Nonlinear control of a photovoltaic converter

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Abstract— **This paper presents a non linear approach for the control of a photovoltaic system. Such system is composed of three distinct elements : a photovoltaic panel which gives the electrical energy, a DC-DC converter which ensures a constant electrical power and a load. For these three components a nonlinear dynamical model is given. A nonlinear control law, using techniques of partial linearization, is developed. Simulation results are presented and different tests, like the reaching of the maximum of power, are discussed.**

Keywords—**Nonlinear control, photovoltaic converter, DC-DC converter**

I. Introduction

Non linear control laws are used in order to increase dynamical performance of a system [8]. Indeed, as these techniques can take into account internal dynamics of a system (it is the case in particular for flat control), generally a more effective control can be expected. The system considered in this paper is a photovoltaic (PV) array which must provide the electrical energy to recharge a battery. Between the panel and the battery exists a controllable system, a DC-DC converter, which ensures the regulation of the tension and/or electric power at the terminal of the battery voltage. A non linear control is developed for this system. In a first part the problem of modelisation is presented. This system being constituted by three subsystems, the different models will be exposed before deducing the non linear state space equations of the whole system. In second part the non linear approach for the control is presented. Based on the flat control proposed by Sira Ramirez [1] for the DC-Dc converter, a technique of partial linearization will be considered. The last part relates to some simulation results to test the performances and the robustness of the proposed control law. At last, as the main objective is to deliver the maximum of electric power (depending on the photoelectrical conditions) at the terminal of the battery, a algorithm of detection of this maximum is presented as well as simulation results.

II. Modelisation

The system studied in this paper is composed of three distinct parts. The fist is a DC-D converter which ensures the regulation of the output voltage. The second is the PV generator which furnishes the electrical energy and the third is the load.

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A. The DC-DC converter

The DC-DC converter has been largely studied by Sira-Ramirez [1], [2], [3]. This circuit, described by the figure 1, is described by the set of equations :

$$\begin{cases} \dot{x}_1 = \dfrac{-1}{L} u x_2 + \dfrac{E}{L} \\ \dot{x}_2 = \dfrac{1}{C_s} u x_1 - \dfrac{x_2}{RC_s} \end{cases} \tag{1}$$

where x_1 represents the inductor current and x_2 the output capacitor voltage. E is an external voltage source. The control U corresponds to a PWM (pulse width modulation) sequence and, by consequence, must verify $u \in \mathcal{D}_u$, $\mathcal{D}_u = [0, 1]$ When the Dc-Dc converter is alone (without PV generator nor load) the external voltage is constante and the load is a simple resistance R.

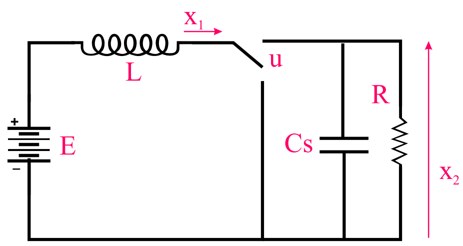

Fig. 1. Equivalent electrical scheme for the DC-DC converter

Such controllable converter are necessary when the external input voltage is supposed not to be constant. It is the case with a solar panel as power generator.

B. The PV generator

A solar panel can be described by the left part of the figure 2. It has been shown [4] that the following set of equations gives the panel output voltage V_p and its associated current I_p :

$$\begin{cases} (a) \quad V_p = n_s V_D - \left(\frac{n_s}{n_p} I_p R_s \right) \\ (b) \quad I_p = n_p \left(I_{cc} - I_D \right) - \left(\frac{n_s V_D}{\frac{n_s}{n_p} R_{sh}} \right) = n_p \left(I_{cc} - I_D - \frac{V_D}{R_{sh}} \right) \\ (c) \quad I_D = I_S \left(e^{\frac{V_D}{\rho V_T}} - 1 \right) \end{cases} \tag{2}$$

where

- V_D : voltage of one cell
- I_p : solar panel current
- V_p : solar panel voltage
- n_p : number of panel
- n_s : number of parallel cells
- ρ : constant depending on the used material.
- I_{cc} : short circuit current of one cell.
- I_S : obscurity current of one cell.
- R_{sh}, R_s : equivalent resistors in series (or parallel) for one cells
- $V_T = k\frac{(273+T)}{q}$: k is the Boltzman constant, q is a panel constant and T is the panel temperature in $^\circ C$

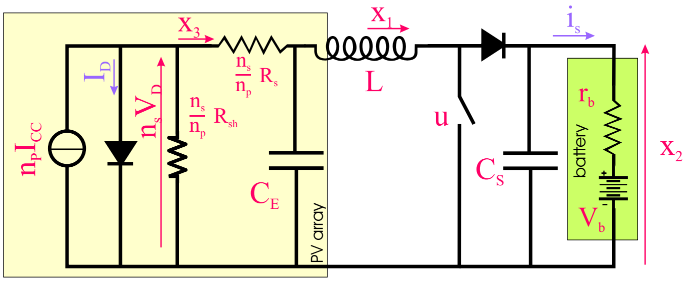

Fig. 2. The system *solar panel + DC-DC* converter

From the two last equations, and by neglecting the term $\frac{V_D}{R_{sh}}$ of (2b), we can deduce :

$$V_D = \rho V_T \ln\left(\frac{I_{cc} - \frac{I_P}{n_p}}{I_S} + 1\right) \tag{3}$$

C. The battery

In this paper only a simple battery model (right part of the figure 2) will be considered. A battery can be approximate by a constant voltage source V_b with a little dissipative resistance r_b. So, if i_s corresponds to the internal current passing through the battery, the voltage x_2 is

$$x_2 = r_B i_s + V_B \tag{4}$$

D. The whole system

The panel, the converter and the battery are now connected. We can seen on the figure 2 that the panel current I_p corresponds to the current noted x_3 and that the PV generator and the Dc-Dc conveter are connected via a capacitor C_e . In the equation (1) the voltage E can be replace by the expression of V_p in (2). The process is described by the following equations:

$$\left\{\begin{array}{l} L\frac{dx_1}{dt} = V_P - ux_2 \\ \\ ux_2 = C_s\frac{dx_2}{dt} + \frac{x_2 - V_B}{r_b} \\ \\ x_3 = C_e\frac{dV_P}{dt} + x_1 \end{array}\right. \tag{5}$$

where

- x_1 : Inductor current
- x_2 : Output voltage
- x_3 : Output current of the panel

V_p from the equations (2) and 3 can be written :

$$V_P = k_1 \ln(\alpha x_3 + \beta) - k_2 x_3 \tag{6}$$

where

- $k_1 = n_s \rho V_T$
- $k_2 = \frac{n_s R_s}{n_p}$
- $\alpha = -\frac{1}{n_p I_s}$
- $\beta = \frac{I_{CC}}{I_S} + 1$

The complete state space representation of this nonlinear system is obtained by derivation of (6) and by using the equations (5) and (4). Finally the whole model is given by:

$$\left\{\begin{array}{l} \dot{x_1} = \frac{k_1}{L}\ln(\alpha x_3 + \beta) - \frac{k_2}{L}x_3 - \frac{ux_2}{L} \\ \dot{x_2} = -\frac{1}{C_s r_B}(x_2 - V_B) + \frac{ux_1}{C_s} \\ \dot{x_3} = \frac{(\alpha x_3 + \beta)(x_3 - x_1)}{c_e(k_1\alpha - k_2\alpha x_3 - k_2\beta)} \end{array}\right. \tag{7}$$

III. Nonlinear control law

The control of this system consists in changing the PWM ratio to obtain wether a desired output voltage, nor a desired output current nor a desired output power depending on control objective.

A. Objective of the control

We consider is this paper that the main objective is to charge the battery. An usual way [5] to do that is to maximize the power absorbed by the battery.

B. Nonlinear control of the whole system

B.1 Flat control of the DC-DC control

Sira Ramirez in [1] has proposed to use the output flat $z = \frac{L}{2}x_1^2 + \frac{C}{2}x_2^2$ to perform a flat control [6] of the DC-DC converter(1). A such control is very interesting because it takes into account the internal dynamic of the system. However, this control cannot be directly applied if the external voltage E is uncertain [7]. In our case, E is issue from the solar panel, and isn't obviously a constant value. Moreover the output power cannot not easily deduce from the flat output z. A different control must be proposed.

B.2 The diffeomorphism

Like the controlled output is the power, it seems to be natural to express the system with respect to this new variable, noted z_1 :

$$z_1 = x_2 i_s = \frac{1}{r_B}\left(x_2^2 - V_B x_2\right) \tag{8}$$

To complete this first coordinate change, we can add the two following variables :

$$\begin{array}{l} z_2 = \frac{L}{2}x_1^2 + \frac{C_s}{2}x_2^2 \\ z_3 = x_3 \end{array} \tag{9}$$

The choice of z_2 an,d z_3 has been dictated by the fact that the diffeomorphism must be invertible and that the control variable u must not appear in the new expression of the system [8], [9]. The inverse of the diffeomorphism is completely defined by the set of equations :

$$\Phi^{-1}:\begin{cases} x_1 = \sqrt{\frac{2}{L}\left(z_2 - \frac{c_s}{2}\left(\frac{V_B+\sqrt{V_B^2+4r_b z_1}}{2}\right)^2\right)} \\ x_2 = \frac{V_B+\sqrt{V_B^2+4r_b z_1}}{2} \\ x_3 = z_3 \end{cases} \quad (10)$$

By using this complete diffeomorphism (8),(9) and (10) we can expressed (7) like :

$$\begin{cases} \dot{z_1} = f_1(z_1,z_2,z_3) + g(z_1,z_2,z_3)u \\ \dot{z_2} = f_2(z_1,z_2,z_3) \\ \dot{z_3} = f_3(z_1,z_2,z_3) \end{cases} \quad (11)$$

.

B.3 The control law

The system (11) is partially linearizable with the control:

$$u = \frac{(v - f_1(z_1,z_2,z_3))}{g(z_1,z_2,z_3)} \quad (12)$$

Applied to the equation (7) and by using the inverse of the diffeomorphism (10), we can express the control in the original state variable set.

$$u = \left[v + \frac{1}{c_s r_B^2}\left(2x_2^2 + V_B^2 - 3x_2V_B\right)\right]\frac{C_s r_B}{x_1\left(2x_2 - V_B\right)} \quad (13)$$

where v is an external input.

The control (13) allows us to have a single integrator between the output power z_1 and the new external input v. A second control stage, corresponding to a simple output feedback enables to stabilize the system with a desired dynamic.

$$v = K(z_1 - P_c) \quad (14)$$

where P_c is the desired power.

The zero dynamics is not completely analyzed and only simulation results are able to concluded that the studied system is not of non-minimum phase type.

IV. Results

Different simulations have been performed to test the proposed control law. The aim is to ensure the performance of the control and to analyze the robustness of the control. At least we will discussed about the reaching the point, depending on the climatic conditions, where the terminal power on the battery is maximal. For all the presented simulations the parameters values are :

• for the panel : $n_p = 1$, $n_s = 36$, $\rho = 1.5$, $I_{cc} = 5\ A$, $I_S = 1.2e^{-10}\ A$, $R_{sh} = 1\ K\Omega, R_s = 0.07\ \Omega$, $k = 1.38e^{-23}$, $T = 50°C$, $q = 1.6e^{-19}C$.

• for the DC-DC converter : $L = 2.2\ mH$, $Cs = 30\ \mu F$, $Ce = 1.2\ \mu F$.

• for the battery : $r_B = 0.5\ \Omega$, $V_B = 48\ Volts$.

It can be note that the different proposed simulation have been made by considering only the average model of the system. Actual works test the proposed control using an exact model of the system in which the converter switching action and the conduction modes are taken into account.

A. Performances

The first presented simulation (fig. 3) consists in a simple step response. In this test the desired output power passed from an initial point 80 *watts* to 90 *watts*. On this figure, we can observe that all is correct. The control law is able to stabilize the system on the desired output power in a time less than 0.1 *ms*. This setting time is directly related to the value of the output feedback gain (equation 14), which has been set for these simulations to $K = 15000$. A

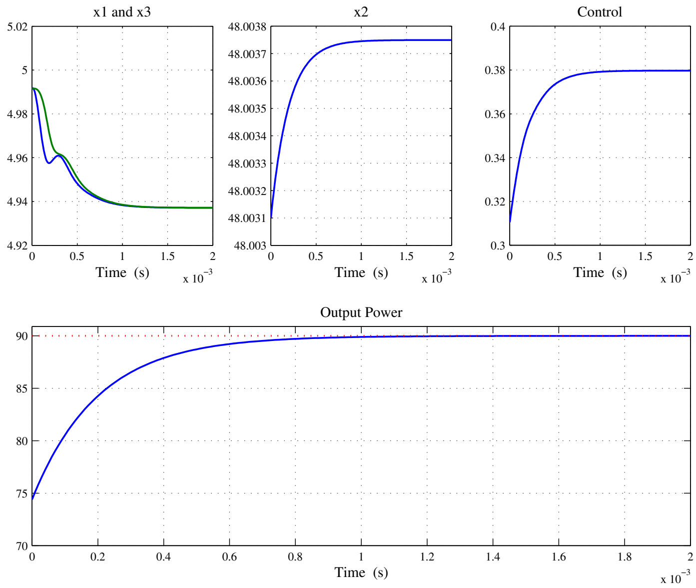

Fig. 3. Performance of the control law

second simulation (fig. 4) is very interesting with regard to the numerical sensitivity of the used model. In this test, a unreachable value for the desired value ($P_c = 110$ *watts*) is given. At first, the control seems able to stabilize the output with the desired power. However at the time $t = 2\ ms$, the decline of the the currents x_1 and x_3 leads the control to its saturation level. When the control reaches the saturation, the voltage x_2 and the output power fall to a minimum value. Moreover when at the time $t = 3\ ms$ the desired power come back to a reachable value ($P_c = 75$ *watts*) the system cannot leave its saturate point. This situation is due to the fact that the validity of the model is lost when currents get small values. Also, no solution was required to solve this point because it seems desirable to await experimental tests to observe what really occurs in this case.

B. Robustness

In this part , the robustness of the control compared to the variations of irradiation of the solar panel will be tested. The aim of this manipulation is to show that variations of irradiation of the solar panel do not affect performances of the control law. The principal parameter which influences the electric power available to the exit of the panel is the irradiation,noted p_s, corresponding to the photoelectrical energy. For all the presented simulation this parameter

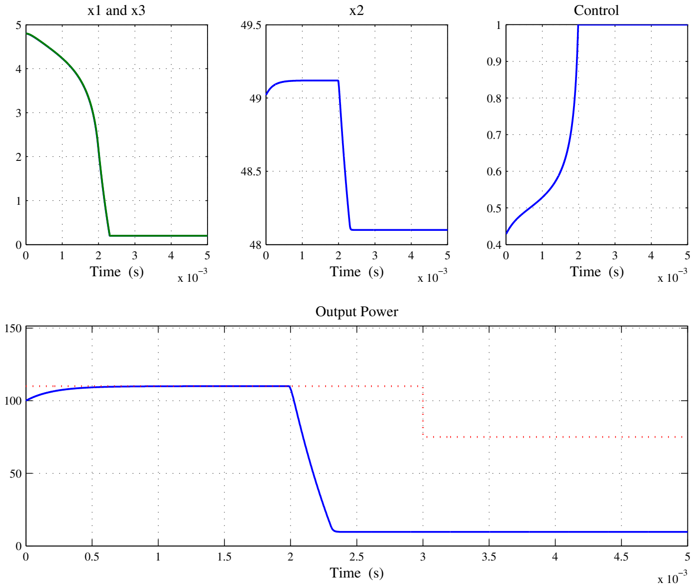

Fig. 4. The problem of the loss of validity of the model

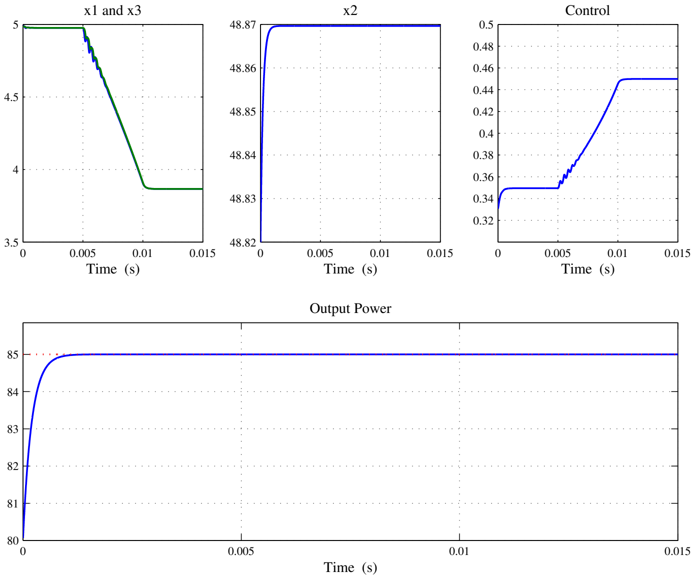

Fig. 5. Test of robustness of the proposed control

has been fixed to $p_s = 1000\ Watt/m^2$. From this values is calculated the value of the current $I_c c$:

$$I_c c = \frac{p_s}{200}$$

and the temperature of the panel

$$T = \frac{3 * p_s}{100} + 30$$

In the case of the simulation presented on the figure 5 the parameter P_s i passed from its initial values $p_s = 1000\ Watts/m^2$ to the final value $p_s = 800\ Watts/m^2$ between the time $t = 5\ ms$ and $t = 10\ ms$. The results f the simulation show that the output power is maintained at its value of reference. Different simulations have been performed to ensure this robustness and all the results found confirm the presented simulation. The only case when the decline of the photoelectrical energy doesn't permit to regulate the output power correspond to the case presented figure 4. Indeed, if the variation leads so that the desired power becomes unreachable then the point of saturation will be reached.

C. The reach of the maximum

The main objective for this control problem is to detect the maximum of electrical power which can be delivered at the battery. The first idea to carry out this research is to apply a ramp of power in entry of the controlled system. When the controlled output does not manage any more to follow this ramp, then the maximum is reached. Unfortunately this simple idea cannot be applied because during a non negligible transient time an unreachable power can be temporality reached and there is no easy way to distinct a transient state with a permanent state. Another strategy was applied to detect the maximum of power which consists in analyzing the speed of variation of the control U. Indeed, when the system falls towards its point of saturation, that correspond to a very fast increasing of the control U. So, if the ramp is stopped when a fixed value of derivation of the control is reached then, with a good precision, the maximum is detected. A example of this algorithm is proposed figure 6. The threshold of detection for the derivation of U has been fixed to 10 and the ramp start from $P_C = 100 Watts$ with a rate of 200 $watts.s^{-1}$. At the time $t = 10.2\ ms$ the threshold is reached and correspond to the power of $P_c = 102.04\ watts$. During the remainder of simulation, P_c keeps this value.

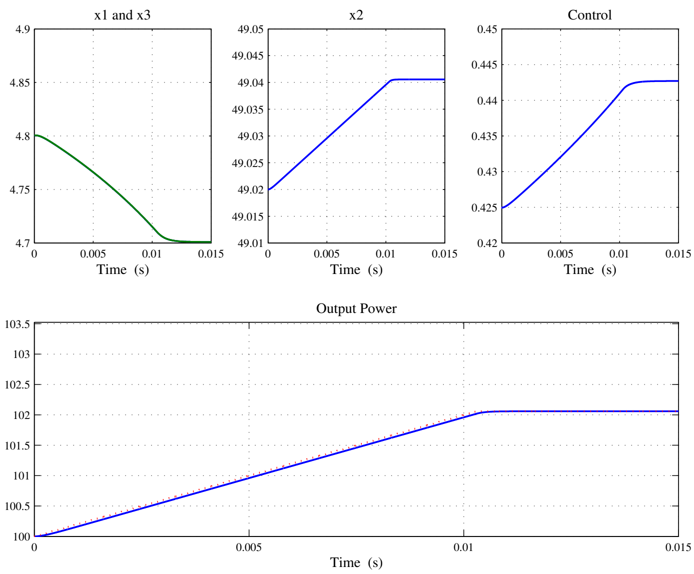

Fig. 6. Research of the maximum of electric power

V. Conclusion

In this paper a non linear approach has been proposed to control the loading of a battery. The electric energy provide from a solar panel. The model of the different subsystem as well as the whole system has been given. Some simulation results are presented and in particular those obtained for founding the the functioning point corresponding to the maximum of electric power at the terminal of the battery. The research of maximum is useful to optimize the load of the battery. The main difficulty of the proposed method is that when the desired power is larger than the electrical possibility, then the system falls to a saturation point and the validity of the model is lost. The actual work consist in test experimentally the command for, on the one hand, validating the demarche and on the other, analyzing the behaviors of the system in the saturated case evoked above. Another work relates to the taking into account of a more complete model of the battery.

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