Application of Controllers Based on Polynomial Methods to Coupled Drives Laboratory Process

Marek Kubalčík, Vladimír Bobál

***Abstract* — Control of a coupled drives apparatus laboratory model as a two inputs – two outputs system is presented. Two control algorithms based on polynomial theory and pole – placement are proposed. The algorithms in adaptive version are then used for control of the model. The results of the real-time experiments are also given.**

***Index Terms*—multivariable control, control algorithms, adaptive control, polynomial methods, pole assignment**

I. Introduction

Many technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The coupled drives apparatus is a typical multivariable nonlinear system with interactions between control loops. The design of a controller able to cope with such a system must be quite sophisticated. There are many different methods of controlling multivariable systems. Several of these use decentralized PID controllers [1], others apply single input-single output (SISO) methods extended into a multivariable case [2]. One possibility is the serial insertion of a compensator ahead of the system to transform the multivariable system into a series of independent SISO loops [3], [4], [5], [6].

In this paper polynomial theory approach [7] is used to control a multivariable system. Two controllers are presented: the first one is based on the configuration given in [8], the second one applies a decoupling method to suppress undesired interactions between control loops.

The paper is organised as follows: section II. contains description of the coupled drives apparatus; section III. presents a mathematical model of the apparatus which was used for controllers design; section IV. describes how feedback control is designed; section V. describes design of the controller with the compensator; section VI. describes the system identification method; section VII. contains the experimental results; section VIII. concludes the paper.

Marek Kubalčík, Department of Control Theory,
Institute of Information Technologies
Tomas BataUniversity
Nám. TGM 275, 762 72 Zlín, Czech Republic
Phone, Fax: +420 57 7543 103, E – mail:kubalcik@ft.utb.cz

II. Description of the Apparatus

Our department has an experimental laboratory model CE 108 - coupled drives apparatus. This apparatus, based on experience with authentic industrial control applications, was developed in cooperation with the University of Manchester and made by a British company, TecQuipment Ltd. It allows us to investigate the ever-present difficulty of controlling the tension and speed of material in a continuous process. The process may require the material speed and tension to be controlled to within defined limits. Examples of this occur in the paper-making industry, strip metal and wire manufacture and, indeed, any process where the product is manufactured in a continuous strip.

The industrial type material strip is replaced by a continuous flexible belt. The principle scheme of the model is shown in Fig. 1. It consists of three pulleys, mounted on a vertical panel so that they form a triangle resting on its base. The two base pulleys are directly mounted on the shafts of two nominally identical servo motors and the apparatus is controlled by manipulating the drive torques to these servo motors. The third pulley, the jockey, is free to rotate and is mounted on a pivoted arm. The jockey pulley assembly, which simulates a material work station, is equipped with a special sensor and tension measuring equipment. It is the jockey pulley speed and tension which form the principle system outputs. The belt tension is measured indirectly by monitoring the angular deflection of the pivoted tension arm to which the jockey pulley is attached.

The manipulated variables are the inputs to the servo motors and the controlled variables are the tension and speed at the work station.

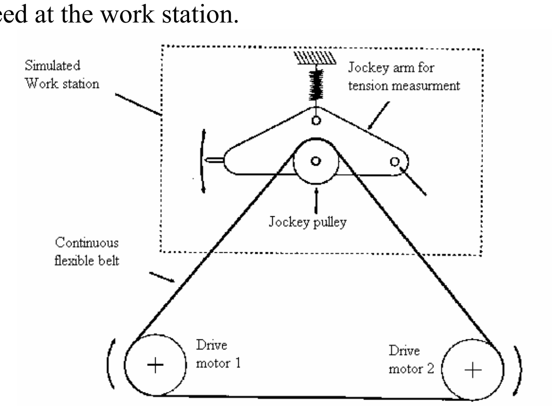

Fig. 1. Principal scheme of CE 108

III. MATHEMATICAL MODEL OF THE APPARATUS

The examined apparatus is a typical example of a two inputs – two outputs system with internal interactions between the control loops. The transfer matrix of the system is

$$\boldsymbol{G}\left(z^{-1}\right)=\frac{\boldsymbol{Y}\left(z^{-1}\right)}{\boldsymbol{U}\left(z^{-1}\right)}=\begin{bmatrix} G_{11}\left(z^{-1}\right) & G_{12}\left(z^{-1}\right) \\ G_{21}\left(z^{-1}\right) & G_{22}\left(z^{-1}\right) \end{bmatrix} \qquad (1)$$

where

$$\boldsymbol{U}\left(z^{-1}\right)=\left[u_1\left(z^{-1}\right), u_2\left(z^{-1}\right)\right]^T \qquad (2)$$

is the vector of manipulated variables (inputs to the servo motors) and

$$\boldsymbol{Y}\left(z^{-1}\right)=\left[y_1\left(z^{-1}\right), y_2\left(z^{-1}\right)\right]^T \qquad (3)$$

is the output vector (tension and speed at the work station). It is possible to assume that the dynamic behaviour of the system can be described in the neighbourhood of steady state by a discrete linear model in the form of the matrix fraction

$$\boldsymbol{G}\left(z^{-1}\right)=\boldsymbol{A}^{-1}\left(z^{-1}\right)\boldsymbol{B}\left(z^{-1}\right)=\boldsymbol{B}_1\left(z^{-1}\right)\boldsymbol{A}_1^{-1}\left(z^{-1}\right) \qquad (4)$$

Where polynomial matrices $\boldsymbol{A}\in R_{22}\left[z^{-1}\right], \boldsymbol{B}\in R_{22}\left[z^{-1}\right]$ are the left indivisible decomposition of matrix $\boldsymbol{G}\left(z^{-1}\right)$ and matrices $\boldsymbol{A}_1\in R_{22}\left[z^{-1}\right], \boldsymbol{B}_1\in R_{22}\left[z^{-1}\right]$ are the right indivisible decomposition of $\boldsymbol{G}\left(z^{-1}\right)$.

At first, the algorithms described bellow were designed for a model with polynomials of the first order. This model turned out to be unsuitable for the coupled drives process and the control algorithms failed. Consequently the polynomial orders were increased and the algorithms were designed for a model with second order polynomials. This model proved to be effective. In case of the controller based on the configuration by [8], a model with nondiagonal matrix $\boldsymbol{A}\left(z^{-1}\right)$ was used. The model has sixteen parameters. In case of decoupling control using the compensator the model was simplified by considering matrix $\boldsymbol{A}\left(z^{-1}\right)$ as diagonal. The reason is explained in section V. This assumption causes reduction of number of parameters. This model has twelve parameters. Particular matrix forms of the models are given in appropriate sections.

IV. DESIGN OF FEEDBACK CONTROL

The configuration of the closed loop, which is shown in Fig. 2, was presented in [8].

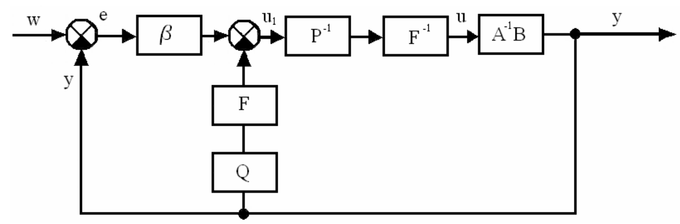

Fig. 2. Block diagram of the closed loop system

The structure of the discrete model matrices was chosen as

$$\boldsymbol{A}\left(z^{-1}\right)=\begin{bmatrix} 1+a_1z^{-1}+a_2z^{-2} & a_3z^{-1}+a_4z^{-2} \\ a_5z^{-1}+a_6z^{-2} & 1+a_7z^{-1}+a_8z^{-2} \end{bmatrix} \qquad (5)$$

$$\boldsymbol{B}\left(z^{-1}\right)=\begin{bmatrix} b_1z^{-1}+b_2z^{-2} & b_3z^{-1}+b_4z^{-2} \\ b_5z^{-1}+b_6z^{-2} & b_7z^{-1}+b_8z^{-2} \end{bmatrix}$$

Polynomial matrices of the right matrix fraction of the system are defined in the form.

$$\boldsymbol{A}_1\left(z^{-1}\right)=\begin{bmatrix} 1+a_9z^{-1}+a_{10}z^{-2} & a_{11}z^{-1}+a_{12}z^{-2} \\ a_{13}z^{-1}+a_{14}z^{-2} & 1+a_{15}z^{-1}+a_{16}z^{-2} \end{bmatrix} \qquad (6)$$

$$\boldsymbol{B}_1\left(z^{-1}\right)=\begin{bmatrix} b_9z^{-1}+b_{10}z^{-2} & b_{11}z^{-1}+b_{12}z^{-2} \\ b_{13}z^{-1}+b_{14}z^{-2} & b_{15}z^{-1}+b_{16}z^{-2} \end{bmatrix}$$

The coefficients of the matrices are given by solving matrix equation

$$\boldsymbol{B}\boldsymbol{A}_1-\boldsymbol{A}\boldsymbol{B}_1=0 \qquad (7)$$

Generally, the vector of input reference signals $\boldsymbol{W}$ is given by

$$\boldsymbol{W}\left(z^{-1}\right)=\boldsymbol{F}_w^{-1}\left(z^{-1}\right)\boldsymbol{h}\left(z^{-1}\right) \qquad (8)$$

Here, the reference signals are considered from a class of step functions. In this case $\boldsymbol{h}\left(z^{-1}\right)$ is a vector of constants and $\boldsymbol{F}_w\left(z^{-1}\right)$ takes the form

$$\boldsymbol{F}_w\left(z^{-1}\right)=\begin{bmatrix} 1-z^{-1} & 0 \\ 0 & 1-z^{-1} \end{bmatrix} \qquad (9)$$

The compensator $\boldsymbol{F}\left(z^{-1}\right)$ is a component formally separated from the controller. It has to be inherent in the controller to fulfil the requirement on the asymptotic tracking. If the reference signals are from the class of step functions, $\boldsymbol{F}\left(z^{-1}\right)$ is an integrator.

It is possible to derive the following equation for the system output (operator z^{-1} will be omitted from some operations for the sake of simplification)

$$\boldsymbol{Y}=\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U}=\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \qquad (10)$$

Where

$$\boldsymbol{U}_1 = \boldsymbol{\beta}(\boldsymbol{W} - \boldsymbol{Y}) - \boldsymbol{QFY} \tag{11}$$

The equation for the controller output, as shown in the block diagram in Fig 2, takes the form

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \tag{12}$$

Substitution of $\boldsymbol{U}_1$ and $\boldsymbol{Y}$ results in

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\left[\boldsymbol{\beta}\left(\boldsymbol{W} - \boldsymbol{A}^{-1}\boldsymbol{BU}\right) - \boldsymbol{QFA}^{-1}\boldsymbol{BU}\right] \tag{13}$$

The equation (13) can be modified using the right matrix fraction of the controlled system to the form

$$\boldsymbol{U} = \boldsymbol{A}_1\left[\boldsymbol{PFA}_1 + (\boldsymbol{\beta} + \boldsymbol{FQ})\boldsymbol{B}_1\right]\boldsymbol{\beta W} \tag{14}$$

The closed loop system is stable when the following diophantine equation is fulfilled

$$\boldsymbol{PFA}_1 + (\boldsymbol{\beta} + \boldsymbol{FQ})\boldsymbol{B}_1 = \boldsymbol{M} \tag{15}$$

Where $\boldsymbol{M} \in R_{22}\left[z^{-1}\right]$ is a stable diagonal polynomial matrix.

$$\boldsymbol{M}\left(z^{-1}\right) = \begin{bmatrix} 1 + m_1 z^{-1} + m_2 z^{-2} + + m_3 z^{-3} + m_4 z^{-4} & 0 \\ 0 & 1 + m_1 z^{-1} + m_2 z^{-2} + + m_3 z^{-3} + m_4 z^{-4} \end{bmatrix} \tag{16}$$

The roots of this polynomial matrix are the ruling factor in the behaviour of the closed loop system. They must be inside the unit circle if the system is to be stable.
The degree of the controller matrices polynomials depends on the internal properness of the closed loop. The structure of the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{\beta}$ was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equation using the uncertain coefficients method.

$$\boldsymbol{P}\left(z^{-1}\right) = \begin{bmatrix} 1 + p_1 z^{-1} & p_2 z^{-1} \\ p_3 z^{-1} & 1 + p_4 z^{-1} \end{bmatrix} \tag{17}$$

$$\boldsymbol{Q}\left(z^{-1}\right) = \begin{bmatrix} q_1 + q_2 z^{-1} & q_3 + q_4 z^{-1} \\ q_5 + q_6 z^{-1} & q_7 + q_8 z^{-1} \end{bmatrix} \tag{18}$$

$$\boldsymbol{\beta}\left(z^{-1}\right) = \begin{bmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{bmatrix} \tag{19}$$

The solution to the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. Using matrix notation we can express the algebraic equations in the form

$$\begin{bmatrix} 1 & 0 & b_9 & 0 & b_{13} & 0 & b_9 & b_{13} \\ a_9 - 1 & a_{13} & b_{10} - b_9 & b_9 & b_{14} - b_{13} & b_{13} & b_{10} & b_{14} \\ a_{10} - a_9 & a_{14} - a_{13} & -b_{10} & b_{10} - b_9 & -b_{14} & b_{14} - b_{13} & 0 & 0 \\ -a_{10} & -a_{14} & 0 & -b_{10} & 0 & -b_{14} & 0 & 0 \\ 0 & 1 & b_{11} & 0 & b_{15} & 0 & b_{11} & b_{15} \\ a_{11} & a_{15} - 1 & b_{12} - b_{11} & b_{11} & b_{16} - b_{15} & b_{15} & b_{12} & b_{16} \\ a_{12} - a_{11} & a_{16} - a_{15} & -b_{12} & b_{12} - b_{11} & -b_{16} & b_{16} - b_{15} & 0 & 0 \\ -a_{12} & -a_{16} & 0 & -b_{12} & 0 & -b_{16} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} m_1 - a_9 + 1 \\ m_2 + a_9 - a_{10} \\ m_3 + a_{10} \\ m_4 \\ -a_{11} \\ a_{11} - a_{12} \\ a_{12} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & b_9 & 0 & b_{13} & 0 & b_9 & b_{13} \\ a_9 - 1 & a_{13} & b_{10} - b_9 & b_9 & b_{14} - b_{13} & b_{13} & b_{10} & b_{14} \\ a_{10} - a_9 & a_{14} - a_{13} & -b_{10} & b_{10} - b_9 & -b_{14} & b_{14} - b_{13} & 0 & 0 \\ -a_{10} & -a_{14} & 0 & -b_{10} & 0 & -b_{14} & 0 & 0 \\ 0 & 1 & b_{11} & 0 & b_{15} & 0 & b_{11} & b_{15} \\ a_{11} & a_{15} - 1 & b_{12} - b_{11} & b_{11} & b_{16} - b_{15} & b_{15} & b_{12} & b_{16} \\ a_{12} - a_{11} & a_{16} - a_{15} & -b_{12} & b_{12} - b_{11} & -b_{16} & b_{16} - b_{15} & 0 & 0 \\ -a_{12} & -a_{16} & 0 & -b_{12} & 0 & -b_{16} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_3 \\ p_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ \beta_3 \\ \beta_4 \end{bmatrix} = \begin{bmatrix} -a_{13} \\ a_{13} - a_{14} \\ a_{14} \\ 0 \\ m_5 - a_{15} + 1 \\ m_6 + a_{15} - a_{16} \\ m_7 + a_{16} \\ m_8 \end{bmatrix} \tag{20}$$

The controller parameters are given by solving these equations.
The control law apparent in the block diagram has the form

$$\boldsymbol{FPU} = \boldsymbol{\beta E} - \boldsymbol{FQY} \tag{21}$$

V. Decoupling Control Using Compensator

There are several ways to control multivariable systems with internal interactions. One possibility is the serial insertion of a compensator ahead of the system [3], [4], [5], [6]. The aim here is to suppress of undesirable interactions between the input and output variables so that each input affects only one controlled variable.

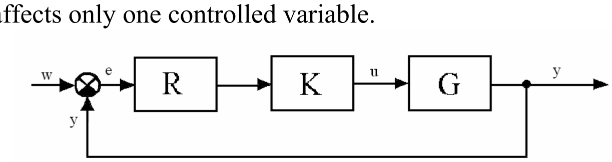

Fig. 3. General scheme of closed loop with compensator

The resulting transfer function $\boldsymbol{H}$ is then given by

$$\boldsymbol{H} = \boldsymbol{KG} \tag{22}$$

The decoupling conditions are fulfilled when matrix $\boldsymbol{H}$ is diagonal.
The matrix $\boldsymbol{B}$ can be written as

$$\boldsymbol{B} = z^{-1}\boldsymbol{B}_{\mathrm{x}} = z^{-1}\begin{bmatrix} b_1 + b_2 z^{-1} & b_3 + b_4 z^{-1} \\ b_5 + b_6 z^{-1} & b_7 + b_8 z^{-1} \end{bmatrix} \tag{23}$$

The compensator, which was used for the control algorithm is adjugated matrix $\boldsymbol{B}_{\mathrm{x}}$. The model was simplified by considering matrix $\boldsymbol{A}$ as diagonal in this case.

$$\boldsymbol{A}\left(z^{-1}\right) = \begin{bmatrix} 1 + a_1 z^{-1} + a_2 z^{-2} & 0 \\ 0 & 1 + a_3 z^{-1} + a_4 z^{-2} \end{bmatrix} \tag{24}$$

The multiplication of matrix $\boldsymbol{B}_\text{x}$ and adjugated matrix $\boldsymbol{B}_\text{x}$ results in diagonal matrix $\boldsymbol{H}$. The determinants of matrix $\boldsymbol{B}_\text{x}$ represent the diagonal elements. When matrix $\boldsymbol{A}$ is supposed as nondiagonal, its inverted form must be placed ahead of the system in order to obtain diagonal matrix $\boldsymbol{H}$, otherwise it may increase the order of the controller and sophistication of the closed loop system.

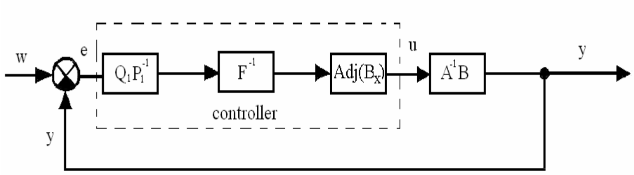

Fig. 4. Closed loop system with chosen compensator

The equation for the system output as shown in this block diagram takes the form

$$\boldsymbol{Y} = \boldsymbol{P}_1(\boldsymbol{AFP}_1 + \boldsymbol{B}_v\boldsymbol{Q}_1)\boldsymbol{B}_v\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{W} \tag{25}$$

where

$$\boldsymbol{B}_v = z^{-1}\boldsymbol{B}_\text{x}\,adj(\boldsymbol{B}_\text{x}) = \begin{bmatrix} det(\boldsymbol{B}_\text{x}) & 0 \\ 0 & det(\boldsymbol{B}_\text{x}) \end{bmatrix} \tag{26}$$

To achieve stability in the closed loop system the following diophantine equation must be fulfilled

$$\boldsymbol{AFP}_1 + \boldsymbol{B}_v\boldsymbol{Q}_1 = \boldsymbol{M} \tag{27}$$

The controller polynomial matrices are chosen as shown below

$$\boldsymbol{P}_1(z^{-1}) = \begin{bmatrix} 1 + p_1 z^{-1} + p_2 z^{-2} & 0 \\ 0 & 1 + p_3 z^{-1} + p_4 z^{-2} \end{bmatrix} \tag{28}$$

$$\boldsymbol{Q}_1(z^{-1}) = \begin{bmatrix} q_1 + q_2 z^{-1} + q_3 z^{-2} & 0 \\ 0 & q_4 + q_5 z^{-1} + q_6 z^{-2} \end{bmatrix}$$

and matrix $\boldsymbol{M}$ is

$$\boldsymbol{M}(z^{-1}) = \begin{bmatrix} \begin{matrix} 1 + m_1 z^{-1} + \\ + m_2 z^{-2} + m_3 z^{-3} + \\ + m_4 z^{-4} + m_5 z^{-5} \end{matrix} & 0 \\ 0 & \begin{matrix} 1 + m_6 z^{-1} + \\ m_7 z^{-2} + m_8 z^{-3} + \\ + m_9 z^{-4} + m_{10} z^{-5} \end{matrix} \end{bmatrix} \tag{29}$$

Solving the diophantine equation defines a set of algebraic equations which we subsequently use to obtain the unknown controller parameters.
For sake of simplification, it was derived det($\boldsymbol{B}_\text{x}(z^{-1})$):

$$\det(\boldsymbol{B}_\text{x}(z^{-1})) = db_3 z + db_2 z^{-1} + db_1 z^{-2}$$
$$\det(\boldsymbol{B}_\text{x}(z^{-1})) = (b_1b_7 - b_5b_3) + z^{-1}(b_1b_8 + b_2b_7 - b_5b_4 - b_6b_3) +$$
$$+ z^{-2}(b_2b_8 - b_4b_6) \tag{30}$$

Algebraic equations have the form

$$\begin{bmatrix} 1 & 0 & db_3 & 0 & 0 \\ a_1 - 1 & 1 & db_2 & db_3 & 0 \\ a_2 - a_1 & a_1 - 1 & db_1 & db_2 & db_3 \\ -a_2 & a_2 - a_1 & 0 & db_1 & db_2 \\ 0 & -a_2 & 0 & 0 & db_1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} m_1 - a_1 + 1 \\ m_2 - a_2 + a_1 \\ m_3 + a_2 \\ m_4 \\ m_5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & db_3 & 0 & 0 \\ a_3 - 1 & 1 & db_2 & db_3 & 0 \\ a_4 - a_3 & a_3 - 1 & db_1 & db_2 & db_3 \\ -a_4 & a_4 - a_3 & 0 & db_1 & db_2 \\ 0 & -a_4 & 0 & 0 & db_1 \end{bmatrix} \begin{bmatrix} p_3 \\ p_4 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} m_6 - a_3 + 1 \\ m_7 - a_4 + a_3 \\ m_8 + a_4 \\ m_9 \\ m_{10} \end{bmatrix} \tag{31}$$

The control law is given by the matrix equation

$$\boldsymbol{FU} = adj(\boldsymbol{B}_\text{x})\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{E} \tag{32}$$

VI. SYSTEM IDENTIFICATION

The algorithms designed here were incorporated into an adaptive control system with recursive identification. The recursive least squares method proved effective for self-tuning controllers [9], [10] and was used as the basis for our algorithm. For our two-variable example we considered the disintegration of identification into two independent parts. We can define difference equations of the models in the vector form

$$\begin{aligned} y_1(k) &= \boldsymbol{\Theta}_1^T(k)\phi_1(k-1) \\ y_2(k) &= \boldsymbol{\Theta}_2^T(k)\phi_2(k-1) \end{aligned} \tag{33}$$

For the case with the first configuration of the closed loop the parameter vectors are completed as shown below:

$$\begin{aligned} \boldsymbol{\Theta}_1^T(k) &= [a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4] \\ \boldsymbol{\Theta}_2^T(k) &= [a_5, a_6, a_7, a_8, b_5, b_6, b_7, b_8] \end{aligned} \tag{34}$$

The data vector is

$$\begin{aligned} \phi_{1,2}^T(k-1) = [&-y_1(k-1), -y_1(k-2), -y_2(k-1), \\ &-y_2(k-2), u_1(k-1), u_1(k-2), u_2(k-1), u_2(k-2)] \end{aligned} \tag{35}$$

For the configuration with the compensator the vectors have following forms:

$$\boldsymbol{\Theta}_1^T(k) = [a_1, a_2, b_1, b_2, b_3, b_4] \tag{36}$$
$$\boldsymbol{\Theta}_2^T(k) = [a_3, a_4, b_5, b_6, b_7, b_8]$$

$$\phi_1^T(k-1) = [-y_1(k-1), -y_1(k-2), \tag{37}$$
$$u_1(k-1), u_1(k-2), u_2(k-1), u_2(k-2)]$$

$$\phi_2^T(k-1) = [-y_2(k-1), -y_2(k-2), \tag{38}$$
$$u_1(k-1), u_1(k-2), u_2(k-1), u_2(k-2)]$$

The parameter estimates are actualised using the recursive least squares method plus directional forgetting.

VII. Experimental Results

The model was connected with PC by the card Advantech PCL – 812. For its control the Matlab and the Real Time Toolbox were used.

The coupled drives apparatus is a nonlinear system with variable parameters which is, therefore, impossible to control deterministically. The nonlinear dynamics was described by the linear model in the neighborhood of steady state. Adaptive control using recursive identification with both controllers was performed.

The right side control matrices, which resulted from a number of experiments, are denoted as follows: without compensator - $\boldsymbol{M}_1$, with compensator $\boldsymbol{M}_2$.

$$\boldsymbol{M}_1(z^{-1}) = \begin{bmatrix} 1-0{,}9z^{-1}+0{,}19z^{-2}- & 0 \\ -0{,}009z^{-3}-0{,}002z^{-4} & \\ 0 & 1-0{,}9z^{-1}+0{,}19z^{-2}- \\ & -0{,}009z^{-3}-0{,}002z^{-4} \end{bmatrix} \tag{39}$$

$$\boldsymbol{M}_2(z^{-1}) = \begin{bmatrix} 1-0{,}7z^{-1}+0{,}01z^{-2}- & 0 \\ -0{,}1z^{-3}-0{,}05z^{-4}+0.0001z^{-5} & \\ 0 & 1-0{,}7z^{-1}+0{,}01z^{-2}- \\ & -0{,}1z^{-3}-0{,}05z^{-4}+0.0001z^{-5} \end{bmatrix} \tag{40}$$

The same initial conditions for system identification were used for both controllers we tested. The initial parameters estimation were chosen to be

$$\boldsymbol{\Theta}_1^T(0) = [0.1, 0.2, 0.3, 0.4, 0.1, 0.2, 0.3, 0.4] \tag{41}$$
$$\boldsymbol{\Theta}_2^T(0) = [0.5, 0.6, 0.7, 0.8, 0.5, 0.6, 0.7, 0.8]$$

and

$$\boldsymbol{\Theta}_1^T(0) = [0.1, 0.2, 0.1, 0.2, 0.3, 0.4] \tag{42}$$
$$\boldsymbol{\Theta}_2^T(0) = [0.2, 0.3, 0.5, 0.6, 0.7, 0.8]$$

The sampling period in both cases was chosen T_0=0,25 s.

The time responses of the control for both cases are shown in Fig. 5, Fig. 6, Fig. 7 and Fig. 8. The controlled variable y_1 is the speed and the controlled variable y_2 is the tension.

VIII. Conclusions

The multivariable adaptive control of the real coupled drives apparatus was realized by means of polynomial theory. The control tests on the laboratory model gave satisfactory results despite the fact that the non-linear dynamics was described by the linear model. With regards to decoupling, it is clear that the controller with the compensator reduces interactions between the control loops.

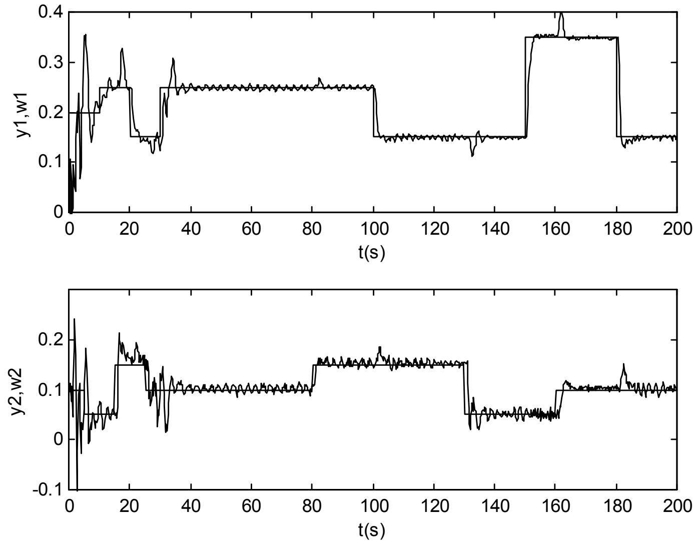

Fig. 5. Adaptive control of the laboratory model without compensator

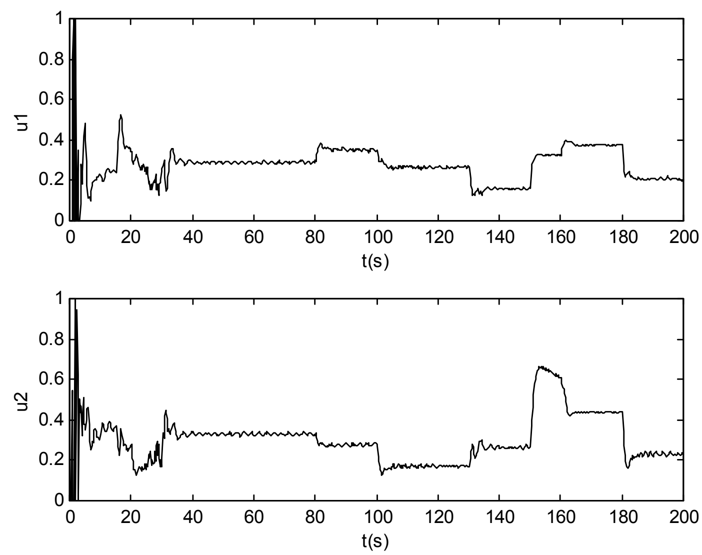

Fig. 6. Adaptive control of the laboratory model without compensator – controller output

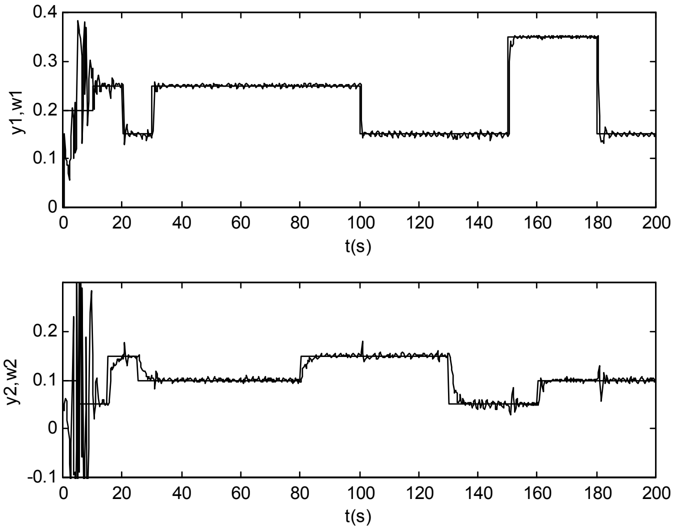

Fig. 7. Adaptive control of the laboratory model with compensator

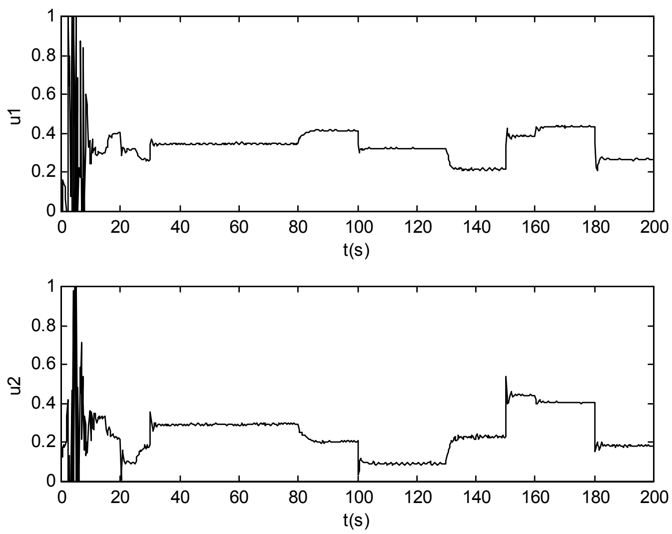

Fig. 8. Adaptive control of the laboratory model with compensator – controller output

ACKNOWLEDGMENTS

The work has been supported by the Grant Agency of the Czech Republic under grants No. 102/03/0070 and 102/01/P013 and by the Ministry of Education of the Czech Republic under grant MSM 281100001. This support is very gratefully acknowledged.

REFERENCES

[1] Luyben, W. L. (1986). Simple Method for Tuning SISO Controllers in Multivariable Systems. *Ing. Eng. Chem. Process Des. Dev.*, 25, pp. 654 – 660.

[2] Chien, I.L., Seborg, D.E., Mellichamp, D. A. (1987). Self-Tuning Control with Decoupling. *AIChE* J., 33, 7, pp. 1079 – 1088.

[3] Krishnawamy, P.R. et al. (1991). Reference System Decoupling for Multivariable Control. *Ind. Eng. Chem. Res.*, 30, pp. 662-670.

[4] Peng, Y. (1990). A General Decoupling Precompensator for Linear Multivariable Systems with Application to Adaptive Control. *IEEE Trans. Aut. Control*, AC-35, 3, pp. 344-348.

[5] Tade, M.O., Bayoumi, M.M., Bacon, D.W. (1986). Adaptive Decoupling of a Class of Multivariable Dynamic Systems Using Output Feedback. *IEE Proc. Pt.D*, 133, 6, pp. 265-275.

[6] Wittenmark, B., Middelton, R. H., Goodwin, G.C. (1987). Adaptive Decoupling of Multivariable Systems. *Int. J. Control*, 46, 6, pp. 1993-2009.

[7] Kučera, V. (1980). Stochastic Multivariable Control: a Polynomial approach. IEEE Trans. of Automatic Control , 5, 913 – 919.

[8] Ortega, V., Kelly R.: (1984). PID self – tuners: Some theoretical and practical aspects. IEEE Trans. Ind. Electron., IE - 3, 332 – 338.

[9] Kulhavý, R. (1987). Restricted exponencial forgetting in real – time identification. *Automatica*, 23, 589 – 600.

[10] Bittanti, S., Bolzern, P., Campi, M. (1990). Convergence and Exponential Convergence of Identification Algorithms with Directional Forgetting Factor. *Automatica*, 26, 5, pp. 929 – 932.