Robust Control of an Underactuated Planar 2R Manipulator

J. Alfredo Rosas-Flores
UPIITA-IPN Depto. de Tecnologías Avanzadas.
Av. IPN # 2580, Col. La Laguna Ticomán, Méx. DF, CP 07340, México
Fax: 52-5-752-9318, e-mail:jarosas@mail.cinvestav.mx

Jaime Alvarez-Gallegos and Rafael Castro-Linares
CINVESTAV-IPN Depto. de Ing. Eléctrica. Sección de Mecatrónica
Av. IPN # 2508, Col. Sn. Pedro Zacatenco Apdo. Postal 14-740, 07300 Méx. DF.
Fax: 52-5-747-3866 e-mail: {jalvarez, rcastro}@mail.cinvestav.mx

Abstract

In this paper the problem of position control of an underactuated planar 2R manipulator is solved by tracking suitable off-line planned optimal trajectories via a robust feedback control. The sliding modes technique is used to synthetize the robust feedback control such that the manipulator's coordinates track the planned trajectories with bounded uniformly tracking error, under error modeling, small deviations on the initial conditions and errors introduced by computing the control law. By experiments, it is illustrated that if the coordinates of the underactuated system track the off-line planned trajectories then the links of the manipulator are placed close to any desired configuration.

Keywords: Underactuated manipulator, position control, robust control law, sliding modes.

1 Introduction

The study of underactuated mechanical systems, that is, mechanical systems with less control inputs than generalized coordinates, is an interesting and growing area. They are subject of research by many authors (see [4],[5], [6], [14]-[17]). Several laboratory prototypes such as the pendubot [14], the acrobot [15], the TORA [9], the inverted pendulum [17], the ball and beam system [8], etc., are useful to prove new control techniques. Underactuated mechanical systems are desirable in view of their saving in energy, reduced cost and weight. However they have the disadvantage that special control techniques must be used. Moreover, fully actuated mechanical systems can become into underactuated systems if some actuator fails, and for this situation it is convenient to have a controller for the underactuated system [7].

An interesting class of underactuated systems are manipulators with rigid links and unactuated joints. The basic control problems are the tracking of oscillatory trajectories and the position control (i. e. to place the links at desired position with zero velocity). The underactuated manipulators can be studied under the gravity force or without it. Underactuated manipulators under the gravity force can be controlled by many techniques with acceptable results [14], [15], [17]. However, the control of manipulators whose dynamics are not affected by gravity force is a challenge, because they are not linear controllable. A manipulator that has its links moving in a horizontal plane, such that the gravity force does not affect their motion, is known as a planar manipulator. In [1] and [3] a class of 3R planar underactuated manipulator is controlled taking advantage of the property of *Small-Time Local Controllability.* To regulate the configuration of a 2R planar underactuated manipulator with an actuator in the first joint only, interesting control strategies are proposed. For example, in [6] the 2R manipulator is stabilized to a desired configuration by using an iterative steering technique, based on the repeated application of an error contractive command. In [2] the same class of manipulator is controlled by tracking time-scaling planned trajectories.

In this paper the configuration for a 2R planar underactuated manipulator is addressed. Because of the Brockett's theorem, [12], the manipulator only can be stabilized at desired positions by discontinuous or by time-varying control laws. Basically, the position control is solved by using algorithmic or open loop controllers, see [6], and there are few time-varying control laws, [2]. The strategy followed here is to use a time-varying control law based on planned optimal trajectories, so the position control problem is solved by making the manipulator's links track suitable optimal trajectories. The planned optimal trajectories are obtained by using the optimal trajectory planner proposed in [13]. Hence, given some off-line planned trajectories it is proposed a robust control law that achieves the objective of tracking the prescribed trajectories. So, the main contribution in paper is to give a time-varying control law that solve the position control problem of an underactuated manipulator of 2dof. Moreover, this control law has robustness property.

This paper is structured as follows. In section 2, the problem statement is addressed. In section 3, the trajectory planning is considered. In section 4, a robust feedback control based on the sliding mode technique is proposed. In the section 5, experimental results are given to illustrate the position control of an underactuated system based on the methodology given in sections 3 and 4. Finally, some conclusions are given in section 6.

2 Problem statement

The manipulator considered here has two degrees of freedom with revolute frictionless joints whose links move in the horizontal plane. See Fig. 1.

If the first joint is active and the second link is passive, the model of the manipulator is given by [12]

$$\begin{array}{c}(a_1+2a_2\cos q_2)\,\ddot{q}_1+(a_3+a_2\cos q_2)\,\ddot{q}_2\\ -a_2\sin q_2\left(2\dot{q}_1\dot{q}_2+\dot{q}_2^2\right)=\tau_1,\\ (a_3+a_2\cos q_2)\,\ddot{q}_1+a_3\ddot{q}_2+a_2\sin(q_2)\dot{q}_1^2=0\end{array}\qquad(1)$$

where $a1=m_1l_{c1}^2+m_2l_1^2+m_2l_{c2}^2+I_{zz1}+I_{zz2}$, $a_2=m_2l_1l_{c2}$, $a_3=m_2l_{c2}^2+I_{zz2}$, m_i denotes the mass of link i, l_i denotes the length of link i, l_{ci} denotes the center of mass of link i with respect to its rotation axis, I_{zzi} is the inertia of link i with respect to its rotation axis, q_1 denotes the position of the first link measured from a reference system, q_2 denotes the position of the second link measured from the first link, $\dot{q}_i$ and $\ddot{q}_i$ denote the angular velocity and angular acceleration, respectively, of link i. τ_1 is the applied torque in the first joint. In this case, the second equation of (1) represents a dynamic constraint which cannot be integrated to obtain an algebraic relation between the coordinates q_1 and q_2, that is, it is a second-order nonholonomic constraint [12]. This dynamic constraint plays an important role to plan the necessary trajectories that make possible the regulation of the underactuated system.

The control goal is stated as: To find a control law u that moves the links of the manipulator close to a desired configuration from an initial configuration, both of them having zero velocity.

There are some control problems that must be considered at the control design process, as: The Taylor linearization of (1) is not controllable at positions with zero velocities, the model (1) is not *Small Time local Controllable* (STLC) in some configurations [11] and the system only can be stabilized at some desired configuration by discontinuous control laws or by time-varying controls, because of the Brockett's theorem [12].

The strategy followed to reach the control goal is to use a time-varying control law based on off-line planned optimal trajectories. This is achieved in two phases,

1. Optimal trajectory Planning.
2. Robust feedback control.

Because of the property STLC implies that the system can reach close configurations in arbitrarily small time, the lack of the STLC in some configurations indicates that sometimes the control law can not eliminate small errors and consequently the system does not track trajectories asymptotically.

3 Trajectory planning

To move the links near the desired positions, by tracking trajectories, it is necessary to plan proper and feasible trajectories that connect the initial configuration with the desired one. So, in this section it is described the procedure to obtain proper planned trajectories, proposed in [13]. The trajectories depend on parameters that must be determined in function of the initial and final configurations.

The problem of trajectory planning to be solved can be stated as follows: find feasible trajectories connecting the initial configuration $(q_1(t_0), q_2(t_0))=(q_{10},q_{20})$ of the system (1), with $(\dot{q}_1(t_0), \dot{q}_2(t_0))=(0,0)$, to the desired configuration, $(q_1(t_f),q_2(t_f))=(q_{1d},q_{2d})$ with $(\dot{q}_1(t_f),\dot{q}_2(t_f))=(0,0)$ in a given time t_f, where t_0 represents the initial time and t_f the final time. A feasible trajectory means a trajectory satisfying all the constraints imposed on the dynamics of the manipulator and performing a desired task. Here, it is neither considered the problem of avoiding obstacles nor the bounded of the states of the system. The constraints to be satisfied by the trajectories of the physical system to reach a static desired configuration are

$$\begin{array}{c}(a_3+a_2\cos q_2)\,\ddot{q}_1+a_3\ddot{q}_2+a_2\sin(q_2)\dot{q}_1^2=0.\\ q_1(t_f)-q_{1d}=0,\\ q_2(t_f)-q_{2d}=0,\\ \dot{q}_1(t_f)=0,\\ \dot{q}_2(t_f)=0,\\ |\tau_1(t)|\le\tau_{\max},\end{array}\qquad(2)$$

where $\tau_{\max}$ is the maximum torque given by the actuator of the system.

To find a suitable planned trajectories, the trajectory planning methodology given in [13] is followed. Since the considered manipulator has one actuator in the first joint, only the coordinate q_1 can be controlled directly. In this manner, by tracking the planned trajectory $q_{1p}(t)$ the links of the manipulator must reach the desired configuration. The planned trajectory $q_{1p}(t)$ is obtained off-line, while the trajectory $q_{2p}(t)$ can be computed in-line by solving the dynamic constraint, where $q_1=q_{1p}$, $\dot{q}_1=\dot{q}_{1p}$ and $\ddot{q}_1=\ddot{q}_{1p}$.

4 Feedback control

The trajectories planned in the previous section can be used to synthesize a feedforward control that allows to reach the control objective. However, if the system is controlled only with feedforward control and there exists some perturbations, modeling errors or incorrect initial conditions, then the system does not reach the desired configuration. So, a feedback control must be used to guarantee the regulation of the system. The sliding mode technique has been chosen to synthesize a feedback control.

There are several feedback control techniques used in underactuated systems. However, there are few control schemes used to solve the trajectory tracking. Here, a sliding mode technique is used the to propose a control law to ensure that coordinates track the off-line planned trajectories. This approach gives a robust control law and guarantees bounded uniformly errors [10]. Because the model of the manipulator does not have the STLC property, it is difficult to design a controller that eliminates the tracking errors without large deviations from the planned trajectories. However, the sliding mode control law ensures that the states of the system are close to the sliding surface such that the links are near of the desired positions.

The control design strategy is as follows. Based on switching surfaces, which are functions of the tracking errors and their time derivatives, a control law is obtained. This control law is derived by using the pseudo-inverse of a non square matrix. Besides, it is proved that the computed control law makes attractive the switching surfaces, making that the tracking errors are ultimately uniformly bounded. So, this ensures that the coordinates of the manipulator are moving about the planned trajectories.

The system model (1) can be written in matrix form as follows

$$D(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = Mu \tag{3}$$

where $\mathbf{q} = [q_1, q_2]^T \in R^2$ is the generalized position vector, the generalized velocity vector $\dot{\mathbf{q}}$ and acceleration vector $\ddot{\mathbf{q}}$ are defined in similar way, $D(\mathbf{q}) \in R^{2\times 2}$ denotes the inertia matrix, $C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \in R^n$ defines the vector of centrifugal and Coriolis forces. In this case the vector $M = [1,0]^T$.

The structure of the control law is proposed from the following relation, which is obtained by solving for $\ddot{\mathbf{q}}$ from (3),

$$\ddot{\mathbf{q}} + D^{-1}(\mathbf{q})C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = B(\mathbf{q})u \tag{4}$$

where $B(\mathbf{q}) = D^{-1}(\mathbf{q})M$, so the controller is proposed with the structure

$$B(\mathbf{q})u = \ddot{\mathbf{q}}_p(t) + D^{-1}(\mathbf{q})C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{v}, \tag{5}$$

with

$$\mathbf{v} = -k\mathbf{s} - k_1 sign(\mathbf{s}) - \Lambda\dot{\mathbf{q}}_e \tag{6}$$

where $\ddot{\mathbf{q}}_p(t) \in R^2$ denotes the vector of planned accelerations, $\mathbf{q}_e = \mathbf{q} - \mathbf{q}_p \in R^2$ is the tracking error vector, $\dot{\mathbf{q}}_e \in R^2$ is the time derivative of the vector of planned trajectories $\mathbf{q}_e$, k and k_1 are positive constants, $\Lambda \in R^{2\times 2}$ is a positive diagonal matrix, and $\mathbf{s} = [s_1, s_2]^T$ is the vector of switching surfaces defined by[1]

$$\mathbf{s} = \dot{\mathbf{q}}_e + \Lambda\mathbf{q}_e. \tag{7}$$

Because of the structure of $\mathbf{s}$, if the control law u, (5), makes stable the surface $\mathbf{s}$, then the errors $\mathbf{q}_e$ are bounded.

To study the behavior of the closed loop system and the stability conditions, the dynamic model (3) is expressed as function of the switching surfaces. The time derivative of (7) is given by

$$\dot{\mathbf{s}} = \ddot{\mathbf{q}}_e + \Lambda\dot{\mathbf{q}}_e. \tag{8}$$

The term $\ddot{\mathbf{q}}_e$ can be put in function of $\mathbf{v}$, i. e. $\ddot{\mathbf{q}}_e = \mathbf{v}$, by substituting directly (5) into (4). So $\dot{\mathbf{s}}$, Eq. (8), takes the form,

$$\dot{\mathbf{s}} = \mathbf{v} + \Lambda\dot{\mathbf{q}}_e$$

and applying (6), it is written as

$$\dot{\mathbf{s}} = -k\mathbf{s} - k_1 sign(\mathbf{s}), \tag{9}$$

where k and k_1 are chosen such that stability is guaranteed.

The reduced system (9) does not consider some approximations made during computation of the control law. In fact, the control law u applied to the manipulator must be explicitly solved from (5), but $B(\mathbf{q})$ is a vector so it is not possible to obtain the control law u in a direct way. For this reason, the left pseudo-inverse of $B(\mathbf{q})$ is used to obtain an approximated control law. Since $rank\ (B(\mathbf{q})) = 1$, the left pseudo-inverse of $B(\mathbf{q})$, denoted by $B^{\#}(\mathbf{q})$, is given as follows

$$B^{\#}(\mathbf{q}) = \left[B^T(\mathbf{q})B(\mathbf{q})\right]^{-1}B^T(\mathbf{q}),$$

so u is given by

$$u = B^{\#}(\mathbf{q})\left[\ddot{\mathbf{q}}_p(t) + D^{-1}(\mathbf{q})C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{v}\right]. \tag{10}$$

Applying (10) to (4), and simplifying, one obtains

$$\ddot{\mathbf{q}}_e = \mathbf{v} + \left[B(\mathbf{q})B^{\#}(\mathbf{q}) - I\right]\left[\ddot{\mathbf{q}}_p(t) + D^{-1}(\mathbf{q})C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{v}\right].$$

The second term is an error of approximation for the ideal relation $\ddot{\mathbf{q}}_e = \mathbf{v}$, which can be considered as part of a vector of uncertainties $\tilde{\mathbf{v}}$. So,

$$\tilde{\mathbf{v}} = \left[B(\mathbf{q})B^{\#}(\mathbf{q}) - I\right]\left[\ddot{\mathbf{q}}_p(t) + D^{-1}(\mathbf{q})C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{v}\right]. \tag{11}$$

So, $\ddot{\mathbf{q}}_e$ is written as

$$\ddot{\mathbf{q}}_e = \mathbf{v} + \tilde{\mathbf{v}},$$

[1]The function $sign(\mathbf{s})$ is defined as $sign(\mathbf{s}) = (sign(s_1), sign(s_2))^T$, where

$$sign(s_i) = \begin{cases} 1, & if \quad s_i > 0 \\ -1 & if \quad s_i < 0 \end{cases}, i = 1, 2.$$

therefore $\dot{\mathbf{s}}$, Eq. (8), takes the form

$$\dot{\mathbf{s}} = \mathbf{v} + \Lambda \dot{\mathbf{q}}_e + \tilde{\mathbf{v}}. \tag{12}$$

Applying the control law (6) to (12), one obtains

$$\dot{\mathbf{s}} = -k\mathbf{s} - k_1 sign\left(\mathbf{s}\right) + \tilde{\mathbf{v}}. \tag{13}$$

The stability of (13) can be ensured if the positive constants k and k_1 are properly chosen. In particular, the value of k_1 depends on the upper bound of the uncertainty $\tilde{\mathbf{v}}$. It is well known that in controlling a real system there exist measurement errors. Besides there are uncertainties on its model and on their parameters. It is supposed that these errors and uncertainties can be put into the vector $\tilde{\mathbf{v}}$.

Lemma 1 *Let consider the system model described by (4) with the control law (5), i.e. the closed loop system (13). If the uncertainty vector $\tilde{\mathbf{v}}$ is bounded, $\|\tilde{\mathbf{v}}\| \leq \alpha$, then the switching surfaces $\mathbf{s}$ are attractive, continuous and uniformly bounded when $k_1 > \alpha$.*

Proof By taking the first time derivative of the following candidate Lyapunov function,

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{s},$$

and using (13), it gives

$$\dot{V} = \mathbf{s}^T\dot{\mathbf{s}} = \mathbf{s}^T\left(-k\mathbf{s} - k_1 sign\left(\mathbf{s}\right) + \tilde{\mathbf{v}}\right). \tag{14}$$

Simplifying and taking upper bounds,

$$\begin{aligned} \dot{V} &= -k\mathbf{s}^T\mathbf{s} - k_1\mathbf{s}^T sign\left(\mathbf{s}\right) + \mathbf{s}^T\tilde{\mathbf{v}} \\ &\leq -k\left\|\mathbf{s}\right\|^2 - k_1\left\|\mathbf{s}\right\| + \left\|\mathbf{s}\right\|\left\|\tilde{\mathbf{v}}\right\| \\ &= -k\left\|\mathbf{s}\right\|^2 - \left(k_1 - \left\|\tilde{\mathbf{v}}\right\|\right)\left\|\mathbf{s}\right\| \\ &\leq -k\left\|\mathbf{s}\right\|^2 - \left(k_1 - \alpha\right)\left\|\mathbf{s}\right\|. \end{aligned}$$

So, by taking $k_1 > \alpha$ the switching surfaces $\mathbf{s}$ are attractive,

$$\dot{V} \leq -k\left\|\mathbf{s}\right\|^2.$$

■

It is important to point out that the vector of uncertainties $\tilde{\mathbf{v}}$ is a bounded function, $\|\tilde{\mathbf{v}}\| < \alpha$, since it depends on uniformly bounded functions, the variables $\dot{\mathbf{q}}$ is physically bounded, and $\dot{\mathbf{q}}_p$ is planned bounded.

Following the Lemma 1, the tracking errors are bounded. However, it is possible to show that these errors are ultimately uniformly bounded. This can be done by using the following Lema to show that $\mathbf{s}$ is ultimately uniformly bounded.

Lemma 2 *[10, pag. 204]Consider the perturbed system*

$$\dot{x} = f\left(t, x\right) + g\left(t, x\right)$$

where $x \in R^n$ and $g\left(t,x\right) \in R^n$ is a nonvanishing perturbation term for the nominal system

$$\dot{x} = f\left(t, x\right).$$

Let $x = 0$ be an exponentially stable equilibrium point of the nominal system. Let $V\left(t,x\right)$ be a Lyapunov function of the nominal system that satisfies

$$\begin{aligned} c_1\left\|x\right\|^2 \leq V\left(t,x\right) \leq c_2\left\|x\right\|^2, \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f\left(t,x\right) \leq -c_3\left\|x\right\|^2, \\ \left\|\frac{\partial V}{\partial x}\right\| \leq c_4\left\|x\right\| \end{aligned}$$

in $[0,\infty] \times D$, where $D = \{x \in R^n \mid \|x\| < r\}$, and c_1, c_2, c_3 and c_4 are positive constants. Suppose the perturbation term satisfies

$$\left\|g\left(t,x\right)\right\| \leq \delta < \frac{c_3}{c_4}\sqrt{\frac{c_1}{c_2}}\theta r$$

for all $t \geq 0$, all $x \in D$, and some positive constant $\theta < 1$. Then, for all $\|x\left(t_0\right)\| < \sqrt{c_1/c_2}\,r$, the solution of the perturbed system $x\left(t\right)$ satisfies

$$\left\|x\left(t\right)\right\| \leq \kappa\left\|x\left(t_0\right)\right\| e^{-\gamma\left(t-t_0\right)}, \forall t_0 \leq t \leq t_1$$

and

$$\left\|x\left(t\right)\right\| \leq b, \forall t \geq t_1$$

for some finite time t_1, where

$$\kappa = \sqrt{\frac{c_2}{c_1}}, \gamma = \frac{\left(1-\theta\right)c_3}{2c_2}, b = \frac{c_4}{c_3}\sqrt{\frac{c_2}{c_1}}\frac{\delta}{\theta}.$$

■

From (13), the perturbed system is identified as

$$\dot{\mathbf{s}} = -k\mathbf{s} + g\left(t, \mathbf{s}\right),$$

where the perturbation term $g\left(t,\mathbf{s}\right)$ is given by $g\left(t,\mathbf{s}\right) = -k_1 sign\left(\mathbf{s}\right) + \tilde{\mathbf{v}}$. So, the nominal system

$$\dot{\mathbf{s}} = -k\mathbf{s}$$

has a exponentially stable equilibrium point at $\mathbf{s} = \mathbf{0}$, since $k > 0$. The perturbation $g\left(t,\mathbf{s}\right)$ is bounded,

$$\left\|g\left(t,\mathbf{s}\right)\right\| \leq k_1 + \left\|\tilde{\mathbf{v}}\right\| \leq k_1 + \alpha \leq \delta.$$

Moreover $V\left(\mathbf{s}\right) = \frac{1}{2}\mathbf{s}^T\mathbf{s} = \frac{1}{2}\left\|\mathbf{s}\right\|^2$, so

$$\dot{V} \leq -k\left\|\mathbf{s}\right\|^2 - k_1\left\|\mathbf{s}\right\| + \alpha\left\|\mathbf{s}\right\| \leq -k\left\|\mathbf{s}\right\|^2 + \delta\left\|\mathbf{s}\right\|$$

from here it is possible to identify

$$\begin{aligned} c_1 &= c_2 = \frac{1}{2}, \\ c_3 &= k, c_4 = 1 \\ b &= \frac{c_4}{c_3}\sqrt{\frac{c_2}{c_1}}\frac{\delta}{\theta} = \frac{1}{k}\frac{\delta}{\theta}, \end{aligned}$$

for some positive constant $\theta < 1$. Besides, an estimated bound of $g(t, \mathbf{s})$ is given by δ,

$$\|g(t, \mathbf{s})\| \leq \delta < \frac{c_3}{c_4}\theta r = k\theta r.$$

Therefore, for a finite time t_1,

$$\|\mathbf{s}(t)\| \leq \frac{1}{k}\frac{\delta}{\theta}, \forall t \geq t_1. \quad (15)$$

Remark 1 *To avoid the chattering phenomena, when the discontinuous function* $sign(\cdot)$ *is used to compute the control law* u*, the function* $sign(s_i)$ *is approximated by the continuous function*

$$sign(s_i) = \tanh\left(\frac{3}{\epsilon_i}s_i\right) \quad (16)$$

where the parameter ϵ_i *gives the width of the switching region near to the switching surface* s_i*. By using the approximated function (16), a sliding mode does not exist but the state vector of the system will move around the switching surface. The introduced error by the use of (16) can be estimated by using Lemma 2, resulting in the same conservative bound (15).*

5 Experimental results

The methodology given in the last sections was applied to one laboratory prototype consisting of an underactuated planar 2R manipulator is obtained. The parameters of this system are: $I_{zz1} = 0.00622489\,kg\ m^2$, $l_1 = 0.2034\,m$, $l_{c1} = 0.156713\,m$, $m_1 = 0.8397135\,kg$, $I_{zz2} = 0.005499\ \ kg\ m^2$, $l_2 = 0.352425\,m$, $l_{c2} = 0.139462\,m$, $m_2 = 0.385286\,kg$. The electronic hardware used to control the manipulator consists of four main parts: a personal computer, a D/A card, a servo amplifier, and a specialized card to read the encoders. The control algorithm is programmed to use the C-language, under a time-interrupt mode such that the sampling period was set to 1 ms.

If the manipulator has the initial configuration $(q_{10}, q_{20}) = (0, 0)$ and one wants to move it to the final configuration $\left(0, \frac{\pi}{2}\right)$, then the parameters of the optimal trajectory are $\alpha_1 = 1.12135677rad$, $\alpha_2 = -0.27875577\,rad$, $t_f = 5.4553\ s$.

The parameters of the control were chosen as $k(t) = 1, k_1(t) = 70, \lambda_{11} = \lambda_{22} = 60, \epsilon_1 = \epsilon_2 = 0.001$. Fig. 2 shows the behavior of the variables q_1 and q_2 together with the planned trajectories q_{1p} and q_{2p}, moreover the control law u is given. The error on the initial configuration is $(0.0065, -0.0005)\,rad$ and the error on the final configuration is $(-0.0064, -0.02353)\,rad$. The robustness of the control law, under uncertainty on the physical parameters of the manipulator, was tested by placing a body of mass $m = 0.05kg$ at the end of the second link. In Fig. 3 is shown the dynamic behavior of variables q_1 and q_2 together with the planned trajectories, q_{1p} and q_{2p}, and the control law u. The error on the initial configuration is $(0.00657, 0.0026)\,rad$ and the error on the final configuration is $(0.0504, -0.00061)\,rad$.

During the experimentation was verified that control law is robust in the sense that it can handle approximations made during the computation of the control law, parametric uncertainties, friction forces and bounded control law. Moreover, it was noted that the performance of the controller is sensitive to large incorrect initial conditions and choices of the values of control parameters.

6 Conclusions

In this paper the control position problem of an underactuated system by means of tracking planned trajectories is solved. The proposed control law results in a robust varying-time control, because the use of sliding modes technique and the use of planned trajectories. This control law guarantees the state of the system to stay into neighborhood of the intersections of the surfaces ($\mathbf{s} = \mathbf{0}$). Besides, it is shown it makes tracking errors to be ultimately uniformly bounded. Experimentally, the control law was verified to be robust although its performance is sensitive to large incorrect initial conditions.

References

[1] Arai, H. K. Tanie and N. Shiroma. *Nonholonomic Control of a Three-DOF Planar Underactuated Manipulator.* IEEE TRA, 1998, Vol. 14, No. 5, pp. 681-695.

[2] Arai, H., K. Tanie and N. Shiroma. *Time-scaling Control of an Underactuated Manipulator.* Proc. of IEEE ICRA, 1998, pp. 2619-2626, Leuven, Belgium.

[3] Bergerman, M. and Y. Xu. *Planning Collision-Free Motions for Underactuated Manipulators in Constrained Configuration Space.* Proc. IEEE ICRA, 1997, pp. 549-555.

[4] Bloch, A. M., N.E. Leonard and J. E. Marsden. *Stabilization of Mechanical Systems Using Controlled Lagrangians.* Proceedings of IEEE of the 36th CDC, 1997, San Diego, California, USA.

[5] Block, D.J. Mechanical Design & Control of the Pendubot. SAE Earthmoving Industry Conference, Peoria, IL. 1995.

[6] De Luca, A., R. Mattone and G. Oriolo. *Control of Underactuated Mechanical Systems: Application to the Planar 2R Robot.* Proc. of the 35th CDC, 1996, pp. 1455-1460, Kobe, Japan.

[7] English, J. E. and A. Maciejewski. *Fault Tolerance for Kinematically Redundant Manipulators: Antic-*

ipating Free-Swinging Joint Failures. IEEE TRA, 1998, Vol. 14, No. 4, pp. 566-575.

[8] Hauser, J., S. Sastry and P. Kokotović. *Nonlinear Control Via Approximate Input-Output Linearization:The Ball and Beam Example.* IEEE TAC, 1992, Vol.37, No. 3.

[9] Janković, M., D. Fontaine, and P. V. Kokotović. *TORA Example: Cascade- and Passivity- Based Control Designs.* IEEE TCST, 1996, Vol. 4, No. 3, pp. 292-297.

[10] Khalil, H. K. *Nonlinear Systems.* Macmillan Publishing Company, N.Y. USA. 1992.

[11] Mullhaupt, Ph, B. Srinivasan and D. Bonvin *Analysis of Exclusively Kinetic Two-Link Underactuated Mechanical Systems.* Automatica, Vol 38, issue 38, 2002, pp. 1565-1573.

[12] Oriolo, G. and Y. Nakamura. *Free-Joint Manipulators: Motion Control Under Second-Order Nonholonomic Constraints.* Proc. IEEE/RSJ Int. Workshop on Intelligent Robots and Systems. 1991, pp. 1248-1253.

[13] Rosas-Flores, J.A., J. Alvarez-Gallegos and R. Castro-Linares. *Control of an Underactuated Planar 2R Manipulator: Experimental Results.* Proccedings of the15th IFAC World Congress, Barcelona, Spain, july 21-26, 2002.

[14] Spong, M. W. *The control of underactuated mechanical systems.* 1st Int. Conf. Mech. Mexico. 1994.

[15] Spong, M. W. *The swingup control problem for the acrobot.*IEEE CSM. 1995, Vol.15, No.1, pp.49-55.

[16] Spong, M. W. *Energy Based Control of a Class of underactuated mechanical systems.* IFAC 13th Triennial World Congress, USA, 1996, pp. 431-435.

[17] Sugie, T. and K. Fujimoto. *Control of Inverted Pendulum Systems Based on Approximate Linerization: Design and Experiment.* Proc. of IEEE of the 33rd CDC. 1994.

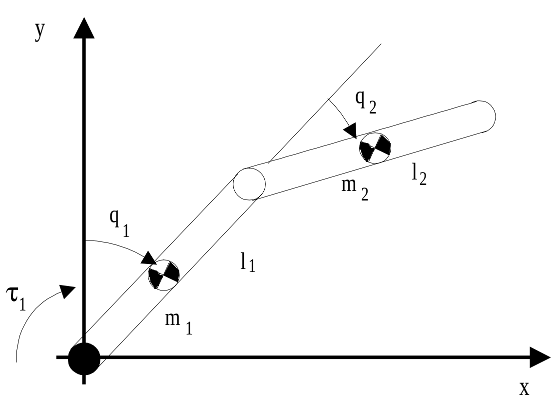

Fig. 1. Two dof underactuated planar manipulator.

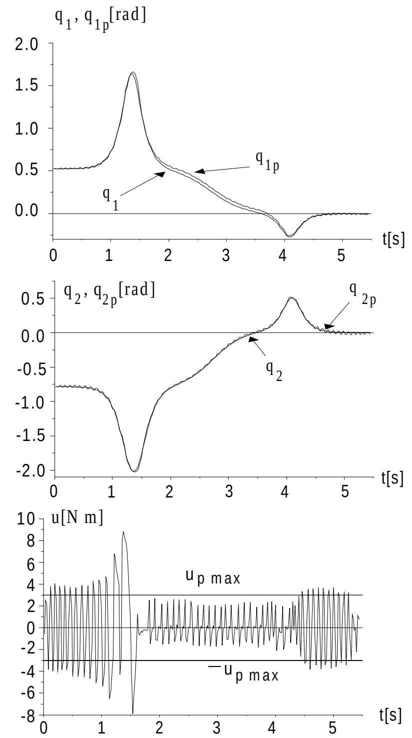

Fig. 2. Experimental closed loop response

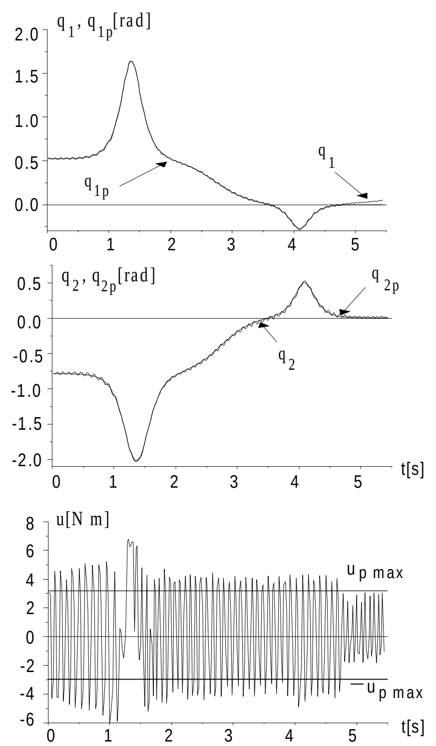

Fig. 3. Experimental closed loop response under parametric uncertainty