NEW APPROACH FOR INDUCTION MOTOR PARAMETERS ESTIMATION

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***Abstract*— A new subspace identification method is proposed for systems that can be described by complex state space realization, namely the induction motor Park's model. Besides all the well known engineering features of the subspace identification, this method allows to reduce the computational burden. The performances of the proposed method are demonstrated throughout an appropriate simulation study. The involved problem consists in determining the electrical parameters of an induction motor model.**

***Index Terms*— Induction motor, subspace identification, complex-formed system, space vector theory.**

I. INTRODUCTION

The most popular control for induction motor drive is the field oriented control. This algorithm is well known to be very sensitive with respect to electrical parameters accuracy. In practice, the constructor specifications are used and completed by no load and locked tests. This off-line method is very simple but extremely approximative. Others off-line methods can be cited, in particular estimation in frequency-domain using standstill frequency response [1]. More recently, some on-line identification schemes have been introduced. In [2], the method is based on a reference adaptive model in a field oriented control scheme. The rotor flux and parameters can be estimated simultaneously using extended Luenberger observer [3] or extended Kalman filter [4]. These algorithms have a very high computational load. Moreover most of these methods are sensitive to initial conditions.

In [5] and [6], the authors have developed identification algorithm based on linear filtering associated to a non linear physical parameters estimation. These two steps methods have been shown efficient. In a first step continuous time transfer functions are estimated using the so-called total least square method. In a second step nonlinear optimization is used to estimate parameters.

In this work, we propose to improve this identification scheme. In system identification we often consider that a model should have as few parameters as possible in order to obtain low variance in the estimated parameters (parsimony principle). Our plug-in consists in the specifying structure of a state space realization of the model to be identify. In the same time, we reduce the (first) identification calculation cost. Indeed, some systems own a particular structure. They can be transformed into an equivalent complex-values system with half number of states, inputs and outputs. These systems called complex-formed systems have been used successfully by [7] and [8]. The electrical behavior of an induction motor can be described by this kind of structure.

To identify a complex state space realization of these systems, we propose a complex subspace identification algorithm. Subspace methods for state space system identification are recent identification techniques. These methods have been mainly developed for linear systems and several algorithms have been proposed such as N4SID, MOESP and CVA. Some interesting overviews on subspace methods are presented in [9], [10] and [11]. More recently some subspace algorithms have been developed for linear time varying systems [12]. Our purpose is to propose a subspace algorithm using complex value signals.

a) Paper outline: The paper is organized as follows. Preliminary definitions and properties concerning the complex-formed systems are provided in section II. In section III, the induction motor model is presented and its complex-formed structure is underlined. The proposed identification method is presented in section IV. An induction motor identification problem is investigated in section V. Some concluding remarks end the paper.

II. PRELIMINARY DEFINITION

The following definitions and notations will be used throughout the paper.

Definition 1: The set $\mathcal{C}_M^{2m\times 2n}$ of complex-formed matrix of size $2m \times 2n$ is given by

$$\mathcal{C}_M^{2m\times 2n} \triangleq \left\{ M = \begin{bmatrix} M_1 & -M_2 \\ M_2 & M_1 \end{bmatrix} \ / \ M_1, M_2 \ \in \mathbb{R}^{m\times n} \right\}$$

Definition 2: The transformation $T_{\mathcal{C}}$ in $\mathcal{C}_M^{2m\times 2n}$ to $\mathbb{C}^{m\times n}$ is defined by

$$T_{\mathcal{C}}\left(\begin{bmatrix} M_1 & -M_2 \\ M_2 & M_1 \end{bmatrix} \right) = M_1 + jM_2$$

Definition 3: The set $\mathcal{C}_S^{2m\times 2p}$ of complex-formed system is defined by

$$\mathcal{C}_S^{2m\times 2p} \triangleq \{ G(s) \ / \quad \begin{array}{l} A \in \mathcal{C}_M^{2n\times 2n}, B \in \mathcal{C}_M^{2n\times 2m}, \\ C \in \mathcal{C}_M^{2p\times 2n}, D \in \mathcal{C}_M^{2p\times 2m} \} \end{array}$$

where $G(s)$ is the underlying transfer matrix of a state space realization (A, B, C, D), i.e.

$$G(s) = \frac{I_{2n}}{s} \star \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Notice that

$$G_c(s) = T_{\mathcal{C}}\left(G(s)\right) = \frac{I_n}{s} \star \begin{bmatrix} T_{\mathcal{C}}(A) & T_{\mathcal{C}}(B) \\ T_{\mathcal{C}}(C) & T_{\mathcal{C}}(D) \end{bmatrix}$$

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III. Induction motor

As stated in the introduction, the proposed identification method will be used to the determination of the induction motor electrical parameters. This identification is carried out from the classical Park's model that will be presented in subsection III-A. In subsection III-B, it is shown that ac machines can be described by complex state space realization thanks to the space vector theory [8]. The representation by complex space vectors has been proposed by [13] and is meanwhile widely used [8], [14], [7]. The rational behind this description is the fact that sinusoidal distribution can be mathematically described by space vectors. This is the case of induction motors as the internal voltages, currents, and flux linkages of a polyphase winding exhibit sinusoidal distributions since the windings themselves are distributed in space. Of particular interest, the proposed identification algorithm uses only stator currents and voltages. This makes it possible to consider the squirrel cage induction motor where the rotor electric measurements are not available.

A. State space Model in stationary frame

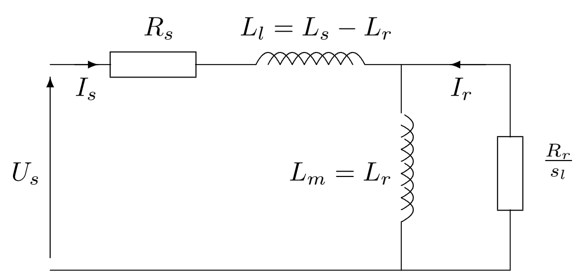

Fig. 1. Equivalent circuit of induction motor in steady-state with leakage inductance on stator side (the slip is defined by $s_l = \frac{\omega_s - \omega_r}{\omega_s}$).

The electrical behavior of an induction motor can be described in the (α, β) coordinate system in stationary reference frame fixed with the stator, with the leakage inductance on stator side as shown on Fig. 1. In this context, the well-known induction motor Park's model [15] is given by the following set of state variables equations

$$\left\{\begin{array}{lcl} \dot{I}_{s\alpha} & = & -\frac{R_s+R_r}{L_l} I_{s\alpha} + \frac{R_r}{L_l L_r}\Phi_{r\alpha} + \frac{\omega_r}{L_l}\Phi_{r\beta} + \frac{1}{L_l}u_{s\alpha} \\ \dot{\Phi}_{r\alpha} & = & -R_r I_{s\alpha} + \frac{R_r}{L_r}\Phi_{r\alpha} - \omega_r \Phi_{r\beta} \\ \dot{I}_{s\beta} & = & -\frac{\omega_r}{L_l}\Phi_{r\alpha} - \frac{R_s+R_r}{L_l} I_{s\beta} + \frac{R_r}{L_l L_r}\Phi_{r\beta} + \frac{1}{L_l}u_{s\beta} \\ \dot{\Phi}_{r\beta} & = & R_r I_{s\beta} + \omega_r \Phi_{r\alpha} - \frac{R_r}{L_r}\Phi_{r\beta} \end{array}\right. \tag{1}$$

where $I_{s\alpha}$, $I_{s\beta}$ are stator currents; $u_{s\alpha}$, $u_{s\beta}$ are stator voltages; $\Phi_{r\alpha}$, $\Phi_{r\beta}$ are rotor flux; $\omega_r = p\Omega$ with Ω the angular speed and p the number of pairs of poles.

As the identification will be performed using only the measurement of stator voltages and currents, only four parameters can be determined [16]. We consider the electrical parameters $(R_s,\ R_r,\ L_l,\ L_r)$. R_s and R_r denote the stator and rotor per-phase resistances. $L_l = L_s - L_r$ is the leakage inductance where L_s and L_r denote the stator and rotor per-phase inductance, respectively. The mutual inductance is assumed to be equal to the rotor inductance, i.e. $L_m = L_r$.

B. Complex-formed Model

Let first define the space vectors for the stator current, i.e. $\overline{I}_s = I_{s\alpha} + jI_{s\beta}$, the stator voltage, i.e. $\overline{U}_s = U_{s\alpha} + jU_{s\beta}$, and the rotor flux, i.e. $\overline{\Phi}_r = \Phi_{r\alpha} + j\Phi_{r\beta}$. Combining these definitions with the electrical model (1), we obtain the following equivalent complex model (This class of systems is described in Section II).

$$\left\{\begin{array}{lcl} \dot{X} & = & (A_1 + jA_2)X + (B_1 + jB_2)U \\ Y & = & (C_1 + jC_2)X + (D_1 + jD_2)U \end{array}\right. \tag{2}$$

with

$$X = \begin{bmatrix} \overline{I}_s \\ \overline{\Phi}_r \end{bmatrix}, \quad U = \begin{bmatrix} \overline{U}_s \\ 0 \end{bmatrix}, \quad Y = \overline{I}_s$$

$$A_1 + jA_2 = \begin{bmatrix} -\dfrac{R_s + R_r}{L_l} & \dfrac{R_r}{L_l L_r} - j\omega_r \dfrac{1}{L_l} \\ R_r & -\dfrac{R_r}{L_r} + j\omega_r \end{bmatrix}$$

$$B_1 + jB_2 = \begin{bmatrix} \dfrac{1}{L_l} \\ 0 \end{bmatrix}, \quad C_1 + jC_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D_1 + jD_2 = 0$$

There are two remarks that are worth to be made. Firstly, the complex-formed model is linear and all its parameters are time invariant except the rotor speed. If the latter is assumed to be constant, we get a linear time invariant model that can be described by the transfer matrix $G_c(s)$ which particularly satisfies $G_c(s) = T_{\mathcal{C}}(G(s))$. $G(s)$ being the transfer matrix of the Park's model when the rotor speed is assumed to be constant. Of fundamental importance, the transformation $T_{\mathcal{C}}$ has the following properties (see [7] for more details).

Property 1:

1) $T_{\mathcal{C}}$ is bijective.
2) $G(s)$ is stable if and only if $G_c(s)$ is stable.
3) $G(s)$ is observable if and only if $G_c(s)$ is observable.
4) $G(s)$ is controllable if and only if $G_c(s)$ is controllable.
5) The discretization of a continuous time complex-formed system by the bilinear transformation ([17]) gives a discrete time complex-formed system.

Secondly, the complex-formed model is more suitable from complexity reduction point of view.

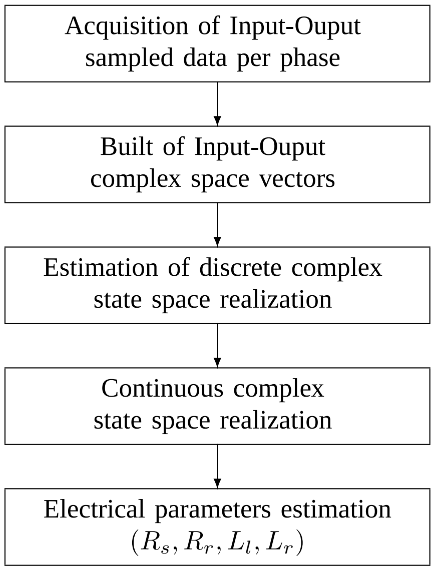

Fig. 2. Identification method description

IV. Subspace Identification of linear complex-formed system

A. Method

Recall that the objective consists in determining the electrical parameters (R_s, R_r, L_l, L_r). We propose the following two steps method.

Step 1 Identification of the complex state space realization

Step 2 Determination of the electrical parameters R_s, R_r, L_l and L_r

In the first step the complex system $G_c(s) = T_{\mathcal{C}}(G(s))$ is identified. The complex-formed structure of the motor model is hence fully exploited to reduce its underlying identification complexity. The second step of the proposed method allows to determine the electrical parameters (R_s, R_r, L_l, L_r) from the identified state space realization. This determination is algebraic thought it can be performed using an appropriate output error identification method. The latter shares all the actual nonlinear optimization problems, namely the wise parameter initialization. Figure 2 shows the main components of the proposed identification method.

B. Complex subspace identification

Let us consider the following class of sampled data time complex system

$$\Sigma : \left\{ \begin{array}{lcl} X(k+1) & = & AX(k) + BU(k) \\ Y(k) & = & CX(k) + DU(k) + V(k) \end{array} \right. \tag{3}$$

where $\{U(k)\} \in \mathbb{C}^m$, $\{Y(k)\} \in \mathbb{C}^p$ and $\{X(k)\} \in \mathbb{C}^n$ respectively denote the input, output and state sequences of the complex system, $\{V(k)\}$ is the noise sequence which is assumed to be uncorrelated with the input sequence. The state space realization (A, B, C, D) can be identified using subspace methods. These methods have shown to be relevant alternative to other identification methods. Indeed, they are more suitable for identification of Multi Input Multi Output systems and require no a priori parameterization nor non linear optimization. Furthermore, they borrow the numerical robustness of QR-factorization and Singular Value Decomposition.

The subspace identification is based on the following set of matrices that fully characterize the complex model. The first set concerns the Hankel matrices

$$U_p = \begin{pmatrix} U(1) & U(2) & \cdots & U(j) \\ U(2) & U(3) & \cdots & U(j+1) \\ \cdots & \cdots & \cdots & \cdots \\ U(i) & U(i+1) & \cdots & U(i+j-1) \end{pmatrix}$$

$$U_f = \begin{pmatrix} U(i+1) & U(i+2) & \cdots & U(i+j) \\ U(i+2) & U(i+3) & \cdots & U(i+j+1) \\ \cdots & \cdots & \cdots & \cdots \\ U(2i) & U(2i+1) & \cdots & U(2i+j-1) \end{pmatrix}$$

with $i > n$ and $j >> 1$. Notice that the Hankel matrices V_f and Y_f are constructed conformably to U_f. The second set concerns the extended observability matrix, the reversed extended controllability matrix and the Markov parameters matrix defined as follows

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{pmatrix} \tag{4}$$

$$\Delta_i = \begin{pmatrix} A^{i-1}B & \cdots & AB & B \end{pmatrix}$$

$$H_i = \begin{pmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{pmatrix}$$

It can be easily shown that the complex system (3) can be rewritten as follows :

$$\left\{ \begin{array}{lcl} X_{i+1} & = & A^i X_1 + \Delta_i U_p \\ Y_f & = & \Gamma_i X_{i+1} + H_i U_f + V_f \end{array} \right.$$

with

$$X_{i+1} = \begin{pmatrix} X(i+1) & X(i+2) & \cdots & X(i+j) \end{pmatrix}$$

The key step in subspace identification methods is to directly estimate the extended observability matrix Γ_i from the Hankel matrices. An instrumental variable matrix $Z_p = \begin{pmatrix} U_p \\ Y_p \end{pmatrix}$ is used together with the orthogonal projection operator Π_u of a matrix into the orthogonal complement of the row space of the matrix U_f, i.e.

$$\Pi_u = I_j - U_f^*(U_f U_f^*)^{-1} U_f$$

Γ_i is computed from the following result.

Theorem 1: . Assume that

– Σ is stable, observable and controllable ;
– $j \to \infty$ and $i > n$;
– the input sequence is persistently exciting of order $2i$;
– the input sequence $\{U(k)\}$ is uncorrelated with the noise sequence $\{V(k)\}$.

Let $O_i \in \mathbb{C}^{pi \times j}$ defined by

$$O_i = Y_f \Pi_u Z_p^* (Z_p \Pi_u Z_p^*)^{-1} Z_p \Pi_u$$

and consider its singular value decomposition

$$O_i = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^* \\ V_2^* \end{pmatrix} \tag{5}$$

Then

– O_i as rank n which is equal to the number of singular values in equation (5) different from zero ;
– the extended observability matrix can be estimated by

$$\Gamma_i = U_1 S_1^{1/2} L \tag{6}$$

where L is a non singular $n \times n$ matrix for a change of basis.

The previous theorem is an extension in the complex framework of theorem 12 in [9] and theorem 2 in [18].

For deriving matrices A and C, a straightforward method is to use the shift invariance structure of Γ_i. (4) implies

$$C = \Gamma_i(1 : p, :)$$

and

$$A = (\underline{\Gamma}_i^* \underline{\Gamma}_i)^{-1} \underline{\Gamma}_i^* \overline{\Gamma}_i$$

with

$$\begin{cases} \underline{\Gamma}_i = \Gamma_i(1 : p \times (i-1), :) \\ \overline{\Gamma}_i = \Gamma_i(p+1 : p \times i, :) \end{cases}$$

The latest two matrices B and D are then computed by minimizing the following linear problem

$$(B, D) = \text{argmin}\Big(\sum_{k=1}^{j} E(k)^* E(k) \Big)$$

with

$$E(k) = Y(k) - (C(qI - A)^{-1} B + D) U(k)$$

.

C. Parameters estimation

Let (A_c, B_c, C_c, D_c) be the continuous time complex state space realization obtained from the identified discrete time complex state space realization (A, B, C, D). This can be obtained easily using zero-order hold on the inputs (ZOH) or bilinear (Tustin) approximation [17].

From the state space matrices of the complex model described by equations (2), we extract the three following relations

$$\begin{cases} (C_1 + jC_2)(B_1 + jB_2) & = & \dfrac{1}{L_l} \\ \det(A_1 + jA_2) & = & \dfrac{R_s}{L_l}\left(\dfrac{R_r}{L_r} - j\omega_r\right) \\ \text{trace}(A_1 + jA_2) & = & \dfrac{R_s + R_r}{L_l} + \dfrac{R_r}{L_r} - j\omega_r \end{cases} \tag{7}$$

These relations are satisfied for any state space basis, in particular, for the continuous time state space realization (A_c, B_c, C_c, D_c) obtained previously. In separating the real part from the imaginary part, we obtain

$$\begin{cases} \Re(C_c B_c) & = & \dfrac{1}{L_l} \\ \Re(\det(A_c)) & = & \dfrac{R_s R_r}{L_l L_r} \\ \Im(\det(A_c)) & = & -\omega_r \dfrac{R_s}{L_l} \\ \Re(\text{trace}(A_c)) & = & \dfrac{R_s + R_r}{L_l} + \dfrac{R_r}{L_r} \\ \Im(\text{trace}(A_c)) & = & -\omega_r \end{cases} \tag{8}$$

We have restricted the number of equations in order to avoid redundancy. This problem can be solved using standard non linear optimization algorithm with Matlab Optimization toolbox [19].

V. APPLICATION TO AN INDUCTION MOTOR

The identification has been tested using simulation data obtained with a 7.5 kW controlled induction motor in constant nominal speed $1450\ rpm$. In [5], the authors note that for low speed the model presents pole-zero cancellation.

The stator voltage space vector applied to the motor is shown on Fig. 3. This signal is a constant frequency sinusoidal signal, allowing to have a constant rotor speed (see Fig. 5), added to a persistent excitation in regard to identify a complete linear model at the specified speed.

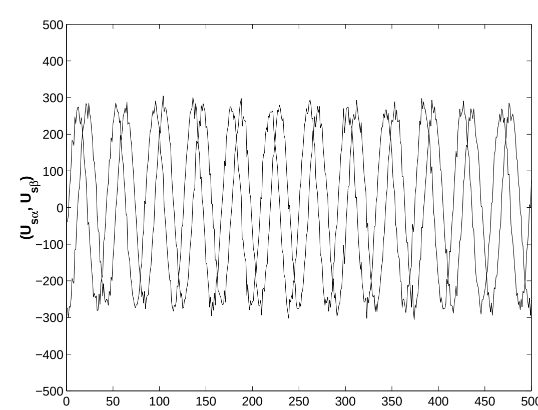

Fig. 3. Space vector of stator voltage in (α, β) reference frame

Extra noise has been added to the simulated (output) signals with a $40dB$ signal-to-noise ratio. The stator current space vector resulting is presented Fig. 4.

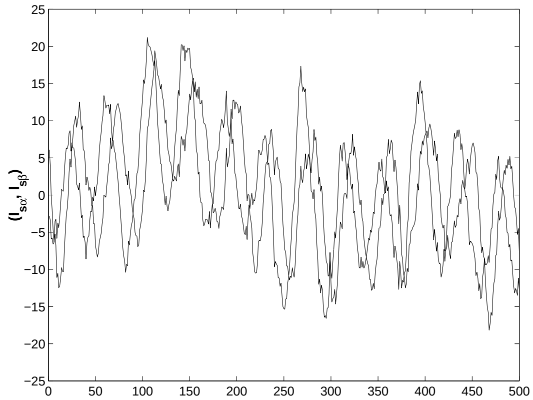

Fig. 4. Space vector of stator current in (α, β) reference frame

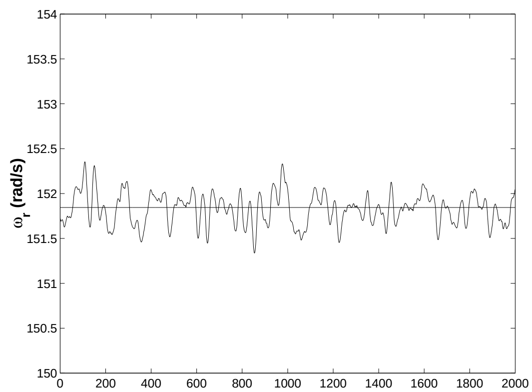

Fig. 5. Rotor speed

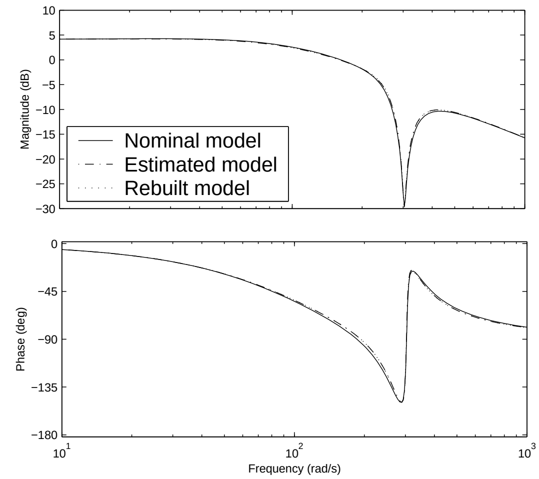

Fig. 6. Bode plots of the nominal, estimated and rebuilt transfers $G(s) = \frac{\overline{I}_s}{\overline{U}_s}$

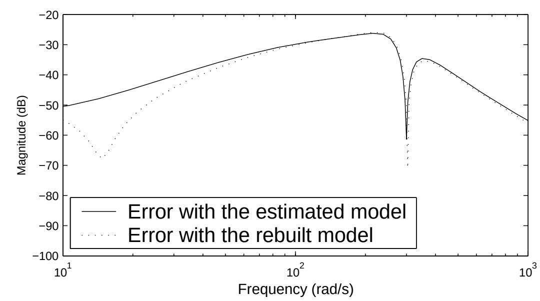

Fig. 7. Magnitude frequency response of estimation error

Identification have been carried out using sequences of length 2000 obtained with a sample frequency of $2\ kHz$. Estimation results are presented on Fig. 6. It is worth noticing that the estimated frequency response fits very well the nominal frequency response. For simple curiosity considerations, we have also given on Fig. 7 the frequency response of the error between the nominal model and respectively the estimated model and the rebuilt model. Notice that the rebuilt model is the model (1) simulated with the estimated parameters. Again, we remark the good agreement between the real transfer and the rebuilt transfer provided by our algorithm.

As shown on Table I, the accuracy of parameters estimation depends on estimation quality of the product $C_c B_c$, the trace and the determinant of matrix A_c. A comparison between exact parameters and estimated parameters is presented on Table II.

VI. Conclusion

The induction motor parameters estimation problem has been solved using a complex subspace identification method. This method is illustrated through an appropriate simulation study. The simulation results are enough promising to develop an engineering methodology around the subspace identification of complex system. Two features have to be taken into account, namely the direct identification of the continuous state space realization and the variations of the rotor speed. The first feature can be handled using the available continuous time identification culture. The second feature could be investigated using the recent results on identification of linear parameter varying systems [12].

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TABLE I

NOMINAL AND ESTIMATED VALUES OF THE PRODUCT C_cB_c, THE TRACE AND THE DETERMINANT OF MATRIX A_c

	$\Re(C_cB_c)$	$\Im(C_cB_c)$	$\Re(det(A_c))$	$\Im(det(A_c))$	$\Re(trace(A_c))$	$\Im(trace(A_c))$
nominal	166.67	0	-176.06	303.69	461.54	-31887
estimate	166.85	-0.15821	-168.9	304.02	418.63	-31855
relative error (%)	0.1115	-	4.067	0.11018	9.2964	0.10163

TABLE II

NOMINAL AND ESTIMATED VALUES OF ELECTRICAL PARAMETERS

Parameter	R_s	R_r	L_l	L_r	ω_r
nominal (S.I)	0.63	0.4	0.006	0.091	303.69
estimate (S.I)	0.62797	0.36037	0.00599	0.09019	304.02
relative error (%)	0.32	9.90	0.11	0.88	0.11

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