On delay-dependent stability in lossless propagation models: A Liapunov-Krasovskii analysis

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Abstract— **This paper addresses the stability problem of a special class of dynamical systems of coupled differential and difference equations arising from the mathematical description of various engineering systems that contain lossless propagation media (pipes or electrical lines).**

More explicitly, we shall derive some *sufficient* stability conditions including delay information using *degenerate* Liapunov-Krasovskii functionals under appropriate *model transformations*. Note that the corresponding model transformations induce additional dynamics that will be also characterized.

Keywords—**lossless propagation; neutral systems; delay effects.**

I. Introduction

It is pointed out in the book of Hale and Verduyn Lunel [18] that neutral functional differential equations (NFDE) are met when dealing with oscillatory systems with some interconnections between them. The time for interaction is important: it is a straightforward way to speak about *propagation phenomena*. *Lossless propagation* is associated to transmission lines without losses; such lines correspond in engineering to LC electrical lines, or to lossless steam, water or gas pipes. Some examples with respect to this topics are to find in Hale and Lunel [18] as well as the paper of Halanay and Răsvan [16].

In general, by *lossless propagation* it is understood the phenomenon associated with long transmission lines for (some) physical signals. In engineering, this problem is strongly related to *electric* and *electronic applications*, e.g. circuit structures consisting of multipoles connected through LC transmission lines (a long list of references may be provided, starting with a pioneering paper of Brayton [4] and going up to a quite recent book of Marinov and Neittaanmäki [22]). Some propagation phenomena may be also met in *power distribution* systems if the distribution area is quite large (see, e.g. Karaev [21]). We shall note that the lossless propagation occurs also for *non-electric* 'signals' as water, steam or gas flows and pressures. With respect to this, we may cite the pioneering (but almost forgotten) papers of Kabakov and Sokolov [19] on steam pipes for combined heat-electricity generation, waterhammer case and many other.

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We shall not insist on the modeling part, but we just point out that a long list of references that were published along the time may be found in the above cited references. The mathematical model is described in all these cases by a *mixed initial* and *boundary value problem* for hyperbolic partial differential equations modeling the lossless propagation. The boundary conditions are of special type being in "feedback connection" with some system described by ordinary differential equations.

This sends to the so-called "derivative boundary conditions" considered by Cooke [8] (see also Cooke and Krumme [9]) but also to the even more general boundary conditions of Abolinia and Myshkis [1], described by Volterra operators. Integration along characteristics of the hyperbolic partial differential equations (which is in fact the method of d'Alembert) mentioned in the cited references allows the association of a certain system of functional equations to the mixed problem; more precisely, a one-to-one correspondence may be established and proved between the solutions of the mixed problem for hyperbolic partial differential equations and the initial value (Cauchy) problem for the associated system of functional equations.

In certain cases, some of them considered in [18], [16], [8], [9], this system of functional differential equations reads as follows:

$$\left\{\begin{array}{rcl} \dot{x}_1(t) & = & Ax_1(t) + Bx_2(t-\tau) \\ & & \quad +f(x_1(t), x_2(t), x_2(t-\tau)) \\ x_2(t) & = & Cx_1(t) + Dx_2(t-\tau) \\ & & \quad +g(x_1(t), x_2(t), x_2(t-\tau)), \end{array}\right. \tag{1}$$

which is a a differential equation coupled with a difference equation.

This system has been treated by Hale and Martinez-Amores [17] by writing the second equation as:

$$\frac{d}{dt}\left[x_2(t) - Cx_1(t) - Dx_2(t-\tau) \right. \\ \left. -g(x_1(t), x_2(t), x_2(t-\tau))\right] = 0,$$

and applying the general results for neutral systems presented in [18].

An earlier approach [26] suggested the treatment of (1) as a special case of neutral systems by letting $x_2(t) = \dot{z}(t)$. This last approach was used in the construction of a Popov-like theory in the input-output approach for absolute stability [26], forced nonlinear oscillations [14] and approximation by ordinary differential equations [15] (which "projected back" on the partial differential equations gave the method of lines).

All these considerations show that (1) represents a type of system that display a self-contained interest. Its linearized version is:

$$\left\{\begin{array}{l} \dot{x}_1(t) = Ax_1(t) + Bx_2(t-\tau) \\ x_2(t) = Cx_1(t) + Dx_2(t-\tau), \end{array}\right. \tag{2}$$

where x_1 and x_2 describe the *differential*, and *difference equations*, $\tau > 0$ is the delay, A, B, C and D are real matrices of appropriate dimensions and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ represents the vector of the state variables, $x \in \mathbf{R^n}$. Note that $x_1 \in \mathbf{R^{n_1}}$ and $x_2 \in \mathbf{R^{n_2}}$ $(n_1 + n_2 = n)$.

Furthermore, in the sequel we assume that the difference operator $\mathcal{D}(\phi) = \phi(\prime) - \mathcal{D}\phi(-\tau)$ is stable, which is equivalent to the *Schur-Cohn* stability of the matrix D (see, for instance, [18] and the references therein). Note also that the Schur-Cohn stability of the D guarantees the stability of the difference operator $\mathcal{D}$ for *all* positive delay values.

The paper extends the time-domain approach prposed in [25] to a more general framework (including various model transformations of the original system), and is organized as follows: Model transformations are presented in section 2, and their corresponding additional eigenvalues characterization in Section 3. Section 4 is devoted to the main results and proof ideas. Various control interpretations are also included. Some concluding remarks end the paper. The notations are standard.

II. Model transformations

One of the methods largely used in the retarded case for deriving *delay-dependent* stability (including information on the delay size) results is based on the Leibniz rule:

$$x_2(t-\tau) \quad = \quad x_2(t) - \int_{-\tau}^{0} \dot{x}_2(t+\theta)d\theta,$$

to transform the original system (2) to a distributed delay system of the form:

$$\left\{\begin{array}{l} \dot{x}_1(t) = Ax_1(t) + Bx_2(t) - B\displaystyle\int_{-\tau}^{0} \dot{x}_2(t+\theta)d\theta \\ x_2(t) = Cx_1(t) + Dx_2(t-\tau). \end{array}\right. \tag{3}$$

Since:

$$\begin{array}{l} \displaystyle\int_{-\tau}^{0} \dot{x}_2(t+\theta)d\theta = C\int_{-\tau}^{0} [Ax_1(t+\theta) + Bx_2(t+\theta-\tau)]d\theta \\ \qquad -D[x(t-\tau) - x(t-2\tau)], \end{array}$$

the system (3) can be rewritten as follows:

$$\left\{\begin{array}{rcl} \dot{x}_1(t) & = & Ax_1(t) + Bx_2(t) \\ & & -BD[x_2(t-\tau) - x_2(t-2\tau)] \\ & & -BCA\int_{-\tau}^{0} x_1(t+\theta)d\theta \\ & & -BCB\int_{-2\tau}^{-\tau} x_2(t+\theta)d\theta \\ x_2(t) & = & Cx_1(t) + Dx_2(t-\tau). \end{array}\right. \tag{4}$$

Such a process is generically called *model transformation*. Note that if the model transformations are largely used in the retarded case [12], [13] (and the references therein) for deriving *delay-dependent* stability results, this method was not sufficiently exploited in the neutral case (see, for instance, [23]). Based on the classification proposed in [23], such a transformation is called a *fixed first-order model transformation*.

Next, let us consider a different model transformation, where the method above will by applied *not* for the *whole* delayed state: $Bx_2(t-\tau)$, but only for some "part" of it. Assume $M \in \mathbf{R^{n_1 \times n_2}}$ be a real matrix, and apply the same procedure as above, but *only* for $Mx_2(t-\tau)$. Then, the system (3) rewrites as follows:

$$\left\{\begin{array}{rcl} \dot{x}_1(t) & = & Ax_1(t) + Mx_2(t) + (B-M)x_2(t-\tau) \\ & & -MD[x_2(t-\tau) - x_2(t-2\tau)] \\ & & -MCA\int_{-\tau}^{0} x_1(t+\theta)d\theta \\ & & -MCB\int_{-2\tau}^{-\tau} x_2(t+\theta)d\theta \\ x_2(t) & = & Cx_1(t) + Dx_2(t-\tau). \end{array}\right. \tag{5}$$

It seems clear that if one takes $M = B$ in (5), we shall recover the previous model transformation (4), and if $M = 0$, (5) reduces to the original system.

This second transformation is called a *parametrized (first-order) model transformation*. The advantage in using (5) will be presented in the sequel and consists in inducing a further degree of freedom in the model (the matrix M), that can be interpreted as an appropriate *control problem*, as seen below.

III. Additional eigenvalues

First at all, let us consider the solutions of the systems (2) and (4). It is easy to see that (2) is defined on $\mathbf{R^{n_1}} \times \mathcal{L}_{\in}([-\tau, \prime], \mathbf{R^{n_2}})$, and (4) on $\mathbf{R^{n_1}} \times \mathcal{L}_{\in}([-\in\tau, \prime], \mathbf{R^{n_2}})$. Furthermore, the construction of the solutions can be done using the 'step-by-step' method in both cases (due to the special form of the corresponding distributed delay in (4)).

It is not difficult to see that each initial condition ϕ for (2), the solution on $[0, \tau]$ is *uniquely* defined. Thus, we can find an initial condition $\bar{\phi}$ defined on $[-\tau, \tau]$ for (4) such that the solutions of both equations are identical for $t \geq t_0 + \tau$, but the reverse is not true.

Note that the same remark holds for the second model transformation (5) with respect to the original system.

As seen in [23] (retarded and neutral cases), the "difference" between the dynamical behaviors of the transformed systems with respect to the original system can be explained by the corresponding *additional eigenvalues* induced by the (fixed or parametrized) transformation under consideration.

In order to analyze these additional eigenvalues, let us focus on the roots of the characteristic equations associated to (2) and (4) and (5).

Thus, we have:

$$\Delta_o(s) \quad = \quad \mathbf{det}\left[\begin{array}{cc} sI_{n_1}-A & -Be^{-s\tau} \\ -C & I_{n_2}-De^{-s\tau} \end{array}\right] \qquad (6)$$

for the original system (2), and

$$\Delta_{t,1}(s) = \mathbf{det}\left[\begin{array}{cc} sI_{n_1}-A+BCA\frac{1-e^{-s\tau}}{s} & -BQ_{t,1}(s) \\ -C & I_{n_2}-De^{-s\tau} \end{array}\right], \qquad (7)$$

with:

$$Q_{t,1}(s) \quad = \quad I_{n_2}-De^{-s\tau}+De^{-2s\tau}-CBe^{-s\tau}\frac{1-e^{-s\tau}}{s},$$

for the transformed system (4), and

$$\Delta_{t,2}(s) = \mathbf{det}\left[\begin{array}{cc} sI_{n_1}-A+MCA\frac{1-e^{-s\tau}}{s} & -Q_{t,2}(s) \\ -C & I_{n_2}-De^{-s\tau} \end{array}\right], \qquad (8)$$

with:

$$Q_{t,2}(s) = M\left(I_{n_2}-De^{-s\tau}+De^{-2s\tau}-CBe^{-s\tau}\frac{1-e^{-s\tau}}{s}\right)+ +(B-M)e^{-s\tau},$$

for the parametrized model transformation (5), respectively.

Some simple computations prove that:

$$\Delta_{t,1}(s) = \mathbf{det}\left[\begin{array}{cc} I_{n_1}-BC\frac{1-e^{-s\tau}}{s} & -B(1-e^{-s\tau}) \\ 0 & I_{n_2} \end{array}\right]\cdot\Delta_{\mathbf{o}}(\mathbf{s}), \qquad (9)$$

and

$$\Delta_{t,2}(s) = \mathbf{det}\left[\begin{array}{cc} I_{n_1}-MC\frac{1-e^{-s\tau}}{s} & -M(1-e^{-s\tau}) \\ 0 & I_{n_2} \end{array}\right]\cdot\Delta_{\mathbf{o}}(\mathbf{s}) \qquad (10)$$

Since the second model transformation includes the first one as a particular case, the results below are directly derived for the parametrized model transformation case. Based on (10), we have:

Proposition III.1 (Additional eigenvalues) Let $s = s_{ik}$, $k = 1, 2, 3, ...$ be all the solutions of the equation

$$1-\lambda_i(MC)\frac{1-e^{-\tau s}}{s} = 0, \qquad (11)$$

where $\lambda_i(MC)$, is the ith eigenvalue of matrix BC. Then s_{ik}, $i = 1, 2, ..., n_1$; $k = 1, 2, 3, ...$ are all the additional eigenvalues of system (5).

The complete set of eigenvalues of (5) consists of the solutions of (11), and the eigenvalues of the original system (2), which are the solutions of $\Delta_0(s) = 0$.

If $M = B$, one recovers the fixed first-order model transformation (4).

A natural consequence is given by the following result:

Corollary III.2: No additional eigenvalues will reach the imaginary axis if the matrix MC and the delay value τ satisfies the condition:

$$\|MC\| \quad < \quad \frac{1}{\tau}. \qquad (12)$$

In conclusion, the stability of systems (2) and (5) are *equivalent* for any delay satisfying

$$\tau \in \left[0, \frac{1}{\|MC\|}\right).$$

Since in the parametrized model transformation case, the parameter M is subject to choice, a natural constraint seems that the stability of the original and transformed systems should be *equivalent*, that is M should be relatively small in norm.

Remark III.3: The results above give the limitations of the model transformation method for deriving *delay-dependent* stability results.

It is clear that if the original and transformed system are equivalent, the delay bound derived using the Liapunov-Krasovskii approach will give the *conservatism* of the method.

Further remarks in the retarded case can be found in [12], [13]. Note also that the same ideas (model transformation construction, additional eigenvalues characterization) hold in the ('standard') neutral case (C invertible) as it was proved in [23].

IV. Main results

We have the following:

Proposition IV.1: The system (2) is asymptotically stable for all delays $\tau \in [0, \tau^*)$ if the following conditions are simultaneously satisfied:
(i) the matrix D is Schur-Cohn stable;
(ii) the system free of delays is asymptotically stable;
(iii) there exist symmetric and positive-definite matrices P and S_i $(i = \overline{1,4})$ of appropriate dimensions, and a real matrix $W \in \mathbf{R}^{\mathbf{n_1}\times\mathbf{n_2}}$ such that the following linear matrix inequality is satisfied:

$$\left[\begin{array}{cc} \mathcal{X}_{11} & \mathcal{X}_{12} \\ \mathcal{X}_{12}^T & \mathcal{X}_{22} \end{array}\right] \quad < \quad 0, \qquad (13)$$

where:

$$\mathcal{X}_{11} = \left[\begin{array}{ccc} \begin{array}{c} A^TP+PA+WC+C^TW^T \\ +C^T(S_1+\tau^*S_4)C \\ +\tau^*S_3 \end{array} & \tau^*WCA & \tau^*WCB \\ \tau^*A^TC^TW^T & -\tau^*S_3 & 0 \\ \tau^*B^TC^TW^T & 0 & -\tau^*S_4 \end{array}\right],$$

$$\mathcal{X}_{12} = \left[\begin{array}{cc} C^T(\tau^*S_4+S_1)D+PB-W & WD \\ 0 & 0 \end{array}\right],$$

$$\mathcal{X}_{22} = \left[\begin{array}{cc} D^T(\tau^*S_4+S_1)D+S_2-S_1 & 0 \\ 0 & -S_2 \end{array}\right],$$

where the zero blocks have appropriate dimensions. Furthermore, the corresponding model transformation is given by:

$$M = P^{-1}W. \tag{14}$$

The proof idea is based on the use of the following Liapunov-Krasovskii functional candidate:

$$\begin{aligned}V(x_1(t), x_{1,t}, x_{2,t}) &= x_1(t)^T P x_1(t)\\ &+ \int_{-\tau}^{0} x_2(\theta)^T S_1 x_2(t+\theta) d\theta\\ &+ \int_{-2\tau}^{-\tau} x_2(\theta)^T S_2 x_2(t+\theta) d\theta\\ &+ \int_{-\tau}^{0}\int_{t+\theta}^{t} x_1(\xi)^T S_3 x_1(\xi) d\xi d\theta\\ &+ \int_{-2\tau}^{-\tau}\int_{t+\theta}^{t} x_2(\xi)^T S_4 x_2(\xi) d\xi d\theta.\end{aligned} \tag{15}$$

for the parametrized model transformation (5).

Remark IV.2: Note that what makes the difference with respect to the "normal" neutral differential equation case is that the *Liapunov-Krasovskii stability theory* [18] can not applied directly to this class of systems excepting the case when C is *invertible*. But as seen in [24] (see also [25]), the main interest of this class of systems related to real problems coming from the case when C is *not* invertible.

The inequality (13) in *(iii)* corresponds to the negativity of the derivative of the candidate V by applying the Schur complement (some tedious, but straightforward computations) as suggested in [23] (see also [3]), but, as said above, such an argument is not *sufficient* for guaranteeing asymptotic stability using a Liapunov-Krasovskii stability argument if C is *not invertible*.

However, the negativity of $\dot{V}$ guarantees that the candidate V is a decreasing function, and thus:

$$V(t) = V(x_1(t), x_{1,t}, x_{2,t}) \leq V(x_1(0), x_{1,t}, x_{2,0}) = V(0), \quad \forall t \geq 0. \tag{16}$$

In conclusion, x_1 is bounded since:

$$\begin{aligned}\|x_1(t)\|^2 &\leq \frac{1}{\lambda_{min}(P)} x_1(t)^T P x_1(t)\\ &\leq \frac{1}{\lambda_{min}(P)} V(x_1(t), x_{1,t} x_{2,t})\\ &\leq \frac{1}{\lambda_{min}(P)} V(0), \quad \forall t \geq 0.\end{aligned} \tag{17}$$

The same argument holds for $x_2(t) - Dx_2(t-\tau)$, and for all $t \geq 0$ by using the difference equation of the original system and (17):

$$\|x_2(t) - Dx_2(t-\tau)\|^2 \leq \frac{\|C\|^2}{\lambda_{min}(P)} V(0), \quad \forall t \geq 0. \tag{18}$$

Furthermore, x_2 is also bounded:

$$\sup_{\theta\in[-\tau,t]} \|x_2(\theta)\|^2 \leq \frac{\|C\|^2}{(1-\|D\|^2)\lambda_{min}(P)} V(0), \quad \forall t \geq 0, \tag{19}$$

since D is Schur-Cohn ($\|D\| < 1$), and x_1 is bounded.

Thus, $\dot{x}_1$ is bounded (from the delay-differential equation of the original system), and we have simultaneously x_1 and $\dot{x}_1$ bounded for all $t \geq 0$.

Using the same ideas as in the proof of Barbălat lemma [2] (see also some discussions in Gopalsamy [11]), it follows that $x_1 \to 0$, and also $x_2(t) - Dx_2(t-\tau) \to 0$ when $t \to +\infty$. In conclusion, the asymptotic stability is guaranteed for all delays $\tau \in [0, \tau^*)$.

Remark IV.3: If the bound $\tau^* = +\infty$, that is a *delay-independent* type stability result, then $W = 0$, $S_i = 0$ ($i = \overline{2,4}$), and (13) rewrites as:

$$\begin{bmatrix} A^T P + PA + C^T S_1 C & PB + C^T S_1 D \\ B^T P + S_1 C D^T & D^T S_1 D - S_1 \end{bmatrix} < 0,$$

which is exactly the linear matrix inequality proposed in [25] with the Liapunov-Krasovskii candidate:

$$\begin{aligned}V(x_1(t), x_{2,t}) &= x_1(t)^T P x_1(t)\\ &\quad + \int_{-\tau}^{0} x_2(t+\theta)^T S_1 x_2(t+\theta) d\theta.\end{aligned}$$

A. Control interpretations

Let us reconsider the model transformation (5) and the result in Proposition IV.1. The procedure for constructing the *suboptimal* delay bound τ^* as a standard LMI based (quasi-convex) optimization problem [3], [**?**] is similar to the *state feedback* construction in [3] (see also [23] for the retarded case).

Furthermore, one can interpret the delay-dependent stability of the above lossless propagation models as a *multi-objective control problem*, since one needs to find some model transformation to guarantee simultaneously the following constraints:
a) the stability equivalence between the original and the transformed systems (see Section 3),
b) the stability of the system free of delay (condition *(ii)* in Proposition IV.1), and
c) the largest value for the delay bound τ^*.
Indeed, as seen in [23] in the retarded case, the constraints on the stability equivalence and the stability of the system free of delay are competitive, which leads to the corresponding "trade-off" on the delay-size.

V. Concluding remarks

This chapter has focused on the *delay-dependent* stability of some linear lossless propagation models. In order to use some simple quadratic Liapunov-Krasovskii functionals for the stability analysis, some *model transformations* of the original system have been proposed. The conservatism of the transformations as well as various ways to improve the delay bound guaranteeing stability have been proposed.

The idea behind was the use of some *control feedback techniques* (state-feedback construction) in order to find the *suboptimal* delay bound.

The advantage of the proposed method lies in its simplicity, numerical tractability and efficiency (see, for instance, the comments in [10] or the monograph [3]).

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