Some Results on Identification of Timed Event Graphs in Dioids

C. A. Maia R. Santos-Mendes L. Hardouin

Abstract This paper deals with parametric SISO timed-event graphs identification. First an appropriate model of the graph is derived from the input-output transfer function. In the following an identification algorithm is developed using Residuation Theory. Some theoretical results are also provided.

Keywords : Discrete event dynamic systems, Timed Petri nets, (max, +) algebra, Systems Identification.

I. Introduction

Discrete Event Systems (DES) appear in many applications in manufacturing, computer and communication systems and are often described by the Petri Net formalism (see [1]). If the concerned systems are characterized by delay and synchronization phenomena, the Timed Event Graphs (TEG) constitute interesting models. TEG are timed Petri nets in which all places have single upstream and single downstream transitions and therefore can be linearly described in dioid algebra ([2], [3], [4]). This formulation has permitted many important achievements on the control of TEG, as for instance the internal model control [5], the closed-loop control via output or state feedback ([6], [7]), and the predictive control [8]. One should remark that the dioid formalism is useful in DES contexts other than TEG control, as for example for the modeling and control of continuous and hybrid Petri nets [9].

A central problem in TEG control is, as in classical control theory for continuous dynamic systems, the identification of the model. Boimond et al. [10] have proposed a parameter identification method based on the system impulse response. The approach considers two ARMA models: one for the transient and another for the periodic behavior. Gallot et *al*. ([11], [12]) have considered the identification of the system impulse response based on a decomposition of the system into a sum of first order sub-systems (the impulse response is split into a sum of so called simple elements). Menguy *et al*. [13] have developed an algorithm for the non-parametric (direct) identification of the system impulse response.

Representation of a dynamic system by an impulse response usually requires an infinite number of parameters. However, given a model structure, parametric models allow to represent a system with a finite number of parameters. This paper proposes a new method for parametric identification based on the knowledge of the model structure. First we develop an appropriate TEG model and then the identification algorithm. As one will notice, the method is not restricted to the case of impulse response estimation, as in the previous cited papers, and it can be used in other input conditions. The main advantage is that the obtained model is closer, in a dioid sense, to the actual system model than the one obtained by a direct calculation of the greatest impulse response.

The paper is organized as follows. Section II introduces some algebraic tools concerning the Dioid theory . Residuation and Linear Systems Theory is presented in sections III and IV respectively. In section V the identification method is developed and section VI gives an illustrative example. A conclusion is given in section VII.

II. Dioid Theory

A *dioid* $\mathcal{D}$ is a set supplied with two internal operations denoted by $\oplus$ and $\otimes$. The operation $\oplus$ is idempotent ($a \oplus a = a$). The neutral elements of $\oplus$ and $\otimes$ exist and are represented by ε and e respectively. The operation $\otimes$ is distributed at left and at right with respect to $\oplus$ and ε is an absorbing element ($\varepsilon \otimes a = a \otimes \varepsilon = \varepsilon, \quad \forall a$). In a dioid, a partial order relation is defined by $a \succeq b$ iff $a = a \oplus b$. As result ε is a bottom element of the dioid because, for all a, $a \succeq \varepsilon$.

A dioid $\mathcal{D}$ is said to be complete if it is closed for infinite sums and if $\otimes$ distributes over infinite sums. The greatest element of a complete diod $\mathcal{D}$ noted by $\top$ is equal to $\bigoplus_{x \in \mathcal{D}} x$.

The greatest lower bound of every set $\mathcal{X}$ of a complete dioid $\mathcal{D}$ always exists and $\mathcal{D}$ is a distributive dioid if it is complete and for all subsets $\mathcal{C}$ of $\mathcal{D}$, $(\bigwedge_{c \in \mathcal{C}} c) \oplus a = \bigwedge_{c \in \mathcal{C}} (c \oplus a)$ and $(\bigoplus_{c \in \mathcal{C}} c) \wedge a = \bigoplus_{c \in \mathcal{C}} (c \wedge a)$, where $x \wedge y$ denotes the greatest lower bound between x and y.

Example 1 ($\overline{\mathbb{Z}}_{\max}$ diod): Consider the set $\overline{\mathbb{Z}} = \mathbb{Z} \cup \{-\infty, +\infty\}$ and define $\oplus$ as the max operator and $\otimes$ as the classical sum $+$. This is a complete dioid in which $\varepsilon = -\infty$, $e = 0$ and $\top = +\infty$.

Theorem 1 ([2]): The implicit equation $x = ax \oplus b$ defined over a complete dioid $\mathcal{D}$ admits $x = a^* b$ as least solution, where $a^* = \bigoplus\limits_{i \in \mathbb{N}} a^i$ (Kleene star operator) with $a^0 = e$.

Example 2 ($\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$ dioid): . The elements are given by $x(\gamma) = \bigoplus\limits_{k \in \mathbb{Z}} x(k) \otimes \gamma^k$ where γ is a variable and $x(k) \in \overline{\mathbb{Z}}_{\max}$. The neutral elements are $\varepsilon(\gamma) = \bigoplus\limits_{k \in \mathbb{Z}} \varepsilon(k) \otimes \gamma^k$ and $e(\gamma) = \bigoplus\limits_{k \in \mathbb{Z}} e(k) \otimes \gamma^k$ where

C. A. Maia is with Departamento de Engenharia Elétrica, Universidade Federal de Minas Gerais, Brazil. On leave at Laboratoire d'Ingénierie des Systèmes Automatisés, Université d'Angers, France. Grant from CAPES, Brazil. E-mail: andrey@dca.fee.unicamp.br

R. Santos-Mendes is with Faculdade de Engenharia Elétrica e de Computação, Universidade Estadual de Campinas, Brazil. E-mail: rafael@dca.fee.unicamp.br

L. Hardouin is with Laboratoire d'Ingénierie des Systèmes Automatisés, Université d'Angers, France . E-mail: laurent.hardouin@istia.univ-angers.fr

$\varepsilon(k) = -\infty, \forall k$, and $e(k) = \begin{cases} 0 & \text{if } k \geq 0 \\ -\infty & \text{otherwise.} \end{cases}$

Remark 1: The variable γ of dioid $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$ can be usually regarded as a backshift operator in the event domain. It plays, in the TEG study, a role similar to the operator z^{-1} in the applications of $Z-$transform to discrete-time linear dynamic systems .

The support of a series $x(\gamma)$ in the dioid $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$ is defined as $supp(x(\gamma)) = \{k | x(k) \neq \varepsilon\}$.

Definition 1: The valuation $val(x(\gamma))$ of a series $x(\gamma)$ is defined as the greatest lower bound of $supp(x(\gamma))$.

III. Residuation Theory and Dioids

Residuation deals with solutions of equations of the type $f(x) = b$ by assuming that f is an isotone map ($a \preceq b \Longrightarrow f(a) \preceq f(b)$). In this section some results on this theory are summarized. Further details can be found in Blyth and Janowitz [14].

Define the subsolutions (supersolutions) of the equation $f(x) = b$ as the elements of the set $\{x | f(x) \preceq b\}$ ($\{x | f(x) \succeq y\}$).

Definition 2 (Residual and residuated mapping): An isotone mapping $f : \mathcal{D} \to \mathcal{E}$, where $\mathcal{D}$ and $\mathcal{E}$ are ordered sets, is a *residuated mapping* if for all $y \in \mathcal{E}$ there exists a greatest subsolution for the equation $f(x) = y$ (hereafter denoted $f^\sharp(y)$). The mapping $f^\sharp$ is called the *residual* of f.

Theorem 2 ([2], Residuation): Let $f : \mathcal{D} \to \mathcal{E}$ be an isotone mapping where $\mathcal{D}$ and $\mathcal{E}$ are ordered sets, then f is residuated iff $f^\sharp$ is the unique isotone mapping such that

$$f \circ f^\sharp(y) \preceq y \quad \text{and} \quad f^\sharp \circ f(x) \succeq x \tag{1}$$

$\forall x \in \mathcal{D}$ and $\forall\, y \in \mathcal{E}$.

Theorem 3 ([2]): Let $f : \mathcal{D} \to \mathcal{E}$ where $\mathcal{D}$ and $\mathcal{E}$ be complete dioids whose zero elements are respectively denoted $\varepsilon_{\mathcal{D}}$ and $\varepsilon_{\mathcal{E}}$. Then f is residuated iff $f(\varepsilon_{\mathcal{D}}) = \varepsilon_{\mathcal{E}}$ and, $\forall \mathcal{A} \subseteq \mathcal{D}$, $f(\bigoplus_{x \in \mathcal{A}} x) = \bigoplus_{x \in \mathcal{A}} f(x)$.

Theorem 4 ([2]): Mappings $L_a : x \mapsto a \otimes x$ and $R_a : x \mapsto x \otimes a$ defined over a complete dioid $\mathcal{D}$ are both residuated. Their residuals are isotone mappings denoted respectively by $L_a^\sharp(x) = a \backslash\!\!\circ\, x = \frac{x}{a}$ and $R_a^\sharp(x) = x /\!\!\circ\, a = \frac{x}{a}$.

Remark 2: These results can be extended to a matrix dioid (see [2]).

It is important to notice that in a commutative dioid $L_a^\sharp(x)$ $=R_a^\sharp(x)$.

The concept of dual residuation can be defined in similar way from the equation $f(x) = b$.

Definition 3 ([2], Dual Residuation): An isotone mapping $f : \mathcal{D} \to \mathcal{E}$, where $\mathcal{D}$ and $\mathcal{E}$ are ordered sets, is a *dually residuated mapping* if for all $y \in \mathcal{E}$ there exists the least supersolution for $f(x) = y$. It is denoted $f^\flat(y)$ and it is called the *dual residual* of f.

Theorem 5 ([2], Dual Residuation): Let $f : \mathcal{D} \to \mathcal{E}$ be an isotone mapping where $\mathcal{D}$ and $\mathcal{E}$ are ordered sets, then f is dually residuated if $f^\flat$ is the unique isotone mapping such that

$$f \circ f^\flat(y) \succeq y \quad \text{and} \quad f^\flat \circ f(x) \preceq x \tag{2}$$

TABLE I
FORMULÆ OF RESIDUATION

$a \frac{x}{a} \preceq x$	(1)
$a \frac{ax}{a} = ax$	(2)
$(x \oplus y) \ominus a = (x \ominus a) \oplus (y \ominus a)$	(3)
$(x \ominus a) \oplus a = (x \oplus a)$	(4)
$(x \oplus a) \ominus a = (x \ominus a)$	(5)
$x \ominus (a \oplus b) = (x \ominus a) \ominus b = (x \ominus b) \ominus a$	(6)

$\forall x \in \mathcal{D}$ and $\forall y \in \mathcal{E}$.

Theorem 6 ([2]): The isotone mapping $T_a : x \mapsto a \oplus x$ from a complete dioid into itself is dually residuated. Its dual residual is denoted $T_a^\flat(x) = x \ominus a$.

Remark 3: $x \ominus a = \varepsilon \Leftrightarrow a \succeq x$.

The table I gives some useful equations involving the residuation ($\ominus$), in dioids (see[2]).

Property 1: In the $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$ dioid, $y(\gamma) \ominus x(\gamma) = \{\bigoplus_{k \in \mathbb{Z}} y(k)\gamma^k\} \ominus \{\bigoplus_{i \in \mathbb{Z}} x(i)\gamma^i\} = \bigoplus_{k \in \mathbb{Z}} (y(k) \ominus x(k))\gamma^k$.

Proof: Directly from table I, formulæ (3) and (6), and observing that $y(k)\gamma^k \ominus \{\bigoplus_{i > k} x(i)\gamma^i\} = y(k)\gamma^k$ and $y(k)\gamma^k \ominus \{\bigoplus_{i \leq k} x(i)\gamma^i\} = (y(k) \ominus x(k))\gamma^k$. ∎

Remark 4: The associated dater (a nondecreasing trajectory) to the series $w(\gamma) = y(\gamma) \ominus x(\gamma)$ is given by $w(k) = \bigoplus_{i \leq k} (y(i) \ominus x(i))$. As a result, if $w(\gamma) \neq \varepsilon(\gamma)$ then $w(val(w(\gamma))) = y(val(w(\gamma)))$.

Property 2: If $y(\gamma) \ominus x(\gamma) \neq \varepsilon(\gamma)$ then $val(y(\gamma) \ominus x(\gamma)) \geq val(y(\gamma))$.

Proof: Obtained by using property 1 and definition 1. ∎

IV. Linear Systems Theory

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Given a TEG, it is possible to associate to each transition a sequence $x = \{x(k)\}_{k \in \mathbb{Z}}$ where $x(k)$ represents the date of the k^{th} firing of the transition x. Such a sequence, usually called a dater, is a nondecreasing function of k. The trajectory of the transition x can also be represented by a formal series $x(\gamma) = \bigoplus_{k \in \mathbb{Z}} x(k) \otimes \gamma^k$ where $x(k) \in \overline{\mathbb{Z}}_{\max}$.

The following example, which represents a workshop with 3 machines (M_1 *to* M_3), illustrates this idea.

Let u and y be respectively the daters of the input and output transistions and x_1 to x_3 be the daters of the internal transitions in TEG of figure 1. The system equations (3) gives the relationship of these variables in the dioid $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$ (when there is no confusion, the operator $\otimes$ will be omitted).

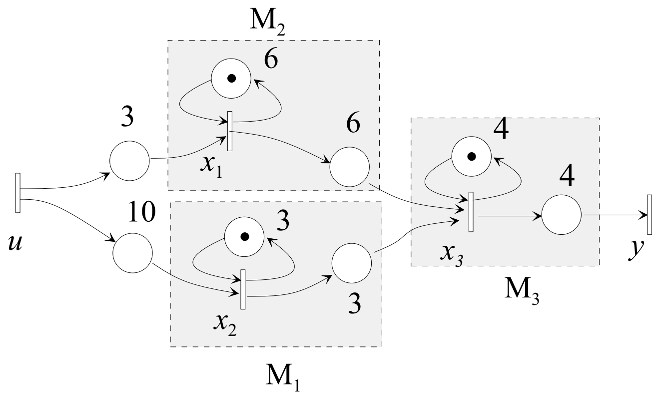

Fig. 1. Example of Timed Event Graph (TEG)

$$
\begin{cases}
x_1(\gamma) &= 6\gamma x_1(\gamma) \oplus 3u(\gamma) \\
x_2(\gamma) &= 3\gamma x_2(\gamma) \oplus 10u(\gamma) \\
x_3\gamma &= 4\gamma x_3(\gamma) \oplus 6x_1(\gamma) \oplus 3x_2(\gamma) \\
y(\gamma) &= 4x_3(\gamma)
\end{cases} \tag{3}
$$

$$y(\gamma) = (17 \oplus 21\gamma \oplus (25\gamma^2)(6\gamma)^*)u(\gamma). \tag{4}$$

This result can be generalized for every TEG. Baccelli et al. [2] have shown that transfer function h has periodic behavior and the output of the system in the $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$ dioid is given by

$$y(\gamma) = h(\gamma)u(\gamma). \tag{5}$$

where

$$h(\gamma) = p(\gamma) \oplus q(\gamma)\gamma^{\nu}(s\gamma^r)^*, \tag{6}$$

with $p(\gamma) = \bigoplus_{i=0}^{\nu-1} \; p_i \; \gamma^i$, $p_i \in \mathbb{N}$, a polynomial which describes the transient behavior of the system and $q(\gamma) = \bigoplus_{j=0}^{r-1} \; q_j \; \gamma^j$, $q_i \in \mathbb{N}$, a polynomial which represents a pattern. This pattern is reproduced for each r events and lasts s time units.

Baccelli *et al*. [2] also have shown that in the set of daters one may write

$$y(k) = \bigoplus_{l=0}^{k} h(l) \otimes u(k-l), \tag{7}$$

where h is the system impulse response ($y = h$ when $u = e$, *i.e.*, transition u fires infinitely many times at $t = 0$).

Property 3 ([2]): Let x and y be two daters, then the dater $x \circ\!\!\!\backslash y$ exists and is given by $[x \circ\!\!\!\backslash y](k) = \bigwedge_{s\in\mathbb{Z}} x(s) \circ\!\!\!\backslash y(k+s)$.

V. Identification method

This paper assumes that there exists a model for a TEG SISO as expressed in equation (5). Its structure, *i.e.* parameters ν and r (see equation (6)) are assumed to be known . The purpose of the identification method is to estimate the unknown polynomials $p(\gamma)$, $q(\gamma)$ and the period duration s.

Expanding equation (5) by using (6), one obtains

$$y(\gamma) = p(\gamma)u(\gamma) \oplus q(\gamma)\gamma^{\nu} z(\gamma), \tag{8}$$

where $z(\gamma) = (s\gamma^r)^* u(\gamma)$. This equation is a solution of the affine equation $z(\gamma) = (s\gamma^r)z(\gamma) \oplus u(\gamma)$. Hence, the system can be represented by the following equations in the $\overline{\mathbb{Z}}_{\max}$ dioid

$$
\begin{aligned}
z(k) &= s \otimes z(k-r) \oplus u(k) \\
y(k) &= p_0 u(k) \oplus \ldots \oplus p_{\nu-1} u(k-\nu+1) \oplus \\
&\quad\; q_0 z(k-\nu) \oplus \ldots \oplus q_{r-1} z(k-\nu-r+1)
\end{aligned} \tag{9}
$$

with initial conditions $z(k) = u(k) = y(k) = \varepsilon$ for $k < 0$.

Taking inspiration from the classical identification theory for the continuous variable dynamic systems [15], $y(k)$ can be rewritten as

$$y(k) = \varphi_k^T \otimes \theta, \tag{10}$$

where $\varphi_k^T = [u(k) \ldots u(k-\nu+1) z(k-\nu) \ldots z(k-\nu-r+1)]$ is the regression vector and $\theta = [p_0 \ldots p_{\nu-1} q_0 \ldots q_{r-1}]^T$ is the parameter vector which will be estimated.

Therefore, for an observation of N input and output transition firings, one gets

$$Y = \Phi \otimes \theta, \tag{11}$$

where $\Phi = [\varphi_0 \ldots \varphi_N]^T$ is the regression matrix and $Y = [y(0) \ldots y(N)]^T$ is the observed output vector.

In order to obtain an estimate of the parameter θ, an error criterion is defined as

$$J(\widetilde{\theta}) = \bigoplus_k (y(k) - \widetilde{y}(k)). \tag{12}$$

Where the output of estimated model ($\widetilde{y}(k) = \Phi \otimes \widetilde{\theta}$) is such that $\widetilde{y}(k) \leq y(k)$. This criterion means that the best model must be as close as possible but less than the observed output, *i.e.*, the greatest $\widetilde{\theta}$ such that $\Phi \otimes \widetilde{\theta} \preceq Y$.

As a first step, variable z will be assumed known. Hence an optimum estimator for the criterion $J(\widetilde{\theta})$ can be obtained by using Residuation Theory,

$$\widehat{\theta} = \bigoplus_{\Phi\otimes\widetilde{\theta}\preceq Y} \widetilde{\theta} = \Phi \circ\!\!\!\backslash Y. \tag{13}$$

Explicitly, the solution to this equation is given by

$$
\begin{aligned}
\widehat{p_i} &= \textstyle\bigwedge_{k=0}^{N} u(k-i) \circ\!\!\!\backslash y(k), & i \in [0 \;\; \nu-1], \\
\widehat{q_j} &= \textstyle\bigwedge_{k=0}^{N} z(k-\nu-j) \circ\!\!\!\backslash y(k), & j \in [0 \;\; r-1].
\end{aligned} \tag{14}
$$

Remark 5: $\widehat{p_i} \geq p_i$ and $\widehat{q_j} \geq q_j$ since $\widehat{\theta}$ is the greatest solution of $\Phi \otimes \widetilde{\widetilde{\theta}} \preceq Y$. Consequently, $\widehat{\theta}$ is a solution to equation (11), *i.e.*, $Y = \Phi \otimes \widehat{\theta}$. This results implies that $\widehat{p_i}$ and $\widehat{q_j}$ satisfies equation (9) for $k = 1, ..., N$. By setting $u(k) = +\infty$ for $k > N$ (This means that no events occur after $k > N$), the equation (9) is satisfied for all $k \in \mathbb{Z}$. So one can also apply the γ transform, which leads to

$$y(\gamma) = \widehat{p}(\gamma)u(\gamma) \oplus \widehat{q}(\gamma)\gamma^{\nu} z(\gamma). \tag{15}$$

.

Proposition 1: If the parameter s is known and the input signal $u(\gamma)$ is sufficiently "rich" (*i.e.*, $e \preceq u(\gamma) \preceq \frac{h(\gamma)}{h(\gamma)}$) then the above estimators will converge to the actual parameters, precisely $\widehat{p}(\gamma) = p(\gamma)$ and $\widehat{q}(\gamma) = q(\gamma)$.

Proof:

If $e \preceq u(\gamma) \preceq \frac{h(\gamma)}{h(\gamma)}$ then $h(\gamma) \preceq y(\gamma) \preceq h(\gamma)\frac{h(\gamma)}{h(\gamma)} = h(\gamma)$(see table I, equation (2)). If s is assumed to be known, z is also known. Moreover the proposed estimator always gives $\widehat{p}_i \geq p_i$ and $\widehat{q}_j \geq q_j$, however $\widehat{p}_i = \bigwedge_{k=0}^{N} u(k-i) \lozenge h(k) \leq u(0) \lozenge h(i) \leq h(i) = p_i$ $(i < \nu)$ because $u(0) \geq 0$. The same reasoning can be applied to $\widehat{q}_j$.

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However, since variable z is unknown, one must estimate it. If an estimate of s (denoted $\widehat{s}$) is available, an estimate of z (denoted $\widehat{z}$) is obtained iteratively, following equation (9), by

$$\widehat{z}(k) = \widehat{s} \otimes \widehat{z}(k-r) \oplus u(k). \tag{16}$$

To estimate the period duration, s, one must remember that the estimates given in equation (14) must satisfy the equation (15) as explained in remark (5). Introducing a new variable $w(\gamma)$, this equation can be rewritten as

$$\begin{cases} w(\gamma) &= \widehat{q}(\gamma)\gamma^{\nu}(s\gamma^r)^* u(\gamma) \\ y(\gamma) &= \widehat{p}(\gamma)u(\gamma) \oplus w(\gamma). \end{cases} \tag{17}$$

A lower bound for $w(\gamma)$ is given by the dual residuation:

$$w(\gamma)_{inf} = y(\gamma) \ominus \widehat{p}(\gamma)u(\gamma). \tag{18}$$

Therefore one has the following inequalities:

$$w(\gamma)_{inf} \preceq w(\gamma) \Rightarrow (s\gamma^r)^* w(\gamma)_{inf} \preceq (s\gamma^r)^* w(\gamma) \preceq y(\gamma). \tag{19}$$

Hence, $(s\gamma^r)^* \preceq w(\gamma)_{inf} \lozenge y(\gamma)$. Then in order to estimate s one must study the set

$$\mathbb{S} = \{s \in \mathbb{N} \mid (s\gamma^r)^* \preceq c(\gamma)\}, \tag{20}$$

where $c(\gamma) = w(\gamma)_{inf} \lozenge y(\gamma)$.

Remark 6: As $w(\gamma)_{inf} \preceq y(\gamma)$ then $c(\gamma) \succeq e$ which implies that $\mathbb{S}$ is nonempty ($s = 0 \in \mathbb{S}$).

Expanding the inequality $(s\gamma^r)^* \preceq c(\gamma)$, one has

$$\begin{cases} 0 & \preceq & c(0) \\ (s\gamma^r)^1 & \preceq & c(r)\gamma^r \\ \vdots & & \\ (s\gamma^r)^i & \preceq & c(ir)(\gamma^r)^i \\ \vdots & & \end{cases} \tag{21}$$

As result, $s \preceq \frac{c(ir)}{i}$, $\forall i \geq 1$, an upper bound for s in $\mathbb{N}$ is $s_{up} = \min\limits_{i=1,\ldots,\infty} \lfloor \frac{c(ir)}{i} \rfloor$, where $\lfloor x \rfloor$ is the integer part of x. Moreover $s_{up} \in \mathbb{S}$ because

$$\begin{cases} (s_{up}\gamma^r)^* &= & (\min\limits_{i=1,\ldots,\infty} \lfloor \frac{c(ir)}{i} \rfloor \gamma^r)^* \\ &= & \bigoplus_{j=0}^{\infty} (\min\limits_{i=1,\ldots,\infty} \lfloor \frac{c(ir)}{i} \rfloor \gamma^r)^j \\ &\preceq & e \oplus \bigoplus_{j=1}^{\infty} (\lfloor \frac{c(jr)}{j} \rfloor \gamma^r)^j \\ &\preceq & e \oplus \bigoplus_{j=1}^{\infty} c(jr)\gamma^{jr} \\ &\preceq & c(\gamma) \end{cases} \tag{22}$$

Finally, the conclusion is that s_{up} is the greatest element of $\mathbb{S}$. Our proposition is to take $\widehat{s} = s_{up}$ as an estimator for s. Some properties of this estimator are given below.

Lemma 1: If $w_{inf}(\gamma) \neq \varepsilon$ then $val(w_{inf}(\gamma)) \geq \nu$.

Proof:

$val(w_{inf}(\gamma)) = val\{y(\gamma) \ominus \widehat{p}(\gamma)u(\gamma)\} = val\{(w(\gamma) \oplus \widehat{p}(\gamma)u(\gamma)) \ominus \widehat{p}(\gamma)u(\gamma)\} = val\{w(\gamma) \ominus \widehat{p}(\gamma)u(\gamma)\} \geq val(w(\gamma)) = \nu$ (by formula (5), table I, and property 2).

■

Proposition 2: If $e \preceq u(\gamma) \preceq \frac{h(\gamma)}{h(\gamma)}$ and $w_{inf}(\gamma) \neq \varepsilon$ then $\widehat{s} = s$.

Proof: $\widehat{s} = \min\limits_{i=1,\ldots,\infty} \lfloor \frac{c(ir)}{i} \rfloor$ and $c(\gamma) = w(\gamma)_{inf} \lozenge y(\gamma)$. By lemma 1, $v_w = val(w_{inf}(\gamma)) \geq \nu$. As $e \preceq u(\gamma) \preceq \frac{h(\gamma)}{h(\gamma)} \Rightarrow y(\gamma) = h(\gamma)$ then $w_{inf}(v_w) = h(v_w)$ (by using property 1 and observing remark 4). Therefore $c(r) = [w(\gamma)_{inf} \lozenge h(\gamma)](r) = \bigwedge_{j=0}^{\infty} \frac{h(r+j)}{w_{inf}(j)} \leq \frac{h(r+v_w)}{w_{inf}(v_w)} = h(r+v_w) - h(v_w) = s$, because $h(r+k) - h(k) = s$ when $k \geq \nu$. Finally, $\widehat{s} = \min\limits_{i=1,\ldots,\infty} \lfloor \frac{c(ir)}{i} \rfloor \leq c(r) \leq s$. As $\widehat{s} \geq s$ (since $\widehat{s}$ is an upper bound for s) the conclusion is that $\widehat{s} = s$.

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Remark 7: If $w_{inf}(\gamma) = \varepsilon$ then $y(\gamma) = \widehat{p}(\gamma)u(\gamma)$, $\widehat{s} = \top$ and $\widehat{q} = \varepsilon$. This means that $\widehat{p}(\gamma)$ is a good model for the TEG.

The following algorithm summarizes the identification method.

Algorithm
begin
Collect N pairs of input and output dates $(u(k), y(k))$;
$\widehat{p}_i = \bigwedge_{k=0}^{N} u(k-i) \lozenge y(k)$ $i = 0, ..., \nu - 1$;
for $k = 0, ..., N$
$(\widehat{p}(\gamma)u(\gamma))(k) = \bigoplus_{i=0}^{\nu-1} (\widehat{p}_i \otimes u(k-i))$;
$w_{inf}(k) = \bigoplus_{i=0}^{k} \{y(i) \ominus (\widehat{p}(\gamma)u(\gamma))(i)\}$;
end
$c(k) = \bigwedge_{i=0}^{N-k} w_{inf}(i) \lozenge y(k+i)$ for $k = 0, ..., N$;
$\widehat{s} = \min\limits_{i=1,\ldots,L} \lfloor \frac{c(ir)}{i} \rfloor$ where $L = \lfloor \frac{N}{r} \rfloor$;
$\widehat{z}(k) = \widehat{s} \otimes \widehat{z}(k-r) \oplus u(k)$ for $k = 0, ..., N$;
$\widehat{q}_j = \bigwedge_{k=0}^{N} \widehat{z}(k-\nu-j) \lozenge y(k)$ for $j = 0, ..., r-1$;
end

VI. ILLUSTRATIVE EXAMPLE

Consider the TEG model depicted in the figure 1, where the structural parameters are $\nu = 2$ and $r = 1$. For an input firing sequence given by $u = [0\ 5\ 9\ 15\ 19\ 21]$ the output firing sequence is $y = [17\ 22\ 26\ 32\ 37\ 43]$. The figure 2 shows those sequences and the behavior in dashed lines when the input is $u_h(\gamma) = h(\gamma) \lozenge h(\gamma)$.

The application to the proposed method to the observed date gives $P(0) = 17$, $P(1) = 21$, $Q(0) = 25$ and $s = 6$, that is, the method converges to actual parameters of the system.

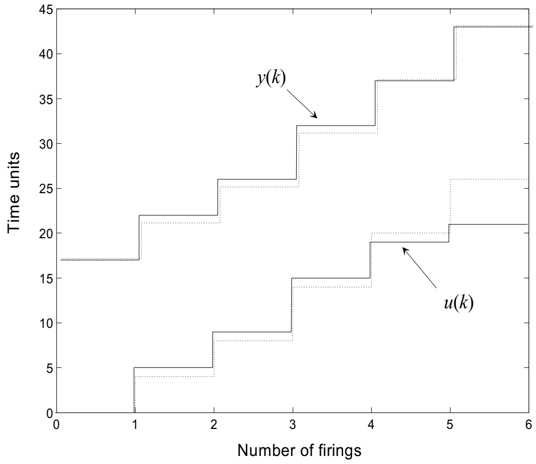

Fig. 2. Example of Timed Event Graph (TEG)

One must note that the input condition does not satisfy the requirements of proposition 2 (*i.e.* $u \npreceq u_h$). This example shows that the proposition is sufficient for the convergence but not necessary.

VII. Conclusion

The paper presents a parametric method for the identification of SISO TEG. The method is not restricted to the case of impulse input and it can be used in other input conditions. Some theoretical results are obtained as the convergence to the actual parameters in case of ”rich” input signal.

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