Nonlinear and Adaptive Control of Buck-Boost Power Converters

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***Abstract*~~ This paper focuses on the problem of controlling DC-to-DC switched power converters of Buck-Boost type. The system nonlinear feature is coped with by resorting to the backstepping control approach. Both adaptive and nonadaptive versions are designed and shown to yield quite interesting tracking and robustness performances. A comparison study shows that backstepping nonlinear controllers perform as well as passivity-based controllers. For both, the choice of design parameters proves to be crucial for achieving robustness with respect to load resistance variations. From this viewpoint, adaptive backstepping controllers are more interesting as they prove to be less sensitive to design parameters.**

***Index terms*~~Switched power converters, Nonlinear control**

I. INTRODUCTION

There are three main types of switched power converters respectively called Boost, Buck and Buck-Boost. These have recently aroused an increasing deal of interest both in power electronics and in automatic control. This is due to their wide applicability domain that ranges from domestic equipments to sophisticated communication systems. They are also used in computers, industrial electronics, battery-operating portable equipments and uninterruptible power sources. From an automatic control viewpoint, a switched power converter constitutes an interesting case study as it is a variable-structure nonlinear system. Its rapid structure variation is accounted for using averaged models [1], [2]. Based on these, different nonlinear control techniques have been developed. These include passivity techniques [2], feedback linearization and, more generally, flatness methods [5]. In this paper, the problem of controlling switched power converters is approached using the backstepping technique [3]. While feedback linearization methods require precise models and often cancel some useful nonlinearities, backstepping designs offer a choice of design tools for accommodation of uncertain nonlinearities and can avoid wasteful cancellations. In this paper, the Backstepping approach is applied to a specific class of switched power converters, namely DC-to-DC Buck-Boost converters. In the case where the converter model is fully known the backstepping nonlinear controller is shown to achieve the control objectives i.e. output voltage tracking and robustness with respect to load resistance uncertainty. In the case of unknown model an adaptive version of the above controller is developed and shown to ensure asymptotically the control objective. Finally, a comparison study shows that a backstepping controller does as well as a passivity-based controller. For both controllers the choice of design parameters turns out to be crucial for achieving robustness with respect to load resistance variations. Inversely, the performances of adaptive backstepping controllers are less sensitive to design parameters.

The paper is organized as follows: in Section 2, the Buck-Boost converter is described and modeled; Section 3 is devoted to designing the backstepping controller whose performances are illustrated and compared to the passivity-type controller; in Section 4 an adaptive version of the backstepping controller is developed and evaluated. A conclusion and a reference list end the paper.

II. BUCK-BOOST CONVERTER PRESENTATION AND MODELING

A Buck-Boost converter is a circuit constituted of power electronics components connected as shown in figure 1. The circuit operates according to the so-called Pulse Width Modulation (PWM) principle. This means that time is shared in intervals of length T (also called sampling period). Within any period, the T_p-switch is closed (conducting) during a period fraction, say μT, for some $0 \le \mu \le 1$. Then, energy is stored in the inductance L and the diode D is blocked. During the rest of the sampling period, i.e. $(1-\mu)T$, the switch T_p is not conducting and, consequently, the inductance discharges in the load resistance R. As this discharge can be total or partial, the absolute value of the output voltage v_s may therefore be superior or lower than the input voltage E. It is worth noting that the value of μ varies from a sampling period to an other. The variation law of μ determines the value of output voltage v_s.

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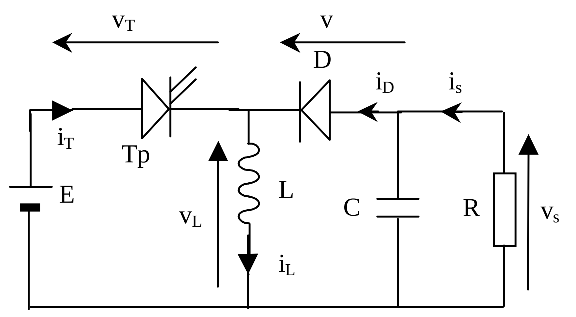

Fig. 1: Buck-Boost converter circuit

The averaged model of such a converter is shown to be the following (see e.g. [1], [2]):

$$\begin{aligned} \dot{x}_1 &= (1-\mu)\frac{1}{L}x_2 + \mu\frac{E}{L} \\ \dot{x}_2 &= -(1-\mu)\frac{1}{C}x_1 - \frac{1}{RC}x_2 \end{aligned} \tag{1}$$

where x_1 and x_2 denote the average input current (i_L) and the average output capacitor voltage (v_s), respectively. The control input for the above model is the function μ, called duty ratio function.

III. BACKSTEPPING NONLINEAR CONTROL OF BUCK-BOOST CONVERTER

The backstepping approach is a recursive design methodology [3]. It involves a systematic construction of both feedback control laws and associated Lyapunov functions. The controller design is completed in a number of steps, which is never higher than the system order.

A. Nonlinear controller design

The aim is to directly enforce x_2, the (average) capacitor voltage output, to track a given reference voltage $V_d<0$. The latter is any bounded and smooth signal. Due to the nonminimum phase nature of Buck-Boost converter a direct output voltage regulation turns out to unfeasible [2], [6]. Therefore, the control problem will be handled resorting to the indirect approach. This consists in forcing output capacitor voltage regulation indirectly through the regulation of the input current. The new control objective is to enforce current x_1 to an appropriate reference I_d. The latter is chosen in such a way that if x_1–I_d vanishes then so does x_2–V_d. From the converter model (1), it follows that: $I_d = \frac{V_d}{R}\left(\frac{V_d}{E}-1\right)$. Following the backstepping technique, a controller is designed in two steps because the controlled system (1) is a second-order.

Step 1. Let us introduce the output error: $z_1 = x_1 - I_d$. Deriving z_1 with respect to time and accounting for (1), implies:

$$\dot{z}_1 = \dot{x}_1 - \dot{I}_d = (1-\mu)\frac{1}{L}x_2 + \mu\frac{E}{L} - \dot{I}_d \tag{2a}$$

In equation (2a), $\frac{x_2}{L}$ behaves as a virtual control input. Such an equation shows that one gets $\dot{z}_1 = -c_1 z_1$ ($c_1>0$ being a design parameter) provided that:

$$\frac{x_2}{L} = \frac{1}{(1-\mu)}\left[-c_1 z_1 - \mu\frac{E}{L} + \dot{I}_d\right] \tag{2b}$$

As $\frac{x_2}{L}$ is just a variable and not (an effective) control input, (2a) cannot be enforced for all $t\geq 0$. Nevertheless, equation (2a) shows that the desired value for the variable $\frac{x_2}{L}$ is:

$$\alpha_1 = \frac{1}{(1-\mu)}\left[-c_1 z_1 - \mu\frac{E}{L} + \dot{I}_d\right] \tag{3}$$

Indeed, if the error:

$$z_2 = \frac{x_2}{L} - \alpha_1 \tag{4}$$

vanishes (asymptotically) then the control objective is achieved i.e. $z_1 = x_1 - I_d$ vanishes in turn. The desired value α_1 is called a stabilization function.

Now, replacing $\frac{x_2}{L}$ by $(z_2+\alpha_1)$ in (2a) yields

$\dot{z}_1 = (1-\mu)(z_2+\alpha_1) + \frac{\mu E}{L} - \dot{I}_d$ which, together with (3), gives:

$$\dot{z}_1 = -c_1 z_1 + (1-\mu) z_2 \tag{5}$$

Step 2. Let us investigate the behavior of error variable z_2. In view of (4) and (1), time-derivation of z_2 turns out to be:

$$\dot{z}_2 = \frac{\dot{x}_2}{L} - \dot{\alpha} = -(1-\mu)\frac{x_1}{LC} - \frac{x_2}{RLC} - \dot{\alpha} \tag{6a}$$

From (3) one gets:

$$\dot{\alpha} = -\frac{\dot{\mu}}{(1-\mu)}\left[\frac{E}{L} - \alpha_1\right] + \frac{1}{(1-\mu)}\left[c_1^2 z_1 - (1-\mu)c_1 z_2 + \ddot{I}_d\right]$$

which together with (6a) implies:

$$\begin{aligned} \dot{z}_2 = &-(1-\mu)\frac{x_1}{LC} - \frac{x_2}{RLC} + \frac{\dot{\mu}}{(1-\mu)}\left[\frac{E}{L} - \alpha_1\right] \\ &- \frac{1}{(1-\mu)}\left[c_1^2 z_1 - (1-\mu)c_1 z_2 + \ddot{I}_d\right] \end{aligned} \tag{6b}$$

In the new coordinates (z_1, z_2), the controlled system (1) is expressed by the couple of equations (5) and (6b). We now need to select a Lyapunov function for such a system. As the objective is to drive its states (z_1, z_2) to zero, it is natural to choose the following function: $V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$. The time-derivative of the latter, along the (z_1, z_2)-trajectory, is:

$$\begin{aligned}\dot V &= z_1 \dot z_1 + z_2 \dot z_2 \\ &= -c_1 z_1^2 - c_2 z_2^2 + z_2\left[c_2 z_2 + (1-\mu) z_1 + \dot z_2\right] \qquad (7)\end{aligned}$$

where $c_2>0$ is a design parameter and $\dot z_2$ is to be replaced by the right side of (6b). Equation (7) shows that the equilibrium (z_1, z_2) = (0, 0) is globally asymptotically stable if the term between brackets in (7) is set to zero. So doing, one gets the following control law:

$$\begin{aligned}\dot\mu = \frac{1}{\left(\frac{E}{L}-\alpha_1\right)}\Big\{&\left[c_1^2-(1-\mu)^2\right]z_1-(1-\mu)(c_1+c_2)z_2 \\ &+(1-\mu)^2\frac{x_1}{LC}+\frac{(1-\mu)x_2}{RLC}+\ddot I_d\Big\} \qquad (8)\end{aligned}$$

Remark 1. The zero-dynamics associated to the control law (8) are determined by letting z_1 = z_2 = 0 (i.e. $x_1 = I_d$, $x_2 = \frac{-\mu E}{(1-\mu)}$)). So doing one gets:

$$\dot\mu = \frac{(1-\mu)}{RCE}\left[(1-\mu)^2 R\, I_d - \mu E\right] \qquad (9)$$

The system (9) has three equilibrium points: $\mu = 1$;

$$\mu = 1+\frac{E}{2R\,I_d}+\sqrt{\left(\frac{E}{2R\,I_d}\right)^2+\frac{E}{R\,I_d}} \qquad \text{and}$$

$$\mu = 1+\frac{E}{2R\,I_d}-\sqrt{\left(\frac{E}{2R\,I_d}\right)^2+\frac{E}{R\,I_d}}\ .$$

Only the third is stable and meaningful since physically $0<\mu<1$. The performance of such a controller is described in the following proposition.

Proposition 1. Consider the control system consisting of the average PWM Buck-Boost model (1) in closed-loop with the controller (8), where the desired output voltage reference V_d is sufficiently smooth and satisfies $V_d<0$. Then, the equilibrium (x_1, x_2, μ)=(I_d, V_d, U) is locally asymptotically stable where $U = \frac{V_d}{V_d - E}$

Remark 2. The local nature of the stability is due to the presence of the saturation element in the control loop (fig. 2). Such an element limits the action of controller (8) and acts as a (unbounded) disturbances.

B. Practical evaluation of the backstepping controller performances

The backstepping controller designed in subsection III.A. has been applied to a Buck-Boost converter according to the experimental setting of fig. 2. The relevant parameters have the following values:

Circuit parameter values: R = 30 Ω, L = 20 mH, C = 68 μF, E = 15 V.

Sampling frequency: F = 5 KHz.

Backstepping controller parameters: c_1=100, c_2=1000.

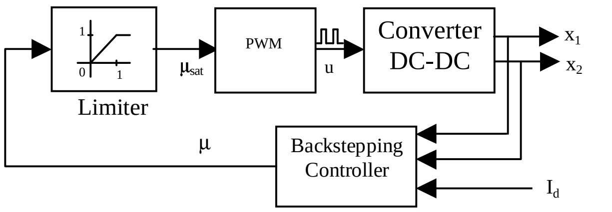

Fig. 2 : Experimental bench for Buck-Boost Converter

Figure 3a shows the controller tracking behavior, the output voltage reference is a filtered square signal that switches between -30 and -10 volts. Such a behavior is quite satisfactory. Figure 3b, illustrates the controller robustness with respect to a load resistance uncertainty; more precisely, the load nominal value (equal to 30 Ω) continues to be used in the control law (8), while the true load is time-varying as it switches between 25 and 30 Ω. One sees that such a load uncertainty and variation generates a chattering phenomenon. Output voltage is more affected by such a chattering that the input current.

C. Backstepping controller versus passivity-based controller

Using the passivity approach [2], a nonlinear controller has been designed in [6] for the Buck-Boost converter, to achieve indirect output voltage regulation. With the notations of [2], the involved design parameter R_1 has been set to 0.0010, this turned out to be the best choice. The controller thus obtained has been applied to the Buck-Boost converter of subsection III.B. The resulting performances are summarized by Figures 4a-b. In the light of Figures 3a and 4a it is seen that, when the converter load is constant and perfectly known, the output reference tracking capabilities of the two controllers (backstepping and passivity) are globally comparable. The signal chattering that can be seen on v_s and i_L is due to sampling (cutting) which is a physical feature of converters. Figures 3b and 4b show that the two controllers are equally robust to load uncertainty and variation.

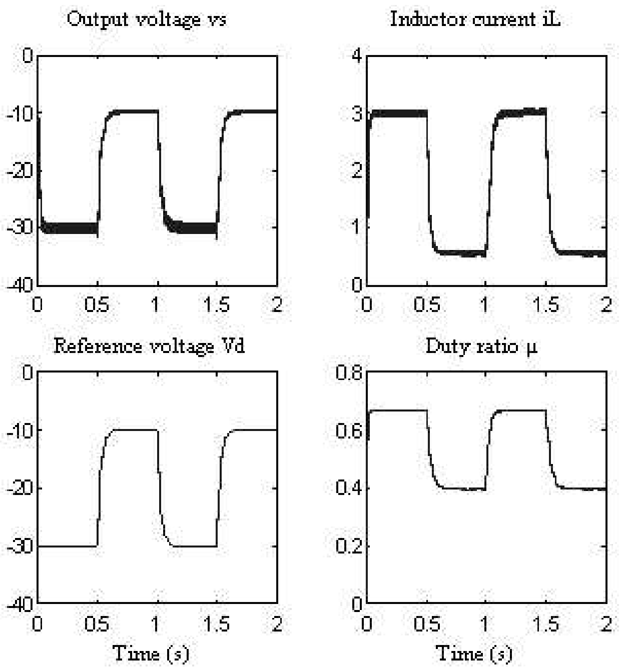

Fig. 3a: Tracking Performances of the Backstepping Controller

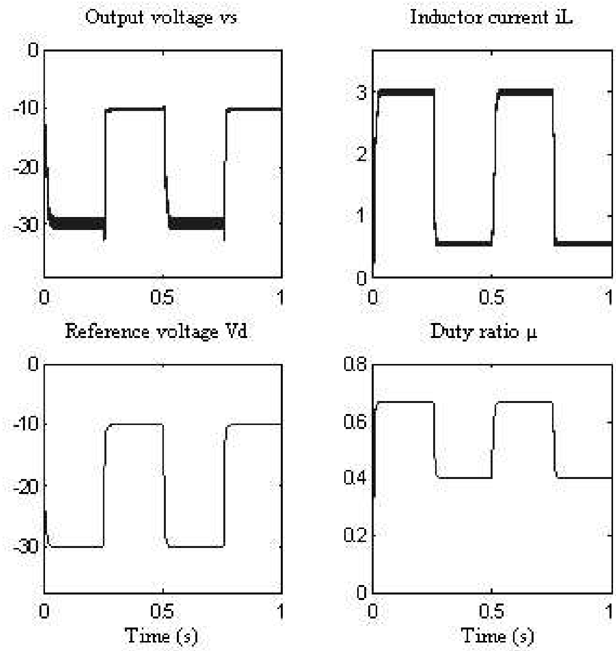

Fig. 4a: Tracking behavior of the passivity-based controller in presence of a time-varying output reference switching between -30V and -10V

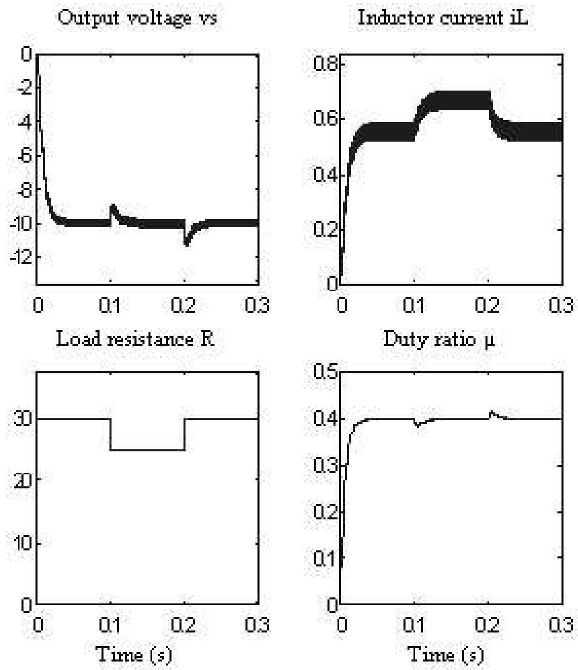

Fig. 3b: Backstepping controller behavior in presence of load resistance variations.

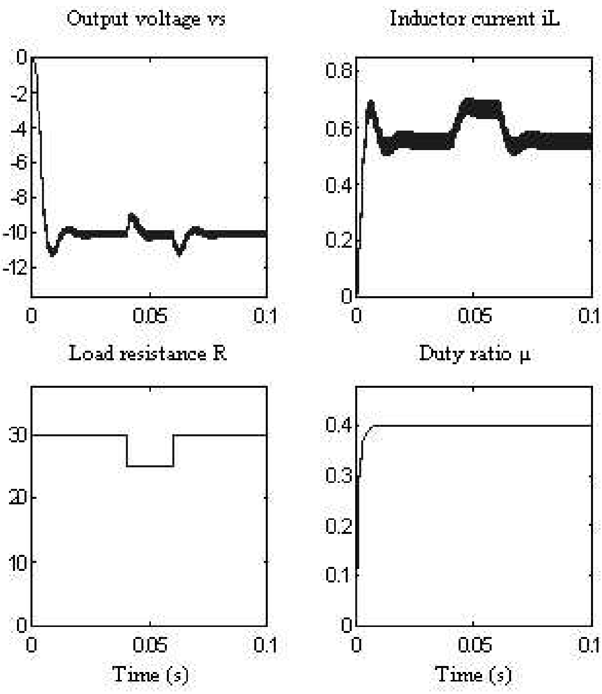

Fig. 4b: Robustness of Passivity-based controller with respect to load variations (between 25 and 30 Ω)

IV. BACKSTEPPING ADAPTIVE CONTROL OF BUCK-BOOST CONVERTER

Controllers of section III perform well only when the converter model is perfectly known. This particularly means that the load resistance is constant and time-invariant. When this is not the case, the controllers may still provide an acceptable behavior provided their design parameters are appropriately chosen. In real situations, finding such an appropriate is time-consuming and necessitates many simulations. Therefore adaptive versions of the above controllers turn out to be interesting alternatives.

A. Backstepping adaptive controller design

For the reasons explained in subsection III.A., the new control design will be performed according to indirect output voltage control principle i.e. the control objective is to enforce the Buck-Boost converter current x_1 to track its desired value $I_d = \frac{V_d}{R}\left[\frac{V_d}{E}-1\right]$. The difference with respect to Section III lies in the fact that load resistance R is not known. To cope with such a model uncertainty the new controller will be given a learning capacity. More specifically, the controller to be designed should involve an on-line estimation of the unknown parameter $\theta = \frac{1}{R}$. The obtained estimate is denoted $\hat{\theta}$. With these notations, one gets:

$$I_d = \theta V_d\left[\frac{V_d}{E}-1\right] \quad \hat{I}_d = \hat{\theta} V_d\left[\frac{V_d}{E}-1\right] \tag{10}$$

where $\hat{I}_d$ denotes the estimate of I_d. Just as for the nonadaptive case (Subsection III.A), the adaptive design procedure includes two steps.

Step 1. Following closely Step 1 of the design in Section III.A, one successively defines the current error $z_1 = x_1 - \hat{I}_d$, the stabilizing function:

$$\alpha_1 = \frac{1}{(1-\mu)}\left[-c_1 z_1 - \mu\frac{E}{L} + \dot{\hat{I}}_d\right] \tag{11}$$

and the error $z_2 = \frac{x_2}{L} - \alpha_1$. In (11), c_1 is a design parameter. With these definitions, one gets from model (1) (similarly to Subsection III.A.):

$$\dot{z}_1 = -c_1 z_1 + (1-\mu) z_2 \tag{12}$$

Step 2. Let $\theta = \hat{\theta} + \tilde{\theta}$, where $\tilde{\theta}$ denotes the parameter estimation error. Then, deriving z_2 with respect to time yields:

$$\dot{z}_2 = -(1-\mu)\frac{x_1}{LC} - \hat{\theta}\frac{x_2}{LC} - \tilde{\theta}\frac{x_2}{LC} + \frac{\dot{\mu}}{(1-\mu)}\left[\frac{E}{L} - \alpha_1\right] - \frac{1}{(1-\mu)}\left[c_1^2 z_1 - (1-\mu) c_1 z_2 + \ddot{\hat{I}}_d\right] \tag{13}$$

Equations (12)-(13) suggests the following Lyapunov function:

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma}\tilde{\theta}^2 \tag{14}$$

where $\gamma>0$ is any real constant, called parameter adaptation gain. Time-derivation of this function, along the trajectory of the system (12)-(13), gives:

$$\begin{aligned}\dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} \\ &= -c_1 z_1^2 - c_2 z_2^2 + z_2\Big[c_2 z_2 + (1-\mu) z_1 - (1-\mu)\frac{x_1}{LC} \\ &\quad - \frac{x_2}{LC}\hat{\theta} + \frac{\dot{\mu}}{(1-\mu)}\left(\frac{E}{L} - \alpha_1\right) - \frac{1}{(1-\mu)}\left\{c_1^2 z_1 - (1-\mu) c_1 z_2 + \ddot{\hat{I}}_d\right\}\Big] \\ &\quad + \tilde{\theta}\left[-\frac{\dot{\hat{\theta}}}{\gamma} - \frac{x_2}{LC} z_2\right]\end{aligned} \tag{15}$$

where c_2 is a design parameter. The control and adaptation laws are respectively obtained by setting to zero the quantities between brackets in (15). So doing, one gets:

$$\begin{aligned}\dot{\mu} = \frac{1}{\left(\frac{E}{L} - \alpha_1\right)}\Big\{&\left[c_1^2 - (1-\mu)^2\right] z_1 - (1-\mu)(c_1 + c_2) z_2 \\ &+ (1-\mu)^2\frac{x_1}{LC} + \frac{(1-\mu)x_2}{LC}\hat{\theta} + \ddot{\hat{I}}_d\Big\}\end{aligned} \tag{16}$$

$$\dot{\hat{\theta}} = -\gamma\frac{x_2}{LC} z_2 \tag{17}$$

Proposition 2. Consider the control system including the average PWM Buck-Boost model (1), where R is the only unknown parameter, in closed-loop with the adaptive controller (16)-(17). If the reference output voltage V_d is smooth enough and satisfy $V_d<0$, then the closed-loop system equilibrium $(x_1, x_2, \mu) = (\hat{I}_d, V_d, U)$ is asymptotically locally stable, where $U = \frac{V_d}{V_d - E}$

B. Practical evaluation of the backstepping controller performances

The components of the controlled Buck-Boost converter have the same values as in Section III. The adaptive controller design parameters have the following values: c_1 = 100; c2 = 1000; $\gamma = 10^{-11}$. The corresponding performances are illustrated by Figure 5. This shows that, despite the load resistance uncertainty, the controller behavior is quite satisfactory. It is worth noting that such a good behavior is preserved when facing different variations of the load resistance i.e. there is no need to tune the design parameters values (c_1 and c_2). Unlikely, the nonadaptive controllers (backstepping and passivity) prove to be very sensitive to the design parameters. That is, when facing a different variation of the load resistance, the performances shown in Fig. 3 and Fig. 4 deteriorate, unless c_1 and c_2 are changed accordingly. The design parameters should be tuned whenever the load resistance variation changes, which is inconvenient in practical applications.

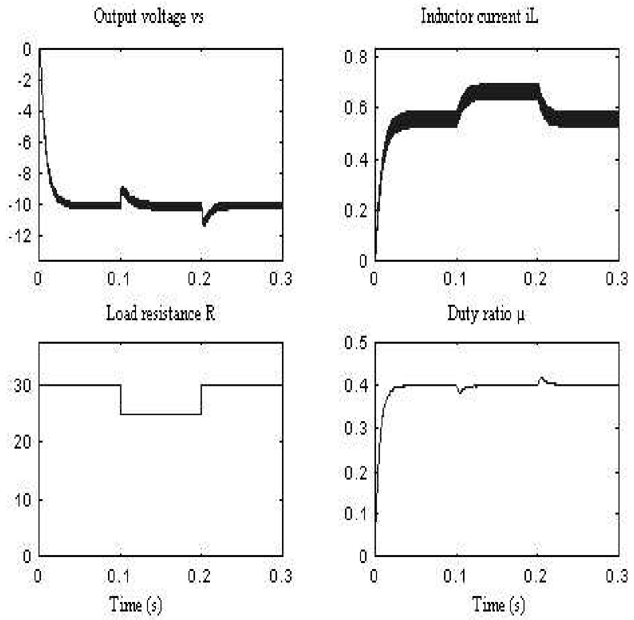

Fig. 5: Backstepping adaptive controller performances

V. CONCLUSION

The problem of Buck-Boost converters has been dealt with using the backstepping approach. The controller design is based on the average PWM model (1). The nonminimum phase feature of the latter makes it impossible to perform a direct output voltage control. Therefore, an indirect voltage control has been resorted-to. Accordingly, the control objective is to enforce the current i_L to track the reference I_d, which in turn implies the convergence of output voltage v_s to its desired value V_d. In the case of perfectly known converter model, the control objective can be ensured using a backstepping nonlinear controller (8). This proved to be quite comparable to passivity-based controllers. In the case of unknown load resistance, an adaptive version of the backstepping controller (16, 17) has been developed to achieve the control objective. The latter proved to be less sensitive to its design parameters, particularly c_1 and c_2, than the nonadaptive controllers.

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