Application of Lifting Scheme to Transmultiplexers

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***Abstract* -- The M-band filter banks applied to M-channel transmultiplexers are presented. The paper considers new type multichannel transmultiplexers. We propose to build transmultiplexers using the lifting scheme. Any filtering with finite filters can be decomposed into a finite sequence of triangular matrices: upper and lower. Such solution provides less multiplying than the traditional structure. Perfect reconstruction can be achieved.**

***Index terms*-- transmultiplexer, filter bank, perfect reconstruction**

I. INTRODUCTION

Filter banks have attracted much attention in the area of signal processing. They find applications mainly in subband coding [8], signal compression and transmutiplexing [8]. Filter banks were originally studied in the context of signal compression where the original signal was split into the subsignals. In this paper our goal is to present the application of lifting scheme to the M-channel transmulitiplexers. This paper is an attempt to increase the variety of filter banks which are used in transmultiplexing.

Transmultiplexers occur in applications such as telephony and television. In television, luminance and chrominance signals are transmultiplexed to form a signal that can be transmitted in single channel. The separation of signals should be perfect and the recovery of each signal should be performed without distortion. The main problem in transmultiplexeres is the leakage of signal from one channel to another. The minimalization of crosstalk is one of the main task for transmultiplexer designs. This paper addresses the design of the new kind of filter bank that reduces the effects mentioned above. We discuss the time and frequency domain requirements for transmultiplexers in case the lifting scheme was applied. Novel spread spectrum transmultiplexers are introduced in this paper. The desired time and frequency properties of the filters are set as the design criteria within the context of subband filtering. The aim of this paper is to present new results on crosstalk-free transmultiplexers. Using this approach, the crosstalk can be reduced to low values and in some cases can be completely eliminated.

II. MULTIRESOLUTION ANALYSIS

The classical multiresolution system [8] is presented in Fig.1. The input signal is split into M subbands by analysis filters and then decimated by M to produce the subband signals. These subband signals are processed depending of applications (analyzed, compressed, coded, etc.). For the case presented in Fig.1, signals are transmited without any change. At the synthesis end, these signals are processed by M-fold expanders, synthesis filters and then summed up to obtain the reconstructed signal. Usually only FIR filters

$$H(z)=\sum_{n=0}^{N} h_n z^{-n} \tag{1}$$

are used , where $h_n \in \Re$ are filter coefficients.

To build the mathematical model for the system presented in Fig.1, let us introduce vector

$$\underline{s}^{in}(z)=\left[\bar{s}^{in}(z) \quad \bar{s}^{in}(w_M z) \quad \cdots \quad \bar{s}^{in}(w_M^{M-1} z)\right]^T \in C^M$$

which consists of z-spectrum of input signal and its scaled versions, where $w_M = \exp(-2\pi j / M)$. For the analysis filters let us define the matrix

$$H(z)=\begin{bmatrix} H_1(z) & H_1(w_M z) & \cdots & H_1(w_M^{M-1} z) \\ H_2(z) & H_2(w_M z) & \cdots & H_2(w_M^{M-1} z) \\ \vdots & \vdots & \cdots & \vdots \\ H_M(z) & H_M(w_M z) & \cdots & H_M(w_M^{M-1} z) \end{bmatrix} \in C^{M\times M} . \tag{2}$$

The synthesis filters let us group in vector

$$G(z)=\left[G_1(z) \quad \cdots \quad G_M(z)\right] \in C^M . \tag{3}$$

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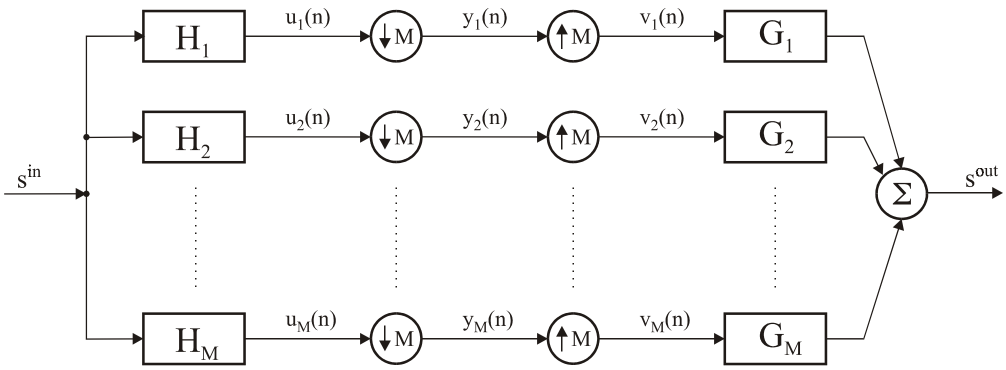

Fig.1. M-channel multirate filter bank

Taking into account the above definitions, the system presented in Fig.1 can be described by equation

$$\bar{s}^{out}(z) = \frac{1}{M} G(z) H(z) \underline{\bar{s}}^{in}(z) \cdot \tag{4}$$

A common requirement in most applications is that s^{out} should be as close to s^{in} as possible. However, some delays carried in by the electronic devices are unavoidable. A perfect reconstruction filter bank is one where

$$\bar{s}^{out}(z) \overset{df}{=} c z^{-k} \bar{s}^{in}(z), \tag{5}$$

that is the output is simply a delayed version of the input. A filter bank has the perfect reconstruction property if and only if

$$\frac{1}{M} G(z) H(z) = \begin{bmatrix} c z^{-k} & 0 & \cdots & 0 \end{bmatrix}. \tag{6}$$

These conditions can be presented as a set of equations

$$\begin{cases} \sum_{m=0}^{M-1} G_m(z) H_m(z) = c z^{-k} M \\ \sum_{m=0}^{M-1} G_m(z) H_m(w_m z) = 0 \\ \qquad \cdots \\ \sum_{m=0}^{M-1} G_m(z) H_m(w_M^{M-1} z) = 0. \end{cases} \tag{7}$$

The requirement for perfect reconstruction imposes a set of bilinear constraints (7) since all operations are linear. The detail treatment of these filter banks can be found in [2].

III. Lifting Scheme

A typical lifting stage consists of three steps [3,7]: splitting, predicting and updating. Signal s^{in} is split into its even s_e^{in} and odd s_o^{in} components, where $s_e^{in}(n) = s^{in}(2n)$ and $s_o^{in}(n) = s^{in}(2n+1)$.

Fig.2 shows the schematic representation of lifting. Lifting procedure predicts the odd signal values $s_o^{in}(n)$ from the neighboring even components s_e^{in}. In the standard multirate filter bank this prediction procedure is equivalent to applying a high-pass filter to s^{in} to obtain s_1^{dec}. The third step transforms the even signal values s_e^{in} into a low-pass filtered and subsampled version of s^{in}. In this way the coarse signal s_2^{dec} is obtained. No information is lost because each lifting step is invertible. Both, the lifting and the inverse lifting schems are shown in Fig.2. It is possible to start from splitting the signal into the add and even components (it is called [3,7] the Lazy wavelet) and use lifting scheme to gradually build up filters with the particular properties.

To extend the well known 2-channel lifting scheme (presented in Fig.2) into *M*-channel realization, let us consider the system presented in Fig.3. At the beginning the input signal s^{in} is delayed *M*-times to split into *M* signals which are next decimated by *M* to produce the subband signals

$$\bar{s}_k^{spl}(z) = \frac{z^{(1-k)/M}}{M} \sum_{m=0}^{M-1} w_M^{m(1-k)} \bar{s}^{in}(w_M^m z^{1/M}) \tag{8}$$

where $k \in \{1, 2, \cdots, M\}$. These signals can be presented in matrix notation

$$\bar{s}^{spl}(z) = \frac{1}{M} \operatorname{diag}\left[1, z^{-1/M}, z^{-2/M}, \ldots, z^{-(1-M)/M}\right] \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 \\ 1 & w_M^{-1} & w_M^{-2} & \vdots & w_M^{1-M} \\ 1 & w_M^{-2} & w_M^{-4} & \vdots & w_M^{2(1-M)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & w_M^{M-1} & w_M^{2(M-1)} & \vdots & w_M^{(M-1)(1-M)} \end{bmatrix} \underline{\bar{s}}^{in}(z) \in \mathbf{C}^M \tag{9}$$

where

$$\underline{\bar{s}}^{in}(z) = \left[\bar{s}^{in}(z^{1/M}) \quad \bar{s}^{in}(w_M z^{1/M}) \quad \cdots \quad \bar{s}^{in}(w_M^{M-1} z^{1/M})\right]^T \tag{10}$$

The output signal

$$\bar{s}^{out}(z) = \begin{bmatrix}1 & z^{-1} & \cdots & z^{1-M}\end{bmatrix} S(z^M) A(z^M)\, \bar{s}^{spl}(z^M) \in \mathbf{C} \tag{11}$$

depends on two filter banks: lifting $A(z)$ and inverse lifting $S(z)$. The perfect reconstruction condition can now be written as

$$S(z^M)A(z^M) = \operatorname{diag}\left[c_1 z^{-\tau_1}, \ldots, c_M z^{-\tau_M}\right]. \tag{12}$$

Determinant of the right side of (12) has form

$$\det \operatorname{diag}\left[c_1 z^{-\tau_1}, \ldots, c_M z^{-\tau_M}\right] = z^{-\sum_{m=1}^{M}\tau_m} \prod_{m=1}^{M} c_m \,. \tag{13}$$

If all filters are FIR type, then the determinants of both matrices $A(z)$ and $S(z)$ must be monomials $c_S z^{-\tau_S}$ and $c_A z^{-\tau_A}$, respectively. Then and only then product of both determinants is equal to (13) and

$$c_A c_S = \prod_{m=1}^{M} c_m \tag{14}$$

$$\tau_A + \tau_S = \sum_{m=1}^{M} \tau_m \,. \tag{15}$$

The problem of finding an FIR filter bank thus amounts to finding nonsingular matrices $S(z)$ and $A(z)$ which determinants are monomials. The lifting matrix $A(z) \in C^{M\times M}$ fulfils condition

$$\begin{bmatrix}\bar{s}_1^{ana}(z)\\ \bar{s}_2^{ana}(z)\\ \vdots\\ \bar{s}_M^{ana}(z)\end{bmatrix} = \begin{bmatrix}A(z)\end{bmatrix} \begin{bmatrix}\bar{s}_1^{spl}(z)\\ \bar{s}_2^{spl}(z)\\ \vdots\\ \bar{s}_M^{spl}(z)\end{bmatrix} \tag{16}$$

and the inverse lifting matrix $S(z) \in C^{M\times M}$ fulfils condition

$$\begin{bmatrix}\bar{s}_1^{syn}(z)\\ \bar{s}_2^{syn}(z)\\ \vdots\\ \bar{s}_M^{syn}(z)\end{bmatrix} = \begin{bmatrix}S(z)\end{bmatrix} \begin{bmatrix}\bar{s}_1^{ana}(z)\\ \bar{s}_2^{ana}(z)\\ \vdots\\ \bar{s}_M^{ana}(z)\end{bmatrix}. \tag{17}$$

After lifting the new modulation matrix can be taken

$$A^{new}(z) = L(z)U(z)\, A^{old}(Z) \tag{18}$$

where $U(z)$ and $L(z)$ are upper and lower triangular matrices respectively, and their determinants are equal to 1. This operation does not change the determinant of the lifting matrix. Any transformation $A(z)$ and $S(z)$ with finite filters can be obtained starting from Lazy wavelet transform followed by a finite number of alternating $U(z)$ and $L(z)$ steps. This does not change the determinant of the lifting $A(z)$ or inverse lifting $S(z)$ matrix.

Every finite filter transform S^{new} can be obtained by starting with the unitary matrix S^{old} followed by triangular lower and triangular upper matrices with all diagonal entries equal to one, so that

$$S^{new} = \begin{bmatrix}1 & s_{1,2} & \cdots & s_{1,M}\\ 0 & 1 & \cdots & s_{2,M}\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 1\end{bmatrix} \begin{bmatrix}1 & 0 & \cdots & 0\\ t_{2,1} & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots\\ t_{M,1} & t_{M,2} & \cdots & 1\end{bmatrix} S^{old} \tag{19}$$

$$A^{new} = \begin{bmatrix}1 & 0 & \cdots & 0\\ -s_{2,1} & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots\\ -s_{M,1} & -s_{M,2} & \cdots & 1\end{bmatrix} \begin{bmatrix}1 & -t_{1,2} & \cdots & -t_{1,M}\\ 0 & 1 & \cdots & -t_{2,M}\\ \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 1\end{bmatrix} A^{old} \,. \tag{20}$$

It follows from the well known matrix property that any matrix with polynomial entries and determinant one can be factored into such elementary matrices.

IV. TRANSMULTIPLEXING

A transmultiplexer [2] combines several signals into a single signal. Transmultiplexers were originally studied in the context of converting Time Division Multiplexing (TDM) into Frequency Division Multiplexing (FDM). Their main application is for simultaneous transmission of several data signals through a single channel. A key point is that the constituent signals should be recoverable from the combined signal. Fig.4 shows the classical structure of a transmultiplexer. The input signals were upsampled, filtered and summed to obtain a composite signal. This composite signal can be transmitted over a single transmission channel. At the receiver end, the signal must be filtered and downsampled to recover the signals.

For well-designed transmultiplexers, s_i^{out} approximates s_i^{in}, where $i \in \{1,2,\cdots,M\}$ is a signal number. Transmultiplexer achieves perfect reconstruction if s_i^{out} is delayed and amplified version of s_i^{in}, namely if there exist nonnegative integers c_i and τ_i such that

$$s_i^{out}(n) = c_i \cdot s_i^{in}(n - \tau_i)\,. \tag{21}$$

In this case, there is no cross-talk, no magnitude and no phase distortions. The goal in transmultiplexer design is a choice of filters that ensure perfect reconstruction (21).

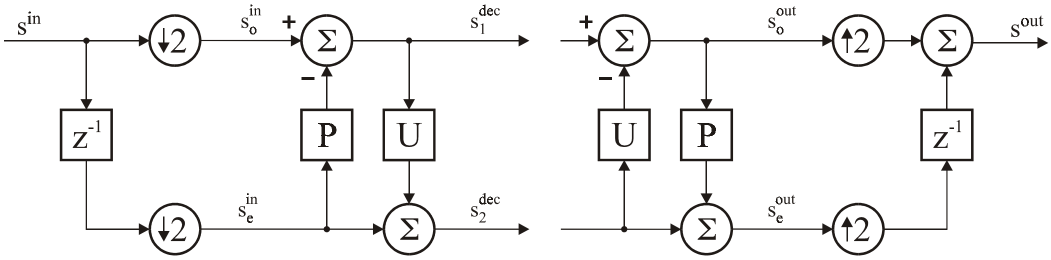

Fig.2. Lifting and inverse lifting scheme

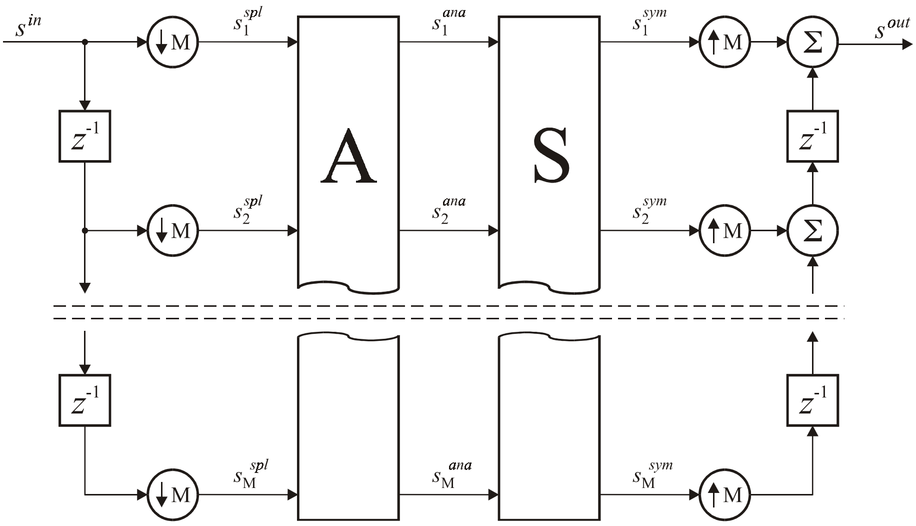

Fig.3. M-channel lifting scheme

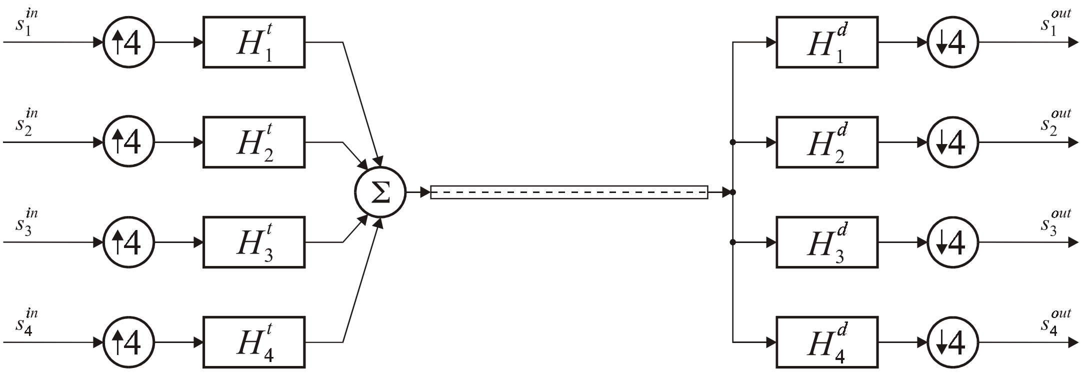

Fig.4. An example of 4-channel transmultiplexer

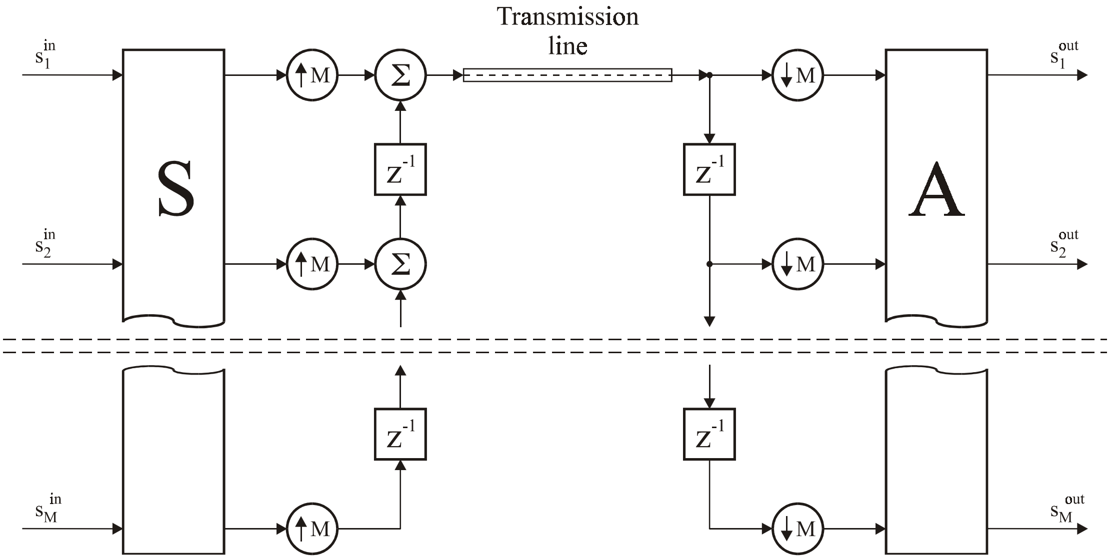

Fig.5. Lifting scheme applied to the transmultiplexer

Transmitted signals are combined into a single channel. In the receiver end, the combined signal is analyzed through analysis subsystems consisting of filters and upsamplers generating the reconstructed signals. The output *i* dependence on inputs $k \in \{1,2,\cdots,M\}$ is described by

$$\bar{s}_i^{out}(z) = \frac{1}{M}\sum_{k=1}^{M}\bar{s}_k^{in}(z)\left[\sum_{m=0}^{M-1}H_i^d(w_M^m z^{1/M})H_k^t(w_M^m z^{1/M})\right] \quad (22)$$

for $i \in \{1,2,\cdots,M\}$. These conditions can be written in matrix notations

$$\bar{s}^{wy}(z) = \frac{1}{M}H^d(z^{1/M})\,H^t(z^{1/M})^T\,\bar{s}^{we}(z) \quad (23)$$

where both, input and output signals have vector representations

$$\bar{s}(z) = [\bar{s}_1(z),\cdots,\bar{s}_M(z)] \quad (24)$$

while the filter banks are described by

$$H(z) = \begin{bmatrix} H_1(z) & H_1(w_M z) & \cdots & H_1(w_M^{M-1}z) \\ H_2(z) & H_2(w_M z) & \cdots & H_2(w_M^{M-1}z) \\ \cdots & \cdots & \cdots & \cdots \\ H_M(z) & H_M(w_M z) & \cdots & H_4(w_M^{M-1}z) \end{bmatrix}. \quad (25)$$

The mixture or crosstalk between signals must be zero. When transmultiplexers with overlaping frequency bands are used for the transmission of signals over non-ideal channels, the intersymbol interference and crosstalk between different data channels may arise. Perfect reconstruction of the input signals with a delay of τ_i samples can be obtained when the following condition hold

$$\sum_{m=0}^{M-1}H_i^d(w_M^m z^{1/M})H_k^t(w_M^m z^{1/M}) = 4c_i z^{-\tau_i}\delta_{i,k}\,. \quad (26)$$

A scheme of the lifting transmultiplexer system is presented schematically in Fig.5. It is to define the subset of filters that are the most suitable for the application under consideration. The perfect reconstruction property is given by (21) and that is why determinants of matrices $A(z)$ and $S(z)$ must be monomial. Both matrices, $A(z)$ and $S(z)$, contain only FIR filters defined by (1).

V. CONCLUSION

Recent advances in the theory of transmultiplexing [1,5,6,9] provide very flexible and applications-oriented tools for the signal transmission.

Lifting leads to speed-up computations when compared to the standard implementation. The cost of the algorithm can be measured in number of multiplications. Under assumption that the orders of all filters are *N* and the length of signals are *L*, we obtain for the *M*-fold upsampled signals *ML*(*N*+1) multiplications in each channel (see Fig.4). Totally, for the both sides of transmultiplexer system we obtain $2M^2L(N+1)$ multiplications. Under assumption that matrices *A* and *S*, for the system presented in Fig.5, have structures presented by (19) and (20), the lifting scheme needs 2*M*(*M*-1)*L*(*N*+1) multiplications, only. It is possible (for example see [4]) to apply matrices *A* and *S* which have less number of elements not equal to 0. In such case the efficiency of lifting scheme presented in Fig.5 is much higher than the efficiency of standard algorithm presented in Fig.4.

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