Largest Lyapunov exponent assignement: A genetic/pole placement approach.

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Abstract− This paper deals with the almost-sure stabilization of discrete-time jump parameter systems. Two novel hybrid algorithms for the almost-sure stabilization test and assignement of the largest Lyapunov exponent are proposed. The first algorithm performs a test of almost-sure stabilization. Using the proposed test, one can conclude whether or not the assignement of the largest Lyapunov exponent is possible. Should the test be conclusive, a disk pole placement design procedure is engaged with the help of a devised genetic algorithm. Numerical examples are given to illustrates the proposed techniques.

Keywords− Lyapunov exponent, almost-sure stabilization, jump parameter systems, genetic algorithms, pole placement.

I. INTRODUCTION

The problem of concern in this paper is to assign the largest Lyapunov exponent, in a desired interval for the class of discrete-time jump parameter systems. We consider particularly the discrete-time jump linear system

$$x(k+1) = A(r(k))x(k) + B(r(k))u(k), \quad (1)$$

where x is the system state vector of dimension n, u is the control input vector of dimension p, and $r(k)$ is the form index which is a stochastic scalar sequence that takes values in the finite index set $\mathcal{N} = \{1, 2, ..., N\}$. In this case $r(k)$ is assumed to be a finite state Markov chain with transition probabilities

$$prob\,\{r(k+1) = j | r(k) = i\} = p_{ij} \quad (2)$$

It is assumed that the finite state Markov chain is irreducible. The system takes the realization $\sum_i = (A_i, B_i)$ when $r(k) = i$, with $i \in \mathcal{N}$. This realization is called the ith form.

Presently, it is not known how to assign the largest Lyapunov exponent, let alone the Lyapunov spectrum of jump parameter systems. In this paper, a new hybrid algorithm for the test of the degree of almost-sure stabilizability of jump parameter systems is proposed. It is via this test that one can conclude whether or not the assignement of the largest Lyapunov exponent is possible. The second algorithm, proposed in this paper, takes advantage of the disk pole placement technique in order to assign the largest Lyapunov exponent, of a jump parameter system, within a desired interval. Indeed, due to the complex nature of this design objective, a desired design interval, within which the largest Lyapunov exponent is to be assigned is chosen.

The paper is organized as follows. In section 2, necessary material needed for the paper is presented. Section 3 develops the corresponding hybrid test stabilization algorithm. In section 4, the problem of the assignement of the largest Lyapunov exponent is formulated and the associated hybrid algorithm is discussed. Section 5 applies the paper's results to an illustrative numerical example. Section 6 concludes and mentions some possible research directions.

II. PRELIMINARIES

A. Leading Lyapunov exponent bounds

In [5], upper bounds on the leading Lyapunov exponent, λ_1, were introduced.

Theorem 1 [5]*:* The largest Lyapunov exponent, λ_1, of a homogeneous N-form DTR dynamical system, with a stationary irreducible FSMC satisfies the following inequality

$$\begin{aligned} &\tfrac{1}{n}\sum_{i\in N} p_i \log|\det(A_i)| \leq \lambda_1 \leq \\ &\tfrac{1}{l}\sum_{i\in N^{[l]}} p_i^{[l]} \log\left\|TA_iT^{-1}\right\| := \lambda^{[l]} \end{aligned} \tag{3}$$

where T is any similarity transformation. Moreover $\lambda^{[ql]} \leq q\lambda^{[ql]}, \qquad q, l \in \mathbb{N}\backslash\{0\}$

The lower and upper bounds provided by this theorem estimate the spread of the Lyapunov spectrum, thus providing valuable information regarding the slowest as well as the fastest dynamics. Although these bounds are expected to be quite consevatrice from almost-sure stability view point, they are simple enough in devising stabilization algorithms for jump parameter systems. This justifies, amongst other things, the use of this formulation for the placement of the largest Lyapunov exponent. In addition, it is important to recall the complex nature of the computation of these latter exponents. Indeed, it is by now well known [4], that the computation of the Lyapunov exponents, even in this relatively simple case, is highly complex. This state of affairs imposes the application of hybrid algorithms such as those proposed in this paper, i.e., genetic algorithms along with LMI techniques.

B. Motivation

Given N desired eigenstructures, or desired Lyapunov spectrum, specified through desired matrices A_{d_i}, $i = 1, \cdots, N$. The design algorithms proposed in [2] and [3] are besed on the idea of making the closed-loop matrices be as close as possible, in some sense, to these matrices. In this way, should the resulting gains succeed in exactly matching the desired matrices, the closed-loop jump parameter system would inherit the desired jump parameter system's dynamical characteristics such as its Lyapunov spectrum. While this approach is reminiscent of eigenstructure assignement, or model following design specification, it remains very difficult to apply since it is very hard to come up with the A_{d_i}, and thus the desired closed-loop dynamics. To circumvent this difficulty, a scheme inspired from the classical pole placement technique was proposed in [3]. That is, instead of providing all the A_{d_i}, the design specifications are limited to the desired almost-sure rate of convergence specified via the largest Lyapunov exponent. In order to fulfil this design requirement, N disks are chosen within which the closed-loop forms are to belong and thus accomplish the design requirements. Since the choice of the needed disks is not known, a genetic algorithm is called for.

III. HYBRID ALMOST-SURE STABILIZATION TEST

Before considering the assignement of the largest Lyapunov exponent, it is important to be able to attest the degree of the almost-sure stabilization of the jump parameter system. It is via this test that one can conclude whether or not the assignement of the largest Lyapunov exponent is possible.

The N desired structures are specified through the disks $D_{d_i}(q_i, r_i)$, $i = 1, ..., N$ of radii r_i and centers q_i. It is then a question of finding the K_i, $i = 1, ..., N$ which satisfy the two following conditions at the same time:

Condition 1: the eigenvalues of the closed-loop matrices $(A_i - B_iK_i)$, $i = 1, ..., N$ belong to D_{d_i}, $i = 1, ..., N$.

Condition 2: the closed-loop jump parametre system is almost-surely stable.

Considering the first condition, the eigenvalues of the matrices in closed-loops $(A_i - B_iK_i)$ belong to $D_{d_i}(q_i, r_i)$ if and only if there exists a symmetric matrix S such that

$$\begin{bmatrix} -r_iS^{-1} & (A_{cl_i} - q_iI) \\ (A_{cl_i} - q_iI)^T & -r_iS \end{bmatrix} \leq 0, \quad i = 1, ..., N. \tag{4}$$

With $A_{cl_i} = A_i - B_iK_i$.

The closed-loop jump parametre system is almost-surely stable provided that the K_i, $i = 1, ..., N$, generated by feasibility problem (4), and the non singular matrix $T := S^{\frac{1}{2}}$ satisfy

$$\sum_{i=1}^{N} p_i \log\|T(A_i - B_iK_i)T^{-1}\| < 0. \tag{5}$$

The conditions 1 and 2 are satisfied with non-switching gains if $K_i = K$, $i = 1, ..., N$.

A solution to the feasibility problem (4) with the constraint (5) can be obtained as follows: while using standard genetic algorithm, we pose v as a vector concatenation of the pairs (q_i, r_i). It consists in moving an initial population P_1 of M_{size} individuals v_{1l}, $l \in \{1, ..., M_{size}\}$ toward a final population P_{kmax} constituted of individuals v_{kmaxl}, $l \in \{1, ..., M_{size}\}$ and having better "quality" in the sense of the closed-loop almost-sure stability constraint. The hybrid stabilization test algorithm (H.S.T.A.) given below describes this heuristic procedure.

HybridStabilizationTestAlgorithm(H.S.T.A.)

1. Initialization. Choose a population size M_{size} (that is an odd number) and a stopping criterion (an desired bound, or maximum number of generations). Randomly generate a population P_1 of individuals v_{1l}, $l \in 1, ..., M_{size}$. Set $k = 1$.

2. While (not stop). Set $P_{k+1} = \phi$

2.1. evaluation and selection:

2.1.1. For each vector v_{kl} of the population P_k:

$$\left\{ \begin{array}{l} \left[\begin{array}{cc} \mathcal{A}_{i_{kl}} & (B_i Y_{i_{kl}}) \\ (B_i Y_{i_{kl}})^T & -X_{kl} \end{array} \right] \leq 0 \\ i = 1, \cdots, N, \end{array} \right. \tag{6}$$

with $\mathcal{A}_{i_{kl}} = -r_{i_{kl}}^2 X_{kl} + (A_i - q_{i_{kl}} I) X_{kl} (A_i - q_{i_{kl}} I)^T - (A_i - q_{i_{kl}} I)(Y_{i_{kl}})^T (B_i)^T - B_i Y_{i_{kl}} (A_i - q_{i_{kl}} I)^T$

2.1.2. calculate

$$\left\{ \begin{array}{l} K_{i_{kl}} = Y_{i_{kl}} (X_{kl})^{-1} \\ T_{kl} = (X_{kl})^{-\frac{1}{2}} \end{array} \right. \tag{7}$$

2.1.3. with $K_{i_{k_m}}$ et , T_{kl}, evaluate the correspondant Lyapunov exponent upper bound:

$$\lambda_{kl} = \sum_{i=1}^{N} p_i \log \|T_{kl}(A_i - B_i K_{i_{kl}}) T_{kl}^{-1}\| \tag{8}$$

2.1.4. Sort the vectors in P_k by increasing order of λ_{kl}.

2.1.5. Include the first $(M_{size} - 1)/2$ solutions (vectors) in P_{k+1}.

2.2. While $(size(P_{k+1}) < (M_{size} - M_m))$

BEGIN (Crossover)

2.2.1. Select two consecutive solutions (in the sens of the λ_{kl} order) v_{kl} and v_{kl+1}

2.2.2. Mate v_{kl} and v_{kl+1} to produce one offspring v_{k+1l} in the following manner:

$$\mathrm{v}_{k+1l} = r_k(\mathrm{v}_{kl} - \mathrm{v}_{kl+1}) + \mathrm{v}_{kl} \tag{9}$$

r_k can be considered as an indice of non real similarity randomly getting in interval $[0 \;\; 1]$ and can change in each generation.

2.2.3. Include v_{k+1l} in P_{k+1}

END (Crossover)

2.3. Mutation : Randomly generte M_m solutions and include them in P_{k+1}.

2.4. Elitism: it consists in enhancing the performance of the algorithm by including in the population P_{k+1} super individual $v_{k_{sup}}$ which is the best one of all the individuals up to generation k, by imposing: $v_{k+1_{l+1}} = v_{k_{sup}}$

End.

Since k_{max} generations are running long, we extricate the best gains $K_{i_{sup}}^{test}$, $i = 1, ..., N$ and the transformation $T_{sup}^{test} := (X_{sup}^{test})^{-\frac{1}{2}}$ corresponding to the minimum λ_{sup}^{test} of all the calculated λ_{kl}, $l = 1, ..., M_{size}$ and $k = 1, ..., k_{max}$. Should the λ_{sup}^{test} be negative, the closed-loop system is made almost-sure stable via the switching gains $K_{i_{sup}}^{test}$.

Note that equation (6) is not other than an LMI formulation, equivalent to equation (4). This one is easily obtained by introducing the variables's change $X = S^{-1}$ and $Y_i = K_i X$.

The genetic operations that were used are initialization, selection, crossover, mutation, along with a specific replacement strategy including an operation of elitism. Further explanation of these operators can be found in [1].

The outcome of the above algorithm could be either positive or negative. That is, should the outcome of the latter algorithm be a positive Lyapunov, i.e., $\lambda_{sup}^{test} > 0$, then one doesn't know how to almost-surely stabilize the given system within the proposed framework. Now, should $\lambda_{sup}^{test} < 0$, then it is possible to assign the closed-loop Lyapunov exponent such that $\lambda_{sup}^{test} < \lambda_{sup}^{desired} < 0$.

IV. LARGEST LYAPUNOV EXPONENT ASSIGNEMENT

A. Problem formulation

The preceeding phase makes it possible to estimate the level of almost-sure stability (λ_{sup}^{test}) made possible via switching gains feedback. If the degree of stability is rather high, it would be possible to target a desired site. This one being defined by a desired interval $In_d = [\overline{\lambda}_{min} \quad \overline{\lambda}_{max}]$, such that $\lambda_{sup}^{test} < \overline{\lambda}_{min}$.

In this case, our objective, for the lagest Lyapunov exponent assignemernt, is formulated as follows:

N disks D_{d_i}, $i = 1, ..., N$ of radii r_i and centers q_i are chosen. It is then a question of finding the adequate K_i, $i = 1, ..., N$ which verify condition 1 (see above), as well as the following condition:

Condition 3: the K_i, $i = 1, ..., N$ and the non singular matrix $T := S^{\frac{1}{2}}$ generated by the feasibility problem (4) satisfy

$$\begin{array}{l} \overline{\lambda}_{max} \leq \sum_{i=1}^{N} p_i \log \\ \|T(A_i - B_i K_i) T^{-1}\| \leq \overline{\lambda}_{min} \end{array} \tag{10}$$

The conditions 1 and 3 are satisfied with non-switching gains if $K_i = K$, $i = 1, ..., N$.

To satisfy the above conditions, the hybrid test stabilization algorithm will be modified in the way detailed in the following paragraph.

B. Hybrid Assignement Algorithm

A solution to the feasibility problem (4) with the design constraint (10) can be obtained as follows: we pose v as a vector concatenation of the pairs (q_i, r_i). It consists in moving an initial population P_1 of M_{size} individuals v_{1l}, $l \in \{1, ..., M_{size}\}$ toward a final population P_{kmax} constituted of individuals v_{kmaxl}, $l \in \{1, ..., M_{size}\}$ and having better "quality" in the sense of the closed-loop largest Lyapunov assignement constraint. The genetic pole placement design algorithm given below describes this heuristic procedure.

Genetic Pole Placement Design Algorithm (G.P.P.D.A.)
1. Initialization. Choose a population size M_{size} (that is an odd number) and a stopping criterion (an desired bound, or a maximum number of generations). Randomly generate a population P_1 of individuals v_{1l}, $l \in 1, ..., M_{size}$. Set $k = 1$.
2. While (not stop). Set $P_{k+1} = \phi$
2.1. evaluation and selection:
2.1.1. For each vector v_{kl} of the population P_k, resolve (6)
2.1.2. calculate (7)
2.1.3. with $K_{i_{k_m}}$ and , T_{kl}, evaluate:

$$\left\{ \begin{array}{l} \lambda_{kl} = \sum_{i=1}^{N} p_i \log \|T_{kl}(A_i - B_i K_{ikl}) T_{kl}^{-1}\|, \\ d_{kl}^{j} = \left| \frac{\overline{\lambda}_{max} + \overline{\lambda}_{min}}{2} - \lambda_{kl} \right| - \left| \overline{\lambda}_{max} - \overline{\lambda}_{min} \right| \end{array} \right. \tag{11}$$

2.1.4. Sort the vectors in P_k by increasing order of d_{kl}.
2.1.5. Include the first $(M_{size} - 1)/2$ solutions (vectors) in P_{k+1}.
2.2. Crossover
2.3. Mutation
2.4. Elitism
End.

Since k_{max} generations are running long, we extricate the best gains $K_{i_{sup}}^{d}$, $i = 1, ..., N$ and the transformation T_{sup}^{d} correspending to the minimum d_{sup} of all the calculated λ_{kl}, $l = 1, ..., M_{size}$ and $k = 1, ..., k_{max}$. Should the d_{sup} be negative, the largest Lyapunov exponent of the closed-loop system is made inside the desired interval via the switching gains $K_{i_{sup}}^{d}$.

I.V. NUMERICAL ILLUSTRATION

In this example a two-dimensional, one-input, three-form jump parameter system, found in [6], is used. The (A_i, B_i), $i = 1, ...3$, are given below.

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.5 & -1 \\ 0 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1.5 & 1 \\ -1 & -0.25 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 1 & -1 \\ 0.5 & -1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

The Markov chain is described by the following transitions matrix Π

$$\Pi = \begin{bmatrix} 0.67 & 0.1 & 0.23 \\ 0.35 & 0.47 & 0.18 \\ 0.06 & 0.3 & 0.64 \end{bmatrix}$$

The eignvalues of the three forms are $(0.5, -2)$, $(-0.8750 \pm 0.7806i)$ and (± 0.7071). That is only the first form is unstable. For the largest Lyapunov exponent, we choose the desired intervall $In_d = [-0.5 \quad -0.45]$.

A. Stabilisation test with switching gains

A.1. Hybrid stabilisation test

Using the (H.S.T.A.), with 51 individuals and a maximum number of generation equal to 10, the system turned out to be almost-surely stabilizable and the Lyapunov exponent upper bound was found to be $\lambda_{sup}^{test} = -0.8569$. The upper Lyapunov exponent lower bound, calculated using the left member in (**??**), is found to be -1.5133. Moreover,

$$\begin{aligned} K_{sup_1}^{test} &= \begin{bmatrix} -0.2678 & -1.4655 \end{bmatrix}, \\ K_{\mathrm{sup}\,2}^{test} &= \begin{bmatrix} -0.2263 & -0.3351 \end{bmatrix}, \\ K_{sup_3}^{test} &= \begin{bmatrix} 1.4005 & -3.7164 \end{bmatrix}, \\ X_{\mathrm{sup}}^{test} &= \begin{bmatrix} 9.1916 & 5.1903 \\ 5.1903 & 2.9967 \end{bmatrix}. \end{aligned}$$

In figure 2, the 'x' marked poles and disk $D_1(0.0126, 0.6353)$ correspond the to first close-loop form. The '*' marked poles and $D_2(-0.3590, 1.0686)$ corresponds to the second close-loop form. The '+' marked poles and $D_3(1.2576, 0.2869)$ correspond to the third close-loop form. The 'o' marked point corresponds to λ_{sup}^{test}.

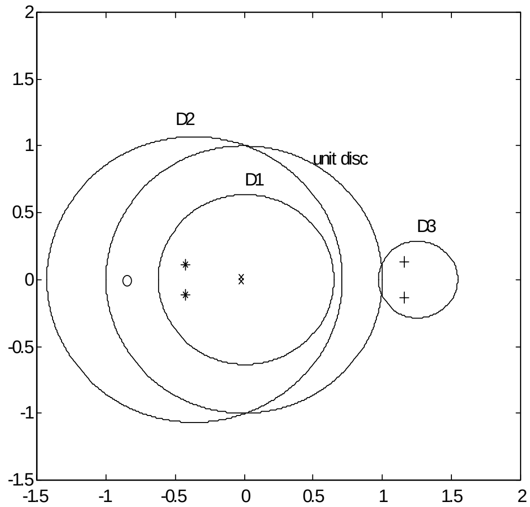

Fig. 1. the pole placement and λ_{sup}^{test} for the (H.T.S.A) with 51 indiviuls.

The obtained value of λ_{sup}^{test} being lower than -0.5, one can consider largest Lyapunov exponent assignement.

A.2. Largest Lyapunov exponent assignement

Using the (G.P.P.D.A.), with 51 individuals and a maximum number of generation equal to 10, the Lyapunov exponent upper bound was found to be $\lambda_{sup}^{d} = -0.4848$. The closed-loop upper Lyapunov exponent lower bound is equal to -1.0299, with

$$\begin{aligned} K_{1_{sup}}^{d} &= \begin{bmatrix} -0.1571 & -1.6566 \end{bmatrix}, \\ K_{3_{\mathrm{sup}}}^{d} &= \begin{bmatrix} -0.2326 & -0.3255 \end{bmatrix}, \\ K_{3_{\mathrm{sup}}}^{d} &= \begin{bmatrix} 0.7415 & -2.5101 \end{bmatrix}, \\ X_{\mathrm{sup}}^{d} &= \begin{bmatrix} 1.3764 & 0.5382 \\ 0.5382 & 0.2778 \end{bmatrix}. \end{aligned}$$

In figure 3, the 'x' marked poles and $D_1(-0.0703, 0.7230)$ correspond to the first close-loop form. The '*' marked poles and $D_2(-0.2770, 1.0240)$ correspond to the second close-loop form. The '+' marked poles and $D_3(1.1509, 0.5172)$ correspond to the third close-loop form. The 'o' marked point corresponds to λ_{sup}^{d}. The '$\triangleright$' marked point corresponds to $\overline{\lambda}_{max}$ and the '$\triangleleft$' marked point corresponds to $\overline{\lambda}_{min}$. Figure 3 illustrates a zoomlens of λ_{sup}^{d} and $\begin{bmatrix}\overline{\lambda}_{min} & \overline{\lambda}_{max}\end{bmatrix}$.

It is worth noting that in order to accomplish the design specification, the applied algorithm "choose" to <u>destabilize</u> the third form, originally stable!, in order to almost-surely stabilize the jump parameter system!

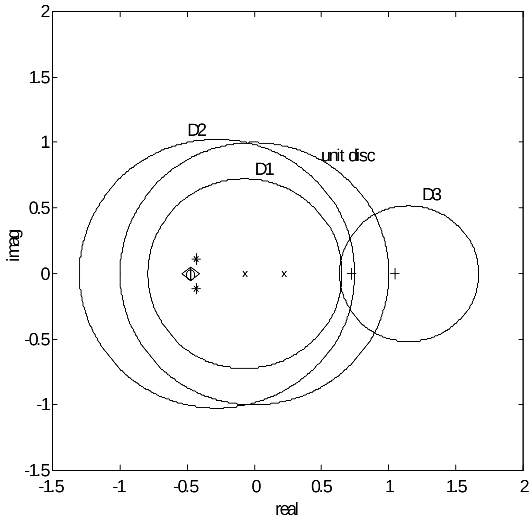

Fig. 2. the pole placement, In_d and λ_{sup}^{d} for the (G.P.P.D.A) with 51 indiviuls.

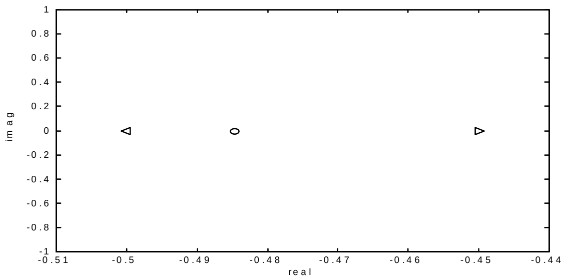

Fig. 3. assignement of λ_{sup}^{d} in $[-0.5 \quad -0.45]$.

B. Stabilisation with non-switching gains

Using the (H.S.T.A.), with 51 individuals and a maximum number of generation equal to 10 for non-switching gains, the largest Lyapunov exponent is turned out to be positive. Indeed, $\lambda_{sup_{NSG}}^{test} = 0.0858$ and the system may be almost-surely not stabilizable with non-switching gains. In this case $D_{d_i}(q_i, r_i)$, $i = 1, ..., N$ are $D_1(-1.0168, 1.6722)$, $D_2(0.3907, 1.1809)$ and $D_3(0.8380, 2.5832)$, with

$$\begin{aligned} K_{sup_{NSG}} &= \begin{bmatrix} -0.1207 & -0.8246 \end{bmatrix}, \\ X_{\mathrm{sup}_{NSG}} &= \begin{bmatrix} 7.2470 & 3.9512 \\ 3.9512 & 3.4155 \end{bmatrix} \end{aligned}$$

On the basis of the fact that for $l = 1$, $\lambda^{test}_{sup_{NSG}}$ constitute a first approximation, it is possible to choose l as large as one needs. To achieve a better accuracy while computing the upper bounds of $\lambda^{test}_{sup_{NSG}}$, a series of increasingly upper bounds are provided using the genetic test algorithm in [1]. The results are illustrated in figure 4.

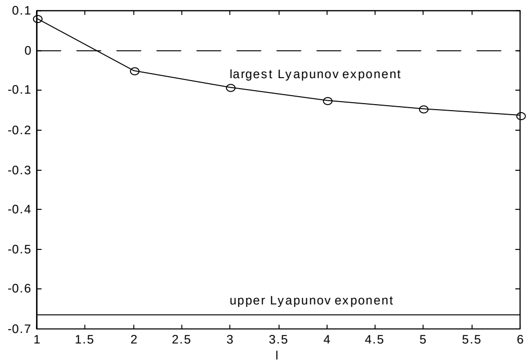

Fig. 4. $\lambda^{test^{[l]}}_{sup_{NSG}}$ for the non-switching gains closed-loop

The circle curve stands for the evolution of $\lambda^{test^{[l]}}_{sup_{NSG}}$ with respect to l. The solid curve corresponds to the upper closed-loop Lyapunov exponent. Althrough $\lambda^{test}_{sup_{NSG}}$ is positive, the GTA [1] succeeds in finding a similarity transformation such that the almost-sure stability of the closed-loop is guaranteed for $l > 1$. Indeed, for $l \geq 2$, $\lambda^{test^{[l]}}_{sup_{NSG}}$s are found to be negative.

Before closing this section, we would like to underline the key role played by the parameters of the genetic algorithms. Let us suppose that the desired design interval is $[-1; -0.9]$. By considering the first test, worked out in section 6.1, one cannot deduce the possibility of this assignement. But if one ncreases the number of individuals in the initial population, using the (H.T.S.A.), with 5001 individuals and a maximum number of generation equal to 10, the system turnes out to be almost-surely stabilizable and the Lyapunov exponent upper bound is found to be $\lambda^{test}_{sup} = -1.2022$. This is attests the importance of the genetic algorithm parameters.

Finally let us note the cases of test and palcement with switching gains. Although the third form in open loop was stable, the almost-sure stability of the jump parameter system was ensured depends on a destabilization of A_3 in closed-loop (figures 1 and 2).

VI. CONCLUSION

In this paper, hybrid algorithms for the closed-loop almost-sure stabilizability test for the class of jump parameter systems was proposed. Should the considered test be achieved, one can apply the hybrid design algorithm to assign the largest Lyapunov exponent in a desired interval. The new proposed algorithms take advantages of the pole placement approach and the genetic techniques for search and optimization of the almost-sure desired specifications.

For instance, to check the achieved closed-loop Lyapunov spectrum it is not enough to assign the upper bound, but it is necessary to assign the complete spectrum. In addition and due to the complexity of the considered problemt, one immediate extension would be to assign the lower Lyapunov exponent bound as wel as the upper one.

REFERENCES

[1] Ben Cheikh, Y., and Ezzine, J. , 2001, *A Genetic algorithm based-computation of the Lyapunov exponent for jump parameter systems.* Proc. IFAC SSSC01, August 29-31, Prague.

[2] Ben Cheikh, Y., and Ezzine, J. , 2001, *Hybrid LMI/genetic algorithms for the stabilization of discrete-time linear jump parameter systems using the Lyapunov exponent.* Proc. MED'01, June 27-29, Dubrovnik, Croatia.

[3] Ben Cheikh, Y., and Ezzine, J. , 1998, *On Almost-sure stabilization of large-scale discrete-time jump parameter systems: a pole placement approach*, Proc. CESA'98, April 1-4, Tunisia.

[4] Blonded, V. D., and Tsitsiklis, J. N., 2000, *A survey of computational complexity results in systems and control*, , Automatica 36, pp. 1249-1274.

[5] Ezzine, J., and Kavranoglu, D., 1997, *On Almost-sure stabilization of discrete-time jump parameter systems: an LMI approach*, Int. J. Control, 68,1129.

[6] Shi, P., Boukas, E., K., and Agarwal, R., K., 1999, *Robust control for Markovien jumping discrete-time systems*, Int. J. of systems science V.30 pp 787-797.