Asymptotic stabilizability of non-linear delay systems by using delayed output

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ABSTRACT

In this note, two main results are introduced in order to deal with asymptotic stabilization of non-linear delayed systems. The first of them investigates algebraic conditions in order to ensure asymptotic stabilizability by using a Lyapunov-Krasovskii approach, while the second result introduces a simple way to algebraically design an asymptotic stabilizing controller for a class of non-linear delayed system.

Index terms – Non linear systems; delay systems; asymptotic stabilizability

I. INTRODUCTION

Models with delay are a useful way to deal with a wide class of physical systems, particularly circuits and electrical systems (see, for instance, (Hou et al, 2000; Komishi et al, 2000). Non-linearities combined with delays are common in the field, as many recent researchers point it out (Cao et al, 2002; Chen et al, 1988). Usually, stabilization is a general problem faced when dealing with this kind of systems. When studying such a question, a number of approaches can be undertaken. These approaches vary from the very role of the delay in the model to the nature of the non-linearity. With respect to the delay, some works consider it as a perturbation (Balachandran and Daner, 1987; Barmish and Shi, 1989), while others consider it as an inherent part of the system (Fu et al, 1989), a fact that constitutes a more general perspective. Regarding the non-linearities' nature, in many cases researchers try to create an ad hoc representation for a particular problem (Maffucci and Miano, 1998). However, sometimes a quite general form is adopted for the non-linearity in order to develop methods for broader sets of systems.

The stability of delay-differential systems with state point delays has been studied in different works (Pandolfi, 1975; Olbrot, 1978; Tadmor, 1988). More specifically, the stabilizability of such systems (i.e., the study of the conditions under which such systems become stable, a study leading to the design of stabilizing devices) has been addressed in a number of papers. Usually, the stability and stabilizability criteria belong to one of two categories: those criteria which are valid irrespectively of the delay numerical value (Kamen, 1982), and those other ones that are dependant of the delay (Jury and Mansour, 1982; Schoen and Geering, 1993).

This note studies the stabilizability of systems with state point delay and with a quite general form of non-linearity. A Lyapunov-Krasovskii approach is employed, which eventually leads to a result providing conditions for a direct design of the stabilizing controller by taking into account specific parameters of the plant and of the non-linearity.

II. MAIN RESULTS

In this section, conditions for a control law to stabilize a class of non-linear delay-differential systems with a point delay in the state is discussed by using an associated extended system. Two main results are introduced.

Let us consider the following state-space mathematical model for a non-linear closed-loop delay system

$$\dot{x}(t) = A_0x(t) + A_1x(t-h) - N\xi(t,h) + B_0u(t) + B_1u(t-h) \quad (1.1)$$

$$y(t) = C_0x(t) + C_1x(t-h) \quad (1.2)$$

where $A_0, A_1 \in \mathrm{R}^{n\times n}, N, B_0, B_1 \in \mathrm{R}^{n\times q}, C_0, C_1 \in \mathrm{R}^{p\times n}$, $x(t) = g(t), \forall t < 0$ and the non-linearity $\xi : \mathrm{R}^+ \times \mathrm{R}^p \to \mathrm{R}^q$ satisfies the growth condition (Bliman 2002)

$$\exists K \in \mathrm{R}^{q\times p} \;/\; \forall t \geq 0, \forall y \in \mathrm{R}^p, \|\xi(t,y)\| \leq \|Ky\| \quad (2)$$

The next lemma will be useful in the sequel.

Lemma 1 (Krasovskii 1963)
System (1) is global asymptotically stable in open loop (i.e., $u(t) \equiv 0$) provided that there exist $P, Q \in \mathrm{R}^{n\times n}$, positive definite, such that

$$R = R(P,Q) \equiv \begin{pmatrix} A_0^TP + PA_0 + Q + C_0^TK^TKC_0 & PA_1 + C_0^TK^TKC_1 & -PN \\ A_1^TP + C_1^TK^TKC_0 & -Q + C_1^TK^TKC_1 & \bar{0} \\ -N^TP & \bar{0} & -I \end{pmatrix} < 0 \quad (3)$$

where $\bar{0}$ is a $n \times n$ block of zeroes. #

Remark 1
The hypothesis in lemma 1 is the condition for success of the Lyapunov-Krasovskii method, when searching for functionals in the class $V(\phi) = \phi^T(0)P\phi(0) + \int_{-h}^{0} \phi^T(s)Q\phi(s)ds$.

Now, based in the above-mentioned lemma, let us introduce a main stabilizability result.

Theorem 1: delay-independent asymptotic stabilizability
System (1) is global asymptotically stabilizable by a control law u(t), being continuous with perhaps bounded discontinuities on a subset $S \subset [-h,0]$, defined by the delay-differential equation

$$\dot{u}(t) = D_0x(t) + D_1x(t-h) + E_0u(t) + E_1u(t-h) \quad (4)$$

and subject to initialization $u(t) = \varphi_u(t)$ on [-h, 0], provided that there exist $P^z, Q^z \in \mathrm{R}^{2n\times 2n}$, positive definite, such that

$$R^z = R^z(P^z,Q^z) \equiv \begin{pmatrix} A_0^{zT}P^z + P^zA_0^z + Q^z + C_0^{zT}K^{zT}K^zC_0^z & P^zA_1^z + C_0^{zT}K^{zT}K^zC_1^z & -P^zN^z \\ A_1^{zT}P^z + C_1^{zT}K^{zT}K^zC_0^z & -Q^z + C_1^{zT}K^{zT}K^zC_1^z & \bar{0} \\ -N^{zT}P^z & \bar{0} & -I \end{pmatrix} < 0 \quad (5)$$

where

$$A_0^z \equiv \begin{pmatrix} A_0 & B_0 \\ D_0 & E_0 \end{pmatrix};\; A_1^z \equiv \begin{pmatrix} A_1 & B_1 \\ D_1 & E_1 \end{pmatrix};\; N^z \equiv \begin{pmatrix} N & \bar{0} \\ \bar{0} & I \end{pmatrix};$$
$$C_0^z \equiv \begin{pmatrix} C_0 & \bar{0} \\ \bar{0} & I \end{pmatrix};\; C_1^z \equiv \begin{pmatrix} C_1 & \bar{0} \\ \bar{0} & I \end{pmatrix};\; K^z \equiv \begin{pmatrix} K & \bar{0} \\ \bar{0} & I \end{pmatrix} \quad (6)$$
#

Proof. The proof follows immediately from Lemma 1.

Define vectors $z(t) \equiv \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}$; $w(t) \equiv \begin{bmatrix} y(t) \\ \bar{0} \end{bmatrix}$ and $\xi^z(t,h) \equiv \begin{bmatrix} \xi(t,h) \\ \bar{0} \end{bmatrix}$. From equations (1), (4) and (6) one gets

$$\dot{z}(t) = A_0^z z(t) + A_1^z z(t-h) - N^z\xi^z(t,h) \quad (7.1)$$

$$w(t) = C_0^z z(t) + C_1^z z(t-h) \quad (7.2)$$

System (7) is called "associated extended system" of system (1) (Alastruey et al 1995). But system (7) is - without loss of generality - formally identical to system (1) in open loop, and then

Lemma 1 is applicable. Therefore system (7) is asymptotically stable. #

Theorem 1 provides a way to evaluate the asymptotic stabilizability of a non-linear delay-differential system. The evaluation is made on the context of a set of algebraic relations, which are very suitable for computer applications. A way of satisfying Theorem 1 is the synthesis of explicit stabilizing controllers fulfilling the following direct corollary.

Corollary 1: controller design

Provided that there exist $P^z, Q^z \in \mathrm{R}^{2n\times 2n}$, positive definite, and $n \times n$ block diagonal, then control signal (4) asymptotically stabilizes system (1) if the following algebraic condition holds

$$R_c < -R_p \qquad (8)$$

where

$$R_p \equiv \begin{pmatrix} A^{zT}_{0p} P^z + P^z A^z_{0p} + C^{zT}_0 K^{zT} K^z C^z_0 & P^z A^z_{1p} + C^{zT}_0 K^{zT} K^z C^z_1 & -P^z N^z \\ A^{zT}_{1p} P^z + C^{zT}_1 K^{zT} K^z C^z_0 & C^{zT}_1 K^{zT} K^z C^z_1 & \bar{0} \\ -N^{zT} P^z & \bar{0} & -I \end{pmatrix} \qquad (9)$$

$$R_c \equiv \begin{pmatrix} A^{zT}_{0c} P^z + Q^z + P^z A^z_{0c} & P^z A^z_{1c} & \bar{0} \\ A^{zT}_{1c} P^z & -Q^z & \bar{0} \\ \bar{0} & \bar{0} & -\frac{1}{2} I \end{pmatrix} \qquad (10)$$

$$A^z_{0p} \equiv \begin{pmatrix} A_0 & B_0 \\ \bar{0} & \bar{0} \end{pmatrix}; \quad A^z_{1p} \equiv \begin{pmatrix} A_1 & B_1 \\ \bar{0} & \bar{0} \end{pmatrix};$$
$$A^z_{0c} \equiv \begin{pmatrix} \bar{0} & \bar{0} \\ D_0 & E_0 \end{pmatrix}; \quad A^z_{1c} \equiv \begin{pmatrix} \bar{0} & \bar{0} \\ D_1 & E_1 \end{pmatrix} \qquad (11)\ \#$$

III. Proof

Note that, provided that $P^z, Q^z \in \mathrm{R}^{2n\times 2n}$ are positive definite and $n \times n$ block diagonal, the following identity holds

$$R = R_p \oplus R_c \qquad (12)$$

where $\oplus$ stands for the direct matrix sum. Then, according to the hypothesis, the next chain of identities holds

$$R < 0 \Rightarrow R_p \oplus R_c < 0 \Rightarrow R_c < -R_p \qquad (13)\ \#$$

Remark 2

Note that Corollary 1 provides a way of satisfying Theorem 1 by way of the synthesis of an explicit stabilizing controller. A very simple algebraic relation, i.e., equation (8), can check such a controller's existence, a fact that is again very suitable for computer applications.

IV. CONCLUSIONS

This paper provides sufficient conditions for testing asymptotic stabilizability of a class of non-linear delay-differential systems with one distributed delay in the state.

Conditions are given in terms of algebraic relations, appropriate for computer applications. The control is given in terms of a control law defined by using a dynamic differential equation which contains a point delay of the same numerical value than that in the original open-loop system.

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VI. REFERENCES

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