MED03 T7-125

Decentralized Control Networks of Multirate Sampled-data Systems

Roberto Ciferri, Sauro Longhi
Dipartimento di Elettronica ed Automatica
Università Politecnica delle Marche, via Brecce Bianche, 60131 Ancona, Italy.
Fax: +39 071 2204835,
Email: r.ciferri@ee.unian.it, sauro.longhi@univpm.it

Abstract— **In this note a set of preliminary results on the control of a large-scale continuous-time plant with decentralized multirate sampled-data control network system is proposed. Sufficient conditions for the solvability of the decentralized control network problem are given in terms of the continuous-time plant. The multirate sampling mechanism and the use of a local network enlarge the class of plants to be stabilized with decentralized controllers.**

Keywords— **Decentralized control, Sample-data systems, Stabilization.**

I. Introduction

THE stabilization problem of a large-scale plant with independent decentralized controllers has been deeply investigated [1]-[6]. Digital solutions such as multirate decentralized controllers have been studied in [7]-[9]. A multirate control scheme of a plant is characterized by digital time-invariant controllers operating on each channel with different sampling rates. A continuous-time plant with such a multirate sampling mechanism can be efficiently modelled by a periodic discrete-time system [10]. Therefore, the results on analysis and control of periodic discrete-time systems can be used for solving different multirate control problems.

The purpose of this note is to present a preliminary set of conditions for the existence of a stabilizing decentralized controller of a large-scale continuous plant. This problem has been analyzed in [7], for the case of single-rate sampling in each input-output plant channel, and in [8], for a discrete-time plant. A different approach has been proposed in [11] with data exchange among the output channels of the plant. A generalization of this solution has been here investigated, a continuous-time plant is considered with the input and the measured output of each plant channel updated and sampled respectively with the same time intervals, the measured outputs connected to a local area network for the data exchange among the output channels of the plant. In general, the input-output channels have sampling rates. The main tools for deducing the existence conditions are based on the algebraic approach developed for the class of periodic systems (see, e.g., [12], [13]) and here specified and adapted to the class of multirate sampled-data systems. A set of sufficient conditions for the solvability of the decentralized control network problem is proposed. This result shows that the multirate sampling mechanism and the use of a local network enlarge the class of plants to be stabilized with decentralized controllers.

This work was supported by Ministero dell'Istruzione dell'Università e della Ricerca

II. Preliminaries

Consider a linear time-invariant continuous-time plant Σ^c, characterized by σ input-output channels and described by

$$\begin{aligned} \dot{x}^c(t) &= A^c\, x^c(t) + \sum_{i=1}^{\sigma} B_i^c\, u_i^c(t) && (1)\\ y_j^c(t) &= C_j^c\, x^c(t), \qquad j = 1,\ldots,\sigma && (2) \end{aligned}$$

where $x^c(t) \in \mathbb{R}^n$ is the state, $u_i^c(t) \in \mathbb{R}^{p_i}, i = 1,\ldots,\sigma$, are the control inputs, $y_j^c(t) \in \mathbb{R}^{q_j}, j = 1,\ldots,\sigma$, are the measured outputs. The *stabilization problem* of Σ^c with a *decentralized continuous-time control system*, constituted by σ independent controllers with input $y_i^c(\cdot)$ and output $u_i^c(\cdot), i = 1,\ldots,\sigma$, has a solution if and only if Σ^c is stabilizable and detectable and all the $2^\sigma - 2$ complementary subsystems are *weakly complete*, i.e. Σ^c has no unstable fixed modes (see, e.g., [1], [3]).

Now, for solving the stabilization problem by a decentralized multirate control system consider a multirate control scheme of system Σ^c, where each input-output channel operates with its own sampling and hold rate, different from the other ones, i.e. the measured output $y_i^c(\cdot)$ of the channel i, with $i \in \{1,\ldots,\sigma\}$, is sampled with a period N_iT_c and the control input $u_i^c(\cdot)$ of the same channel is connected with a zeroth order circuit whose hold interval is N_iT_c, with $N_i \in \mathbb{Z}^+$ and $T_c \in \mathbb{R}$. Denote with ω the least common multiple of the integers $N_i,\ i = 1,\ldots,\sigma$. Without loss of generality, it is assumed that the greatest common divisor of the integers $N_i,\ i = 1,\ldots,\sigma$, is equal to 1 and all the samplers and hold circuits are synchronized at time $t = 0$.

The corresponding discrete-time state-space model Σ^d of the multirate sampled-data system is characterized by σ input-output channels and given by the series connection of ω-periodic systems $\widetilde{\Sigma}_i,\ i = 1,\ldots,\sigma$, which describe the mechanism of zeroth hold circuits, with $\widehat{\Sigma}$, which represents the sample-data system associated to Σ^c ([14], [15]). The ω-periodic system $\widetilde{\Sigma}_i$ of the channel i, with $i \in \{1,\ldots,\sigma\}$, has the following form:

$$\begin{aligned} \widetilde{x}_i((k+1)T_c) &= \overline{S}_i(k)\, \widetilde{x}_i(kT_c) + S_i(k)\, u_i(kT_c) && (3)\\ u_i^c(kT_c) &= \overline{S}_i(k)\, \widetilde{x}_i(kT_c) + S_i(k)\, u_i(kT_c) && (4) \end{aligned}$$

where $k \in \mathbb{Z}^+$, $\widetilde{x}_i(kT_c) \in \mathbb{R}^{p_i}$ is the state, $u_i(kT_c) \in \mathbb{R}^{p_i}$ is the input of channel i of Σ^d, $\overline{S}_i(k) := (I_{p_i} - S_i(k))$, I_{p_i} denotes the identity matrix of dimension p_i, and $S_i(\cdot)$ is an ω-periodic matrix given by:

$$\begin{aligned} S_i(k) &:= \operatorname{diag}\{\sigma_i(k)\}, & (5)\\ \sigma_i(k) &:= \begin{cases} 1, & k = jN_i, \\ 0, & k \neq jN_i, \end{cases} \quad j \in \mathbb{Z}^+. & (6) \end{aligned}$$

The ω-periodic system $\widehat{\Sigma}$, with σ input-output channels, has the following form:

$$\begin{aligned} x^c((k+1)T_c) &= e^{A^c T_c}\, x^c(kT_c) + \sum_{i=1}^{\sigma} B_i^d\, u_i^c(kT_c) & (7)\\ y_j(kT_c) &= T_j(k)\, C_j^c\, x^c(kT_c),\ j = 1, \ldots, \sigma & (8) \end{aligned}$$

where $k \in \mathbb{Z}^+$, $B_i^d := \int_0^{T_c} e^{A^c(T_c-\theta)} B_i^c d\theta$ and $T_j(\cdot)$ is an ω-periodic matrix given by:

$$\begin{aligned} T_j(k) &:= \operatorname{diag}\{\tau_j(k)\}, & (9)\\ \tau_j &:= \begin{cases} 1, & k = iN_j, \\ 0, & k \neq iN_j, \end{cases} \quad i \in \mathbb{Z}^+. & (10) \end{aligned}$$

Then, the ω-periodic discrete-time model Σ^d of the multirate sampled-data system is given by:

$$\begin{aligned} x((k+1)T_c) &= A(k)\, x(kT_c) + \sum_{i=1}^{\sigma} B_i(k)\, u_i(kT_c) & (11)\\ y_j(kT_c) &= C_j(k)\, x(kT_c),\ j = 1, \ldots, \sigma & (12) \end{aligned}$$

where $k \in \mathbb{Z}^+$,

$$x(kT_c) := [\widetilde{x}_1(kT_c)'\, \widetilde{x}_2(kT_c)' \, \ldots \, \widetilde{x}_\sigma(kT_c)'\, x^c(kT_c)']' \in \mathbb{R}^{\tilde{n}},$$

with $\tilde{n} := n + \sum_{i=1}^{\sigma} p_i$, is the state, and the ω-periodic matrices $A(\cdot)$, $B(\cdot)$ and $C(\cdot)$ have the following form:

$$A(k) = \begin{bmatrix} \overline{S}_1(k) & 0 & \cdots & 0 & 0 \\ 0 & \overline{S}_2(k) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \overline{S}_\sigma(k) & 0 \\ B_1^d \overline{S}_1(k) & B_2^d \overline{S}_2(k) & \cdots & B_\sigma^d \overline{S}_\sigma(k) & e^{A^c T_c} \end{bmatrix}$$

$$B_i(k) = \begin{bmatrix} 0 & \cdots & 0 & S_i(k)' & 0 & \cdots & 0 & (B_i^d S_i(k))' \end{bmatrix}'$$

$$C_j(k) = [\ 0 \quad 0 \quad \cdots \quad 0 \quad T_j(k) C_j^c\ ].$$

III. Problem statement

Given the plant Σ^c and the set of sampling and hold circuits corresponding to the σ channels, consider a control network scheme characterized by linear discrete-time local controllers $\mathcal{C}_i$, for $i = 1, \ldots, \sigma$, making use not only of the measured output $y_i(\cdot)$ of the local channel i, but also of the other outputs connected by a local network, as shown in Fig.1. The idea is that each controller takes information from all the σ channels, in order to avoid the possible lack of the structural properties required for the decentralized solution to the stabilization problem of Σ^d.

The different sampling periods of the σ channels and the time delay of the data transmission on the network

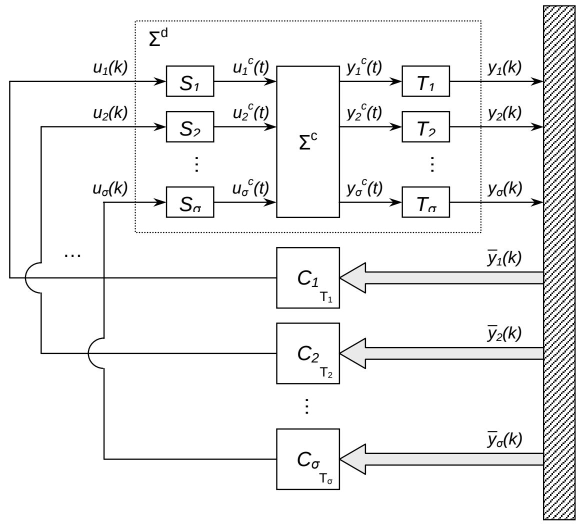

Fig. 1. Decentralized control network architecture.

make impossible, for each controller $\mathcal{C}_i$, with $i \in \{1, \ldots, \sigma\}$, the real-time acquisition of the measured outputs $y_j(\cdot)$, for $j = 1, \ldots, \sigma$ and $j \neq i$. Controller $\mathcal{C}_i$, at sampling time t_i, acquires the last sample $y_j(t_j)$ of output channel j, with $t_j < t_i$, $j = 1, \ldots, \sigma$ and $j \neq i$. The delay on the acquisition is modelled keeping memory of the samples of each channel for a time delay $d := max_{i,j=1,\ldots,\sigma,\, k=1,\ldots,\omega-1}(d_{ij}(k))$, where $d_{ij}(k)$ is the ω-periodic time shift, at time k, between the sampling time at channel i and the sampling time at channel j, for $i, j = 1, \ldots, \sigma$. The extended ω-periodic system $\overline{\Sigma}$, whose representation let model this mechanism, is given by

$$\begin{aligned} \bar{x}((k+1)T) &= \bar{A}(k)\, \bar{x}(kT) + \sum_{i=1}^{\sigma} \bar{B}_i(k)\, u_i(kT) & (13)\\ \bar{y}_j(kT) &= \bar{C}_j(k)\, \bar{x}(kT), \quad j = 1, \ldots, \sigma & (14) \end{aligned}$$

with the extended state and outputs

$$\bar{x}(kT_c) := \begin{bmatrix} x(kT_c) \\ x^c((k-1)T_c) \\ \vdots \\ x^c((k-d)T_c) \end{bmatrix},\ \bar{y}_j(kT_c) := \begin{bmatrix} y_j(kT_c) \\ y_1(l_1^k T_c) \\ \vdots \\ y_{j-1}(l_{j-1}^k T_c) \\ y_{j+1}(l_{j+1}^k T_c) \\ \vdots \\ y_\sigma(l_\sigma^k T_c) \end{bmatrix}$$

$$l_i^k := \begin{cases} k - N_i, & k = hN_i, \\ N_i[k/N_i], & k \neq hN_i, k = hN_j, \end{cases}$$
$$h \in \mathbb{Z}^+, \quad i, j = 1, \ldots, \sigma, \quad i \neq j \qquad (15)$$

where $[\cdot]$ is the integer part function, $\bar{x}(kT_c) \in \mathbb{R}^{\bar{n}}$, with $\bar{n} := (d+1)n + \sum_{i=1}^{\sigma} p_i$, $\bar{y}_j(kT_c) \in \mathbb{R}^{\bar{q}}$, with $\bar{q} := \sum_{j=1}^{\sigma} q_j$, and

$\bar{A}(k) :=$

$$\begin{bmatrix} \overline{S}_1(k) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \overline{S}_2(k) & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \overline{S}_\sigma(k) & 0 & 0 & \cdots & 0 & 0 \\ B_1^d\overline{S}_1(k) & B_2^d\overline{S}_2(k) & \cdots & B_\sigma^d\overline{S}_\sigma(k) & e^{A^c T_c} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & I_n & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & I_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & I_n & 0 \end{bmatrix}$$

$$\bar{B}_i(k) := \begin{bmatrix} 0 & \cdots & 0 & S_j(k)' & 0 & \cdots & 0 & (B_i^d S_i(k))' & 0 & \cdots & 0 \end{bmatrix}'$$

$$\bar{C}_j(k) := \begin{bmatrix} 0 & \cdots & 0 & T_i(k)C_j^c & 0 \\ 0 & \cdots & 0 & 0 & \widetilde{C}_1(k) \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \widetilde{C}_{j-1}(k) \\ 0 & \cdots & 0 & 0 & \widetilde{C}_{j+1}(k) \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \widetilde{C}_\sigma(k) \end{bmatrix}$$

with

$$\begin{aligned} \widetilde{C}_h(k) &:= \begin{bmatrix} \widetilde{T}_1(l_h^k)C_h^c & \widetilde{T}_2(l_h^k)C_h^c & \cdots & \widetilde{T}_d(l_h^k)C_h^c \end{bmatrix} \\ \widetilde{T}_m(l_h^k) &:= \begin{cases} 1, & l_h^k = k-m, \\ 0, & l_h^k \neq k-m, \end{cases} \\ & \qquad h = 1,\ldots,\sigma, \quad m = 1,\ldots,d. \end{aligned} \tag{16}$$

Making use of the notations above introduced, the considered control problem is here stated.

Decentralized Multirate Control Network Problem (DMCNP) *The stabilization problem of Σ^c by a decentralized multirate control network system consists of finding for each input-output channel a linear discrete-time periodic local controller $\mathcal{C}_i$ with period ω/N_i and of the form*

$$\widehat{x}_i((h+1)N_iT_c) = F^i(h)\widehat{x}_i(hN_iT_c) + G^i(h)\bar{y}_i(hN_iT_c) \tag{17}$$

$$u_i(hN_iT_c) = H^i(h)\widehat{x}_i(hN_iT_c) \tag{18}$$

such that the ω-periodic closed-loop system given by Σ^d and independent controllers $\mathcal{C}_i$, for $i = 1,\ldots,\sigma$, is asymptotically stable.

In order to analyze the solvability conditions of the introduced problem, the time-invariant representation of the extended multirate sampled-data system $\overline{\Sigma}$ is here recalled. The state transition matrix of $\overline{\Sigma}$ is expressed by $\bar{\Phi}(k,k_0) := \bar{A}(k-1)\bar{A}(k-2)\ldots\bar{A}(k_0)$ with $k > k_0$, $k, k_0 \in \mathbb{Z}^+$, and $\bar{\Phi}(k,k) := I_n$ for all $k \in \mathbb{Z}^+$. For any initial time $k_0 \in \mathbb{Z}^+$, the output response of the ω-periodic system $\overline{\Sigma}$ for $k \geq k_0$, to given initial state $\bar{x}(k_0)$ and control functions $u_i(\cdot)$, can be expressed throughout the time-invariant associated system of $\overline{\Sigma}$ at time k_0, denoted by $\overline{\Sigma}^{k_0}$ [16]. This time-invariant state-space representation of $\overline{\Sigma}$ is similar to the lifted representation of an input-output periodic operator considered in [17] and [18]. For an arbitrary time k, system $\overline{\Sigma}^k$ is represented by

$$\bar{x}^k(h+1) = \bar{E}^k\,\bar{x}^k(h) + \sum_{i=1}^{\sigma} \bar{J}_i^k\,u_i^k(h) \tag{19}$$

$$\bar{y}_j^k(h) = \bar{L}_j^k\bar{x}^k(h) + \sum_{i=1}^{\sigma}\bar{M}_{ji}^k u_i^k(h), \quad j = 1,\ldots,\sigma \tag{20}$$

where

$$\begin{aligned} \bar{E}^k &:= \bar{\Phi}(k+\omega,k), \\ \bar{J}_i^k &:= [\bar{\Delta}_i^k(0) \ \ \ldots \ \ \bar{\Delta}_i^k(\omega-1)], \\ \bar{\Delta}_i^k(\ell) &:= \bar{\Phi}(k+\omega,k+\ell+1)\bar{B}_i(k+\ell), \\ \bar{L}_j^k &:= [\bar{\Gamma}_j^k(0)' \ \ \ldots \ \ \bar{\Gamma}_j^k(\omega-1)']', \\ \bar{\Gamma}_j^k(\ell) &:= \bar{C}_j(k+\ell)\bar{\Phi}(k+\ell,k), \\ \bar{M}_{ji}^k &:= \begin{bmatrix} \bar{\Theta}_{ji}^k(0,0) & \cdots & \bar{\Theta}_{ji}^k(0,\omega-1) \\ \vdots & \ddots & \vdots \\ \bar{\Theta}_{ji}^k(\omega-1,0) & \cdots & \bar{\Theta}_{ji}^k(\omega-1,\omega-1) \end{bmatrix}, \\ \bar{\Theta}_{ji}^k(\ell,r) &:= \begin{cases} 0, & \ell \leq r, \\ \bar{C}_j(k+\ell)\bar{\Phi}(k+\ell,k+r+1)\bar{B}_i(k+r), & \ell > r, \end{cases} \\ & \quad \ell, r = 0,1,\ldots,\omega-1, \ i,j = 1,\ldots,\sigma. \end{aligned}$$

It is easy to see that, if $\bar{x}^k(0) = \bar{x}(k)$ and $u_i^k(h) = [\,u_i(k+h\omega)' \ldots u_i(k+\omega-1+h\omega)'\,]'$, $i = 1,\ldots,\sigma$, for all $h \in \mathbb{Z}^+$, where $\mathbb{Z}^+$ denotes the set of non-negative integers, then $\bar{x}^k(h) = \bar{x}(k+h\omega)$ and $\bar{y}_j^k(h) = [\,\bar{y}_j(k+h\omega)' \ldots \bar{y}_j(k+\omega-1+h\omega)'\,]'$, $j = 1,\ldots,\sigma$, for all $h \in \mathbb{Z}^+$. Thus, $\bar{y}_j^k(\cdot), j = 1,\ldots,\sigma$, gives $\bar{y}_j(\cdot)$ in a lifted form over each period, provided that $u_i^k(\cdot), i = 1,\ldots,\sigma$, coincides with the lifted form of $u_i(\cdot)$ over each period. Moreover, the characteristic polynomial of $\bar{E}^k$ is independent of k, and, by the periodicity of $\overline{\Sigma}$, it characterizes the stability of $\overline{\Sigma}$ [19]. For this reason the eigenvalues of $\bar{E}^k$ are called the eigenvalues of $\overline{\Sigma}$.

IV. Main results

The solvability conditions for the decentralized problem of periodic systems have been stated in [6]. These results can be used for introducing the solvability conditions of DMCNP in terms of periodic representation of the multirate sampled-data system $\overline{\Sigma}$. In order to introduce such conditions, the following notations are needed.

A complementary subsystem $\overline{\Sigma}_{\mathcal{I}\mathcal{J}}$ of $\overline{\Sigma}$, associated to the sets $\mathcal{I} := \{i_1,\ldots,i_\mu\}$ and $\mathcal{J} := \{j_1,\ldots,j_\nu\}$, with $\mathcal{I} \cap \mathcal{J} = \emptyset$ and $\mathcal{I} \cup \mathcal{J} = \{1,\ldots,\sigma\}$, has the form

$$\bar{x}((k+1)T_c) = \bar{A}(k)\bar{x}(kT_c) + \bar{B}_{\mathcal{I}}(k)u_{\mathcal{I}}(kT_c) \tag{21}$$

$$\bar{y}_{\mathcal{J}}(kT_c) = \bar{C}_{\mathcal{J}}(k)\bar{x}(kT_c) \tag{22}$$

where $\bar{B}_{\mathcal{I}}(k) := [\bar{B}_{i_1}(k)\,\bar{B}_{i_2}(k)\,\ldots\,\bar{B}_{i_\mu}(k)]$ and $\bar{C}_{\mathcal{J}}(k) := [\bar{C}_{j_1}(k)'\,\bar{C}_{j_2}(k)'\,\ldots\,\bar{C}_{j_\nu}(k)']'$.

The time-invariant representation $\overline{\Sigma}_{\mathcal{I}\mathcal{J}}^k$ of $\overline{\Sigma}_{\mathcal{I}\mathcal{J}}$ has the same structure of $\overline{\Sigma}^k$ with matrices $\bar{J}_{\mathcal{I}}^k, \bar{L}_{\mathcal{J}}^k$ and $\bar{M}_{\mathcal{J}\mathcal{I}}^k$ defined as $\bar{J}^k, \bar{L}^k$ and $\bar{M}^k$, with $\bar{B}(k)$ and $\bar{C}(k)$ substituted with $\bar{B}_{\mathcal{I}}(k)$ and $\bar{C}_{\mathcal{J}}(k)$ respectively.

Lemma 3.1. [6] *The DMCNP admits a solution if and only if:*

(i) *system $\overline{\Sigma}$ is stabilizable and detectable, i.e. for an arbitrary $k \in \mathbb{Z}^+$ and for all z outside the open unitary disk,*

$$rank\,[\,\bar{E}^k - zI_{\bar{n}} \quad \bar{J}^k\,] = \bar{n}, \tag{23}$$

$$rank \begin{bmatrix} \bar{E}^k - zI_{\bar{n}} \\ \bar{L}^k \end{bmatrix} = \bar{n}; \tag{24}$$

(ii) *the* $2^\sigma - 2$ *complementary subsystems* $\bar{\Sigma}_{\mathcal{I}\mathcal{J}}$ *are weakly complete, i.e., for all* $\mathcal{I}$ *and* $\mathcal{J}$*, for an arbitrary* $k \in \mathbb{Z}^+$ *and for all* z *outside the open unitary disk,*

$$rank \begin{bmatrix} \bar{E}^k - zI_{\bar{n}} & \bar{J}^k_{\mathcal{I}} \\ \bar{L}^k_{\mathcal{J}} & \bar{M}^k_{\mathcal{J}\mathcal{I}} \end{bmatrix} \geq \bar{n}. \qquad (25)$$

Denoting with $B^c_{\mathcal{I}} := [B^c_{i_1}\ B^c_{i_2}\ \dots\ B^c_{i_\mu}]$ and $C^c := [{C^c_1}'\ {C^c_2}'\ \dots\ {C^c_n}']'$, the solvability condition can be stated in terms of the given continuous time-invariant system Σ^c if the sampling rates are chosen appropriate to system Σ^c.

Theorem 3.1. *Given a continuous-time plant* Σ^c *which is stabilizable and detectable, the DMCNP has a solution if:*

(i) *every pair* $(\lambda^c_a, \lambda^c_b)$ *of distinct eigenvalues of* A^c*, with* $\mathrm{Re}[\lambda^c_a] = \mathrm{Re}[\lambda^c_b] \geq 0$*, has* $\mathrm{Im}[\lambda^c_a - \lambda^c_b] \neq \pm 2h\pi/\omega T_c$*, for all* $h \in \mathbb{Z}^+$*;*

(ii) *the* $2^\sigma - 2$ *conditions*

$$rank \begin{bmatrix} A^c - \lambda I & B^c_{\mathcal{I}} \\ C^c & 0 \end{bmatrix} \geq n \qquad (26)$$

are verified for all $\mathcal{I}$ *and for each unstable eigenvalue* λ *of* A^c*.*

The proof of this theorem has been performed making use of some results on the analysis of linear periodic discrete-time systems [13], Jordan form of matrix A^c and elementary operations on matrices of conditions (23), (24) and (25).

The conditions of Theorem 3.1 are not related to the multirate mechanism but only to the least common multiple of the sampling and hold intervals. The condition (i) of Theorem 3.1 preserves the stabilizability and determinability of system $\overline{\Sigma}$ and the fulfillment of condition (ii) of Lemma 3.1 if condition (ii) of Theorem 3.1 is verified.

The design of the controllers $\mathcal{C}_i$, for $i = 1, \dots, \sigma$, is performed in three steps.

(Step 1) Making use of classical algorithms (see, e.g., [1], [2], [4], [5], [20]), compute time-invariant decentralized controllers $\overline{\mathcal{C}}^0_i$ for the stabilization of the time-invariant system $\overline{\Sigma}^0$ (the time-invariant representation of $\overline{\Sigma}$ at time $t = 0$).

(Step 2) Making use of the algorithm proposed in [21], compute an ω-periodic realization $\overline{\mathcal{C}}_i$ associated to $\overline{\mathcal{C}}^0_i$.

(Step 3) Compute the discrete-time system $\mathcal{C}_i$ with sampling rate $N_i T_c$ corresponding the ω-periodic system $\overline{\mathcal{C}}_i$.

V. Numerical example

Consider a linear time-invariant continuous-time plant Σ^c, characterized by $\sigma = 2$ input-output channels and described by

$$\begin{aligned} \dot{x}^c(t) &= A^c\, x^c(t) + B^c_1\, u^c_1(t) + B^c_2\, u^c_2(t) \\ y^c_1(t) &= C^c_1\, x^c(t) \\ y^c_2(t) &= C^c_2\, x^c(t) \end{aligned}$$

where

$$A^c = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},\ B^c_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ B^c_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$C^c_1 = [\,0 \quad 1 \quad 0\,], \quad C^c_2 = [\,0 \quad 0 \quad 1\,]$$

and characterized by different sampling and updating periods, $T_1 = 2\,sec$ and $T_2 = 3\,sec$.

For this plant the conditions for the existence of a simple decentralized multirate control system designed without information exchange between the two output channels are not verified, missing a solution for the decentralized control problem.

On the contrary, the conditions of Theorem 3.1 for the existence of a solution to the DMCNP are verified and a decentralized control multirate system can be implemented. In this case, it is possible to design two independent singlerate controllers $\mathcal{C}_1$ and $\mathcal{C}_2$, which guarantee the asymptotical stability of the closed-loop system.

VI. Conclusions

The problem of stabilizing a large-scale continuous-time plant characterized by different sampling and updating intervals for each input-output channel of the plant has been here analyzed.

A preliminary result for the development of a decentralized digital control network system is introduced. The possibilities for stabilizing a large-scale continuous plant by a decentralized digital control system are improved throughout the output data exchange by using local networks. Each local controller of the digital control scheme can make use of the local sampled-data output measures and of someone or of all output measures of the other channels to avoid the lack of structural properties.

This work introduces a set of preliminary results useful for a future research on the analysis of the performances of a decentralized control network in terms of robustness and/or parameters uncertainties. Moreover, an interesting problem to be analyzed is the problem related to the probable asynchronism of data sampling and of data updating among the various channels of a real digital decentralized control network scheme, due to physical and/or technological constraints.

References

[1] S.H. Wang and E.J. Davison, *On the Stabilization of Decentralized Control Systems*, IEEE Trans. Autom. Contr., vol. AC-18, pp. 473-478, 1973.

[2] J.P. Corfmat and A.S. Morse, *Decentralized Control of Linear Multivariable Systems*, Automatica, vol. 12, pp. 479-495, 1976.

[3] B.D.O. Anderson and D.J. Clements, *Algebraic characterization of fixed modes in decentralize control*, Automatica, vol. 17, pp. 703-712, 1981.

[4] A.B. Özgüler, *Decentralized Control: A Stable Proper Fractional Approach*, IEEE Trans. Autom. Contr., vol. 35, pp. 1109-1117, 1990.

[5] E.J. Davidson and T.N. Chang, *Decentralized stabilization and pole assignment for general proper systems*, IEEE Trans. Autom. Contr., vol. 35, pp. 652-664, 1990.

[6] P.P. Khargonekar and A.B. Özgüler, *Decentralized Control and Periodic Feedback*, IEEE Trans. Autom. Contr., vol. 39, pp. 877-882, 1994.

[7] M.E. Sezer and D.D. Siljak, *Decentralized multirate control*, IEEE Trans. Autom. Contr., vol. 35, pp. 60-65, 1990.

[8] R. Scattolini and N. Schiavoni, *Decentralized Control of Multirate systems Subject to Exogenous Signals*, IEEE Trans. Autom. Contr., vol. 41, pp. 1540-1544, 1996.

[9] H. Ito, *Worst-case performance and stability of multirate sampled-data systems with nonsynchronous decentralized controllers*, Proc. American Contr. Conf., vol. 1, pp. 778-783, 1997.

[10] S. Longhi, *Structural Properties of Multirate Sampled-Data Systems*, IEEE Trans. Autom. Contr., vol. 39, pp. 692-696, 1994.

[11] H. Ishii and B.A. Francis, *Stabilization with control networks*, Automatica, vol. 38, pp. 1745-1751, 2002.

[12] O.M. Grasselli and S. Longhi, *Zeros and poles of linear periodic multivariable discrete-time systems*, Circ. Syst. Sign. Processing, 7, pp. 361-380, 1988.

[13] O.M. Grasselli and S. Longhi, *The finite zero structure of linear periodic discrete-time systems*, Int. Journal of Syst. Science, vol. 22, pp. 1785-1806, 1991.

[14] M.C. Berg, N. Amit and J.D. Powell, *Multirate digital control system design*, IEEE Trans. Autom. Contr., vol. AC-33, pp. 1139-1150, 1988.

[15] P. Colaneri and G. De Nicolao, *Optimal stochastic control of multirate sampled-data systems*, Proc. European Contr. Conf., Grenoble, France, pp. 2519-2523, 1991.

[16] R.A. Meyer and C.S. Burrus, *A unified analysis of multirate and periodically time-varying digital filters*, IEEE Trans. Circ. Syst., vol. CAS-22, pp. 162-168, 1975.

[17] M.A. Dahleh, P.G. Voulgaris and L.S. Valavani, *Optimal and robust controllers for periodic and multirate systems*, IEEE Trans. Autom. Contr., vol. AC-37, pp 90-99, 1992.

[18] P.P. Khargonekar, K. Poolla and A. Tannenbaum, *Robust control of linear time-invariant plants using periodic compensation*, IEEE Trans. Autom. Contr., vol. AC-30, pp. 1088-1096, 1985.

[19] D.S. Evans, *Finite-dimensional realization of discrete-time weighting patterns*, SIAM Appl. Math., vol. 22, pp. 45-67, 1972.

[20] F.M.Jr. Brasch and J.B. Pearson, *Pole placement using dynamic compensator*, IEEE Trans. Autom. Contr., vol. AC-15(1), pp. 34-43, 1970.

[21] P. Colaneri and S. Longhi, *The realization problem for linear periodic systems*, Automatica, vol. 31, pp. 775-779, 1995.