Stochastic Model of Irradiation for the Analysis of Solar System

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Abstract--In this paper a short overview of a method for the stochastic analysis of solar system is given. A brief overview of a stochastic process is presented. On the basis of the insolation measurement for the Island of Lastovo for the period of 1981 to 1997, the correlation function of quasi-ergodic period is derived. The density spectrum function is also shown. The approximation of correlation and spectrum function is given. According to relation between the density spectrum function of input and output signals, the parameters of colored filter are determined. The comparison between measured correlation function and its approximation is given. Also, the comparison of spectrum functions is made. One possible result of simulation is presented. At the end, the comparison between simulated and measured correlation functions is also presented.

Index Terms--Stochastic model, insolation, correlation function, spectral density function, colored filter

I. Introduction

Since both the development and the application of solar energy system have increased in the last decade, it was necessary to develop the methods for design and analysis of such a system. Taking in the consideration the (un)reliability of the Sun's irradiation as a source of energy, it is important to have a good and reliable estimation of capacity and also reliability of a technical system for transformation of solar energy. There are many methods and algorithms for analyzing a solar system. The simplest one is based on the monthly average value of irradiation on a horizontal surface. The results of such analysis are unreliable because they do not represent possible deviation from average values. On the other side, the model is practical when quick and simple estimation of the capacity of proposed solar system on the considered location is needed. Using a stochastic model allows us to get an insight of possible variation of irradiation. This model prefers long series of measurment of irradiation at considered location. Liu and Jordan have defined the distribution function of daily clearness index K_t. The function does not depend of a location. It depends only on long-term average value of clearness index [1]. Similar. Tovar *et. al.* in [2] give a distribution function based on one-minute values of a clearness index. In this case, the distribution function is influenced by an air mass in the considered moment of the day. But in many cases one-minute values of clearness index are not available. Morf [3] suggests two-state irradiance model based on the recurrent homogeneous Markov process. Perić *et. al.* [4] have created a stochastic model which includes all variability of climate at considered location through an empirical distribution matrix. The matrix has dimension of 12x100. Each column represents empirically created distribution function for corresponding months. Components in specified column are the values of the empirical distribution function in uniformly distributed knots. The interval of the clearness index is divided into 100 gaps. A knot is the value of the clearness index in the middle of a gap. The stochastic model described here is based on colored filter of second order in Laplace's domain. The input to filter is band limited white noise, which can be simulated in most mathematic programming languages. The output of such a simulation is a time-sequenced value of *the daily relative sunshine duration*. Using the Page-Angstroem correlation [1], the clearness index can be easily derived from the relative sunshine duration. It is important to mention that generated daily value of clearness index can be used as hourly value because of similarity of both distribution function $F_{Kt}(x) \cong F_{kt}(x)$ as it mentioned in [1].

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II. The Methodology of Analisys of Measured Data

For statistic analysis the long series of sunshine duration records have to be accessible. The recording of irradiation in the Republic of Croatia has quite long tradition. But mostly, these are indirect measurements such as the sunshine duration measurement. Based on this data, the daily clearness index can be estimated. For the purpose of this work, the daily sunshine duration measurements for the Island of Lastovo for the period of 1981 to 1997 are used. More reliable results could be obtained if the values of clearness index were available. But, in many cases this approximation satisfies.

A. Irradiation as a stochastic process

If the relations between the planet Earth and Sun are known, the extraterrestrial irradiation can be easily determined. However, there are many unknown (or less known) processes in the atmosphere with high influence on the level of the global irradiation. Some of them are air and cloud motion variation of the air pressure and content of vapor etc. Because of the influence of these naturally stochastic processes, the irradiation can be also presented as stochastic process. The global irradiation[1] can be written as:

$$H = H_0 \cdot K_t \, , \quad (1)$$

where H_0 is extraterrestrial irradiation. This is well-defined time function. The parameters of this function depend only on geographic parameters of a location. K_T is the clearness index and that is also time function but more stochastic than deterministic. Liu-Jordan's correlation [1] links the clearness index with sunshine duration *S:*

$$K_T = a + bK_S , \; K_S = \frac{S}{S_0} , \quad (2)$$

where S_0 is length of day (time from sunrise to sunset), *a* and *b* are the coefficients which are dependent of a location and part of year.[2] Since it is impossible to find deterministic mathematical model, which covers all influences of a variation of clearness index, that variable will be treated as a stochastic one. Instead of the clearness index K_T, the relative sunshine duration K_S is considered.

III. The Theoretical Basis

A. The stochastic processes

If the realization of some process at the considered moment has random (stochastic) nature, this process is called a stochastic process [4]. Every realization of stochastic processes can be described by the aid of the cumulative distribution function and the density of distribution. The (cumulative) distribution function is defined as follows:

$$F_X(x,t) = P(X(t) \le x) \, . \quad (3)$$

The density of distribution function is the derivation of distribution. This is the probability that the value of the random variable in moment *t X(t)* falls into the interval *[x, x+dx]. The expected value* is an average value of the random variable over the infinite set of realizations.

$$E[X(t)] = \overline{X}(t) = \int_{-\infty}^{\infty} X(t) \frac{dF_X}{dx} dx \quad (4)$$

The deviation of the value of random variable from its average is described by the *dispersion*.

$$D(t) = \int_{-\infty}^{\infty} \left(X(t) - \overline{X}(t) \right)^2 \frac{dF_X}{dx} dx \quad (5)$$

[1] The amount of Jouls of solar energy fallen on the considered surface during considered time

[2] The monthly values of parameters *a* and *b* for the Island of Lastovo are available.

However, the functions of expected value and dispersion give us insufficient information about interdependency between random variable in different moments. This characteristic is described by the correlation function $R_X(t_1.t_2)$.

$$R_X(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(t_1)X(t_2) \frac{\partial F_X(x_1,t_1,x_2,t_2,))}{\partial x_1 \partial x_2} dx_1 dx_2 \, , \quad (6)$$

where the value of the function $F_X(x_1,t_1,x_2,t_2)$ is the probability $P(X(t_1) \le x_1, X(t_2) \le x_2)$. The stochastic processes where the expected value and dispersion are time independent functions and the correlation depends only on difference $\tau = t_1 - t_2$ are called stationary stochastic processes.

B. The ergodic stochastic processes

The ergodic processes are those stationary stochastic processes where all statistical characteristics normally obtained by averaging over the infinite set of realizations are equal to those obtained by averaging over the time [6] (See Fig. 1.) In this case, the expressions for the expected value, dispersion and correlation are:

$$E[X(t)] = \int_{-\infty}^{\infty} X(t) \frac{dF_X}{dx} dx = \overline{X} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} X(t) dt \, , \quad (6)$$

$$D_X = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left(X(t) - \overline{X} \right)^2 dt \quad (7)$$

$$R_X(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} X(t) X(t+\tau) dt \quad (8)$$

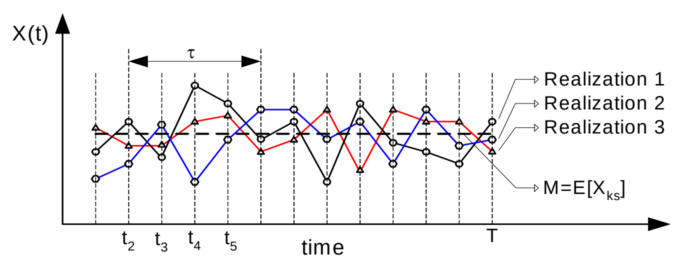

Fig. 1. An example of the ergodic process

C. Power density spectrum

Very often in practice many characteristic processes can be analyzed in frequency domain. Power density spectrum of stochastic process is defined as the Fourier transformation of the correlation function *$R_X(\tau)$.*

$$S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j\omega\tau} d\tau \quad (9)$$

Taking in the consideration that $R_X(\tau)$ is even function it follows:

$$S_X(\omega) = 2\int_0^{\infty} R_X(\tau)\cos(\omega\tau)d\tau \qquad (10)$$

IV. STOCHASTIC MODEL OF RELATIVE SUNSHINE DURATION

A colored filter of the second order realizes the model shown here. The filter is described by its transfer function in the Laplace's domain, where a white noise signal is the input. The output in time domain from such a filter is a time-sequenced value of the relative sunshine duration. In this case, an equality of density spectrum of the filter output and the density spectrum of considered stochastic process has to be accomplished. Let $F(j\omega)$ be transfer function of filter, $X(t)$ be the input signal and $Y(t)$ be the output signal. According to [6] it follows:

$$S_X(\omega) = |F(j\omega)|^2 S_Y(\omega)\,, \qquad (11)$$

where $S_X(\omega)$ and $S_Y(\omega)$ are density spectrums of input and output signals. If the input signal is white noise, which density spectrum is unity, the formula for design of colored filter is following.

$$S_{K_S}(\omega) = |F_{K_S}(j\omega)|^2\,, \qquad (12)$$

where $S_{Ks}(\omega)$ is density spectrum of the stochastic process of relative sunshine duration.

A. *The density spectrum of relative sunshine duration*

For the model presented in this work the daily sunshine duration measurements for the Island of Lastovo for the period of 1981 to 1997 are analyzed. Every single measurement of relative sunshine duration can be treated as a single realization of stochastic variable for the considered day in the year. Also every set of measurements of relative sunshine duration in one year can be treated as a realization of a stochastic process. The next step of analysis is to group the recorded data into sets with constant or as small as possible gradient of the expected value function. Ordinary non-stationary stochastic process over a year is divided into series of quasi-ergodic processes. In this case, a considered quasi-ergodic period is March. Several sets of measurements of the considered variable, the average value function and its linear approximation are shown in Fig. 2. The figure shows that the average function tends to be constant value, **0.5346.** The average function is obtained by averaging over the sets of 17 measurements for each day in March. On the other hand, the average value of TMY (the Typical Meteorological Year) is **0.523.** TMY is one characteristic realization in sets of realizations with the average value nearest to the overall average. In this case TMY is 1993. So the assumption of ergodic process for the period of March is valid. According to the definition of ergodic process the analysis of input data is independent of its average value. On the other side the computer simulation of white noise considers zero average value. So it is more practical to deal with centralized stochastic variable of relative sunshine duration:

$$K_{sc} = K_s - \overline{K}_s \qquad (13)$$

Hence, the design of colored filter is carried on the data of relative sunshine duration for March in the year 1993 (Table 1). By the statistical analysis of input data in MatLAB™ the correlation function (Fig. 3) and the density spectrum (Fig. 4) are obtained.

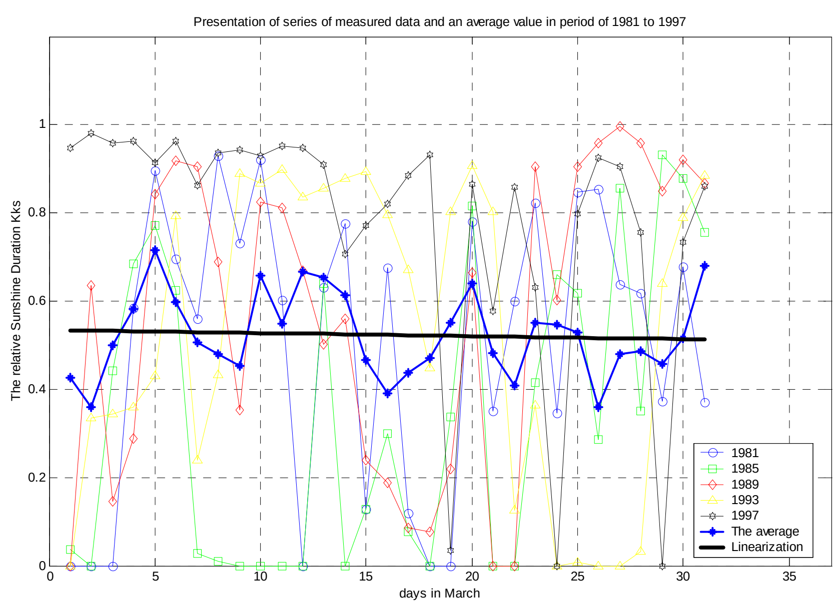

Fig. 2. Presentation of several sets of measurement, the average value function and its linear approximation

Table 1. Recorded data of relative sunshine duration for March in the year of 1993.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
K_S	0.00	0.34	0.34	0.36	0.43	0.79	0.24	0.43	0.89	0.87	0.90	0.83	0.86	0.88	0.89
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0.79	0.67	0.45	0.80	0.91	0.80	0.13	0.37	0.00	0.01	0.00	0.00	0.03	0.64	0.79	0.88

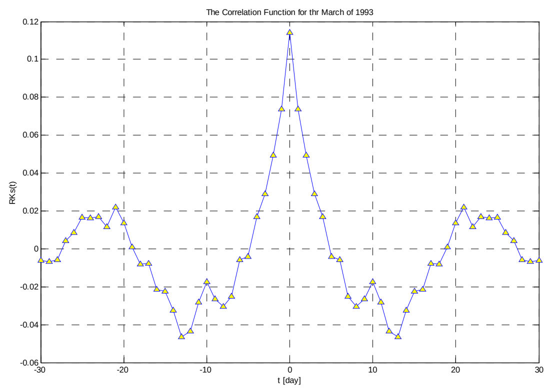

Fig. 3 The correlation function

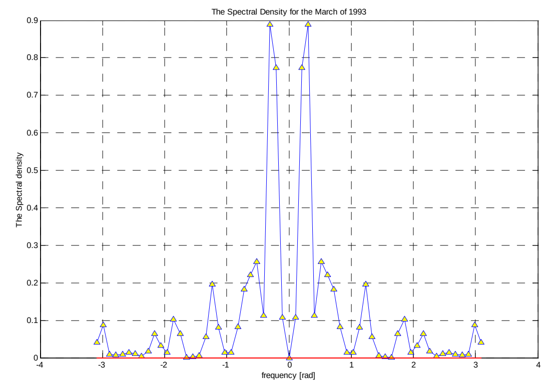

Fig. 4. The power density spectrum

The density spectrum function is calculated by equation (9) or (10). However, taking into consideration that the sets of considered data are finite and time is discrete, the density spectrum can be found as it follows:

$$S_{Ks}(\omega)=\sqrt{\left[\sum_{t=-(N-1)}^{(N-1)} R_{Ks}(t)\cos(\omega t)\right]^2+\left[\sum_{t=-(N-1)}^{(N-1)} R_{Ks}(t)\sin(\omega t)\right]^2} \tag{14}$$

where is:

N	number of elements in TMY realization.
$\boldsymbol{R_{Ks}}$	correlation function.
t	time [days]
$\boldsymbol{\omega=k\Delta\omega}$	frequency
$\boldsymbol{\Delta\omega=2\pi/N}$	

k=-(N-1),...-1,0,1,..,N-1.

The correlation function in Fig. 3 can be approximated with the following equation, [7]:

$$R(\tau)=De^{-\alpha|\tau|}\cos(\omega_0\tau) \tag{15}$$

The coefficient D in eq. (15) has a meaning of the dispersion of the considered set of data. In this case:

$$D=\sum_{i=1}^{N} {K_{SC}}^2=0{,}11403 \tag{16}$$

The other coefficients, α and ω_0 can be determined by the method of least squares. For the considered example, these coefficients are: **α=0.092, ω_0=0.25133.** The comparison between the correlation function and its approximation eq. (15) is shown in Fig. 5.

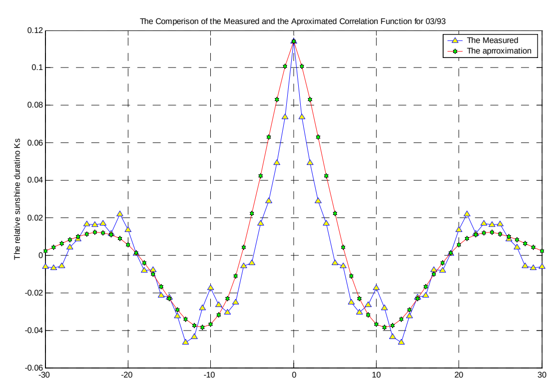

Fig. 5. The comparison of the correlation function and its approximation

There are two ways for determination of mathematical expression for the density spectrum function. The first way is the well-known method of polynomial approximation, directly from the values of the density spectrum, (Fig. 4). The second one is more practical approach by applying (15) into (9) or (10). Thus, the following is obtained:

$$S(\omega) = D\frac{2\alpha\left(\omega_0{}^2+\omega^2+\alpha^2\right)}{\omega^4+2(\alpha^2-\omega_0{}^2)\omega^2+(\omega_0{}^2+\alpha^2)^2} \tag{17}$$

An important characteristic of density spectrum function is expressed by the following equation:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)d\omega = D \tag{18}$$

The result obtained by numerical integration of density spectrum approximation over the interval of [-π, π] is 0.11109. The error according to (16) is 2.5 %.

B. Design of colored filter of the second order

The goal of design is to determine a filter, where density spectrum of the response in the case when white noise is input corresponds to the density spectrum function of process, which realization is simulated by the colored filter. Taking into consideration, the polynomial orders of numerator and denominator of density spectrum approximation in (17), the mathematical expression of filter is:

$$F(j\omega) = K\frac{A+(j\omega)}{(j\omega)^2+B(j\omega)+C} \tag{19}$$

According to the criteria in (11) and taking into account (17) and (19), finally we obtain:

$$K = \sqrt{2D\alpha} = 0.1448$$

$$A = \sqrt{\omega_0{}^2+\alpha^2} = 0.7596$$

$$B = 2\alpha = 0.184$$

$$C = \omega_0{}^2+\alpha^2 = 0.577$$

V. Results and Verification of Results

The simulation of the stochastic process of the relative sunshine duration can be realized in most mathematical programming languages, which are capable to generate white noise signals. For this instance a series of 31 values of relative sunshine duration is simulated using MatLAB™. The block scheme of the simulation is shown in Fig. 6., and the results are presented in Fig. 7. The correlation function and density spectrum function of such obtained data series are shown and compared to measured functions in Fig. 8 and Fig.9.

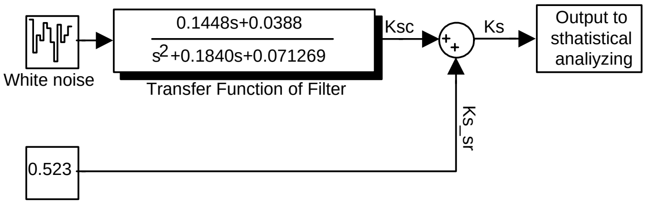

Fig. 6. Block scheme of simulation for March

Some simulated realizations exceed the expected interval [0, 1], Fig. 7. That is consequence of dissipation of simulated value in the area of expected interval. For practical use, only the output from filter, which falls into interval [0, 1], has to be considered.

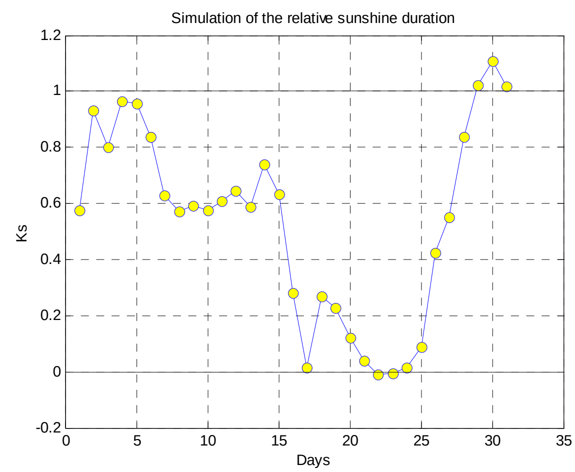

Fig. 7. The result of simulation

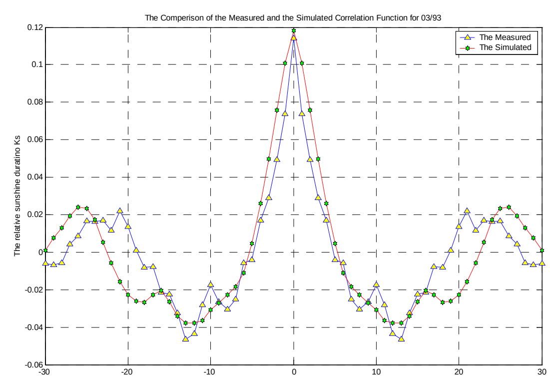

Fig. 8.The comparison of the correlation functions with its simulation

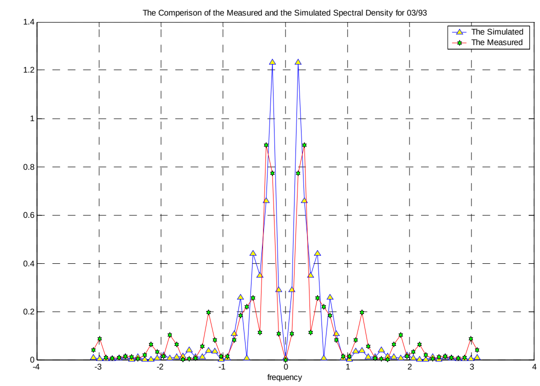

Fig. 9.The comparison of the density spectrum function with its simulation

The resembling procedure can be carried out for the other periods in a year. For example; the comparison of the correlation function with its simulation for November is shown in Fig. 10. Thereby it should be emphasized that the TMY for November is the year of 1997. It can be shown that the quasi-ergodic process in this part of year is shorter than one month. But for the purpose of demonstration, the parameters of filter are given in Table 2.

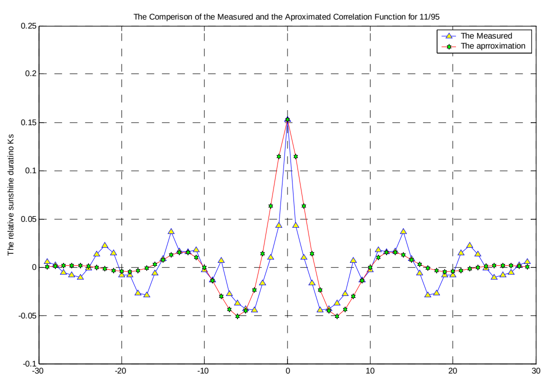

Fig. 10. The comparison of the correlation functions with its simulation for November

Table 2. The parameters of colored filter for November

K	A	B	C
0,2322	0,503	0,352	0,253

VI. CONCLUSION

The comparison between the correlation function and density spectrum of measured and simulated data in Figs. 8 & 9 justifies the described procedure. In other words, the colored filter of second order can simulate the stochastic nature of irradiation. The obtained result is useful, because it provides simpler statistic analysis of the work of a solar system. In this way, it is possible to estimate important statistic indicators of considered technical system such as: reliability and availability. By using the Page-Angstroem equation, the clearness index can be derived. Other parameters of irradiation (global irradiation, beam and diffusion part of radiation) can be also estimated. Taking into account the similarity between the distribution function of daily and hourly clearness index, the usage of this filter can also provide the simulation of the hourly irradiations. The described model can be improved in two ways. The first way concerns the measurement technique; greater measurement density, much longer series, direct measurement of all components of irradiations instead of indirect (the clearness index). The second one concerns determination of quasi-ergodic portion of year. This procedure enables more effective design of a solar systems.

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