Friction Model for Adaptive Compensation

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Abstract— **The paper introduces a new model to describe the friction phenomenon at low velocities. The model has the useful property that it is linear in parameters. Using this model robust adaptive control algorithm is developed for high precision positioning. Simulations were performed to show the performances of the applied control algorithm.**

Keywords— **Friction modelling, Friction Compensation, Robust Adaptive Control.**

I. Introduction

Friction is a nonlinear phenomenon that causes the performances of servomechanisms to deteriorate. In high precision positioning systems it is inevitable to know or estimate the exact value of the friction force to assure good control characteristics and to avoid some undesired effects such as limit cycle and steady state error.

Many models were developed to explain the friction phenomenon, most of them based on experimental results. Generally, these models represent the friction force in function of velocity and three (or less) terms of the friction force are considered: the static friction (the value of the friction force at zero velocity), the Coulomb friction (that depends on the sign of velocity and the viscous term (proportional with velocity). Tribological experiments showed that this simple model cannot explain some phenomena in the low velocity regime, such as the Striebeck effect (decreasing friction force with increasing velocity), friction lag (delay between a change in velocity and the corresponding change in friction), presliding displacement. Moreover the friction may change as a function of normal forces in contact, temperature variations, position etc. so the parameters of the friction models should be considered time varying. This is why new models were introduced to explain this phenomenons [1]. The friction models for low velocities are discussed in detail in the next Section.

Making allowance for friction force in mechanical control systems is inevitable, if fine tracking is desirable. To compensate the effect of the friction, generally a feedforward term is introduced in the structure of the controller, which aim is the cancellation of the effect of friction force. In previous works that deals with friction compensation, two trends can be separated: model-oriented friction compensation techniques and friction modelling using soft computing methods. Being a nonlinear mapping between the velocity and friction force, many papers tries to model the friction phenomenon using universal approximators such as neural networks or fuzzy systems. Using feedforward type neural networks, a direct compensation of the friction force was proposed in [2] for servo-systems with unknown dynamics. In [3] the friction phenomena is modelled using RBF type networks. Fuzzy logic based model, describing the friction present in a DC motor is derived in [4] through offline fuzzy clustering techniques. Adaptive observers were developed for friction compensation in [5] [6] [7] with guaranteed stability. This papers takes into consideration only the Coulomb friction term. A novel dynamic friction model was also introduced which explains some dynamic behavior in the friction phenomena as Dahl-effect and presliding displacement [8]. But both the states and the parameters of that type of dynamic models are unknown so the tractability and the usage of this model are difficult. However, adaptive estimation of some parameters and state estimation techniques for the states of this model were developed [9], [10], [11].

Independently of the applied model or compensation strategy we always will have some model uncertainty that should be taken into consideration at the design of control strategy. At the other hand, as it was mentioned before the friction parameters are not constant so on-line estimation of the parameters is necessary. For these reasons the present paper uses robust adaptive techniques [12] to compensate the friction force. Recent results in robust adaptive control are presented in [13]. An early result in robust adaptive friction compensation is presented in [14].

The rest of the paper is organized as follows: Section II presents the introduced friction model. In Section III a robust adaptive control algorithm is developed for linear positioning system in which the friction force is modelled using the previously introduced model. The stability of the close loop system is also discussed. Simulation results are presented in Section IV.

II. Linearly Parameterized Friction Model

Many servo-controlled machines are lubricated with oil or grace (hydrodynamic lubrication). To describe the friction phenomenon generally, four regimes of lubrications are treated. Static Friction (1): the junctions deform elastically and there is no excursion until the control force do not reach the level of static friction force. Boundary Lubrication (2): this is also solid to solid contact, the lubrication film is not yet built. A sliding of friction force occurs in this domain of low velocities. Partial Fluid Lubrication (3): the lubricant is drawn into the contacts. Until the fluid film is not thicker than the height of aspirates in the contact regime, some solid-to-solid contacts will influence the motion. Full Fluid Lubrication (4): The viscous term dominates the friction phenomenon, the solid-to-solid contact is eliminated. The value of the friction force can be

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considered as proportional with the velocity. From these domains results a highly nonlinear behavior of the friction force. Appearing to zero velocity the friction force decreases in function of velocity and at higher velocities the viscous term will be dominant so the friction force increases with velocity. Moreover the friction also depends on the sign of velocity with an abrupt change when the velocity pass through zero.

To explain this nonlinear behavior, several models were introduced such as the exponential model, Lorentzian model or Gaussian model. These models represent the friction force in function of velocity. Let us mention that all this models were experimentally introduced in such way to fit measurement data. In spite of the fact that they describe well qualitatively the friction phenomena but the exact values of the parameters for these models hardly can be determined. The most wide spread model is the exponential model because its relative simplicity.

$$F_f(v) = \begin{cases} \alpha_{0_+} + \alpha_{1_+} v + \alpha_{2_+} e^{-\alpha_{3_+} v}, for\ v \geq 0 \\ \alpha_{0_-} + \alpha_{1_-} v + \alpha_{2_-} e^{-\alpha_{3_-} v}, for\ v < 0 \end{cases} \quad (1)$$

Note that this model has eight parameters, four for each velocity domain (positive and negative) but there are parameters, α_3, that do not multiply linearly a known function, these parameters are included in a nonlinear function.

The model, introduced in this paper, was developed based on this exponential model. For the simplicity only the positive velocity domain is considered, but same study can be made for the negative velocities. Let us consider that our mechanical system moves in $0 \dots v_{max}$ velocity domain.

Let us consider a linear approximation for the exponential curve represented by two lines: d_{1_+} which cross through the $(0, F_f(0))$ point and it is tangent to curve and d_{2_+} which passes through the $(v_{max}, F_f(v_{max})$ point and tangential to curve. These two lines meet each other at the v_{sw} velocity. In the domain $0 \dots v_{sw}$ the d_{1_+} can be used for the linearization of the curve and d_{2_+} is used in the domain $v_{sw} \dots v_{max}$. If these approximations are used for linearization of the nonlinear curve, easily can be seen that *the approximation error is always positive and bounded*. The maximum value of approximation error occurs at the velocity v_{sw} for both linearizations.

If we consider the positive part of the friction model (1), the obtained equations for the d_{1_+} and d_{2_+}, using Taylor expansion, are:

$$d_{1_+} : F_{L1f_+}(v) = \alpha_{0_+} + \alpha_{2_+} + \left.\frac{\partial F_f(v)}{\partial v}\right|_{v=0} \quad (2)$$
$$= \alpha_{0_+} + \alpha_{2_+} + (\alpha_{1_+} - \alpha_{2_+}\alpha_{3_+})v$$
$$d_{2_+} : F_{L2f_+}(v) = F_f(v_{max}) + \left.\frac{\partial F_f(v)}{\partial v}\right|_{v=v_{max}} \quad (3)$$
$$= F_f(v_{max}) + (\alpha_{1_+} - \alpha_{2_+}\alpha_{3_+} e^{-\alpha_{3_+} v_{max}})v$$

The value of switching velocity v_{sw} can be determined form (2) and (3):

$$v_{sw} = \frac{\alpha_{0_+} + \alpha_{2_+} - F_f(v_{max})}{\alpha_{2_+}\alpha_{3_+}(1 - e^{-\alpha_{3_+} v_{max}})} \quad (4)$$

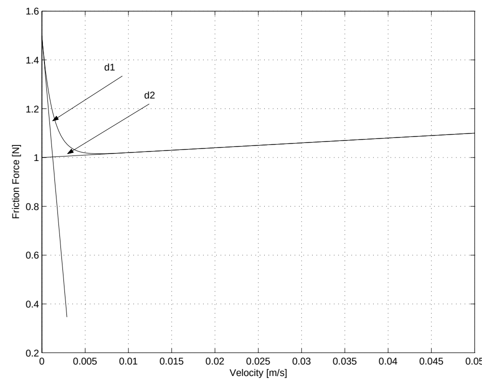

Fig. 1. Linearization of exponential friction model

So in the linearization of the exponential friction model with bounded error can be done with two lines in the $0 \dots v_{max}$ velocity domain:

$$d_{1_+}: F_{L1f_+}(v) = a_1 + b_1 v, for\ 0\ \leq v\ \leq\ v_{sw} \quad (5)$$
$$d_{2_+}: F_{L2f_+}(v) = a_2 + b_2 v, for\ v_{sw}\ \leq v\ \leq\ v_{max} \quad (6)$$

Now let us consider two exponential membership functions parameterized in the following way:

$$\phi_{1_+}(v) = \frac{e^{-\beta(v - v_{sw})}}{1 + e^{-\beta(v - v_{sw})}}\ for\ v \geq 0, \quad otherwise\ \ \phi_{1_+}(v) = 0 \quad (7)$$
$$\phi_{2_+}(v) = \frac{1}{1 + e^{-\beta(v - v_{sw})}}\ for\ v \geq 0, \quad otherwise\ \ \phi_{2_+}(v) = 0 \quad (8)$$

where β a large positive number and v_{sw} is defined in (4).

If we apply the F_{L1f_+} from (5) on the membership function ϕ_1 from (7) and F_{L2f_+} on ϕ_2 we can obtain a new model that has the same behavior as the exponential friction model, moreover it is linearly parameterized if we consider the parameters of the lines. So for the positive velocity domain we have:

$$F_{f_+}(v) = a_1\phi_{1_+}(v) + b_1 v\phi_{1_+}(v) + a_2\phi_{2_+}(v) + b_2 v\phi_{2_+}(v) \quad (9)$$

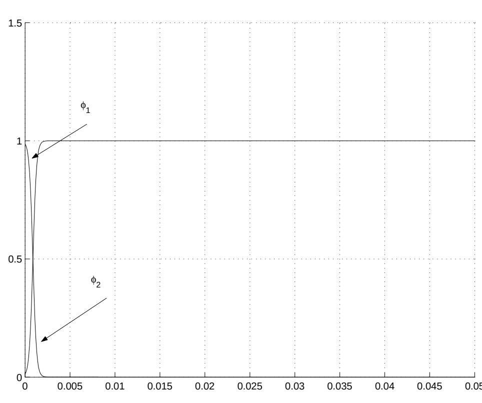

Fig. 2. Membership functions

With same train of thoughts a similar model can be determined for the negative velocity domain. Combining the negative and positive velocity domains the obtained friction model reads as:

$$F_f(v) = \underline{\theta}^T \underline{\xi}(v) \tag{10}$$
$$where:\ \underline{\theta} = (a_1\ b_1\ a_2\ b_2\ a_3\ b_3\ a_4\ b_4)$$
$$\underline{\xi}(v) = (\phi_{1_+}\ v\phi_{1_+}\ \phi_{2_+}\ v\phi_{2_+}\ \phi_{1_-}\ v\phi_{1_-}\ \phi_{2_-}\ v\phi_{2_-})$$

The shape of the obtained model is presented on Fig. 3.

Let us mention that the membership functions depends on v_{sw} which should be known *a priori* if we would like to use this model as a linearly parameterized one. So it should be determined from measurements. The effect of v_{sw} on the control characteristics is discussed in the Section IV.

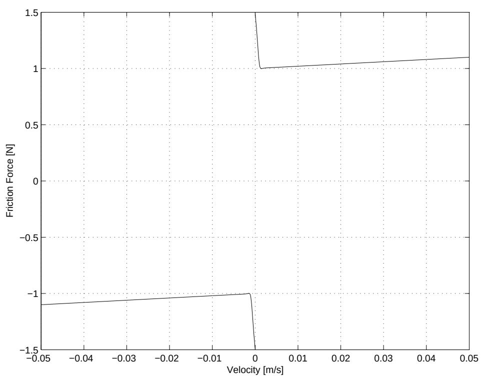

Fig. 3. Linearized Friction Model

III. Robust Adaptive Method for Friction Compensation

To develop an adaptive compensation method that uses the previously introduced friction model, a 1 Degree Of Freedom dynamics were considered, in which the friction force is modelled using (10). The dynamics of the system reads as:

$$m\dot{v} + \underline{\theta}^T \underline{\xi}(v) = ku + d \tag{11}$$

where $m > 0$ is the mass of the load, $k \geq k_m > 0$ is the amplification of the drive with known minimum value k_m and d is unmeasurable bounded disturbance, i.e. $|d| < D_M$ with D_M known. The position output x of the plant (11) is defined as $\dot{x} = v$.

Let us define the tracking error $e(t) = x_d(t) - x(t)$ and the tracking error metric $S(t) = (\frac{d}{dt}+\lambda)e(t)$ with $\lambda > 0$. x_d is the prescribed trajectory, a smooth, twice differentiable function in time.

The control problem can be formulated as follows: design a control law u such as that the tracking error metric $S(t)$ satisfies $|S(t)| < \Phi$ for $t \to \infty$ where Φ is a given precision.

Differentiating $S(t)$ with respect to time we obtain:

$$m\dot{S}(t) = m(\ddot{x}_d + \lambda\dot{e}(t)) + \underline{\theta}^T \underline{\xi}(v) - ku - d \tag{12}$$

The parameters $\underline{\theta}$, m, k are unknown, consequently the control law can be developed using estimated parameters, that are generated on-line by an adaptation rule. Let us denote the estimation errors and the estimated parameters as follows:

$$\underline{\widetilde{\theta}} = \underline{\theta} - \underline{\widehat{\theta}} \qquad \widetilde{m} = m - \widehat{m} \qquad \widetilde{k} = k - \widehat{k} \tag{13}$$

Define the following control law:

$$u = \frac{1}{\widehat{k}(1+\rho(k_m - \widehat{k}))}(-\widehat{m}(\ddot{x}_d + \lambda\dot{e}(t)) - \underline{\widehat{\theta}}\xi(v) - k_S S_\Delta(t) - D_M sat(S/\Phi)) \tag{14}$$

with $k_S > 0$ and ρ is defined as:

$$\rho = \begin{cases} 1, & \text{if } \widehat{k} < k_m \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

With this modification can be assured that the denominator of the control law will never be smaller than k_m for any $\widehat{k}$.

$S_\Delta(t) = S(t) - sat(S(t)/\Phi)$ where $sat(\cdot)$ denotes the saturation function. The following propriety can easily be verified:

$$\dot{S}_\Delta = \dot{S} \text{ for } |S_\Delta| \geq \Phi \text{ and } \dot{S}_\Delta = 0 \text{ otherwise.} \tag{16}$$

The most common adaptation rule in the adaptive control systems is the gradient method [12], [15]. To increase the robustness of the control system a modified gradient algorithm, the switching-σ adaptation law [12] is applied in this paper:

$$\begin{aligned} \dot{\widehat{\theta}}_i &= -\gamma_{\theta_i}\xi(v)S_\Delta(t) - \sigma(\widehat{\theta}_i)\gamma_{\theta_i}\widehat{\theta}_i \\ \dot{\widehat{m}} &= -\gamma_m(\ddot{x}_d + \lambda\dot{e}(t))S_\Delta(t) - \sigma(\widehat{m})\gamma_m\widehat{m} \\ \dot{\widehat{k}} &= -\gamma_k u(1+\rho(k_m - \widehat{k}))S_\Delta(t) - \sigma(\widehat{k})\gamma_k\widehat{k} \end{aligned} \tag{17}$$

with γ_{θ_i}, γ_m, γ_k strictly positive adaptation gains.

The function σ is defined as:

$$\sigma(\widehat{\theta}) = \begin{cases} 0, & \text{if } \Theta \leq |\widehat{\theta}| \\ \sigma_0(|\widehat{\theta}|/\Theta - 1), & \text{if } |\widehat{\theta}| < \Theta \leq 2|\widehat{\theta}| \\ \sigma_0, & \text{otherwise} \end{cases} \tag{18}$$

where $\sigma_0 > 0$ and $\Theta > |\theta|$.

The closed loop system for $S \geq \Phi$ with this control law can be written as:

$$m\dot{S}(t) = \widetilde{m}(\ddot{x}_d + \lambda\dot{e}(t)) + \underline{\widetilde{\theta}}^T\xi(v) + \widetilde{k}u - k_S S_\Delta(t) - (D_M sat(S/\Phi) - d) \tag{19}$$

To examine the behavior of the closed loop system let us consider the following Lyapunov like cost function:

$$V(t) = mS_\Delta(t)^2 + \frac{1}{\gamma_m}\widetilde{m}^2 + \frac{1}{\gamma_k}\widetilde{k}^2 + \sum_{i=1}^{8}\frac{1}{\gamma_{\theta_i}}\widetilde{\theta}_i^2 \tag{20}$$

Due to adaptation laws (17) and the propriety (16) $\dot{V}(t) = 0$ for $S(t) < \Phi$. Otherwise we have:

$$\dot{V}(t) = m\dot{S}_\Delta(t)S_\Delta(t) + \frac{1}{\gamma_m}\dot{\widehat{m}}\widetilde{m} + \frac{1}{\gamma_k}\dot{\widehat{k}}\widetilde{k} + \sum_{i=1}^{8}\frac{1}{\gamma_{\theta_i}}\dot{\widehat{\theta}}_i\widetilde{\theta}_i \tag{21}$$

The equation of the closed loop system can be introduced in (21) using the propriety (16). If we also introduce the adaptation laws (17) we obtain:

$$\dot{V}(t) = -k_S S_\Delta(t)^2 - S_\Delta(t)(D_M sat(S/\Phi) - d) + \sigma(\widehat{k})\widehat{k}\widetilde{k} + \sigma(\widehat{m})\widehat{m}\widetilde{m} + \sum_{i=1}^{8} \sigma(\widehat{\theta}_i)\widehat{\theta}_i\widetilde{\theta}_i \qquad (22)$$

Because it was assumed that $S(t) \geq \Phi$ we have $sat(S/\Phi) = sign(S) = sign(S_\Delta)$. Using the following simple relation that if $|d| < D_M \Rightarrow dS_\Delta \leq D_M\,|S_\Delta|$.

At the other hand if $\sigma_0 > 0$ and $\theta_0 > |\theta|$ from the definition (18) results $\sigma(\widehat{\theta})\widehat{\theta}\widetilde{\theta} \leq 0$. From these observations yields:

$$\dot{V}(t) \leq -k_S S_\Delta(t)^2 \qquad (23)$$

Notice that (23) is also valid for $|S(t)| < \Phi$. Since $V(t)$ is a positive and nonincreasing function, therefore $V(\infty)$ is finite and well defined.

Thus, if $S_\Delta(0)$, $\widetilde{m}(0)$, $\widetilde{k}(0)$ and $\underline{\widetilde{\theta}}(0)$ is bounded $\Rightarrow S_\Delta(t)$, $\widetilde{m}(t)$, $\widetilde{k}(t)$ and $\underline{\widetilde{\theta}}(t) \in L_\infty\ \forall\ t > 0$.

If $S_\Delta(t)$, $e(0)$ and $\dot{e}(0)$ is bounded $\Rightarrow e(t)$ and $\dot{e}(t) \in L_\infty$.

If $e(t)$, $\dot{e}(t)$, $x_d(t)$ and $\dot{x}_d(t) \in L_\infty \Rightarrow x(t)$, $\dot{x}(t) \in L_\infty$.

From (14) results that if $\widehat{m}$, $\widehat{k}$, $\underline{\widehat{\theta}}$, $\ddot{x}_d$, $\dot{e}(t)$ and $S_\Delta \in L_\infty \Rightarrow u(t) \in L_\infty$.

From:

$$\int_0^\infty S_\Delta(t)^2 dt \leq \frac{-1}{k_S}\int_0^\infty \dot{V}(t) = \frac{V(0) - V(\infty)}{k_S} < \infty \qquad (24)$$

results that $S_\Delta(t) \in L_2$.

From (12) results that if $S_\Delta(t)$, $\widetilde{m}(t)$, $\widetilde{k}(t)$, $\underline{\widetilde{\theta}}(t)$, $\ddot{x}_d(t)$, $\dot{e}(t)$ $u(t) \in L_\infty \Rightarrow \dot{S}_\Delta(t) \in L_\infty$.

Because $S_\Delta(t)$ and $\dot{S}_\Delta(t) \in L_\infty$ and the relation (24) holds, by Barbalat's lemma $S_\Delta(t) \to 0$ when $t \to \infty$, consequently the inequality $|S(t)| \leq \Phi$ is obtained asymptotically. Thus the control law (14) with the adaptation law (17) solves the formulated control problem.

IV. Simulations and Results

In order to test the applicability of our theoretical results we performed simulations on a linear positioning table described by the equation (11) with $m = 1kg$ and $k = 1$. The disturbance d was modelled by an additive random signal. The prescribed velocity is a sinusoidal one. The prescribed position was obtained by integrating this signal.

$$v_d(t) = 0.012 sin(16t) + 0.06 \qquad (25)$$

$$x_d(t) = \int_0^t v(\tau)d\tau; \quad \ddot{x}_d(t) = \frac{dv(t)}{dt}$$

The control objective is to track this prescribed position, such that the tracking error metric $S(t) \leq 10^{-3}$. The parameters of the controller were chosen as follows: $\lambda = 10,\ k_d = 8,\ D_M = 10^{-4},\ \Phi = 10^{-3}$.

All parameters of the mechanical system and the friction model was departed with 50% from its real values. The simulation results in Fig. 4 shows that the control law guarantees very precise tracking for position output. The convergence of the two friction parameters during the adaptation is also presented. Due to membership functions (7) it can be observed that the parameters determining the behavior of friction force are tuned only when the plant is in the corresponding velocity regime.

To show the effect of the switching velocity v_{sw} used in the membership functions, its value were departed with 50% from its real value and at the same time all the other controller parameters remained the same. The results from Fig. 5 shows that the parameters and the error metric converges much slower but in the long run position tracking error shows the same behavior.

Acknowledgement

Support for the research of adaptive friction compensation is provided by the Hungarian National Research program under grant No. OTKA T 042634 and by the Control System Research Group of the Hungarian Academy of Sciences.

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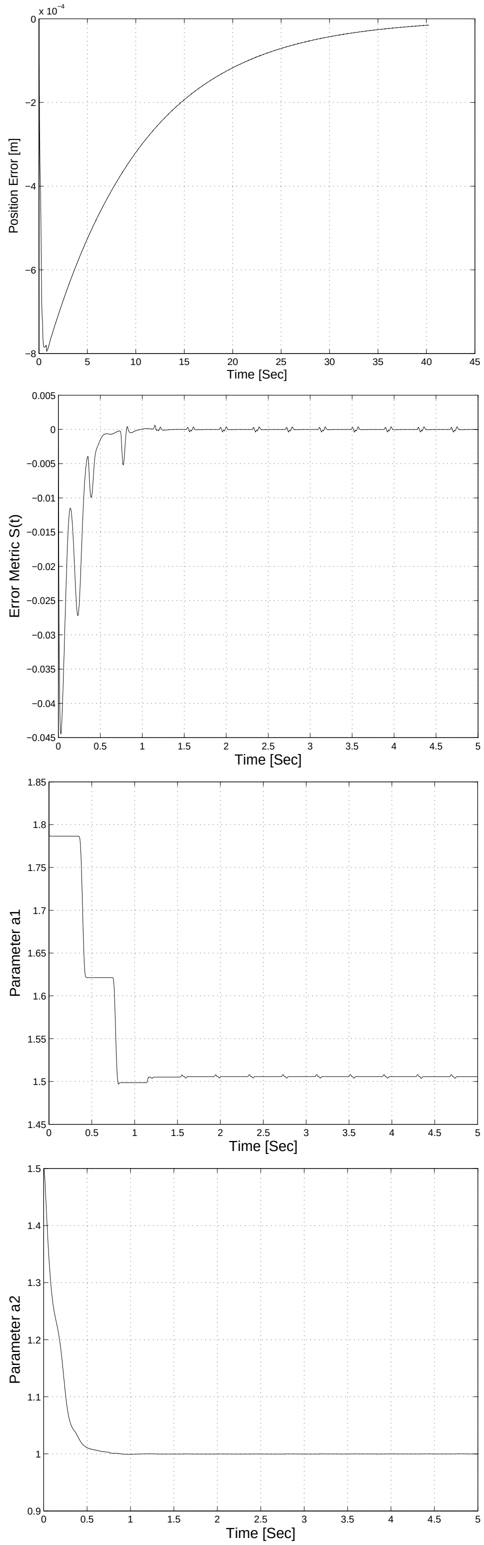

Fig. 4. Simulation Results

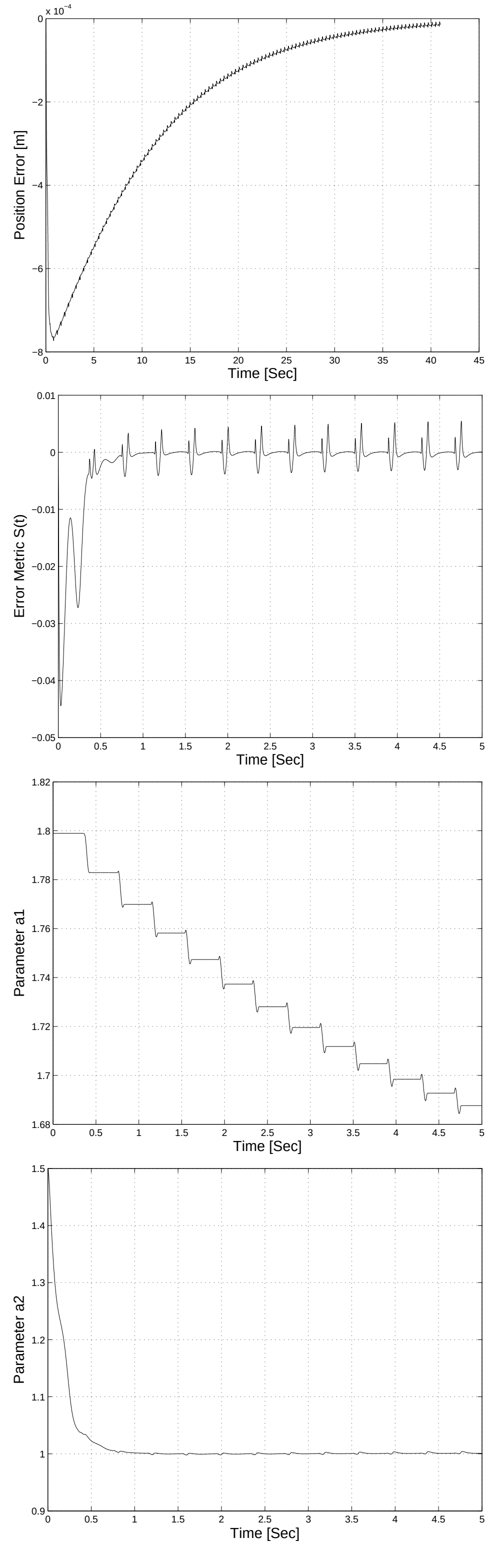

Fig. 5. Simulation results with modified v_{sw}