# Multi Criteria Decision Making Using Dynamics of criteria

S. Khanmohammadi, *R. Almeida ribeiro, J. Jassbi

I.A.U., Science & Research campus, Iran, * Unversidade Lusiada, Portugal

***Abstract*— In traditional decision making problems the weights of criteria and values of alternatives are considered to be fixed. Also in the fuzzy decision making procedure, the values are fixed (fuzzy values with fixed membership functions). In this paper a new methodology is used for decision making where instead of fixed (crisp or fuzzy) weightings for criteria their dynamical behaviors versus the independent variable such as time or cost, are considered to calculate the decision values of different alternatives. The introduced method is used for decision making in an electrical power dispatching system as a case study.**

***Key words*— MCDM, dynamical weights, decision matrix, auxiliary decision table**

## I. Introduction

Decision making based on scientific methodologies is the main goal of organizational managers and system experts. In a traditional Multi Criteria Decision Making (MCDM), each criterion is weighted by a fixed value and the decision maker uses these values to calculate a decision value for each alternative and prioritizes the alternatives based on the calculated decision values, normally in descending order[1]-[3]. Even in the application of MCDM for dynamic systems, the weights of criteria are considered to be static [4]. Also in the fuzzy decision making procedure the values are fixed (fuzzy values with fixed membership functions) [5],[6]. In this paper a new tautology called dynamic multi criteria decision making (DMCDM) is introduced where the dynamics of weightings of criteria versus a main factor (independent variable) such as time, cost … is considered for decision making calculations. In section 2 a general dynamic model of MCDM is reviewed. In Section 3 a new decision matrix is developed on the basis of the main decision matrix were three new criteria are generated and used for decision-making. The introduced criteria are calculated based on the behaviors of the main criteria. Section 4 deals with the application of the introduced methodology to a decision making problem in an electrical power dispatching system in the case of power shortages. This methodology is useful for online decision making in dynamic systems such as job shop scheduling, material handling, electrical power dispatching as well as management of robot end effectors in hybrid systems.

## II. A MCDM with dynamic weights of criteria

Table I represents a common MCDM model as a decision matrix. Where $A_i$, i=1…n represents the ith alternative for decision making, $C_j$, j=1…m is the weight of the jth criterion and $a_{ij}$ is the decision value of the ith alternative corresponding to the jth criterion.

TABLE I
A COMMON DECISION MATRIX

| | $C_1$ | $C_2$ | … | $C_n$ |
|---|---|---|---|---|
| $A_1$ | $a_{11}$ | $a_{12}$ | | $a_{1n}$ |
| $A_2$ | $a_{21}$ | $a_{22}$ | | $a_{2n}$ |
| … | | | | |
| Am | $a_{m1}$ | $a_{m2}$ | | $a_{mn}$ |

In this model the decision value of the ith alternative is calculated by:

$$dA_i = \sum_{j=1}^{n} C_j a_{ij}\,, \quad i=1,2,\ldots m \tag{1}$$

where $dA_i$, $C_j$ and $a_{ij}$ are crisp and fixed values. In the dynamical model of MCDM introduced in this paper, the dynamics of each criterion $C_j(x)$, as a function of a main factor, $x$, is considered for decision making. In this way the calculated decision value $dA_i(x)$ is a function of the main factor $x$ (time, cost...).

$$dA_i(x) = \sum_{j=1}^{n} C_j(x) a_{ij}, \quad i=1,2,...m \qquad (2)$$

Hence decision functions, instead of decision values, must be compared for decision making.

## III. New criteria for comparison of decision functions

The following three new criteria are introduced for comparing decision functions:

1) Integral of decision function (IDF)
2) Maximum value of decision function (MDF)
3) Distance of maximum point and expected maximum point (DMP)

Each of these criteria are studied in the following subsections:

### A. Integral of decision function

The integral of decision function $dA_i(x)$, representing the total utility of alternative $A_i$, in the rang $[x_{min},x_{max}]$ (defined by decision maker), is obtained by

$$IDF_i = \int_{x_{min}}^{x_{max}} dA_i(x)dx, \quad i=1,2,...m \qquad (3)$$

A typical decision function and its integral is shown in Fig. 1.

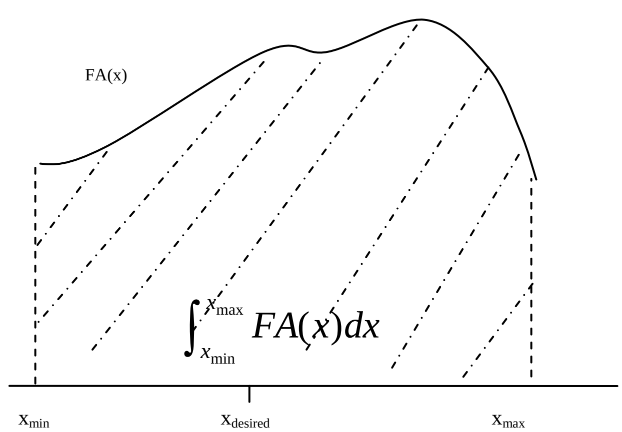


Fig. 1. A typical decision function and its integral in the range $[x_{min}, x_{max}]$

### B. Maximum value of decision function

Maximum value of the decision function is potentially a reasonable criteria for decision making between different alternatives. It can be obtained by differentiation the decision function for each alternative

$$\left.\frac{d[dA_i(x)]}{dx}\right|_{XMDF_i} = 0 \Rightarrow MDF_i = dA_i(XMDF_i) \qquad (4)$$

where $XMDF_i$ is the maximum point and $MDF_i$ is the maximum value of decision function for the ith alternative.

Figure 2 shows the maximum point and the maximum values of a typical decision function $dA_i$.

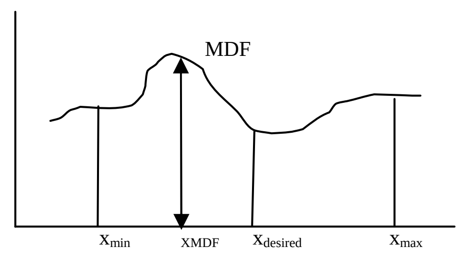


Fig. 2. Maximum value and maximum point of a typical decision function.

### C. Distance of maximum point and expected maximum point (DMP)

When the decision value is function of a main criterion, a preferred decision point may be considered by the decision maker. For example in a commercial problem, the producer will prefer that the maximum number of clients will reach 4 months after introducing the new product. In this case the distance of maximum point from the preferred maximum point

$$DMP_i = \left| X_{desired} - XMDF_i \right| \qquad (5)$$

as shown in Fig. 2, may be considered as a decision making criterion. These three criteria are then gathered in an auxiliary new decision table for decision making calculations.

### D. Auxiliary decision table

In this table, the values of *IDF*, *MDF* and *DMP* are considered as the decision criteria and the obtained values for each alternative are normalized and filled in the table as shown in table II.

TABLE II
AUXILIARY TABLE

| | *IDF* | *MDF* | *DMP* |
|---|---|---|---|
| $A_1$ | $IDF_1$ | $MDF_1$ | $DMP_1$ |
| … | … | … | … |
| $A_I$ | $IDF_I$ | $MDF_I$ | $DMP_I$ |
| … | … | … | … |
| $A_M$ | $IDF_M$ | $MDF_M$ | $DMP_M$ |

Now the decision value $dA_i$ can be calculated using the following formula:

$$dA_i = IDF_i + MDF_i + DMP_i \ , \quad i = 1,2,\ldots,m \qquad (6)$$

## IV. CASE STUDY

A part of the decision matrix of current research on an electrical power dispatching system is presented in table III. This is a part of a general decision matrix with four alternatives and five criteria for an electrical power dispatching system in the case of shortage of electrical power. In this study four regions of city are considered as different alternatives for prioritizing. The time period of 24 hours is divided to four periods of times with appropriately similar conditions (0-8, 8-14, 14-18, 18-24). The models of predictions of dynamics of weights of different criteria:

$c_1$: Residential area
$c_2$: Shopping centers
$c_3$: Clubs and recreation centers
$c_4$: Educational centers
$c_5$: Medical urgent care centers

based on historical data are presented in Fig. 3 for the period 18-24. It is noted that the periods of times and the dynamics of criteria may differ in different applications. The elements of decision matrix represent the normalized quantitative values of criteria in different regions.

TABLE III
DECISION MATRIX OF ELECTRICAL DISPATCHING SYSTEM FOR TIME PERIOD 18-24

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $A_1$ | 0.5 | 0.7 | 0.3 | 0.1 | 0.3 |
| $A_2$ | 0.1 | 0.3 | 1.0 | 0.7 | 0.1 |
| $A_3$ | 1.0 | 0.8 | 0.4 | 1.0 | 0.1 |
| $A_4$ | 0.3 | 0.3 | 0.2 | 0.6 | 1.0 |

In Fig. 3, x and $C_i(x)$ represent the independent variable (time), and the dependent variable (weight of criteria) respectively.

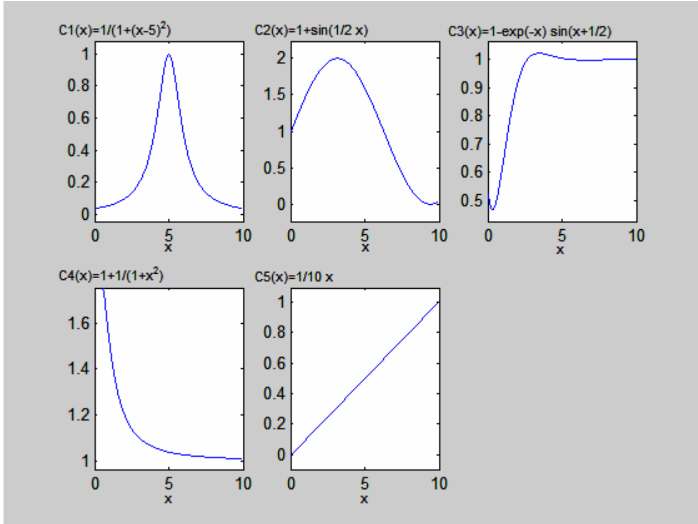


Fig. 3. Dynamic weights of criteria

Figure 4 shows the decision values $dA_i(x)$ for different alternatives as the functions of independent variable x using Eq. (2).

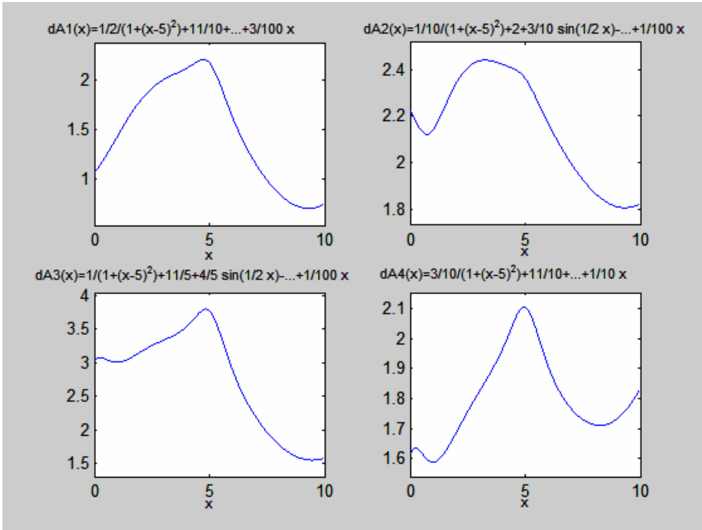


Fig. 4. Decision values dAi(x) for different alternatives

The auxiliary table for decision making can be obtained as tableIV based on decision values $dA_i(x)$ of different alternative of Fig. 4.

TABLE IV
NORMALIZED ($\in [1,10]$) AUXILIARY TABLE OF TABLE III BASED ON DYNAMICS OF FIG. 4

| New criteria→<br>Alternatives ↓ | IDF | MDF | DMP | Final decision values |
|---|---|---|---|---|
| $A_1$ | 1.5367 | 2.1053 | 1.0000 | 4.6419 |
| $A_2$ | 2.7832 | 10.000 | 5.7797 | 18.5629 |
| $A_3$ | 10.000 | 1.5789 | 10.000 | 21.5789 |
| $A_4$ | 1.0000 | 1.0000 | 3.3062 | 5.3062 |

according to the final decision values, the priority list of alternatives is :$A_3$, $A_2$, $A_4$, $A_1$ for period 18-24.

## V. CONCLUSION

A new tautology for application of dynamics of weights of criteria is introduced.

Three new criteria called, integral of decision function (IDF), maximum value of decision function (MDF) and distance of maximum point and expected maximum point (DMP) are defined and calculated based on the dynamics of weights of criteria to generate and auxiliary decision table for ranking different alternatives. Because of dynamical characteristic of decision procedure, the introduced methodology can be applied to real time decision making in hybrid systems such as flexible manufacturing systems, transportation, robotics management as well as intelligent evacuation systems in disasters.

## REFRENCES

[1] E. Ballestero, c. Romero, *Multiple criteria decision making and its applications to economic problems*. Boston, Dordrecht. London: Kluwer Academic publisher, 1998.

[2] E.U.CHOO, B. Schoner, W. C. Wedley, ”Interpretation of criteria weights in multi criteria decision making,” Computer & Industrial Engineering, Vol. 37(3), pp. 527-541, November 1999.

[3] J. Sarkis, “A comparative analysis of DEA as a discrete alternative multiple criteria decision tool,” European Journal of Operational Research, Vol. 123(3), pp.543-557, June 2000.

[4] P. J. Agrell, J. Wikner, “An MCDM framework for dynamic systems,” Int. J. Production Economics, Vol. 45(1-3) pp.279-292, August-1996.

[5] S. Khanmohammadi, I. Hassanzadeh, R. M. Mathur, K. V. Patil, “A new fuzzy decision making procedure applied to emergency electric power distribution scheduling,” Engineering Applications of Artificial Intelligence, Vol. 13(6), pp. 731-740 , December2000.

[6] E. Triantaphyllou, *Multi criteria decision making method: a comparative study*, Kluwer academic publishers, Applied optimization series, Vol. 44, 2002.