Adaptive Control Law for Uncertain System with Static Nonlinearity

Alexey Bobtsov, Darina Romasheva

Abstract – **The task of synthesis of a control law for a system represented as linear stationary and static nonlinear parts is considered. It is supposed that parameters of the linear block are unknown, and its transfer function from an input to output is strictly minimum-phase. As about nonlinear part of the system we suppose, that it is known inaccurately, is not reduced to an input of the linear block and, generally, does not satisfy sector restrictions. The adaptive regulator ensuring an asymptotic stability with regard to variables of linear part is synthesized. The output of a control system, but not its derivatives is used as a measured variable.**

Index terms – **Adaptive control; nonlinear stabilization; complex systems; nonlinear control; output control**

I. Introduction

Within the scope of the control theory special interest is paid to the description of plants, represented as structures, including linear dynamic blocks and nonlinear static blocks [1]. The large number of works is devoted to analysis and synthesis of a control law for the case, when nonlinearity is reduced to an input of control of the system linear part (see papers [1], [2]). As about the linear part, for it the assumption of a strict real positiveness of its transfer function from an input of control to an output is given [1, 2]. For nonlinearity, as a rule, it is necessary, that it is an output function and belongs to some sector. Also it is necessary to mark results, in which the case of nonlinear system with a strictly minimum - phase uncertain linear part and nonlinearity of input of control [5] is considered.

Today it is considered, that the problem of analysis of nonlinear systems, from the mentioned above class, for the case of input nonlinearity is trivial [1, 3, 4]. Therefore directions of researches now mainly concern the tasks of analysis and synthesis of control for the systems, in which nonlinearity is not reduced to input (see Fig. 1). To the last publications devoted to the given themes it is necessary to refer works by Kokotovic group and, in particular, papers [3, 4]. In [3, 4] the conditions of existence of a static feedback which ensures an asymptotic stability of the system are offered. The disadvantages of methods considered in [3, 4] are the following: synthesis of a control law on measurements of state variables, and also parametrical determinacy of a linear part.

Developing results, represented in [3-4], in the given work we suppose, that only an output variable of a system is measured, parameters of the linear block are unknown, and its transfer function from an input to output is strictly minimum - phase. Let's consider also, that nonlinear part of the system is known inaccurately, is not reduced to an input of the linear block and does not satisfy sector restrictions. The purpose of control is the solution of the task of synthesis of adaptive regulator, ensuring asymptotic stability with regard to state variables of the linear system part.

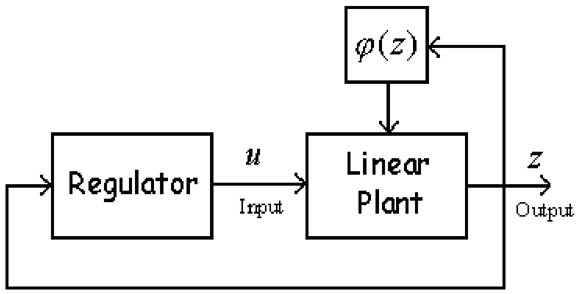

Fig. 1. The closed-loop nonlinear system.

II. Statement of the problem

Let's consider a nonlinear system of the form [3, 4]

$$\dot{x} = Ax + G\varphi(z) + Bu\,, \tag{1}$$

$$z = Hx\,, \tag{2}$$

where $x \in R^n$ is a vector of state variables; $u \in R$ is a control; $z \in R$ is an output; transfer function from an input u to an output z - has a relative degree $\rho = 1$ and its numerator is Hurwitz; A, B, G, H - unknown matrices; the pair (A, B) is assumed to be controllable; the pair (A, H) is assumed to be observable; $\varphi(z)$ - unknown nonlinearity.

Let $\varphi(z)$ satisfy the following assumptions:

1) $\varphi(0) = 0$;

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2) $|z| \to \infty \;\Rightarrow\; |\varphi(z)| \to \infty$ (see work [2]);

3) function $\psi(z)$ is known and is such that

$$\psi(z)z^2 \geq [\varphi(z)]^2 \quad \text{for any} \quad z\,; \tag{3}$$

4) there exists a number $C_0 \geq 0$, such that

$$\lim_{z\to 0}\frac{|\varphi(z)|}{|z|} \leq C_0 < \infty\,. \tag{4}$$

The purpose of control consists in calculation of the function $u(z,t)$ from model (1), ensuring an asymptotic stability of equilibrium position $x=0$ of the system (1), for assumptions of nonlinearity $\varphi(z)$.

III. Synthesis of Control Law

Let's choose a control law of the form (see Figure 2):

$$u = -\hat{k}z - \hat{\gamma}\,\psi(z)z\,, \tag{5}$$

$$\dot{\hat{k}} = \mu_1 z^2\,, \tag{6}$$

$$\dot{\hat{\gamma}} = \mu_2\psi(z)z^2\,, \tag{7}$$

where $\hat{k}$ and $\hat{\gamma}$ are feedback coefficients being tuned, $\mu_1 > 0$ and $\mu_2 > 0$ are constant parameters.

Using known results by Fradkov in passification of linear systems [5], it is possible to show, that for linear part of the system (1) there exists a positive constant $k_0 > 0$ such, that for all $k > k_0$ it is possible to specify a symmetrical positive definite matrix P satisfying two matrix inequalities

$$(A-kBH)^T P + P(A-kBH) = -Q \leq -\lambda P\,, \tag{8}$$

$$PB = H\,, \tag{9}$$

where $\lambda > 0$ is a positive number.

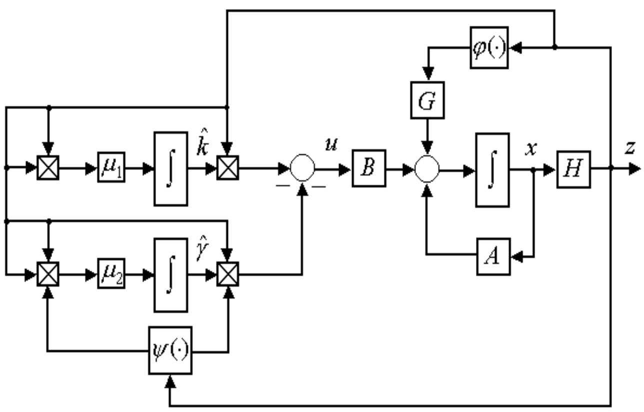

Fig. 2. The closed-loop nonlinear system (1), (2), (5)-(7).

Let's consider parametrical errors of the form

$$\tilde{k} = k - \hat{k}\,, \tag{10}$$

$$\tilde{\gamma} = \gamma - \hat{\gamma}\,, \tag{11}$$

where coefficient k ensures realization of the property (8), (9), and constant parameter $\gamma > 0$ satisfies the inequality

$$-\lambda P + \gamma^{-1} PGG^T P \leq -\bar{\lambda} P\;, \tag{12}$$

where $\bar{\lambda} > 0$ is a positive number.

Then it is possible to transform the model of closed system (1), (5), (6), (7):

$$\dot{x} = A_c x + G\varphi(z) - B\gamma\psi(z)z + B(\tilde{k}z + \tilde{\gamma}\psi(z)z)\,, \tag{13}$$

$$\dot{\tilde{k}} = -\mu_1 z^2\,, \tag{14}$$

$$\dot{\tilde{\gamma}} = -\mu_2\psi(z)z^2\,, \tag{15}$$

where matrix $A_c = A - kBH$.

Theorem. The control law of the form (5) - (7) ensures for the system (1), (5), (6), (7) boundedness of all its trajectories, and also asymptotic stability of equilibrium position $x = 0$.

Proof. Let's consider Lyapunov function of the following form:

$$V = x^T Px + \frac{1}{\mu_1}\tilde{k}^2 + \frac{1}{\mu_2}\tilde{\gamma}^2\,. \tag{16}$$

Differentiating (16) and using the equations (1), (5), (6), (7), (13), (14) and (15)

$$\dot{V} = x^T(A_c^T P + PA_c)x + 2x^T PG\varphi(z) - 2x^T PB\gamma\psi(z)z + $$
$$+ 2x^T PB(\tilde{k}z + \tilde{\gamma}\psi(z)z) - 2(\tilde{k}z^2 + \tilde{\gamma}\psi(z)z^2)\,. \tag{17}$$

By force of expressions (8), (9) for (17) we obtain

$$\dot{V} \leq -\lambda x^T Px + 2x^T PG\varphi(z) - 2z\gamma\psi(z)z + $$
$$+ 2z(\tilde{k}z + \tilde{\gamma}\psi(z)z) - 2(\tilde{k}z^2 + \tilde{\gamma}\psi(z)z^2) \leq $$
$$\leq -\lambda x^T Px + 2x^T PG\varphi(z) - 2z\gamma\psi(z)z\,. \tag{18}$$

Let's introduce the following variables

$$a = x^T PG \text{ and } b = \varphi(z)$$

Then, using the following inequality

$$2ab \leq \gamma^{-1}a^2 + \gamma b^2\,, \tag{19}$$

for (18) we obtain

$$\dot{V} \leq -\lambda\, x^T Px + \gamma^{-1} x^T PGG^T Px + \gamma[\varphi(z)]^2 - 2\gamma\psi(z)z^2\,. \tag{20}$$

As for the number γ the inequality (12) is carried out, and also taking into consideration condition (3), for expression (20) we obtain

$$\dot{V} \le -\bar{\lambda} x^T P x \le 0 . \tag{21}$$

From (21) it follows:

1) stability of equilibrium position $x = 0$;
2) boundedness of all trajectories of the system (1), (5), (6), (7);
3) boundedness of the right part of equation (13);
4) realization of a condition

$$\lim_{t\to\infty} \int_0^t (x^T P x) d\tau \le C_1 < \infty . \tag{22}$$

From boundedness of the right part of equation (13) and relation (22), by force of known Barbalat lemma, follows an asymptotic stability of equilibrium position $x = 0$.

IV. Example

Let's consider the following nonlinear system:

$$\dot{x}_1 = x_2 + \varphi(z) , \tag{23}$$
$$\dot{x}_2 = 10 x_2 + u , \tag{24}$$
$$z = x_1 + x_2 , \tag{25}$$

where $\varphi(z) = z^3$.

Choose the control according to equations (5)-(7)

$$u = -\hat{k} z - \hat{\gamma}\, \psi(z) z , \tag{26}$$
$$\dot{\hat{k}} = \frac{1}{2} z^2 , \tag{27}$$
$$\dot{\hat{\gamma}} = 2 \psi(z) z^2 . \tag{28}$$

Choose the function $\psi(z) = z^3$ and simulate control system (23)-(28). The results of computer simulation for variables x_1, x_2, $\hat{k}$ and $\hat{\gamma}$ are presented in the fig. 3, 4, 5 and 6. As can be seen from the figures, presented control provides boundedness of all variables of the system (23) – (28) and asymptotic stability of equilibrium position $x_1 = 0$ and $x_2 = 0$.

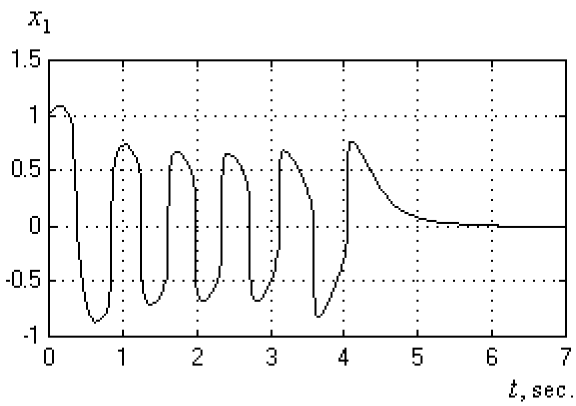

Fig. 3. Transients in control system (23) - (28) for variable x_1.

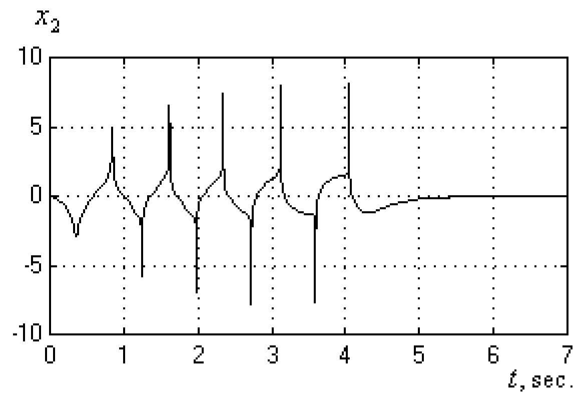

Fig. 4. Transients in control system (23) - (28) for variable x_2.

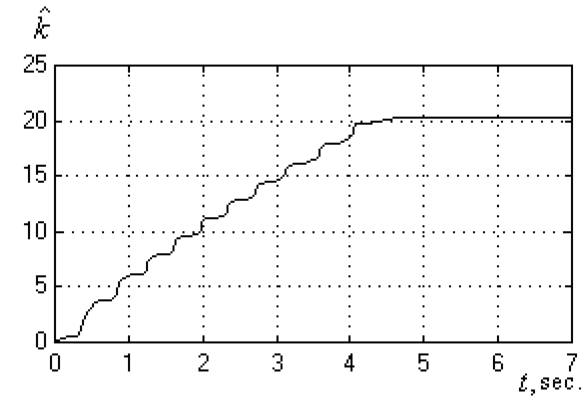

Fig.5. Transients in control system (23) - (28) for variable $\hat{k}$.

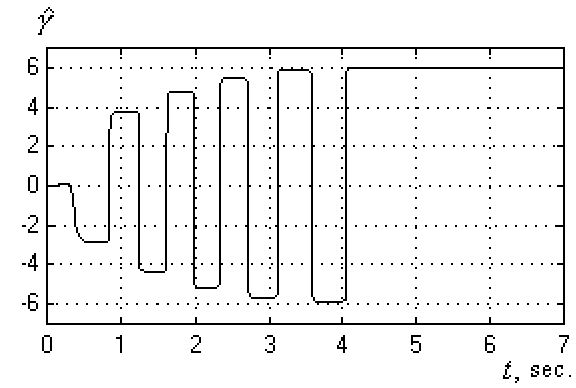

Fig. 6. Transients in control system (23) - (28) for variable $\hat{\gamma}$.

V. CONCLUSION

Developing methods represented in [3-5], in the paper the task of synthesis of adaptive regulator for stabilization of nonlinear system with static nonlinearities is considered. Assuming, that linear part of the system is strictly minimum - phase and static nonlinearity is not reduced to an input of control, the adaptive regulator of the form (5) - (7), ensuring achievement of asymptotic stability with regard to state variables of the linear block is developed. Thus the indicated task was solved for the case of unknown parameters of the linear block and indeterminacy of a nonlinear part. As a measured variable only the output variable, but not its derivatives was used.

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