoning fuzzy, but certain structural properties of the system can be used to make the analysis and design of the fuzzy controller into a systematic rather than try and error procedure.

Table 1. The first set of FLS control rules

If e is pos_1 and $\dot{e}$ is pos_2 then u is neg_3
If e is pos_1 and $\dot{e}$ is neg_2 then u is $zero_3$
If e is neg_1 and $\dot{e}$ is pos_2 then u is $zero_3$
If e is neg_1 and $\dot{e}$ is neg_2 then u is pos_3

4.2. The control Law

The Lyapunov function candidate can be chosen based on several objectives. Some possible approaches are as follow:

1. Define a Lyapunov function candidate to guarantee the stability of the internal dynamics and to meet other performance measures. This is often very complicated.
2. Tune the parameters of the Lyapunov function candidate according to the other performance measures and/or internal stability.
3. Add other control terms to the main controller term such that each term satisfies one of the other performance measures and/or internal stability.

One has to be careful in selecting the last two approaches such that the main control term is effective.

In this section two Lyapunov function candidates suggested in [14], are used to derive the FLS control rules. Although these two functions, are simple functions of output error, its derivative and its integrals, they give good physical insight to the problem from the viewpoint of a PID controller. These functions and their derived FLS control rules in [14] are briefly reviewed and modified then to improve the performance and stabilize the internal dynamics. The first function and its time derivative are

$$V(e) = \frac{1}{2}(e^2 + \dot{e}^2) \quad , \dot{V}(e) = \dot{e}(e + \ddot{e}) \qquad (8)$$

In [14] four FLS control rules were derived assuming $\ddot{e} \propto u$ with e and $\dot{e}$ as premise variables as summerized in Table 1. Using $\ddot{e} = \ddot{y} - \ddot{y}_r$ and (4), $\dot{V}$ can be rewritten as:

$$\dot{V}(e) = \dot{e}e + \dot{e}(\ddot{y} - \ddot{y}_r) = (e - \ddot{y}_r)\dot{e} + \dot{e}(A + Bu) \qquad (9)$$

Now, recalling from section 3 that the sign of B in (4) is positive, the same rules of Table 1 are used with $e - \ddot{y}_r$ and $\dot{e}$ as premise variables. Note that the term $\dot{e}A$ in (9) was not considered in [14] and behaves like an uncertainty in this formulation whose effect should be dominated by the effect of the FLS control term only if the internal dynamics is stable. This problem is discussed later. The FLS rules based on (8), do not generate effective control signals in case of

Table 2. The second set of FLS control rules

Rule No.	1	2	3	4	5	6	7	8
e	+	+	-	-	+	+	-	-
$\dot{e}$	+	+	+	+	-	-	-	-
$\int e d\tau$	+	-	+	-	+	-	+	-
u	NB	NM	Z	NS	PS	Z	PM	PB

small $(e-\ddot{y}_r)\dot{e}$. This can be well understood when tracking step-like references where despite the large steady state errors the controller behaves slowly. As a remedy to this problem, the second Lyapunov function candidate may be chosen. In [14], this function has been used to track step references with no reason stated, but it seems quite true to apply it in this case. This function is

$$V(e)=\frac{1}{2}(e^2+\dot{e}^2+(\int_0^t ed\tau)^2) \quad (10)$$

where the related control rules are given in Table 2 with e, $\dot{e}$ and $\int edt$ as premise variables. Substituting e with $e-\ddot{y}_r$, the same rules are used here.

Another goal is to stabilize the internal dynamics of the system. To this end, the following approaches can be taken

I. The control signal can be further limited:

If abs (deflection) is high, Then (LM)$_i$ is 0

If abs (deflection) is small, Then (LM)$_i$ is 1

*(control signal)$_i$ = (LM)$_i$ * u_i* $\quad i=1,\dots,n$ (11)

The cost paid for less vibration is a slower response.

II. Rearranging the terms in (4), we have

$$\phi(\ddot{q}+Kq+f_2\dot{q})=Bu-(N_1+\phi N_2^T)(h_1+f_1+g_1)$$
$$-(N_2+\phi N_3)h_2-N_2(Kq+f_2\dot{q})-\ddot{\theta} \quad . \quad (12)$$

Qualitatively speaking, adding a state feedback of the vibration modes may help the damping of the internal modes. This is similar to the term used in [17,18] based on a sensitivity analysis. The new term is added as

$$u_s=-[(Z_1*\dot{q}+Z_2*q)*Z_3*abs(u_{FLS})] \quad (13)$$

where Z_1 and Z_2 are determined according to the modal damping considerations and the bounds on positive-definite matrices f_2 and K . $Z_3*abs(u_{FLS})$ is added so that the effect of the FLS control term holds. $Z_3\in[0,1]$ can be determined using some fuzzy rules based on the tip deflection. Due to damping time constant of the modes, this term is effective for slow trajectories.

III. Output redefinition techniques may be used to choose a suitable minimum phase output for the system [2]. This approach has an indirect view to output error.

5. Simulation results

Tracking of both the sinusoidal and step trajectories were considered and the results are shown in figures 1 to 3. In each set of figures, from left to right, top to bottom the signals of the actuator (Nm), tip deflection (m), hub angle (rad) and tip pseudo angle (rad) have been shown respectively without/with the hub viscous friction [2]. For a planar single-link flexible manipulator with two pseudo-clamped [16] vibration modes, the system model was derived [15] using the parameters of Table 3. The output

Table 3. Model parameters

Length (m),Width (mm), Heigth (mm)	0.6, 50, 1
Hub Radius (mm),Hub Inertia (Kgm^2)	40, 0.63
Young Modulus $E(P)$, Stiffness Matrix $K(Nm)$	200e+6, $diag(11.604,121.74)$
Tip Load and Inertia (kg, Kgm^2)	1.5, .06
First Frequency , Second Frequency (Hz)	0.5476, 1.837
Viscous Friction (Nms/rad), Coulomb Friction (Nm)	0.0018, $\begin{pmatrix}4.8 & (\dot{\theta}>0)\\ -4.55 & (\dot{\theta}<0)\end{pmatrix}$
Structural Damping (Nms)	$diag(0.4,4)$

was selected to be the tip pseudo angle [2]. The second approach stated in Section 4.2 was also used for improving the internal stability. Simulation results for a joint-PD controller in tracking the sinusoidal trajectory are given in Figure 1. Figures 2 and 3 show the system responses for sinusoidal and step trajectories. The membership functions are of the gaussian type [14]. As seen, the results show considerable improvements for the fuzzy controllers comparing to the joint-PD controller.

6. Conclusions

The idea of FLS was investigated and extended through its application to the tracking control of the nonminimum phase nonlinear system of a flexible manipulator. Only the output relative degree and some structural properties of the model were used. This methodology provides a framework for stability and performance analysis of the system. Simulation results show the effectiveness of the FLS despite the nonlinearity and uncertainties of the model.

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Appendix

It is well known that the M matrix is independent of the joint angle for a single-link flexible manipulator [2]. Therefore N and its blocks are independent of θ too. Hence the claim 1 is proved. To prove the claim 2, note that the mass matrix M can be written as [1,18].

$$\begin{pmatrix} M_{11}(q) & \mu_1 & \cdots & \mu_n \\ \mu_1 & 1+\mu_1^2/J & \mu_1\mu_2/J & \mu_1\mu_n/J \\ \vdots & \vdots & \ddots & \vdots \\ \mu_n & \mu_1\mu_n/J & \cdots & 1+\mu_n^2/J \end{pmatrix} \quad (14)$$

$$M_{11}(q) = J + g(q) \quad (15)$$

where $g(q)$ for two modes are as follows:

$$g(q) = \alpha_1 q_1^2 + \alpha_2 q_2^2 + 2\alpha_{12} q_1 q_2$$

$$\alpha_1 = 1 + \mu_1^2/J - J_p p_1'(L_1), \alpha_2 = 1 + \mu_2^2/J - J_p p_2'(L_1)$$

$$\alpha_{12} = \mu_1\mu_2/J - J_p p_2'(L_1) p_1'(L_1) \quad (16)$$

By partitioning M we have:

$$M(q) = \begin{pmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{pmatrix}, M_1 \in R, M_2^T \in R^{n\times 1}, M_3 \in R^{n\times n} \quad (17)$$

The inverse of M always exists and is given by:

$$N = M^{-1} = \begin{pmatrix} M_1^{-1} + M_1^{-1} M_2 X^{-1} M_2^T M_1^{-1} & M_1^{-1} M_2 X^{-1} \\ X^{-1} M_2^T M_1^{-1} & X^{-1} \end{pmatrix}$$

$$X = M_3 - M_2^T M_1^{-1} M_2 \quad (18)$$

Using (17) and handy manipulations, we would have:

$$X = \begin{pmatrix} 1+\mu_1^2\zeta & \cdots & \mu_1\mu_n\zeta \\ \vdots & \ddots & \vdots \\ \mu_1\mu_n\zeta & \cdots & 1+\mu_n^2\zeta \end{pmatrix} = I + \xi M_2^T M_2$$

$$\xi = 1/J - 1/M_{11} \quad (19)$$

$$X^{-1} = (I + \xi M_2^T M_2)^{-1} = I - I\xi M_2^T (I + M_2 \xi M_2^T)^{-1} M_2$$

$$= I - \xi M_2^T (I + \xi M_2^T M_2)^{-1} M_2 = 1 - \frac{\xi M_2^T M_2}{1+\xi M_2 M_2^T} \quad (20)$$

$$X^{-1} M_2^T = \frac{M_2^T}{1+\xi(\mu_1^2 + \cdots + \mu_n^2)} \quad (21)$$

Using (4) and substituting from (18) and (17) we have:

$$B = M_1^{-1} + M_1^{-1} M_2 X^{-1} M_2^T M_1^{-1} + \phi(-X^{-1} M_2^T M_1^{-1})$$

$$= M_1^{-1}[1 + (M_1^{-1} M_2 - \phi)(X^{-1} M_2^T M_1^{-1})] \quad (22)$$

$$B = M_1^{-1}\left[1 + (M_1^{-1} M_2 - \phi)\frac{[\mu_1 \cdots \mu_n]^T}{1+\xi(\mu_1^2 + \cdots + \mu_n^2)}\right] =$$

$$\frac{1 + (\mu_1^2 + \cdots + \mu_n^2)/J - \phi M_2^T}{M_1[1+\xi(\mu_1^2 + \cdots + \mu_n^2)]} \quad (23)$$

Defining the hub angle as the output $\phi = 0$ and using (4):

$$B = B_0 = \frac{1 + (\mu_1^2 + \cdots + \mu_n^2)/J}{M_1[1+\xi(\mu_1^2 + \cdots + \mu_n^2)]} = N_1, \quad \forall\xi \quad (24)$$

N and hence N_1 are positive definite and as a result:

$$M_1[1+\xi(\mu_1^2 + \cdots + \mu_n^2)] > 0, \quad \forall\xi \quad (25)$$

since ϕM_2^T is independent of ξ we have:

$$sign\left(\frac{-\phi M_2^T}{M_1[1+\xi(\mu_1^2 + \cdots + \mu_n^2)]}\right) = const., \quad \forall\xi \quad (26)$$

Thus the sign of B is independent of the vibration modes and depends on ϕ statically $\therefore$

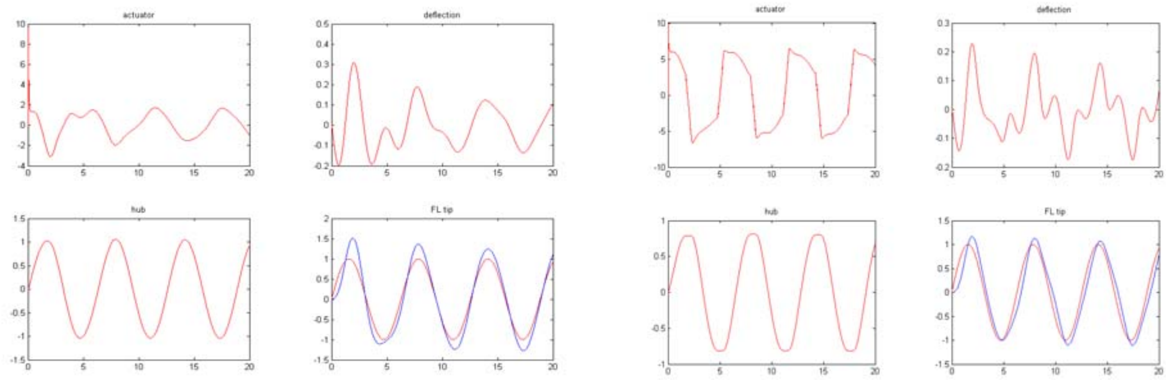

Figure 1. Tracking of the sin(t) using the joint-PD controller without/with the coulomb friction at the hub

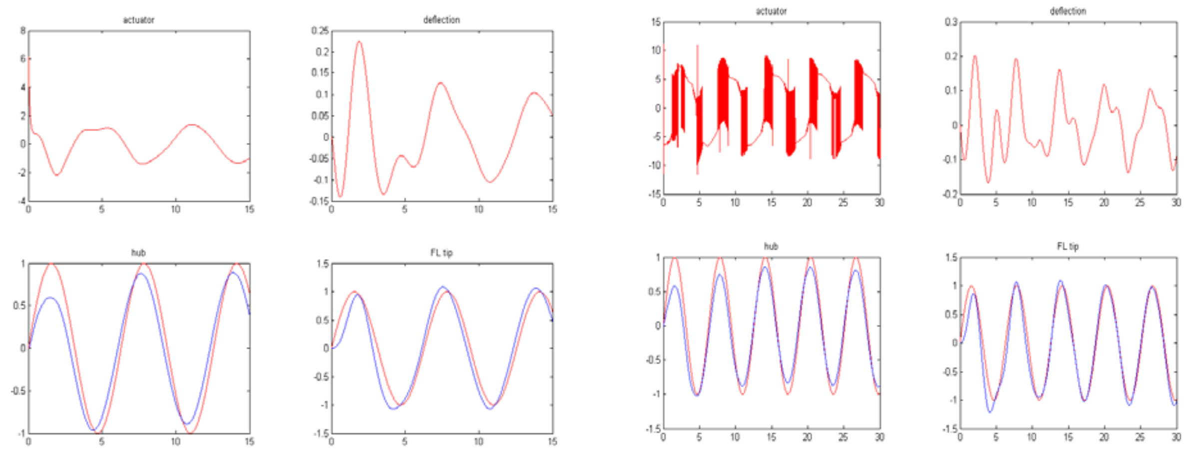

Figure 2. Tracking of the sin(t) using the first set of FLS control rules without/with the coulomb friction at the hub

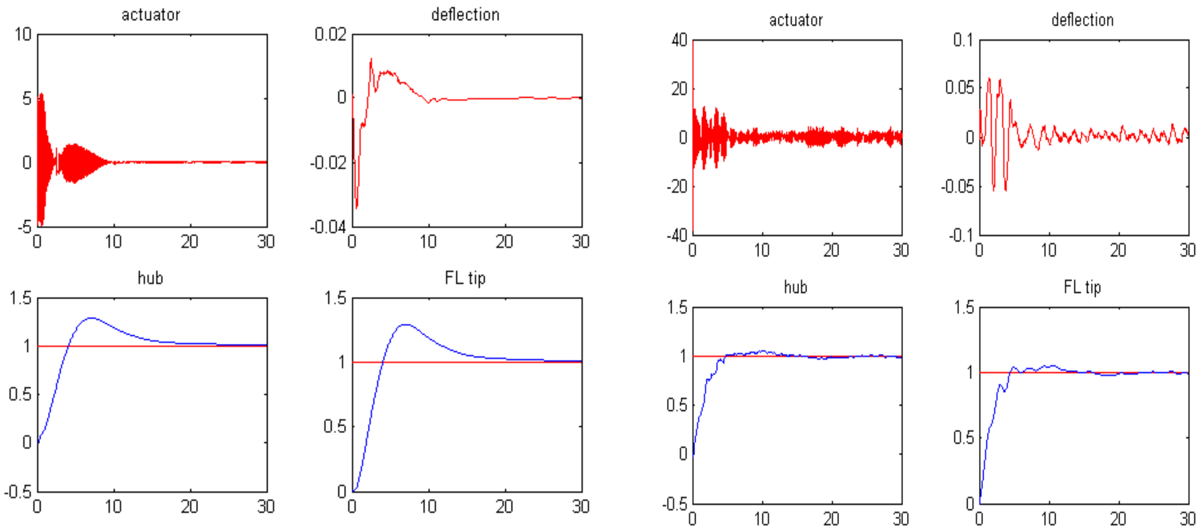

Figure 3. Tracking of the step using the second set of FLS control rules without/with the coulomb friction at the hub