Identification of the Continuous-time Transfer Function from Sampled Data Measurements with Application to Disk Drive Dynamics

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***Abstract*—A method for identifying the continuous-time transfer function of an unknown linear time-invariant system by processing the sampled system output data is developed. The novelty of the method is that it can identify the system transfer function at frequencies that violate *Shannon*'s Sampling theorem. The method is based on exciting the system with two orthogonal sinusoids and constructing a linear complex equation that relates the sampled output points with the unknown values of the transfer function. The equation is independent of aliasing and has always a solution. An important application of the method is for identifying the continuous-time dynamics of a Hard Disk Drive system where the sampling frequency of the output is fixed by the number of servo bursts and the spinning speed of the disk.**

***Index Terms*—identification, disk drives, signal sampling, transfer functions**

I. INTRODUCTION

The natural world operates in an analog domain, but information signals may frequently be processed, measured or otherwise manipulated more efficiently in the digital domain. The conversion from the analog domain to the digital domain is accomplished with analog-to-digital converter (ADCs) or it may happen naturally due the sensing process. An ADC receives as input an analog or continuous-time signal and produces as output a digital signal. The digital signal is a sequence of numbers that equal to the value of the analog signal at the sampling times. For example if $y(t)$ is the analog signal, $y(nT)$, n=0,1,2,…, will be the digital signal and $y(nT) = y(t)\,|_{t=nT}$ where T is the sampling period of the ADC. It is clear that some information present in the analog signal is lost during the conversion process. The amount of the information lost depends on how fast the sampling process takes place, i.e., how close to each other the points nT are on the time axis. This implies that the smaller the sampling period T or equivalently the higher the sampling frequency $\omega_s = 2\pi / T$ is, the smaller the amount of information lost during the conversion [1]-[2].

In some applications, such as in the case of hard disk drives the analog-to-digital conversion occurs naturally. For example in the case of the disk drive the position of the head relative to the center of the track is generated in the digital form with a sampling frequency that depends on the number of prewritten sectors on the disk with position information data and the spinning speed of the disk. In the case of the disk drive and other applications it is of interest to identify the continuous time transfer function of the system to be used for modeling and control design purposes [1],[6].

According to *Shannon*'s sampling theorem the recovery of the lost information during the digital conversion is not possible if the sampling frequency is less than half the maximum frequency present in the signal because of aliasing [1]-[2]. In practice, aliasing is minimized or avoided by increasing the sampling frequency well above the marginal one dictated by *Shannon*'s Sampling Theorem. In the case of the disk drive and other applications the sampling rate is fixed. This implies that for system identification and signal reconstruction to be possible, the highest frequency in the analog signal should be much less than the sampling frequency. Ideally according to *Shannon*'s sampling theorem the limiting frequency is half the sampling frequency. The implication is that dynamics and resonant modes in the system corresponding to frequencies close and above half of the sampling frequency cannot be identified with current methods due to aliasing. The current method of identifying the transfer function of a system from sampled data is to excite the system with a sinusoidal sweep, record the sampled output data and used a Fast Fourier Transform method to calculate the values of the transfer function at different frequency points [7]-[9]. This method however breaks down at frequencies close and above half of the sampling frequency. Consequently the value of the transfer function at high frequencies cannot be calculated with current methods unless the sampling frequency is increased further. In the case of a disk drive this is not possible unless additional position data sectors are added and/or the spinning speed is increased further.

In this paper, the deficiencies of the prior works are overcome by the proposed method. It would be beneficial to identify the transfer function of a continuous time system using the digitized output of the system. The identification of the system is essential for control design purposes and/or modeling and appears in almost every practical control system

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problem. In the case of the disk drive, where the sampling frequency of the output is fixed by the design, identification of the high frequency dynamics and resonant frequencies helps design or tune the servo controller parameters in order to improve the performance of the servo controller. This in turn will lead to a more precise tracking of the center of the tracks, which in turn implies smaller in size disk drives for the same capacity of data [3]-[6].

The proposed technique is composed of several exemplary operational components. The system transfer function is calculated at a wide range of specified frequencies by exciting the system sequentially with two orthogonal sinusoidal signals with a frequency that varies over the range of interest and using the corresponding sampled output data to construct an algebraic complex equation that relates the unknown values of the transfer function at the frequency of interest with known data. This equation is independent of aliasing and has always a solution. The estimate of the transfer function at the frequency point of interest is simply the average of all the output samples weighted by the values of the input excitation signal. This process is repeated for each frequency and the data are used to construct a Bode Diagram for the system from which the transfer function of the system can be obtained using standard curve fitting techniques. The use of two orthogonal sinusoids may be reduced to a single sinusoid if the excitation frequency is not close to be equal to half or integer multiple of the sampling frequency and the noise level is low. In this case the inversion of a large matrix is required to be calculated. When the transfer function has unstable poles, the excitation signal is passed thought a specially designed pre-filter before applied to the system. When the unstable system is in closed loop, in addition to the pre-filter, the output of the controller is passed thought a specially designed filter whose output is used to modify the measured digitized output of the system.

An important technical advantage of the proposed method is that it enables Bode diagram construction at all frequencies of interest without having to increase the sampling frequency. This in turn allows the identification of the transfer function of the continuous time system over a wide frequency range where current methods break down.

Another important technical advantage of the proposed method is the ability to use it in any control feedback system where the sampling frequency is limited in order to occasionally check the validity of the model (generated from the Bode diagram) of the system on which the control design is based on in order to adjust the controller parameters and improve performance. An important application of the proposed technique is to identify the high resonant frequencies of a disk drive system where the position error signal is measured at a fixed sampling rate determined by the number of position sectors on the disk and the spinning speed. The knowledge of the resonant frequencies allows the tuning of the servo controller in order to improve the tracking performance of the hard disk servo controller in addition to other benefits [1],[6].

II. IDENTIFICATION OF CONTINUOUS-TIME TRANSFER FUNCTIONS IN OPEN LOOP

Consider the system described by Fig. 1.

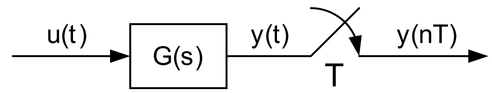

Fig. 1. System Block Diagram

where $G(s)$ is an unknown transfer function, $u(t)$ is the input, and $y(nT)$ is the output sampled every T seconds. The output $y(t)$ is not available for measurement but $y(nT)$ and $u(t)$ are measurable. According to *Shannon*'s Sampling Theorem the output $y(t)$ cannot be reconstructed from $y(nT)$ even in the ideal case unless the sampling frequency $\omega_s = 2\pi / T$ is greater than twice the maximum frequency in $y(t)$.

In practice, due to computational errors $y(t)$ can be reconstructed provided its maximum frequency is well below $\omega_s / 2$. For frequencies close to and above $\omega_s / 2$ aliasing takes place in which case ideal filtering or any kind of signal reconstruction cannot be used to reconstruct $y(t)$ from $y(nT)$. This problem prevents the generation of a Bode diagram for $G(s)$ using conventional methods, for frequencies close and above $\omega_s / 2$.

The proposed method bypasses the limitations of the *Shannon*'s Sampling Theorem, by designing the input excitation signal $u(t)$ so that the calculation of $G(j\omega_i)$ at any finite frequency ω_i from the sampled output data $y(nT)$ is computationally feasible despite aliasing.

A. *Identification of Continuous-Time Transfer Function of Stable Systems*

The identification of $G(s)$ in Fig. 1 involves the excitation of the system over a certain frequency range, construction of a Bode diagram that represents the frequency response of the system and calculation of $G(s)$ from the Bode diagram by using standard curve fitting techniques. The conventional method for generating a Bode diagram is to excite the system with a rich frequency signal and record the output sequences. Then compute the Discrete Time Fourier Transform (DTFT) or Fast Fourier Transform (FFT) for both input and output sequences. The value of the transfer function at a particular frequency is computed from the knowledge of the spectrums of the output and input signal at that frequency. In Fig. 1, however, only $y(nT)$ is available for measurement and at frequencies $\omega_i \geq \omega_s / 2$ the sequence $y(nT)$ does not imply a unique $y(t)$ due to aliasing. In fact an infinite number of $y(t)$ signals are equal to $y(nT)$ at $t=nT$. In the frequency domain this means that at high frequencies that violate *Shannon*'s Sampling Theorem, folding takes place where the digitized output spectrum consists of overlapping spectrums of the continuous-time output that are shifted by multiples of the sampling frequency. The continuous-time output spectrum cannot be calculated from the digitized output spectrum due to aliasing. Consequently the current FFT methods used for identification break down in this frequency region.

The proposed method for estimating $G(j\omega_i)$ at any given frequency is as follows:

The system is excited by $u(t) = \sin\omega_i t$ and $u(t) = \cos\omega_i t$ over non-overlapping intervals of time. Let

$$Y_s = [y_s(T) \quad y_s(2T) \quad \cdots \quad y_s(KT)]^T$$

be the vector with the values of $y(nT)$ when $u(t) = \sin\omega_i t$ is used and

$$Y_c = [y_c(T) \quad y_c(2T) \quad \cdots \quad y_c(KT)]^T$$

is the vector with the values of $y(nT)$ when $u(t) = \cos\omega_i t$ is used. Let

$$Y = Y_c + jY_s$$

$$H = [e^{j\omega_i T}, e^{j\omega_i 2T}, \cdots, e^{j\omega_i KT}]^T$$

Then $\hat{G}(j\omega_i)$ is calculated from

$$\hat{G}(j\omega_i) = H^+ Y / K$$

where H^+ is the complex conjugate of H. The above equation is independent of aliasing and has always a solution. The number of data collected in Y depends on the accuracy required. Since only Y depends on the real data, the presence of noise gets averaged and the estimation of $\hat{G}(j\omega_i)$ is unbiased. The process of excitation is repeated for different values of ω_i and a Bode plot is constructed using the pairs $\hat{G}(j\omega_i)$, ω_i. The transfer function $G(s)$ is then calculated from the Bode diagram using standard techniques.

The process of excitation using two orthogonal sinusoids can be simplified for frequencies of interest whose value is away from half or multiple integer of the sampling frequency. In this case the system in Fig. 1 is excited by the input $u(t) = \sin\omega_i t$, where ω_i is kept constant over the time internal [0, KT], where $K \geq 2$ is the number of data to be obtained from $y(nT)$ that reflects the effect of excitation of the system by $\sin\omega_i t$. Then ω_i is changed to the next value of interest over the interval [KT, $2KT$]. The process continues for all ω_i that are not close to $m\omega_s$ with $m=\pm1/2,\pm1,\pm2,\ldots$. For each frequency ω_i we form

$$Y = 2j[y(1) \quad y(2) \quad \cdots \quad y(KT)]^T$$

$$H = \begin{bmatrix} e^{j\omega_i T} & e^{j\omega_i 2T} & \cdots & e^{j\omega_i KT} \\ e^{-j\omega_i T} & e^{-j\omega_i 2T} & \cdots & e^{-j\omega_i KT} \end{bmatrix}^T$$

Then $\hat{G}(j\omega_i)$ is calculated using the equation

$$[\hat{G}(j\omega_i) \quad -\hat{G}(-j\omega_i)]^T = (H^+H)^{-1}H^+Y$$

From $\hat{G}(j\omega_i)$ we obtain $\left|\hat{G}(j\omega_i)\right|$ and $\angle\hat{G}(j\omega_i)$ that give one point on the Bode diagram at the frequency ω_i. By repeating the above calculations for each ω_i more points are obtained and the Bode diagram is constructed.

The above method will fail in the presence of high level of aliasing. For example when $\omega_i = m\omega_s$, $m=\pm1/2,\pm1,\pm2,\ldots$, the inverse of H^+H does not exist. In practice the inverse of H^+H may not be computable when ω_i is close to $m\omega_s$ or if it is computable it may lead to inaccurate estimates of $G(j\omega_i)$ due to noise effects and other inaccuracies. For this reason, when $\omega_i \geq \omega_{max}$ where ω_{max} is the maximum frequency of excitation without aliasing the first method involving the use of two orthogonal sinusoids is more appropriate. The method can also be used using excitation signals whose phase difference is less than 90 degrees. Orthogonality however simplifies calculations considerably as no inverse of a large matrix needs to be computed.

The above method is based on the assumption that the system transfer function has stable poles which means a bounded input leads to a bounded output. The present method however can be used to handle also cases where the transfer function has unstable poles as described next.

B. *Identification of Continuous-Time Transfer Function of Unstable Systems*

If the system is unstable then the excitation signals of the form $u(t) = \sin(\omega_i t)u_{-1}(t)$ and $u(t) = \cos(\omega_i t)u_{-1}(t)$ over non-overlapping intervals of time, where $u_{-1}(t)$ is a unit step function will lead to an unbounded output. In this case the excitation signal is passed through a specially designed pre-filter before applied to the system as shown in Fig. 2.

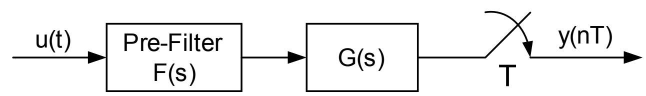

Fig. 2. Open-loop identification for an unstable system

The pre-filter $F(s)$ is designed such that $F(s)G(s)$ has a bounded impulse response. In other words $F(s)$ is designed to have the unstable poles of the system as zeros, which are assumed to be known. In practice the unstable poles of $G(s)$ may not be cancelled exactly by the zeros of $F(s)$ leading to $F(s)G(s)$ with unstable poles. The presence of $F(s)$ however reduces the effect of the instability in which case the output does not grow unbounded as fast giving sufficient time to collect output data. The output data collected are used to form a vector

$$Y_s = [y_s(T) \quad y_s(2T) \quad \cdots \quad y_s(KT)]^T$$

be the vector with the values of $y(nT)$ when $u(t) = \sin\omega_i t$ is used and

$$Y_c = [y_c(T) \quad y_c(2T) \quad \cdots \quad y_c(KT)]^T$$

be the vector with the values of $y(nT)$ when $u(t) = \cos\omega_i t$ is used. Let

$$Y = Y_c + jY_s$$

$$H = [e^{j\omega_i T}, e^{j\omega_i 2T}, \cdots, e^{j\omega_i KT}]^T$$

Then $\hat{GF}(j\omega_i)$ is calculated from

$$\hat{GF}(j\omega_i) = H^+ Y / K$$

However, for frequencies of interest whose value is away from half or multiple integer of the sampling frequency, the excitation signal can be simplified by using a single sinusoid $u(t)=\sin(\omega_i t)u_{-1}(t)$. In this case, the output data collected are use to form the vector

$$Y = 2j[y(1) \quad y(2) \quad \cdots \quad y(KT)]^T$$

$$H = \begin{bmatrix} e^{j\omega_i T} & e^{j\omega_i 2T} & \cdots & e^{j\omega_i KT} \\ e^{-j\omega_i T} & e^{-j\omega_i 2T} & \cdots & e^{-j\omega_i KT} \end{bmatrix}^T$$

Let $\hat{G}F(j\omega_i)$ denote the estimate of $G(j\omega_i)F(j\omega_i)$. $\hat{G}F(j\omega_i)$ is calculated using the following equation.

$$[\hat{G}F(j\omega_i) \quad -\hat{G}F(j\omega_i)]^T = (H^+H)^{-1}H^+Y$$

Then the magnitude and phase of $\hat{G}(j\omega_i)$ is obtained as:

$$20\log\{|\hat{G}(j\omega_i)|\} = 20\log\{|\hat{G}F(j\omega_i)|\} - 20\log\{|F(j\omega_i)|\}$$

$$\angle\hat{G}(j\omega_i) = \angle\hat{G}F(j\omega_i) - \angle F(j\omega_i)$$

III. IDENTIFICATION OF CONTINUOUS-TIME TRANSFER FUNCTION OF UNSTABLE SYSTEMS IN CLOSED-LOOP

Fig. 3 shows the system with transfer function *G*(*s*) controlled by a discrete time controller with transfer function *C*(*z*). *r*(*t*) is a reference input. The controller is designed *a priori* based on some approximate knowledge of *G*(*s*) in order to achieve close loop stability. Let $G_R(s)$ be an approximation of *G*(*s*) on which the design of *C*(*z*) is based on and it is therefore known.

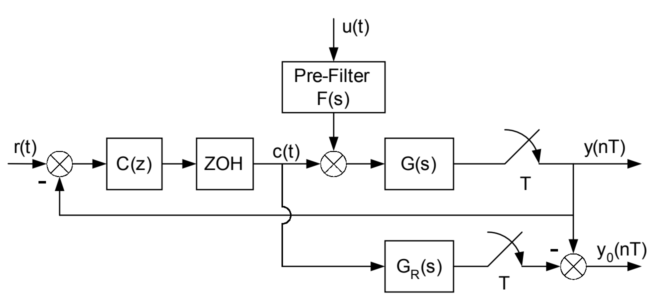

Fig. 3. Block diagram of identification in a close loop

Identifying *G*(*s*) in the configuration shown in Fig. 3 is as follows: The system is excited by the input *u*(*t*) that is passed through a pre-filter *F*(*s*) designed so that *F*(*s*)*G*(*s*) has stable poles. It is assumed that the locations of the unstable poles of *G*(*s*) are known *a priori*. The output *y*(*nT*) is constructed as shown in Fig. 3 by subtracting from the measured output signal $y_0(nT)$ the effect of *c*(*t*) generated by the controller.

The system is excited by $u(t)=\sin\omega_i t$ and $u(t)=\cos\omega_i t$ over non-overlapping intervals of time. Let

$$Y_{0s} = [y_{0s}(T) \quad y_{0s}(2T) \quad \cdots \quad y_{0s}(KT)]^T$$

be the vector with the values of $y_0(nT)$ when $u(t)=\sin\omega_i t$ is used and

$$Y_{0c} = [y_{0c}(T) \quad y_{0c}(2T) \quad \cdots \quad y_{0c}(KT)]^T$$

be the vector with the values of $y_0(nT)$ when $u(t)=\cos\omega_i t$ is used. Let

$$Y = Y_{0c} + jY_{0s}$$

$$H = [e^{j\omega_i T}, e^{j\omega_i 2T}, \cdots, e^{j\omega_i KT}]^T.$$

Then $\hat{G}F(j\omega_i)$ is calculated from

$$\hat{G}F(j\omega_i) = H^+Y/K$$

Then the magnitude and phase of $\hat{G}(j\omega_i)$ is obtained as:

$$20\log\{|\hat{G}(j\omega_i)|\} = 20\log\{|\hat{G}F(j\omega_i)|\} - 20\log\{|F(j\omega_i)|\}$$

$$\angle\hat{G}(j\omega_i) = \angle\hat{G}F(j\omega_i) - \angle F(j\omega_i)$$

By repeating these tests for each frequency of interest a Bode diagram can be constructed that could be then to identify *G*(*s*) by following standard curve fitting or other techniques.

IV. APPLICATION TO A HARD DISK DRIVE (HDD) SYSTEM

Fig. 4 shows a closed loop configuration of a hard disk drive servo system. In Fig. 4 *G*(*s*) represents the dynamics of the disk drive system, *C*(*z*) the controller transfer function in the *z*-domain, *d*(*t*) represents the output disturbances due to noise, higher order harmonics etc. The output *y*(*nT*) represents the position error signal (PES), which is the deviation of the position of the head from the center of the track the head is trying to track. Since the position error data are stored only at designed sectors on the disk, referred to as servo bursts, the values of the sampling period *T* depends on the number of the servo bursts and the spinning speed of the disk and cannot be reduced below a certain value [1],[6]. This implies that the sampling frequency cannot increase beyond a certain value. This in turn implies that the resonant modes of the disk drive system that are above half of the sampling frequency cannot be identified using the conventional methods. Current approaches to bypass this problem include an increase in the sampling rate by adding more servo bursts and/or increasing the spinning speed. Both of these approaches have their limitations and are costly [3]-[5]. It will be very desirable to have a method of identifying the resonant modes of the system, periodically and use that knowledge to update the controller parameters for better tracking. Since a small percentage change in the resonant frequencies may affect the performance of the controller the occasional monitoring of the location of the resonant frequencies and tuning of the controller parameters is essential for better tracking control performance.

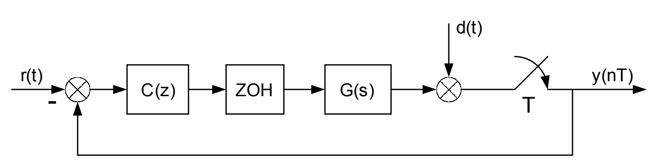

Fig. 4. Closed-loop servo system for a hard disk drive

The proposed method is applied to the hard disk drive system as shown in Fig. 5 and explained below. For clarity of presentation we consider a particular model of a disk drive system with transfer function *G*(*s*) given by

$G(s) = G_R(s)\overline{G}(s)$, where $\overline{G}(s)$ represents the high resonant frequencies and $G_R(s)$ the low frequency dynamics. In disk drives $\overline{G}(s)$ has stable poles but $G_R(s)$ may have unstable poles. In this case the controller $C(z)$ is designed based on the low frequency approximation of $G(s)$ namely $G_R(s)$. The block diagram of this method is described in Fig. 5. For the purpose of illustration we assume that $G_R(s) = 1.0337/s^2$. The pre-filter F(s) is chosen as $F(s) = s^2/(s+10)^2$ such that $F(s)G(s)$ has stable poles. The disturbance $d(t)$ is of the form

$$\begin{aligned} d(t) &= D_1 \sin(\omega_d t + \phi_1) + D_2 \sin(2\omega_d t + \phi_2) \\ &\quad + D_3 \sin(3\omega_d t + \phi_3) + n_w \end{aligned}$$

where $\omega_d = 754$ (rad/s) is the angular speed of the disk. n_w is noise and D_i, ϕ_i, i=1,2,3 are unknown constants.

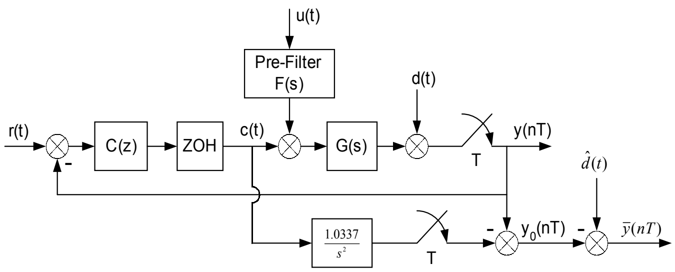

Fig. 5. HDD closed-loop identification using pre-filter and estimated values of disturbance

The proposed method applies to this problem as follows: the disturbance $d(t)$ is estimated using standard techniques, at the sampling instances and subtracted from the estimated output $y_0(nT)$. Let $\hat{d}(t)$ be the estimate of $d(t)$ at t=nT. The excitation signals $u(t)$ is chosen as $u(t) = \sin\omega_i t$ and $u(t) = \cos\omega_i t$ over non-overlapping intervals of time as described in previous sections. Let

$$\overline{Y}_s = [\bar{y}_s(T) \quad \bar{y}_s(2T) \quad \cdots \quad \bar{y}_s(KT)]^T$$

be the vector with the values of $\bar{y}(nT)$ when $u(t) = \sin\omega_i t$ is used and

$$\overline{Y}_c = [\bar{y}_c(T) \quad \bar{y}_c(2T) \quad \cdots \quad \bar{y}_c(KT)]^T$$

be the vector with the values of $\bar{y}(nT)$ when $u(t) = \cos\omega_i t$ is used. Let

$$Y = \overline{Y}_c + j\overline{Y}_s$$

$$H = [e^{j\omega_i T}, e^{j\omega_i 2T}, \cdots, e^{j\omega_i KT}]^T$$

Then $\hat{GF}(j\omega_i)$ is calculated from

$$\hat{GF}(j\omega_i) = H^+ Y / K$$

Then the magnitude and phase of $\hat{G}(j\omega_i)$ are obtained as described in section III.

The proposed method can also be used to simultaneously estimate $d(t)$ and $G(j\omega_i)$. Let the estimate of $\hat{d}(t)$ be

$$\hat{d}(t) = \sum_{k=1}^{3} \hat{D}_{ks} \sin(k\omega_d t) + \hat{D}_{kc} \cos(k\omega_d t)$$

where $\hat{D}_{ks}$, $\hat{D}_{kc}$, k=1,2,3, are estimated so that $\hat{d}(t)$ is very close to $d(t)$ (within the noise level). Select the input excitation as $u(t) = \sin\omega_i t$ and $u(t) = \cos\omega_i t$ over non-overlapping intervals of time. Let

$$Y_{0s} = [y_{0s}(T) \quad y_{0s}(2T) \quad \cdots \quad y_{0s}(KT)]^T$$

be the vector with the values of $y_0(nT)$ when $u(t) = \sin\omega_i t$ is used and

$$Y_{0c} = [y_{0c}(T) \quad y_{0c}(2T) \quad \cdots \quad y_{0c}(KT)]^T$$

be the vector with the values of $y_0(nT)$ when $u(t) = \cos\omega_i t$ is used. Let

$$Y = Y_{0c} + jY_{0s}$$

$$H = \begin{bmatrix} e^{j\omega_i T} & \sin(\omega_d T) & \cos(\omega_d T) & \sin(2\omega_d T) & \cos(2\omega_d T) & \sin(3\omega_d T) & \cos(3\omega_d T) \\ e^{j\omega_i 2T} & \sin(\omega_d 2T) & \cos(\omega_d 2T) & \sin(2\omega_d 2T) & \cos(2\omega_d 2T) & \sin(3\omega_d 2T) & \cos(3\omega_d 2T) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{j\omega_i KT} & \sin(\omega_d KT) & \cos(\omega_d KT) & \sin(2\omega_d KT) & \cos(2\omega_d KT) & \sin(3\omega_d KT) & \cos(3\omega_d KT) \end{bmatrix}$$

$\hat{GF}(j\omega_i)$ denotes the estimate of $G(j\omega_i)F(j\omega_i)$ and is estimated using the equation

$$[\hat{GF}(j\omega_i), \hat{D}_{1s}, \hat{D}_{1c}, \hat{D}_{2s}, \hat{D}_{2c}, \hat{D}_{3s}, \hat{D}_{3c}]^T = (H^+ H)^{-1} H^+ Y$$

V. SIMULATION RESULTS

The properties of the proposed method are demonstrated using simulations as presented below:

A. Identification of the Dynamics of HDD System in Open-Loop

For simulation purpose, assume that the dynamics of the HDD system are $G(s) = (1.0337/s^2)\overline{G}(s)$. For the purpose of demonstration, let us consider $\overline{G}(s)$ to be of the form $\overline{G}(s) = \overline{N}(s)/\overline{D}(s)$, where

$$\begin{aligned} \overline{N}(s) &= 0.0205s^8 - 17545s^7 + 7.7716\times10^8 s^6 - 1.6578\times10^{14} s^5 + 9.4397\times10^{18} s^4 \\ &\quad - 4.5821\times10^{23} s^3 + 3.5921\times10^{28} s^2 - 3.7757\times10^{32} s + 4.3497\times10^{37} \end{aligned}$$

$$\begin{aligned} \overline{D}(s) &= s^8 + 4502.0s^7 + 1.1553\times10^7 s^6 + 3.9934\times10^{13} s^5 + 4.6555\times10^{19} s^4 \\ &\quad + 1.1323\times10^{23} s^3 + 7.6438\times10^{28} s^2 + 1.0242\times10^{32} s + 4.21\times10^{37} \end{aligned}$$

The output of the dynamics is the position error signal that is measured at a frequency of 12.72 kHZ or 79,922.1 rad/sec. The stable pre-filter $F(s) = s^2/(s+10)^2$ is constructed such that $F(s)G(s)$ has stable poles. In this case the feedback loop is disconnected as shown in Fig. 6.

The proposed method is used to estimate $G(j\omega_i)$ at any frequency of interest ω_i. The input to the system is chosen as $u(t) = \sin(\omega_i t)$, where ω_i is varied from 0 to $\omega_{\max}$ where $\omega_{\max} << \omega_s/2$ and $\omega_s = 2\pi\times12.72\times10^3$ rad/sec. In this case aliasing is not present and $G_h(j\omega_i)$, $\omega_i \in [0, \omega_{\max}]$ is calculated using conventional methods. For $\omega_i > \omega_{\max}$, two inputs are applied sequentially $u(t) = \sin(\omega_i t)$ and $u(t) = \cos(\omega_i t)$ over selected intervals of time that do not overlap and are large enough to generate sufficient number of output samples. For each ω_i the method is used to generate

$\hat{G}_h(j\omega_i)$ from which the magnitude $|G_h(j\omega_i)|$ and phase $\angle G_h(j\omega_i)$ are calculated. Table 1 shows a comparison of the actual values and estimated ones for frequencies well above the sampling frequency. In this example, a Gaussian noise with zero mean and standard deviation of 0.2 is included in the measurements of $y(nT)$. Table 1 demonstrates that the proposed method is efficient.

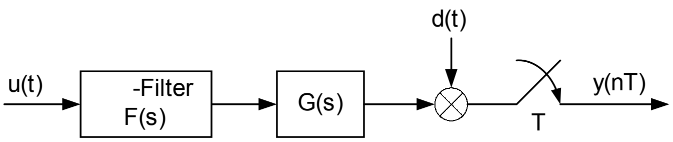

Fig. 6. Block Diagram of Open loop Identification

TABLE 1
BODE DATA OF THE HDD MODEL AND ITS OPEN-LOOP IDENTIFIED MODEL AT DIFFERENT FREQUENCY POINTS

Freq. Points (rad/s)	HDD Model		Open-loop Estimation	
	Mag. (db)	Phase (deg)	Mag. (db)	Phase (deg)
100	-79.7164	-180.0637	-79.7235	-180.0921
1000	-119.7076	-180.6366	-119.7009	-180.6605
20000	-167.4917	-192.1355	-167.4697	-192.0469
30000	-164.3153	-203.3778	-164.3314	-203.2680
33853	-154.3241	-278.0598	-154.3475	-277.6308
48000	-158.5398	-71.9127	-158.5904	-69.1207
58146	-160.8905	-313.8451	-160.9610	-311.3364
68666	-171.5536	-104.1586	-171.5547	-102.3250
70000	-180.1188	-161.3407	-179.9732	-161.0215
10^5	-213.2273	-92.4926	-210.5540	-116.5133

B. *Identification of the Dynamics of HDD System in Closed-loop*

The proposed method is used for the HDD system with $G(s)$ as given in part A. Table 2 shows a comparison of the actual values and estimated ones for the closed-loop estimation at specific frequency points. As shown in Table 1 and 2, open-loop identification achieves better results.

TABLE 2
BODE DATA OF THE HDD MODEL AND ITS CLOSED-LOOP IDENTIFIED MODEL AT DIFFERENT FREQUENCY POINTS

Freq. Points (rad/s)	HDD Model		Closed-loop Estimation	
	Mag. (db)	Phase (deg)	Mag. (db)	Phase (deg)
100	-79.7164	-180.0637	-79.7117	-179.9984
1000	-119.7076	-180.6366	-119.7101	-179.9484
20000	-167.4917	-192.1355	-167.6362	-196.8404
30000	-164.3153	-203.3778	-164.9278	-204.1158
33853	-154.3241	-278.0598	-154.9899	-268.9914
48000	-158.5398	-71.9127	-159.3689	-64.6243
58146	-160.8905	-313.8451	-162.1194	-305.1481
68666	-171.5536	-104.1586	-171.2437	-98.5231
70000	-180.1188	-161.3407	-179.6061	-151.4613
10^5	-213.2273	-92.4926	-200.3397	-164.0347

VI. EXPERIMENT RESULTS

The proposed open-loop identification method is used to identify the transfer function of an actual Western Digital WD300ABRTL 30 GB disk drive. The raw Bode data is collected based on 50Hz interval from 50Hz – 15000Hz as shown as red line in Fig. 7. The identified Bode diagram is shown in Fig. 7. The transfer function $G(s)=N(s)/D(s)$ is obtained by using complex least square curve fitting techniques and given by:

$$\begin{aligned}N(s) &= 5.842\times10^{6}s^{14} - 3.855\times10^{11}s^{13} + 1.659\times10^{17}s^{12} - 1.386\times10^{22}s^{11}\\ &+ 2.08\times10^{27}s^{10} - 1.86\times10^{32}s^{9} + 1.562\times10^{37}s^{8} - 1.204\times10^{42}s^{7}\\ &+ 7.711\times10^{46}s^{6} - 3.983\times10^{51}s^{5} + 2.428\times10^{56}s^{4} - 6.357\times10^{60}s^{3}\\ &+ 4.264\times10^{65}s^{2} - 3.851\times10^{69}s + 3.085\times10^{74}\end{aligned}$$

$$\begin{aligned}D(s) &= s^{16} + 6268s^{15} + 3.086\times10^{10}s^{14} + 1.699\times10^{14}s^{13} + 3.886\times10^{20}s^{12}\\ &+ 1.861\times10^{24}s^{11} + 2.575\times10^{30}s^{10} + 1.051\times10^{34}s^{9} + 9.624\times10^{39}s^{8}\\ &+ 3.223\times10^{43}s^{7} + 2.009\times10^{49}s^{6} + 5.08\times10^{52}s^{5} + 2.135\times10^{58}s^{4}\\ &+ 3.218\times10^{61}s^{3} + 8.641\times10^{66}s^{2}\end{aligned}$$

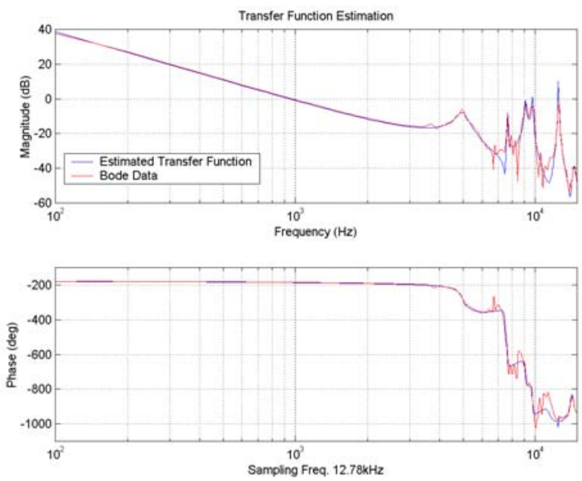

Fig. 7. The identified Bode diagram in open loop. The red line is raw Bode diagram data from the experiment. The blue line is the continuous-time transfer function obtained by using curve fitting.

REFERENCES

[1] G. F. Franklin, J. D. Powell, and M. Workman, *Digital Control of Dynamic Systems*, Addison Wesley, 1998.

[2] A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signal and Systems*, 2nd edition, Prentice Hall, Upper Saddle River, NJ 07458, 1996.

[3] B. Frank, "Rigid disk drive price trends," *IDEMA Insight*, X(4), 1997.

[4] Grochowski and R. Hoyt, "Future trends in hard disk drives," *IEEE Trans. Magn.*, vol. 3 1850 – 1854, 1996.

[5] K.K. Chew, "Control system challenges to high track density magnetic disk storage," *IEEE Trans. Magn.*, vol. 32, no. 2, pp. 1799-1804, 1996.

[6] Abramovitch, D. and Franklin, G., "A brief history of disk drive control", *IEEE Control Syst. Mag.*, Vol. 22, No. 3, Jun 2002, pp. 28–42.

[7] I. Kollar, G. Franklin, and R. Pintelon, "On the equivalence of z-domain and s-domain models in system identification", *Instrumentation and Measurement Technology Conference*, 1996. Conference Proceeedings. 'Quality Measurements: The Indispensable Bridge between Theory and Reality', Vol. 1, 4-6 Jun 1996, pp. 14-19.

[8] H. S. Yang, et. al., "Identification of contributors to HDD servo errors by measuring PES only", *IEEE Trans. Magn.*, Vol. 37, No. 2, Mar. 2001, pp. 883–887.

[9] H. S. Lee, "Controller optimization for minimum position error signals of hard disk drives", *IEEE Trans. Ind. Electron.*, Vol. 48, No. 5, Oct 2001, pp. 945–950.