Loop-Gain Optimization of Unstable–NMP SISO plants

Marcel J. Sidi
Holon Academic Institute of Technology
Electrical and Electronics Engineering Department
52 Golomb Str., Holon 58102, Israel
e-mail: sidi@hait.ac.il; fax: 972-3-5365285

Abstract– **The paper presents a design method for optimizing the loop-gain of unstable - nonminimum phase feedback systems. Due to obvious gain-band-width limitations existing in such systems, the proposed best loop optimization goal is maximization of the stability gain and phase margins, which are low inherently in such feedback systems. The design procedure is based on coprime factorization of the plant and controller, and the Bezoult identity. An optimization process varies iteratively the observer-based controller parameters until maximization of the gain and phase margins is achieved.**

Index Terms-**Coprime factorization, Bezout identity, controller parameterization, stability, relative stability margins, optimal loop-gain, uncertain plants, robustness.**

I. INTRODUCTION

The problem of stabilizing unstable -nonminimum phase (NMP) plants has been subject to intensive research. In the 1970's, a great advance of the matter was attained by introducing the principle of coprime factorization of unstable- NMP plants and of the controllers, thus accomplishing internal stabilization of the closed-loop feedback system by means of the Bezout identity, [1]-[3] and many other papers treating this topic. An exhaustive treatment of the Factorization Approach to feedback control synthesis can be found in [4]. The central advantage of this approach is that internal stability of the closed-loop system is guaranteed for any unstable -NMP plant, with no restrictions on the number of right half plane (RHP) zeros and poles, and of their relative locations in the *s*-plane. Unfortunately, for such complex systems, the resulting performances of the design are quite poor: the obtained phase and gain margins are very small, there is no way to control the bandwidth of the open-loop system, the gain peaking of the closed-loop transfer function (TF), and so on. There have been found general analytical constraints on the maximum achievable crossover frequency for a feedback-loop in which the NMP plant contains one RHP zero, or the minimum crossover frequency when the plant comprises one unstable pole [5]. The achievable maximum bandwidth when the plant contains several RHP zeros, or the achievable minimum bandwidth when the plant contains several RHP poles was also analytically estimated, [6]. In the two references above, it is clearly shown that tradeoffs exist between achievable crossover frequencies and gain and phase margins. It was also possible to derive analytical relations between achievable crossover frequencies, and gain and phase margin for the case when the plant comprises one RHP zero preceding one RHP pole, and vice versa, [7]. The analytical approach in the last three references was not attempted for plants comprising simultaneously multiple RHP poles and zeros, because of obvious theoretical and practical difficulties.

The coprime factorization and controller parameterization approaches gained a vast popularity in the development of the H_∞-control paradigm for robust feedback systems, and is, in some sense, the mile-stone of this so important design approach for uncertain robust systems, [8], [9] and others.

The theoretical problem of stabilizing a feedback system when the plant comprises numerous RHP poles and zeros is completely solved by the conventional coprime factorization theory, and the Bezout identity. Unfortunately, as far as the author knows, no serious attempt has been made to characterize a suitable engineering definition of "goodness" to such stabilized feedback system. The usual desired design specifications, as relative stability margins, bandwidth, maximum closed-loop gain peaking, steady state error coefficients, etc., are not compatible when the plant includes several RHP poles and zeros. If defined, these specifications cannot be all achieved completely. What remains to the designer is to attain the "best possible performances" of the feedback controlled system, in general, the best stability margins.

In order to get an idea of the difficulties emerging out of the problem at hand, the following illustrative example is considered. On the Nichols chart (NC) in figure 1a are displayed two stabilizing open-loop TFs pertaining to the unstable- NMP plant $P(s) = (s\text{-}1)/s(s\text{-}2)$. $L_1(s)$ was achieved in [3] by using the technique of coprime factorization of the plant and controller, and satisfying the Bezout identity. $L_2(s)$, that also stabilizes internally the same plant, was obtained by using the design technique in which the theoretical approximate maximum phase and gain margins are calculated first, and then achieved by conventional classical loop-shaping, [7]. It is clear from the figure that the $L_2(s)$ design improves significantly the gain and phase margins, thus ensuring improved stability robustness to larger plant uncertainties. The $L_2(s)$ design is also not satisfactory from an engineering viewpoint; unfortunately, this is the best that can be achieved with such an unfriendly plant.

The problem to be solved in this paper is maximization of the phase and gain margins for unsta-

.

ble- NMP feedback systems, by means of plant coprime factorization, and optimization of the controller parameters satisfying the Bezout identity, thus guaranteeing internal stability.

For lack of space, and in order to simplify the explanation of the optimization procedure, the single input-single output (SISO) case is treated here.

II. Definition Of The Optimal $L(j\omega)$ On The Nichols Chart.

We propose here an optimality characteristic to the open-loop TF $L(j\omega)$ displayed on NC, where the relative stability phase and gain margins are clearly exhibited, see Fig. 1. The closed loop TF gain of the canonic ODOF (one- degree- of- freedom) unity-feedback structure in Fig. 2 is:

$$|T(s)| = |Y(s)/R(s)| = |L/(1+L)|; \; L(s)=G(s)P(s) \qquad (1)$$

Looking at contours of constant $|T|$ on NC, (not shown here) it is apparent that the larger $|T|_{max}$ is, the smaller the phase and gain margins are. There exist analytic relations that express minimum gain and phase margins in terms of the highest $|T|$ constant gain contour touched by the open-loop TF $L(j\omega)$, [7], [11].

$$GM \geq 1 + 1/|T(j\omega)|_{max}; \qquad (2a)$$
$$PM \geq 2\sin^{-1}[1/2|T(j\omega)|_{max}] \qquad (2b)$$

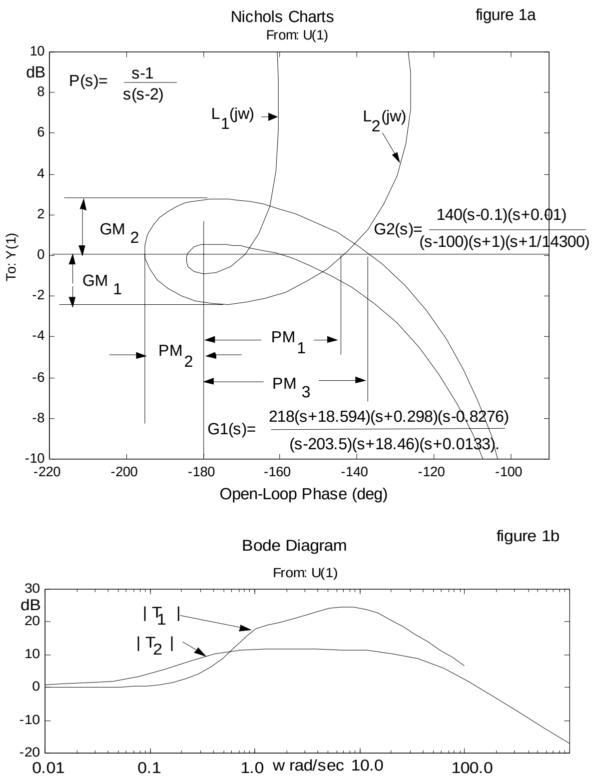


Figure 1. a- $L_1(j\omega)$ and $L_2(j\omega)$ on the Nichols chart.
b- Bode plots of $|T_1(j\omega)|$ and $|T_2(j\omega)|$.

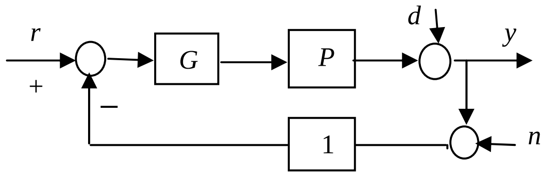


Figure 2. Canonic ODOF unity-feedback system.

We conclude that, by lowering $|T|_{max}$, the gain and phase margins are both increased. This fact is clearly perceived in Fig. 1b, where $|T_1|_{max}$ = 25dB, while $|T_2|_{max}$ = 12dB only. Moreover, according to the Nyquist stability criterion, the number of negative encirclements of the stability critical point (0dB, -180^o on the NC) must equal the number of unstable open-loop poles. Hence, for unstable plants, two, or more gain-margins, phase-margins and crossover frequencies should be observed, as shown in Fig. 1a. Accordingly, we may define an optimal $L(j\omega)$ for an unstable- NMP plant **as that $L(j\omega)$ for which the stability phase and gain margins are maximized.**

Remark 1. It is important to remind the reader that in cases the plant contains RHP zeros only, or RHP poles only, any desired phase and gain margins can be achieved, of course, under obvious restrictions on their crossover frequencies. They will be, accordingly, low for the former, and high for the later case.

III. Maximization Of Stability Margins

The procedure for obtaining a stable solution together with maximal phase and gain margins for unstable-NMP SISO plants is as follows:

3.1- Coprime factorization of plant P(s) and controller G(s).

Define

$$P(s) = \frac{np(s)}{dp(s)}; \quad G(s) = \frac{ng(s)}{dg(s)} \qquad (3a,b)$$

For coprime factorization of both $P(s)$ and $G(s)$, define polynomials $xp(s)$ and $xg(s)$, with stable roots, to obtain

$$Np(s) = \frac{np(s)}{xp(s)}; Dp(s) = \frac{dp(s)}{xp(s)} \qquad (4a,b)$$

$$Ng(s) = \frac{ng(s)}{xg(s)}; Dg(s) = \frac{dg(s)}{xg(s)}$$

(4c,d)

Since $Np(s)$ and $Dp(s)$ are chosen to be relatively prime, there exist $Ng(s)$ and $Dg(s)$ for which

$$Np(s)Ng(s) + Dp(s)Dg(s) = 1 \qquad (5)$$

Insert the definitions of (4a,b,c,d) into (5) to obtain:

.

$$\frac{np(s)ng(s)}{xp(s)xg(s)}+\frac{dp(s)dg(s)}{xp(s)xg(s)}=1 \qquad (6)$$

3.2- Calculation of the order of the controller polynomials ng(s) and dg(s).

For a proper, or a strictly proper $P(s)$, δ_p is the order of $dp(s)$, but hence, also of $xp(s)$. For a proper $G(s)$, δ_g is the order of $dg(s)$, but hence, also of $xg(s)$. In order to satisfy (6), it is clear that the following identity must be held:

$$\delta_p + \delta_g + 1 = 2(\delta_g + 1) \qquad (7)$$

or, finally:

$$\delta_g = \delta_p - 1 \qquad (8)$$

For instance, suppose that $\delta_p = 3$ (the plant has three poles), then $\delta_g = \delta_p - 1 = 2$, and the controller $G(s)$ will take the form of

$$G(s)=\frac{ng(s)}{dg(s)}=\frac{q_1 s^2+q_2 s+q_3}{g_1 s^2+g_2 s+g_3} \qquad (9)$$

3.3- Derivation of the controller polynomials ng(s) and dg(s).

$G(s)$ in (9) insinuates that for this special case, six parameters are to be derived, so that the optimized $L(j\omega)$ will come out with maximal gain and phase margins. In the general case, the parameters q_i and g_i cannot be arbitrarily assigned. They must satisfy equation (6), in which the coefficients of the polynomials $xp(s)$ and $xg(s)$ are freely chosen, under the restriction that the two polynomials will be Hurwitz, (with no roots in the closed RHP). In section 3.4, it will be shown how to derive coefficients of $xp(s)$ and $xg(s)$ optimizing $L(j\omega)$, in the sense of achieving the 'maximal obtainable' gain and phase margins.

Derivation of the controller $G(s)$ for the arbitrary choice of the coefficients of $xp(s)$ and $xg(s)$ is best explained by solving a real example.

Example 1. Find a controller $G(s)$ that stabilizes the plant $P(s)= (s-5)/[s(s-2)(s-10)]$.

Solution:

1. Define the plant in the common form:

$$P(s)=\frac{np(s)}{dp(s)}=\frac{k(s+z)}{(s+p_1)(s+p_2)(s+p_3)}$$

$$=\frac{ks+kz}{s^3+t_1 s^2+t_2 s+t_3}\text{, where}$$

$t_1 = p_1 + p_2 + p_3$, $t_2 = p_1 p_2 + p_1 p_3 + p_2 p_3$; $t_3 = p_1 p_2 p_3$

2. Define the polynomials:

$xp = (s + a)(s^2 + b\, s + c) = (s^3 + r_1 s^2 + r_2\, s + r_3)$;

$r_1 = a+b$, $r_2 = a\, b + c$; $r_3 = a\, c$;

$xg = (s^2 + d\, s + f)$.

Next, arbitrarily assign the positive design parameters a, b, c, d, and f. (The roots of xp and xg are stable for positive a, b, c, d, and f.)

3. The controller G(s) is defined as in Eq.(9).

4. Substitute the so defined polynomials $np(s)$, $dp(s)$, $ng(s)$, $dg(s)$, $xp(s)$ and $xg(s)$ into (6) to get: $np(s)\, ng(s) + dp(s)\, dg(s) = xg(s)\, xp(s)$. The obtained polynomials in both sides of the equation will be of order 5. After equating coefficient expressions of the same order, we obtain six linear equations in six unknowns, which are the coefficients of the controller polynomials $ng(s)$ and $dg(s)$, namely, q_1, q_2, q_3, g_1, g_2, and g_3. These equations can be put in matrix form:

$$\mathbf{Mv} = \mathbf{u} \qquad (10)$$

in which

$\mathbf{v} = [g_1\ g_2\ g_3\ q_1\ q_2\ q_3]^T$,

$\mathbf{u} = [1\ \ (d + r_1)\ \ (f + d\, r_1 + r_2)\ \ (f\, r_1 + d\, r_2 + r_3)\ \ (f\, r_2 + d\, r_3)\ \ f\, r_3)]^T$, and

$$\mathbf{A}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ t_1 & 1 & 0 & 0 & 0 & 0 \\ t_2 & t_1 & 1 & k & 0 & 0 \\ t_3 & t_2 & t_1 & kz & k & 0 \\ 0 & t_3 & t_2 & 0 & kz & k \\ 0 & 0 & t_3 & 0 & 0 & kz \end{bmatrix} \qquad (11)$$

Finally,

$$\mathbf{v} = [g_1\ g_2\ g_3\ q_1\ q_2\ q_3]^T = \mathbf{M}^{-1}\mathbf{u}. \qquad (12)$$

In this example, $z = -5$, $p_1 = -2$, $p_2 = -10$, $p_3 = 0$. a, b, c, d, and f are arbitrarily assigned as:

$a = 2$, $b = 2$, $c = 4$, $d = 4$, $f = 9$.

The derived controller is:

$$G(s)=\frac{574.6s^2-1309.9s-14.4}{s^2+20s-321.56}$$

The obtained open-loop TF, named $L_{\mathrm{init}}(j\omega)$, is shown in Fig. 3a. The unstable -NMP plant has been stabilized, however, L_{init} is quite poor from sensitivity to plant uncertainty viewpoint. Two of the phase margins are very small, about ± 1 deg, the two gain margins are also very small, -0.2dB and +0.01dB. Such a design is of no practical use. The solution can in theory be improved by varying the design parameters a, b, c, d, and f with the hope of increasing the gain and phase margins, but, due to their large number, five in this example, the task seems to be unrealistic. To overcome this difficulty, we shall define a compatible cost function, and mini-mize it by an iterative optimization process.

3.4- Definition of a cost function optimising $L(j\omega)$.

The clue in choosing a cost function to our problem is found in Fig. 3b, where the unity-feedback $|T_{\mathrm{init}}|$ is shown (this TF was achieved by arbitrarily assigning

.

the design parameters a, b, c, d, and f). This unity-feedback gain is very high, as is also observed in Fig.3a. In Fig. 3b, $|T_{init}|$ reaches numerically a peak value of 44dB. We suggest improving this initial solution by using a "minimax algorithm" that will decrease $|T_{init}|$ to its achievable minimum, $|T_{opt}|$. According to (2a,b), minimization of $|T|$ maximizes the phase and gain margins. Consequently, let us define the cost function

$$CF = |T(j\omega)| - 1 \qquad (13)$$

We define next the vector of optimizing parameters:

$$par = [a\ b\ c\ d\ f]^T \qquad (14)$$

The maximization of the stability margins is then achieved by:

$$\frac{\min imize}{par} \max\left[\left|T(j\omega)\right| - 1\right] \qquad (15)$$

($|T(j\omega)|$ is in arithmetic units in this context.)

The rationale behind the proposed CF is as follows: as long as the difference $|T|$ -1 is positive, the optimization process tends to decrease $|T|$ toward unity gain. When the difference $|T|$ -1 is negative, the optimization process tends to make $|T|$ smaller. After performing the optimization by use of any standard "minimax" algorithm, for instance, FMINIMAX in *Optimization toolbox*, Matlab, (see Appendix), the peaking in $|T|$ was decreased by more than 25dB, see $|T_{opt}|$ in Fig. 3b. The phase and gain margins, as expected, are drastically increased, see L_{opt} in Fig. 3a.

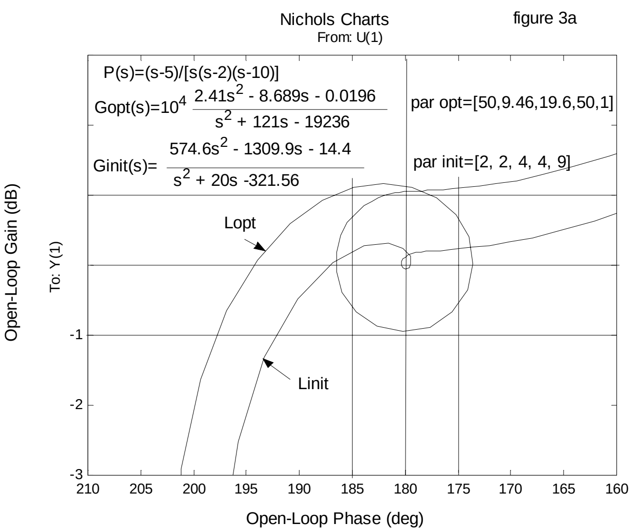


Figure 3a-L_{init} and L_{opt} on NC.

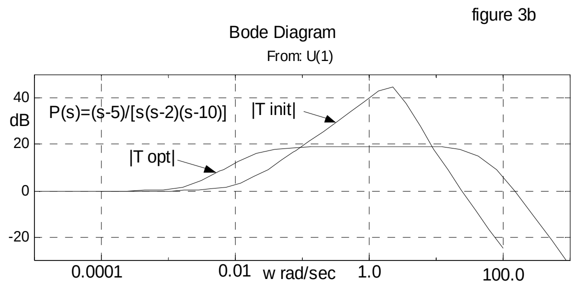


Figure 3b.-Bode plots of $|T_{init}|$ and of $|T_{opt}|$.

IV. Simulation Results And Discussions.

The presented optimization procedure was checked with numerous difficult examples, and the resulting solutions are very satisfying. The achieved stability margins were also compared with examples in the scientific literature, and they have always showed a better solution. The following Example 2 is presented in [9].

Example 2: The plant is $P(s) = (s\text{-}2)/(s\text{-}3)(s\text{+}1)$.

Solution:

The solution in [9] is performed by using the H_∞ paradigm. It is marked L_3 in Fig. 4a. The controller is $G_3(s) = \dfrac{7.85(s+1)(s+100)(s-0.1545)}{s(s+32)(s-20)}$.

The same problem was solved by hand calculation, based on the theory in [7], page 140. The resulting L_2 is also shown in Fig. 4a. The controller is $G_2(s) = \dfrac{62.1*10e6(s-0.1)(s+1)}{s(s-120)(s+500)(s+1000)}$. By using the optimization procedure presented in this paper, L_1 in Fig. 4a is obtained. The controller is

$$G_1(s) = \frac{13965(s+1)(s-0.003508)}{s(s+169.0988)(s-65.05)}.$$

It is clear from the shown $L(j\omega)$'s that L_1 is the best solution. It is worth remarking that this solution is very close to the theoretically predicted optimal L_2 in [7].

Several remarks are in order:

1- A feedback control system is expected to fulfill several control tasks, such as a good input –output behavior in the time domain, immunity to external disturbances, satisfactory relative stability margins, stability and sensitivity robustness in spite of plant parameter ignorance. These basic performances are to be achieved with the lowest possible sensor noise amplification. Such conflicting requirements are solved by trading off between them, which is a common procedure in control design. With plants that are minimum-phase and stable, the above require-

ments can be attained at will. With unstable-NMP plants, most of these basic requirements cannot be
achieved for *a priory* defined design specifications. For instance:

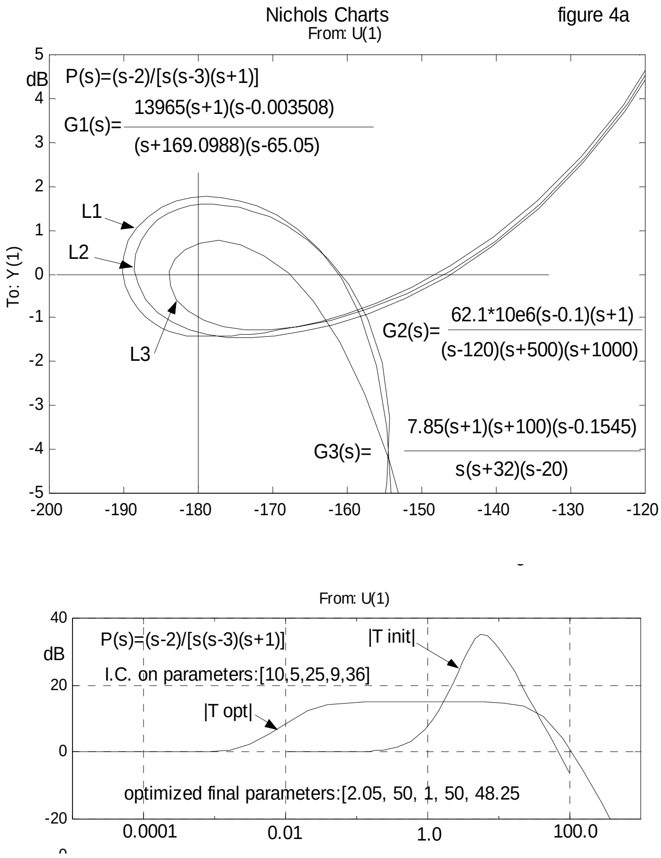


Figure 4. a -L's of 3 design methods; b- Bode plots of $|T_{\mathrm{init}}|$ and of $|T_{\mathrm{opt}}|$ (optimization design method).

2- A good input–output behavior in the time domain cannot be achieved because of the inherently expected excessive peaking in $|T(j\omega)|$.
3-Immunity to external disturbances cannot be achieved at will, because the crossover frequencies are determined by the location of the NMP zeros and unstable poles, thus limiting the frequency range where $|L(j\omega)|$ can be large.
4-Robustness to plant uncertainties that can be achieved is very restricted. Two gain margins and three phase margins are apparent in figure 1a. The plant ignorance that this kind of $L(j\omega)$ can tolerate is very limited.
The conclusion is that the best the control designer can do is to maximize the phase and gain margins.

V. The MIMO Case.

The MIMO case is not treated here in detail. However
a similar procedure as for the SISO case, using doubly coprime factorization and the Bezoult identity, shows also excellent results for MIMO systems. The relevant optimizing cost function was changed to:

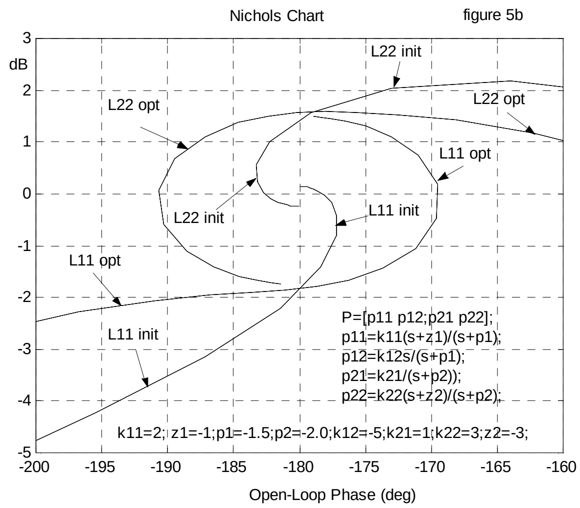
$$\frac{\min imize}{par} \max\left[\sqrt{\sum_{i=1}^{q}|T_{ii}(j\omega)|^2} - 1\right]$$

where $|T_{ii}|$'s are the unity- feedback gains of the direct channels, and q is the number of inputs to the plant. The results of a design example follow.

Example 3. The plant is given by:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$p_{11} = k_{11}(s+z_1)/(s+p_1)$; $p_{12} = k_{12}s/(s+p_1)$;
$p_{21} = k_{21}/(s+p_2)$; $p_{22} = k_{22}(s+z_2)/(s+p_2)$.
$k_{11} = 2$; $z_1 = -1$; $p_1 = -1.5$; $p_2 = -2.0$; $k_{12} = -5$; $k_{21} = 1$; $k_{22} = 3$; $z_2 = -3$.
The initial vector *par* is chosen as $par_{init} = [-1\ -1.5\ -0.5\ -1]^T$. The obtained initial open- loop TFs $L_{11\mathrm{init}}$ and $L_{22\ \mathrm{init}}$ are shown in figure 5, telescoped near the Nyquist stability point, 0dB and –180 deg on the Nichols chart. After performing the optimization by use of any minimax algorithm, for instance, MINIMAX, in *Optimization toolbox*, Matlab, the optimizing vector *par* changed to $par_{opt} = [-50\ -50\ -1.726\ -1.7188]^T$. The optimized $L_{11\mathrm{opt}}(j\omega)$ and $L_{22\ \mathrm{opt}}(j\omega)$ are shown on the same figure 5. It is evident that the initial solution, based on an arbitrary choice of vector *par*, was substantially improved. The two gain margins of $L_{11}(j\omega)$ increased from (0.15dB and 1.8dB) to (1.5dB and 1.8dB), and the phase margin increased from 2^0 to 11^0. The two gain margins of $L_{22}(j\omega)$ inc-



reased from (0.2dB and 1.4dB) to (1.6dB and 1.8dB), and the phase margin increased from 3^0 to 11^0.

Figure 5. Initial and optimised L_{11} and L_{22}.

.

VI. SUMMARY.

A procedure for stabilizing unstable- NMP plants, while achieving maximal gain and phase margins, has been presented in this paper. With such difficult

plants, the achievable stability margins are low, and no analytical procedures exist to predict them, except for plants comprising a very limited number of RHP poles and zeros. The design procedure presented in this paper achieves open-loop $L(j\omega)$*'s* with improved phase and gain margins, as compared to solutions ba-sed on arbitrary choice of controlling parameters, that satisfy the Bezout identity. The presented design tec-hnique in this paper can be applied to an unstable -NMP plant of any order by modifying accordingly the plant and controller structures.

APPENDIX

(Comments into m- files are printed in italic).Run with Matlab 5.3

Run "minimaxt" to obtain the controller *G*(*s*)=*ng*(*s*)/*dg*(*s*).

```
%------------------------------------
%listing of minimaxt.m to be used
%with calct.m revised 13 Jan. 2003
%par(1)=a1; par(2)=b1; par(3)=c1;
%par(4)=d1; par(5)=f1;%init. cond.
global np dp ng dg;%
par0 = [2   2  2 2 2 ];%I.C. on
%optimizing parameters a1 b1 c1 d1
%f1;
LB=[1 1 1 1 1];UB=[50 50 50 50 50];
%low and upper bounds on optimizing
%parameters
[par,fval,maxfval,exitflag,output,la
mbda]=fminimax('calct',par0,[],[],[]
,[],LB,UB)
a1=par(1);b1=par(2);c1=par(3);
d1=par(4); f1=par(5);
%-----------------------------------
%in this part:checking the resulting
%closed-loop roots
%np=k[1  z]; dp=[1  p1+p2+p3
%p1*p2+p1*p3+p2*p3  p1*p2*p3];
%t1 = p1+p2+p3;t2=p1*p2+p1*p3+p2*p3;
%t3= p1*p2*p3;
%np = k*[1 z]; dp=[1  t1   t2  t3];
%check of closed loop roots
nt1 = conv(np,ng);nt=[0 0 nt1];
dt = nt+conv(dp,dg);
closedloop_roots=roots(dt)
%===================================
%Listing of calct.m
%================================
%calct.m 29 December 2002, to be
%used with minimaxt.m, rev.13/1/03
function F=calct(par)
%P(s)=k(s+z)/(s+p1)(s+p2)(s+p3);
%enter plant parameters
k = 1;z = -5;p1= -10;p2= -2; p3 = 0;
%input data
global np dp ng dg;%
a1=par(1);b1=par(2);c1=par(3);
d1=par(4); f1=par(5);%optimizing
%parameters
t1= p1+p2+p3;t2=p1*p2+p1*p3+p2*p3;
t3= p1*p2*p3;

np=k*[1  z]; dp=[1  t1   t2  t3];
r1=a1+b1; r2=c1+a1*b1; r3=a1*c1;
mat=[ 1    0    0     0    0     0;
     t1    1    0     0    0     0;
     t2   t1    1     k    0     0;
     t3   t2   t1   k*z    k     0;
      0   t3   t2     0  k*z     k;
      0    0   t3     0    0   k*z];

% mat vout = vinp
vinp=[1 d1+r1  f1+r1*d1+r2
r1*f1+r2*d1+r3  r2*f1+r3*d1 r3*f1]';
vout=inv(mat)*vinp;g1=vout(1);
g2=vout(2); g3=vout(3);
q1=vout(4);q2=vout(5);q3=vout(6);
ng=[q1 q2 q3]; dg=[g1 g2 g3];
%calculation of |T|;
%T=ng*np/(ng*np+dg*dp);
nt1=conv(np,ng);nt=[0 0 nt1];
dt=nt + conv(dp,dg);roots(dt);
[mag ph w] = bode(nt,dt);[mag ph w];
F = mag - 1;
%==================================
```

REFERENCES

[1] D. C. Youla, J. J. Bongiorno, and H. A. Jabr, "Modern Wiener-Hopf Design of Optimal Cont-rollers. Part I: The ingle-Input- Output Case", IEEE Transactions on Automatic Control, Vol. AC-21,No. 1, pp.3-13, February, 1976.

[2] D. C. Youla, J. J. Bongiorno, and H. A. Jabr., "Modern Wiener -Hopf Design of Optimal Cont-rollers. Part II: The Multivariable Case", IEEE Transactions on Automatic Control, Vol. AC-21, No. 3, pp.319-338, June, 1976.

[3] J. Bongiorno and D. Youla, "On the design of Sin-gle-Loop Single-Input – Output Feedback Control Systems in the Complex - Frequency Domain", IEEE Transactions on Automatic Control, Vol. AC-22, pp.416-423, June, 1977.

[4] M. Vidyasagar, *Control System Synthesis- A Fac-torization Approach*, MIT Press Series, 1987.

[5] M. Sidi, "Gain-bandwidth limitations of feedback systems with non-minimum phase Plants", INT.J. Control, vol. 67, NO 5, pp. 731-743.,1997.

[6] M. Sidi, and O. Yaniv, " Margins and Bandwidth Limitations of NMP SISO Feedback Systems", Proceedings of the 7th Mediterranean Conference on Control and Automation (MED99), Haifa, Is-rael,674-688, June 28-23, 1999.

.

[7] M. Sidi, *Design of Robust Control Systems*, Krieger Publishing Company, 2001.
[8] J. Doyle, B. Francis, and A. Tannenbaum, *Feedback Control Theory*, Macmillan Publishing Company, 1992.
[9] K. Zhou with J. Doyle, *Essentials of Robust Control*, Prentice Hall, New Jersy, 1998.
[10] F. Fairman, *Linear Control Theory*, John Wiley & Sons, New York, 1998.
[11] S. Skogestad and I. Postletwaite, *Multivariable Feedback Control*, John Wiley & Sons, Chiochester, New York, 1996.