Fuzzy-Sugeno Lateral Tyre Modelling of a Mobile Robot

J.T. Economou, A.Tsourdos, P.C.K. Luk, B.A. White
Department of Aerospace, Power and Sensors
Cranfield University-RMCS
Shrivenham,SN6 8LA
England,UK
E-mail J.T.Economou@rmcs.cranfield.ac.uk

Abstract— **Wheeled mobile robot and land vehicle handling is greatly influenced by the non-linear phenomena which are experienced by the wheel at the tyre/ground contact patch. It is well accepted that the tyre, although in proportion is a small part of a vehicle, is a complex and non-linear component. In this paper an alternative mathematical modelling approach is shown which complements the traditional approaches reported in the literature. The tyre is modelled using a first order Sugeno fuzzy model. The blended Sugeno fuzzy tyre model retains its initial non-linear characteristics and the model does have an explicit mathematical formulation which originates from the linguistic tyre model description formulation. The methodology shown here describes accurately the lateral force generation with respect to the tyre slip angle over the entire slip angle envelope. The suggested method has been validated from experimental data.**

Keywords— **Fuzzy Logic, Sugeno, Tyre Modelling, Mobile Robot**

NOMENCLATURE

f	Front wheel
r	Rear wheel
$F_y^{[f,r]}$	Lateral wheel force
$\alpha_{[f,r]}$	Tyre slip angle
$\dfrac{\partial F_y}{\partial \alpha}$	Tyre cornering stiffness
$C_{\alpha_{[f,r]}}$	Linearised tyre cornering stiffness
q	Fuzzy rule number
k	Fuzzy variable index
c_q	Membership function centre
d_q	Membership function spread
A_q	Fuzzy set
μ_{A_q}	Membership function
λ_q	Multivariable blender
γ	Sugeno fuzzy offset

I. Introduction

VEHICLE stability issues are directly related to the lateral forces which are generated at the wheel tyre patches. Hence the modelling of these forces is necessary for accurately predicting the mobile robot's trajectory. For the conventional steered vehicles the traditional approach has been used to consider manoeuvres of low to moderate severity. However there have been attempts to model the vehicle's response while the forces approach values near the friction limit at the road/tyre interface. Further work relates the lateral forces and the longitudinal tyre forces using the wheel slip which includes the tyre elasticity properties. In [3] an extension of the LuGre dynamic [2] friction model from longitudinal motion to longitudinal/lateral motion is developed.

In this paper an alternative analytical approach has been considered which allows the lateral tyre modelling using an "intelligent based method" which belongs to the class of 1-st order fuzzy logic Sugeno model [10].

The proposed approach can be applied to robotic vehicles [1] in which an explicit mathematical lateral tyre model description is necessary with respect to the front and rear tyre slip angles. Some of the advantages of using the fuzzy-Sugeno tyre model, instead of a linearised model is that the complete lateral force envelope can be studied over the entire front and rear slip angle which can influence the vehicle stability.

In [9] fuzzy logic Mamdani modelling of vehicles which are entering the force saturation region have been presented together with experimental results. Two linguistic variables were chosen, hardness and slipperiness which produced a satisfactory tyre-ground interface model.

Other researchers [4] have used a Mamdani fuzzy logic method to derive the tyre force for a vehicle which has all the wheels driven from a permanent magnet machine.

In the case of a fuzzy tyre model for fine tuning or even assistance in automatically generating the rules which best describe the tyre lateral force and slip angle surface several techniques exist in the literature [5], [6], [7], [8]. Further research in this area [11] has related for the class of Takagi-Sugeno-Kang fuzzy systems the optimum number of rules and maximum allowed error between the experimental data and the predicted.

The paper has been organised as follows: in Section 2 the tyre lateral force generation summary is given together with the experimental set-up; in Section 3 the Sugeno lateral tyre model is given; Section 4 gives the experimental and the estimated fuzzy Sugeno results; finally the conclusions are given.

II. Tyre Lateral Force Generation

The basic tyre-axle-ground system is shown in Figure 1. In this case the camber angle which is generated by rotating the wheel with respect to the vertical axis form the ground is zero. The tyre is considered to be elastic and therefore flexible.

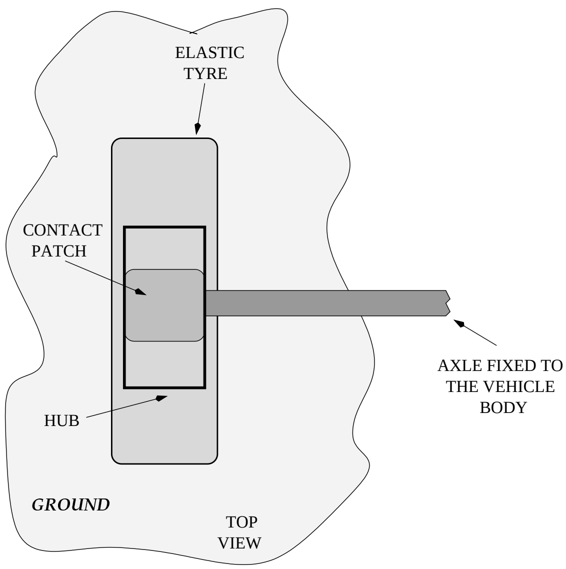

Fig. 1. Basic tyre-axle system , (top view)

When a vehicle yaw request is initiated then a turning moment is induced which causes the slip angle to be generated between the hub and the elastic tyre. This angle is usually less than 12 rotational degrees but sufficiently large to cause a smooth rotation compared to a maximum Ackerman vehicle wheel rotation of approximately 30 rotational degrees. The maximum slip angle depends on the friction limit between the tyre and the ground, the tyre elastic characteristics, the tyre pressure and finally on the differential demand between the two wheel sides. Figure 2 illustrates the tyre slip angle generation. At time $t = \delta t$ the tyre contact patch partially reaches

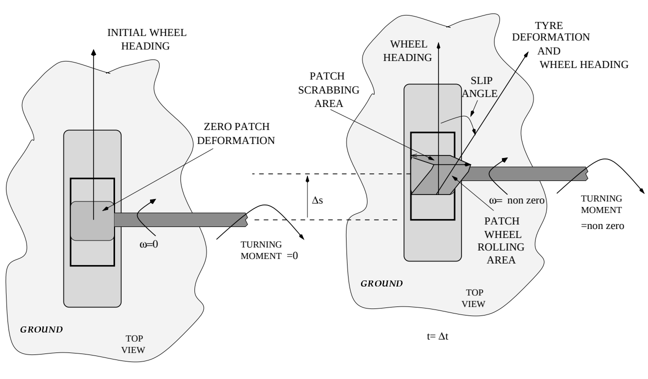

Fig. 2. Wheel slip angle graphical generation

the friction limit and therefore tyre scuffing occurs and the remaining of the tyre patch temporarily deforms and creates an Ackerman equivalent angle. As the wheel rotates the two combined effects of wheel rolling and tyre deformation causes the overall vehicle to turn smoothly by requesting a differential demand. The tyre force is a complex variable to model due to the large number of conflicting parameters. For example a large patch area would offer less sinkage on a soft surface and therefore reduces losses. It would also cause greater friction for the tyre on hard tarmac grounds and therefore difficulty in turning.

A. Experimental Setup

The tyre rig used for obtaining data consisted of the travelling road which simulates a free rolling wheel on tarmac, the vertical load system, the tyre slip angle setting, the camber angle setting, and the lateral force transducers as shown in Figure 3. Firstly the rig was cal-

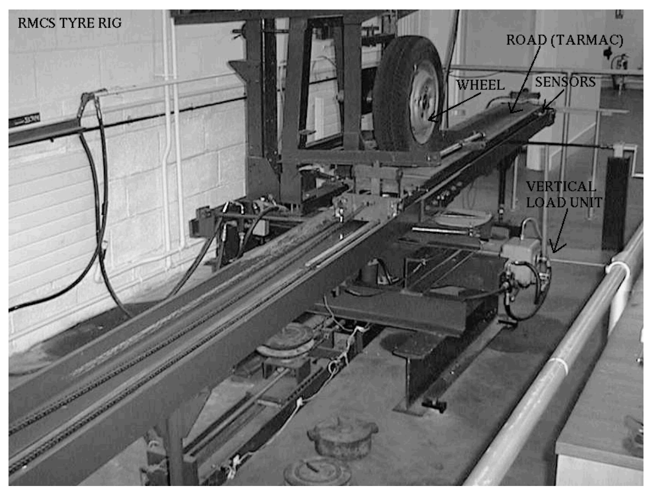

Fig. 3. Tyre-rig at Cranfield University

ibrated using the appropriate equipment and after placing the wheel with the required camber and slip angle and vertical load the experiment may start. The rig then provides the induced cornering force in real time and therefore a mapping between the several slip angles and cornering forces for different vertical loads.

III. Sugeno Tyre Model

The proposed Sugeno-type tyre model piecewise linear section which corresponds to the fuzzy $q-th$ rule is given in Equation 1.

$$\left.\begin{array}{c} \mathit{Rule}\ q \in [1, q_{max}] :\ \text{IF}\ \alpha_{[f,r]}\ \text{is}\ A_q \ldots \\ \\ \text{THEN}\ F_y^{[r,f](q)} = \frac{\partial F_y^{[r,f]}}{\partial \alpha_{[r,f]}}|^{(q)} \alpha_{[r,f]} + \gamma^{(q)} \\ \text{where}\ q \in N^{*+} \end{array}\right\} \quad (1)$$

For this tyre there can be three piecewise linear sections which correspond to $q = 2$ fuzzy rules. Experimental data using the tyre rig which was discussed in Section II-A are shown in Figure 4.

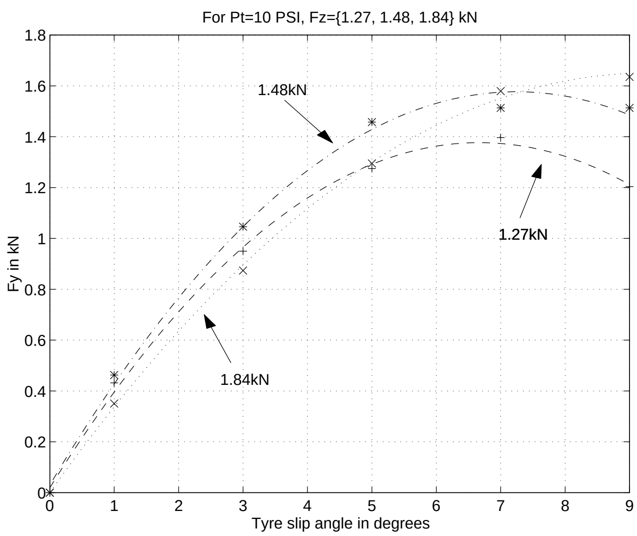

Fig. 4. Typical plot shows for a constant tyre pressure and various loads, the variation of the tyre cornering force with respect to the tyre slip angle: The data were obtained from the Tyre Rig at Cranfield University. The tyre hub velocity is $u = 0.12m/s$ and the camber angle is 0 degrees.

The experimental results of Figure 4 show the significance of the radial tyre load on the cornering force. A progressive increase in the load results in an expected increase of the Coulomb friction limit. For all data the regime of approximate linearity is up to around 5 degrees on a hard surface.

In Figure 5 a typical lateral force variation is shown with respect to the slip angle variation. The fuzzy logic methodology allows the user to define the regions of linearity from observation or alternatively via the use of fuzzy specific optimisation tools such as the subtractive clustering method [7], [8]. The individual $q-th$ fuzzy rule applied here is of a 1st order Sugeno type and therefore each individual rule is piecewise linear. However from the properties shown in Section III-A more than two rules are partially activated. The resulting rule blending produces the lateral tyre force which retains the non-linear tyre characteristics.

The selected membership functions are shown in Equations 2.

$$\left.\begin{aligned} &\mu_{A_q}(\alpha_{[f,r]}) = e^{\left(\frac{-(\alpha_{[f,r]}-c_q^{[f,r]})^2}{d_q^{[f,r]}}\right)} \\ &\text{with } q=\{1,2\}. \end{aligned}\right\} \qquad (2)$$

A. *Tyre Sugeno fuzzy properties*

In this section the fuzzy Sugeno properties are given which are valid for a tyre which exerts non-linear behaviour which requires a large number of fuzzy rules q_{max}.

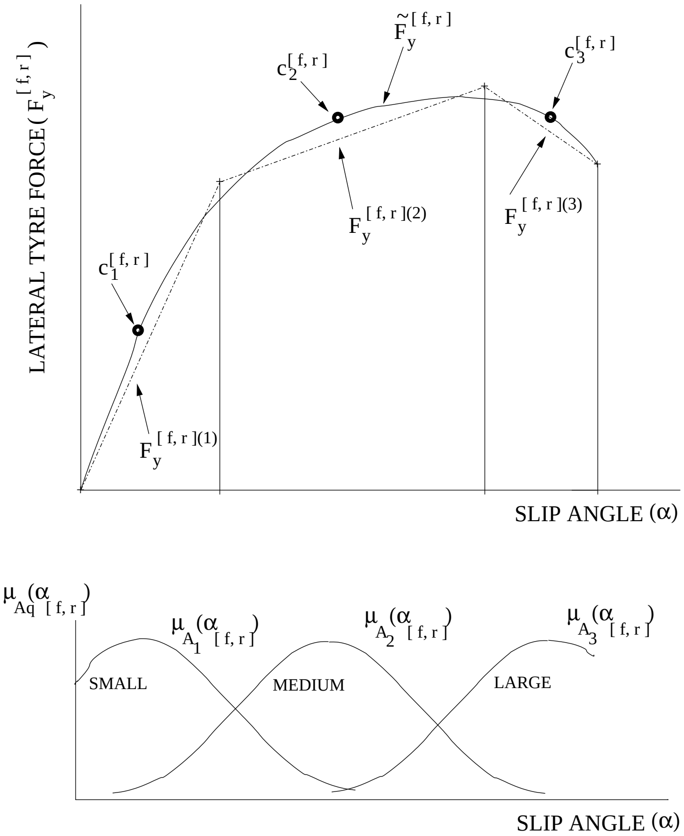

Fig. 5. Lateral force fuzzyfication

Property 1: Fuzzifier: Let $(\alpha_{[f,r]})$ be objects in the Universes $\Omega_{\alpha_{[f,r]}} \subset \Omega$. The fuzzifier is a function which maps the variable $(\alpha_{[f,r]})$ to values into interval $[0,1]$ (normal sets), Equations (2).

Property 2: Consistency: A set of fuzzy rules is consistent if there are no rules with the same antecedent but different Sugeno consequent.

Property 3: Rules Completeness: A set of fuzzy rules is complete if for any $(\alpha_{[f,r]}) \in \Omega$ there exists at least one rule $q \in [1, q_{max}]$ such that $\mu_{A_q}(\alpha_{[f,r]}) \neq 0$.

Property 4: Rule Influence : The $q-th \in [1, q_{max}]$ rule influence λ_q which combines the prepositions of the antecedent for each rule and is given by: $\lambda_q = \left(\prod_{k=1}^{k_{max}} \mu_{A_k^{j_k}}(x_k)\right)_q$. Where x_k for $k=1$ is given by $\underline{x} \triangleq [x_1] = [\alpha_{[f,r]}]$.

Property 5: Membership Functions: The fuzzy sets for the Sugeno variable are: $A_q = \{\Omega_{\alpha[f,r]} \subset \Omega | \mu_{A_q}(\alpha_{[f,r]})\}$.

Property 6: The parallel fuzzy firing of all $q-$rules result to the defuzzyfication Equation (3).

$$\left.\begin{aligned} &\tilde{F}_y^{[f,r]}(\alpha) = \frac{\sum_{q=1}^{q_{max}} \lambda_q\left(\frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}\Big|^{(q)}\alpha_{[f,r]}+\gamma^{(q)}\right)}{\sum_{q=1}^{q_{max}} \lambda_q} \\ &\text{Where } \lambda_q = \prod_{k=1}^{k_{max}} \mu_{A_q}(\alpha_{[f,r]}) = \mu_{A_q}(\alpha_{[f,r]}) \end{aligned}\right\} \qquad (3)$$

The tyre non-linearity is captured in Equation 3 because all the piecewise models have been blended. For this specific tyre, Equation 3 is expanded to Equation 4.

$$\left.\begin{aligned}
\tilde{F}_y^{[f,r]}(\alpha) &= \frac{\lambda_1 \frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}|^{(1)}\alpha_{[f,r]}+\gamma^{(1)}}{\sum_{q=1}^{q_{max}}\lambda_q}+ \\
&+\frac{\lambda_2 \frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}|^{(2)}\alpha_{[f,r]}+\gamma^{(2)}}{\sum_{q=1}^{q_{max}}\lambda_q}\dots \\
&+\frac{\lambda_{q_{max}}(\frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}|^{(q_{max})})\alpha_{[f,r]}+\gamma^{(q_{max})}}{\sum_{q=1}^{q_{max}}\lambda_q}
\end{aligned}\right\} \quad (4)$$

The resulting fuzzy Sugeno lateral force generation is shown in Equation 5. The non-linear Sugeno type fuzzy augmented coefficients $(\xi_1^{[f,r]}, \xi_2^{[f,r]}, \dots, \xi_{q_{max}}^{[f,r]})$ are given in Equations 6.

$$\tilde{F}_y^{[f,r]}(\alpha) = \xi_1^{[f,r]} + \xi_2^{[f,r]} + \dots + \xi_{q_{max}}^{[f,r]} \Big\} \quad (5)$$

$$\left.\begin{aligned}
\xi_1^{[f,r]} &= \frac{e^{(\frac{-(\alpha_{[f,r]}-c_1^{[f,r]})^2}{d_1^{[f,r]}})}(\frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}|^{(1)}\alpha_{[f,r]}+\gamma^{(1)})}{g^{[f,r]}}+ \\
\xi_2^{[f,r]} &= \frac{e^{(\frac{-(\alpha_{[f,r]}-c_2^{[f,r]})^2}{d_2^{[f,r]}})}(\frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}|^{(2)}\alpha_{[f,r]}+\gamma^{(2)})}{g^{[f,r]}} \\
&\vdots \\
\xi_{q_{max}}^{[f,r]} &= \frac{e^{(\frac{-(\alpha_{[f,r]}-c_{q_{max}}^{[f,r]})^2}{d_{q_{max}}^{[f,r]}})}(\frac{\partial F_y^{[f,r]}}{\partial \alpha_{[f,r]}}|^{(q_{max})}\alpha_{[f,r]}+\gamma^{(q_{max})})}{g^{[f,r]}}
\end{aligned}\right\} \quad (6)$$

In Equation 7 the coefficient $g^{[f,r]}$ is given.

$$\left.\begin{aligned}
g^{[f,r]} &= e^{(\frac{-(\alpha_{[f,r]}-c_1^{[f,r]})^2}{d_1^{[f,r]}})} + e^{(\frac{-(\alpha_{[f,r]}-c_2^{[f,r]})^2}{d_2^{[f,r]}})} \\
&\dots + e^{(\frac{-(\alpha_{[f,r]}-c_{q_{max}}^{[f,r]})^2}{d_{q_{max}}^{[f,r]}})}
\end{aligned}\right\} \quad (7)$$

IV. Experimental Results

The data acquired provided the dynamic tyre cornering force and the distance travelled by the wheel. Figure 6 shows the experimental data which were obtained from the tyre rig for different tyre slip angles. The wheel rolls at a constant speed while the slip angle is set initially by varying the travelling surface with respect to the hub heading. Figure 6 shows that as the tyre slip angle increases with the cornering force, which has been converted to the wheel lateral force.

However for the static fuzzy Sugeno model analysis the steady-state data were obtained by considering the data set $\mathcal{Z}_i \subset \mathcal{P}_i$ where $\mathcal{P}_i$ represents the dynamic data set for each tyre slip angle $\alpha_i = \{1^o, 3^o, 5^o, 7^o, 9^o\}$. The steady-state data sets $\mathcal{Z}_i$ were obtained by selecting the

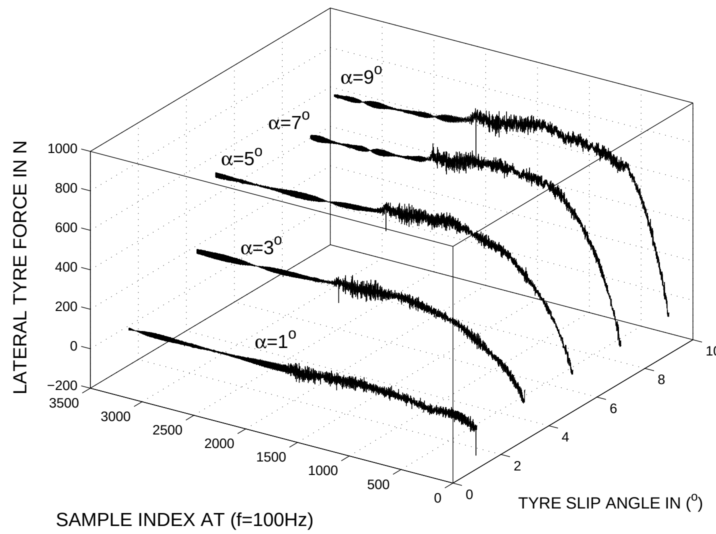

Fig. 6. Experimental data obtained from the Tyre Rig for different tyre slip angles.

lateral tyre force $(F_{yi}(kT_s))$ data points which satisfy the property:

$$k \geq k'|^{0.9 F_y^{(p)}{}_i(k^{(p)}T_s)}. \quad (8)$$

Where $F_y^{(p)}{}_i(k^{(p)}T_s)$ represents the first peak lateral force value for each slip angle data set (i). The resulting data sets $\mathcal{Z}_i$ are shown in Figures 7(a),(b),(c),(d) and (e).

The cluster's mean for each tyre slip angle in combination with the subtractive clustering method have generated for these data sets two piecewise linear fuzzy Sugeno rules [7], [8]. The two Sugeno models were combined using the fuzzy properties shown in Section III-A which maintained the non-linear lateral tyre characteristics. The resulting estimated fuzzy Sugeno estimated lateral tyre force is shown together with the mean cluster data points in Figure 8.

V. Conclusions

In this paper an analytical approach has been shown for modelling the non-linear behaviour of the tyre lateral component of a mobile robot. The force was modelled using a Sugeno 1-st order type fuzzy system. Without the loss of generality the tyre slip angle was the influential variable while the camber angle, the tyre pressure and the vertical load were constant during the analytical formulation.

Further research will extend these results and report for the case of a multivariate Sugeno fuzzy tyre model and will be supported from experimental data. The proposed technique allows the use of a model which is applicable to both conventional and differentially steered wheeled mobile robots. The theoretical procedure has been validated from experimental data and is independent of the

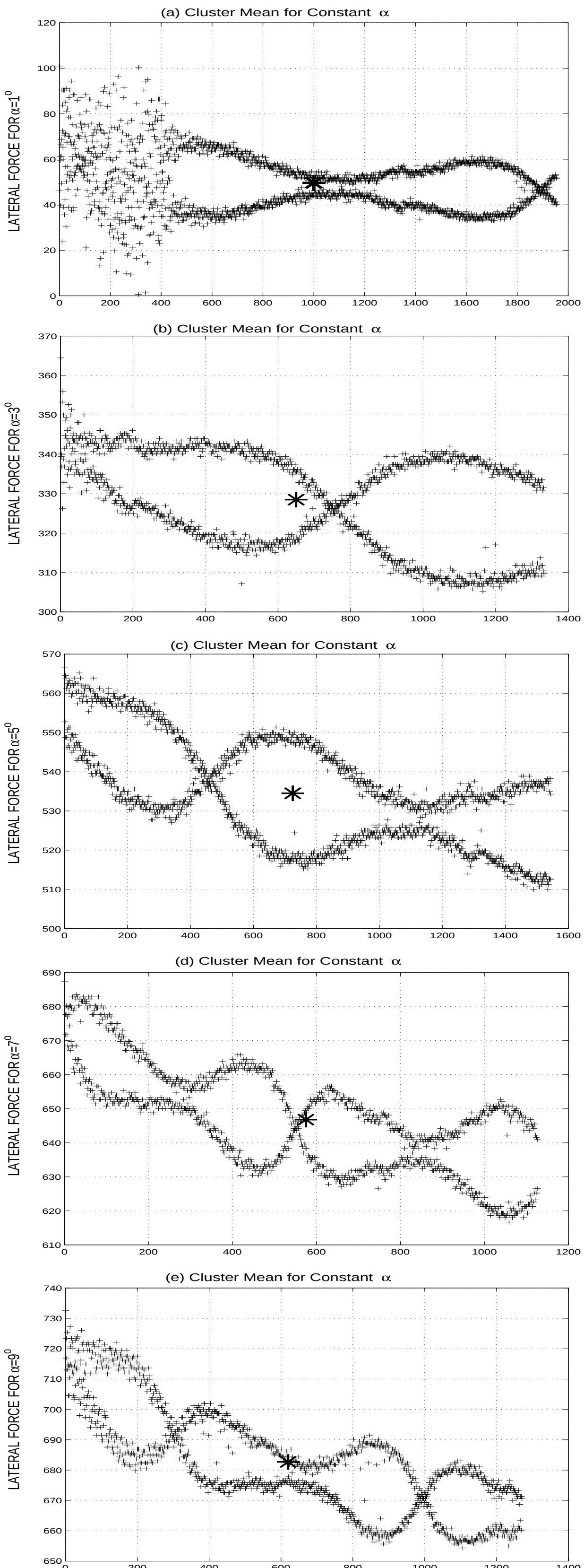

Fig. 7. Experimental $\mathcal{Z}_i$ Data Clusters

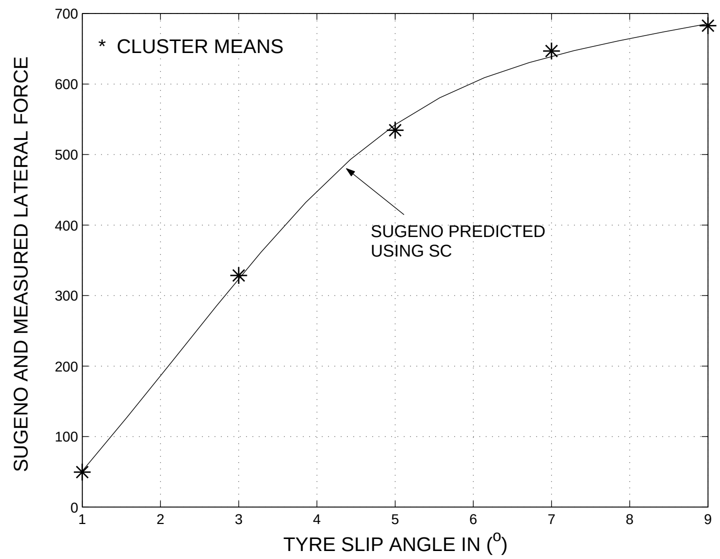

Fig. 8. Fuzzy Sugeno Lateral Force Estimated and Mean Data Clusters

large influence of the parameter variations when compared to polynomial based models. In addition the fuzzy methodology allows the inclusion of tyre expert knowledge into the modelling procedure thus allowing the capturing of all the essential tyre behaviour.

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