A New Continuous Time Adaptive Backlash Inverse Controller and Applications to Output Backlash Compensation

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Abstract

This paper presents an adaptive control scheme for systems with unknown output backlash. A new compact dynamical backlash inverse model is used in conjunction with a PD controller to improve the tracking performance of systems with a backlash hysteresis affecting the output of a plant. The new backlash inverse model is implemented in continuous-time and is continuously differentiable. The suggested scheme handles the case where the value of backlash spacing is known, as well as the case when the backlash spacing is unknown. For the latter case, an adaptive update law is developed to estimate the unknown backlash spacing. The stability of the closed-loop system is shown using Lyapunov arguments. Simulation results show that the control methodology greatly improves tracking performance over a PD type controller.

I. Introduction

In many systems, performance is greatly compromised by the presence of hard nonlinearities such as friction and backlash. An essential task in designing controllers is to counter the undesirable effects of such nonlinearities. For instance, one of the major problems that arise when using gears is the problem of backlash. Backlash in gears caused by the fact that gears do not mesh exactly leaving some spacing between gears teeth. During motion reversal the spacing causes the load gear to momentarily lose contact with the driving gear, hence causing tracking error. Combating backlash can be achieved by either tightly meshing gears, which inherently increases friction and may lead to jamming, or by using very precise gears which, in many cases, is cost prohibitive. If gear backlash is not accounted for, it will result in the degradation of system performance and will reduce positioning accuracy. In addition, the fact that backlash is a multi-valued nonlinearity, it usually leads to internal energy storage which frequently causes instability and self sustained oscillations. Backlash like hysteresis can appear in actuators, such as hydraulic servo-valves, give rise to limit cycling and instability. Adaptive compensation is the most appropriate technique to handle the uncertainty in the backlash parameters; however, since backlash is not a differentiable nonlinearity, recent nonlinear control design methodologies such as backstepping cannot be applied. Recently, there have been newly proposed control schemes that address the problem of backlash. The majority of previous work concentrated on compensating for input backlash whereby the backlash nonlinearity precedes the plant input [1, 2, 3, 4, 5]. One exception is the discrete time implementation of the backlash inverse proposed in [6] for the case where the backlash nonlinearity preceded the actual output of the plant.

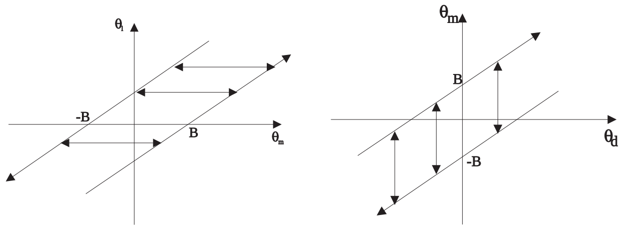
In this paper, we present a new continuous-time approximate backlash inverse to handle system with output backlash. A continuous-time adaptive backlash inverse controller is developed to handle the case where the backlash parameter is unknown. The control methodology is simple and can be combined with standard control methods. The results obtained are the global boundedness of the tracking error of a one degree-of-freedom (DOF) system. The use of a single adaptation gain without the need to know apriori an estimate of the backlash parameter makes the method efficient. The advantage of the new adaptive backlash inverse scheme is its continuous differentiability thereby making it an attractive choice to be combined with advanced control methods such as adaptive backstepping. Simulations results for the case where the backlash parameter is known and the case where it is unknown show great improvement in the reduction of the tracking error.

II. Backlash and the Exact Right Backlash Inverse

Consider a gear mounted directly on a DC motor driving another gear. The two gears are meshed with backlash spacing equal to $2B$. Backlash nonlinearity can be described as

$$\dot{\theta}_l = \begin{cases} m\dot{\theta}_m & \text{if } \dot{\theta}_m > 0 \quad \& \quad \dot{\theta}_l > 0 \quad \& \quad \theta_l = m(\theta_m - B), \\ & \text{if } \dot{\theta}_m < 0 \quad \& \quad \dot{\theta}_l < 0 \quad \& \quad \theta_l = m(\theta_m + B) \\ 0 & otherwise \end{cases} \tag{1}$$

where $\theta_l(t) = \theta_m(t) - B$ means that the gears teeth are in contact for the upward direction and $\theta_l(t) = \theta_m(t) + B$ for the downward motion (see Figure 1 (a)). Meanwhile, m is the slope of the lines which captures the gears ratio. In many applications



(a) (b)

Figure 1: (a)Backlash hysteresis. (b) Ideal backlash inverse.

an encoder that measures the position (i.e. the angle θ_m) of the motor are mounted directly on the motor before the measurement gets affected by the backlash nonlinearity. Similarly, another encoder would be mounted on the load side to measure the angle of the load (i.e. θ_l) which is affected by backlash. Therefore, it is reasonable to assume that the plant states are measurable and available for feedback.

The proper right backlash inverse is described in [4] and is restated here as follows:

$$\dot{\theta}_m = \begin{cases} \dot{\theta}_{ld} & \text{if } \dot{\theta}_{ld} > 0 \quad \text{and} \quad \theta_m = \theta_{ld} + B, \\ & \text{if } \dot{\theta}_{ld} < 0 \quad \text{and} \quad \theta_m = \theta_{ld} - B, \\ 0 & \dot{\theta}_{ld} = 0 \\ g(\tau, t) & \text{if } \dot{\theta}_{ld} > 0 \quad \text{and} \quad \theta_m = \theta_{ld} - B, \\ -g(\tau, t) & \text{if } \dot{\theta}_{ld} < 0 \quad \text{and} \quad \theta_m = \theta_{ld} + B, \end{cases} \tag{2}$$

where $g(\tau, t)$ represents an instantaneous jump from one line to the opposite one by an amount equal to $2B\delta(\tau, t)$, the full backlash distance. The effect of $g(\tau, t)$ is to force θ_m to instantaneously skip the backlash spacing thereby eliminating the damaging effect of loosing information when the driving gear is reversing direction while the load one is not. By examining equation 1 (b) we can summarize the backlash inverse function as follows:

$$\theta_m(t) = \theta_{ld}(t) + f(t) = \mathcal{BI}(\theta_{ld}) \tag{3}$$

where $f(t)$ is given by

$$f(t) = \begin{cases} B & \dot{\theta}_{ld} > 0 \\ -B & \dot{\theta}_{ld} < 0 \\ f(t-) & \dot{\theta}_{ld} = 0 \end{cases} \tag{4}$$

where $f(t-)$ indicates that no change should occur in the value of $f(t)$. This can be described by $\dot{f}(t) = 0$. By designing $f(t)$ we can make sure that the condition producing backlash will be avoided [4]. The problem with the exact backlash inverse is that it is discontinuous and nondifferentiable which is not desirable when performing trajectory pattern matching. Therefore, in [4] suggestions for a smooth approximations to $f(t)$ were presented and have been shown to add to the error in the mismatch between the backlash and its inverse.

Lemma 1 [4]: The characteristic $\mathcal{BI}(\cdot)$ defined by 1 (b) is the right inverse of the characteristic $\mathcal{B}(\cdot)$ in the sense: $\mathcal{B}(\mathcal{BI}(\theta_{ld}(t_0))) = \theta_{ld}(t_0)$ gives $\mathcal{B}(\mathcal{BI}(\theta_{ld}(t))) = \theta_{ld}(t) \quad \forall \quad t > t_0$.
Lemma 1 above states that if for some time t_0 the backlash inverse function is able to invert the backlash then it serves a backlash inverse for all time thereafter.

III. Smoothly Differentiable Backlash Inverse Model

The model that will be presented here is based on developing a continuous-time differentiable approximation to the right backlash inverse discussed above. Let θ_{ld} be the desired position to be tracked; moreover, let θ_m^* be an approximate inverse backlash signal of θ_{ld}(i.e. $\theta_m^* = \hat{\mathcal{BI}}(\theta_{ld})$), whereby the circumflex is used to distinguish it from the ideal $\mathcal{BI}(\cdot)$ presented previously in [4]. The idea here is to design a controller that makes the motor angle θ_m tracks θ_m^* which is going to be the input to the backlash nonlinearity at the output of the motor. Hence, when tracking is achieved then $\theta_m = \theta_m^*$ and the following is obtained

$$\theta_l = \mathcal{B}(\theta_m) = \mathcal{B}(\theta_m^*) = \mathcal{B}(\hat{\mathcal{BI}}(\theta_{ld})) = \theta_{ld} + e(t) \tag{5}$$

where $e(t)$ is the error in the mismatch between $\mathcal{B}(\cdot)$ and $\hat{\mathcal{BI}}(\cdot)$ which will be shown to be bounded. Define the following dynamics

$$\dot{W} = \tanh(k_s\dot{\theta}_{ld}) - \frac{\sigma}{B_n}W \tag{6}$$

where B_n is the nominal backlash distance, $k_s > 0$ used to ensure that the hyperbolic tangent behaves like a signum function, and σ is a positive gain constant used for stability purpose (will be shown shortly.) By examining the equilibrium points of $W(t)$, which can be obtained by setting $\dot{W}(t) = 0$ as follows

$$W = \frac{B_n}{\sigma}\tanh(k_s\dot{\theta}_{ld}). \tag{7}$$

Noting that the behaviour of hyperbolic tangent function $\tanh(k_s\theta_{ld)}$ for a sufficiently large k_s can be approximated by

$$W = \frac{B_n}{\sigma}\mathrm{sgn}(\dot{\theta}_{ld}). \tag{8}$$

It is evident that $\sigma W(t)$ satisfies the first and second cases of $f(t)$ defined in 4. Meanwhile, by setting $\dot{\theta}_{ld} = 0$ in 6 results in

$$\dot{W} = -\frac{\sigma}{B_n}W. \tag{9}$$

This shows that the $W(t)$ is exponentially stable with $W(t) = 0$ as its equilibrium point. Therefore, the third condition in 4 is satisfied since $\sigma W(t)$ exponentially approaches zero whenever $\dot{\theta}_{ld} = 0$. Therefore, the inverse hysteresis model may simply be written as

$$\begin{aligned} f(t) &= \sigma W(t) \\ \theta_m^*(t) &= \theta_{ld}(t) + f(t) = \mathcal{BI}(\theta_{ld}). \end{aligned} \tag{10}$$

If the value of the actual backlash distance B is not exactly equal to the nominal value B_n then an adaptation will be used to correct for the mismatch (This will be shown later.) It should be pointed out that the above backlash inverse is approximate to the ideal inverse backlash model which involves a $\delta(\tau, t)$ which is captured by the constant σ in the new model. The larger the σ-parameter the sharper is the slope of transitions in $W(t)$. Ideally a delta function of amplitude $2B$ in the proper direction would be necessary to exactly match the backlash behavior. Simulation results shows that the new backlash inverse model improves performance greatly and achieves excellent reduction in the tracking error.

Property of Backlash Inverse State: Assume that $0 < \frac{B}{\sigma} < \alpha$. If $|W(0)| \leq \alpha$ for

any α real constant, then $|W(t)| \leq \alpha \; \forall \; t > 0$.
Proof. Consider the following Lyapunov function

$$V = \frac{W^2}{2}.$$

Differentiating V along the trajectories of its dynamics, we obtain

$$\begin{aligned} \dot{V}_W &= W(\tanh(k_s \dot{\theta}_{ld}) - \frac{\sigma}{B_n} W) \\ &= W \tanh(k_s \dot{\theta}_{ld}) - \frac{\sigma}{B_n} W^2. \end{aligned} \tag{11}$$

In addition, applying the inequality

$$ab \leq \frac{\varsigma}{2} a^2 + \frac{1}{2\varsigma} b^2$$

to the first term in 11, then the term can be bounded as

$$\dot{V}_W \leq -(\frac{\sigma}{B_n} - \frac{\varsigma}{2}) W^2 + \frac{1}{2\varsigma} \tanh^2(k_s \dot{\theta}_{ld}). \tag{12}$$

Since the hyperbolic tangent function can be upper bounded by 1 then by properly choosing the values of σ and ς to satisfy the inequality

$$\frac{\sigma}{B_n} \;\; > \;\; \frac{1}{2}(\varsigma + \frac{1}{\varsigma})$$

ensures that $\dot{V}_W < 0$. As a result all solutions of $W(t)$ are bounded by a ball which can be made arbitrarily small by a proper choice of σ and ς. This concludes the proof.

IV. Backlash Inverse for a System with a Known Output Backlash Nonlinearity

Backlash distance in gears is usually supplied by the manufacturer of gears as part of the specification of the device. Based on that an exact backlash inverse is developed in this section to counter the effects that backlash has on tracking. The system considered here consists of a DC motor with no friction driving a gear which is connected to another (see Figure **??**.) It is assumed that both θ_m and ω_m, the motor angle and velocity respectively, are available for feedback and that they are the input to the backlash function. Meanwhile, θ_l, the load angle, is the output of the backlash function and is also measurable. The motor with backlash dynamics may be written as

$$\begin{aligned}
\dot{\theta}_m &= \omega \\
\dot{\omega}_m &= \tau \\
y(t) &= \theta_l = Backlash(\theta_m) = \mathcal{B}(\theta_m). \qquad (13)
\end{aligned}$$

Let the position tracking error be defined as $\tilde{\theta} = \theta_m - \theta_m^*$ where θ_m^* is defined as follows:

$$\theta_m^* = \hat{\mathcal{BI}}(\theta_{ld}) = \theta_{ld} + \sigma W(t) \qquad (14)$$

where θ_{ld} is the desired load trajectory and $W(t)$ is the inverse backlash dynamics given by

$$\dot{W} = \tanh(k_s \dot{\theta}_{ld}) - \frac{\sigma}{B_n} W, \qquad (15)$$

where $2B_n$ is the backlash spacing. Defining the following control law for the system in Figure 13

$$\tau = \ddot{\theta}_m^* - k_v \dot{\tilde{\theta}} - k_p \tilde{\theta} \qquad (16)$$

with a proper choice of k_p and k_v will ensure that θ_m will asymptotically track θ_m^*. As a result the stability of the closed-loop system is ensured.
Claim. We claim that the control law τ will assure the closed loop stability and boundedness of tracking error and hence minimizing the effects of backlash.

Proof. By applying the control laws in 16 to the system described by 13 we get

$$\ddot{\theta}_m = \ddot{\theta}_m^* - k_v \dot{\tilde{\theta}} - k_p \tilde{\theta} \qquad (17)$$

which leads to

$$\ddot{\tilde{\theta}} + k_v \dot{\tilde{\theta}} + k_p \tilde{\theta} = 0. \qquad (18)$$

Hence, by choosing k_v and k_p to ensure that 18 is strict Hurwitz then $\tilde{\theta}$ will asymptotically approach zero. As a result, θ_m will track θ_m^* exactly. Moreover, since θ_m^* is the inverse backlash of θ_{ld} we get the following:

$$y(t) = \theta_l = \mathcal{B}(\theta_m) = \mathcal{B}(\theta_m^*) = \mathcal{B}(\mathcal{BI}(\theta_{ld})) = \theta_{ld} + \Delta \qquad (19)$$

Proposition: The unparameterizable part $\Delta = \mathcal{B}(\hat{\mathcal{BI}}(\theta_{ld})) - \theta_{ld}$ of the control error is bounded $\forall\ t \geq 0$.

Proof. Because the closed-loop system is asymptotically stable, the motor angle will track the backlash inverse trajectory in finite time. When tracking is achieved, one of the following three situations can occur:

1. If θ_m^* and θ_l lie on the upward line of the backlash hysteresis, then $\sigma W(t) = Bsgn(\dot{\theta}_{ld}) = B$ and

$$\begin{aligned} \theta_l &= \theta_m^* - B = \theta_{ld} + \sigma W(t) - B \\ \Delta &= \theta_l - \theta_{ld} = \sigma W(t) - B \\ &= Bsgn(\dot{\theta}_{ld}) - B = 0. \end{aligned} \tag{20}$$

2. If θ_m^* and θ_l lie on the downward line of the backlash hysteresis, then $\sigma W(t) = Bsgn(\dot{\theta}_{ld}) = -B$ and

$$\begin{aligned} \theta_l &= \theta_m^* + B = \theta_{ld} + \sigma W(t) + B \\ \Delta &= \theta_l - \theta_{ld} = \sigma W(t) + B \\ &= Bsgn(\dot{\theta}_d) + B = 0 \end{aligned} \tag{21}$$

3. If θ_m^* and θ_l lie on the inner segment of the backlash hysteresis, then

$$\begin{aligned} \theta_l &= \theta_m^* + B_s \\ &= \theta_d + \sigma W(t) + B_s \longrightarrow B_s \in (-B, B) \\ \Delta &= \theta_l - \theta_{ld} = \sigma W(t) + B_s \\ |\Delta| &= |Bsgn(\dot{\theta}_{ld}) + B_s)| \leq 2B. \end{aligned} \tag{22}$$

This shows that in all three cases Δ is bounded since $W(t)$ as shown earlier is BIBO stable. Simulation results for the case of known backlash spacing are shown in section VI.

V. Adaptive Backlash Inverse Control of a D.C. Motor with Unknown Output Backlash

In this section, we propose an adaptive backlash inverse controller for the case where the backlash spacing B is not known exactly and only an approximate measurement B_n is available. The right backlash inverse discussed earlier is modified here

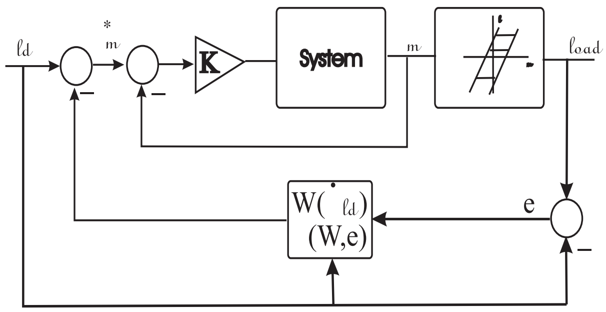


Figure 2: System block diagram.

to generate a trajectory pattern that will reduce the backlash effect hence reducing the position tracking error. An adaptive update law will be derived and used to correct for the mismatch between the actual B and the initial estimate B_n. The state space model of a D.C. motor can be written as

$$\begin{aligned} \dot{\theta}_m &= \omega_m \\ \dot{\omega}_m &= \tau \end{aligned} \tag{23}$$

Let the position tracking error be defined as $\tilde{\theta} = \theta_m - \theta_m^*$ where θ_m^* is defined as follows:

$$\theta_m^* = \hat{\mathcal{BI}}(\theta_{ld}) = \theta_{ld} + \hat{\beta} f(t) \tag{24}$$

where $\hat{\beta}$ is a scaling factor which is used to adjust the value of B_n by an adaptive update law which will be discussed later. Furthermore, $f(t)$ is equal to $\sigma W(t)$ with $W(t)$ being the inverse backlash state given by

$$\dot{W} = \tanh(k_s \dot{\theta}_{ld}) - \frac{\sigma}{B_n} W \tag{25}$$

Defining the following control law for the system in 23:

$$\tau = \dot{\omega}_m^* - k_v \tilde{\omega} - k_p \tilde{\theta} \tag{26}$$

will result in an inner closed loop system dynamics as follows

$$\ddot{\tilde{\theta}} + k_v \dot{\tilde{\theta}} + k_p \tilde{\theta} = 0. \tag{27}$$

By choosing k_v and k_p to make 27 strictly Hurwitz will ensure system stability and results in θ_m asymptotically tracking θ_m^* the backlash inverse trajectory. Meanwhile, the adaptation for $\hat{\beta}$ is chosen to be

$$\dot{\hat{\beta}} = -k_s e(t)\sigma W(t) \tag{28}$$

with $e(t) = \theta_l - \theta_{ld}$ which will be shown later to equal

$$e(t) = \tilde{\beta}\sigma W(t). \tag{29}$$

Equation 28 is chosen based on the analysis presented in [7] for the error model stated in 29. The effect of the control law 26 and the adaptation law 28 is to stabilize the closed loop system and ensures the reduction and boundedness of the tracking error thereby greatly improving performance. To prove the previous statement consider the following Lyapunov function

$$V = k_p\frac{\tilde{\theta}^2}{2} + \frac{\tilde{\omega}^2}{2} + \frac{W^2}{2} + \frac{\tilde{\beta}^2}{2\Gamma} \tag{30}$$

Differentiating V along the trajectories of the closed loop system results in

$$\begin{aligned}\dot{V} &= k_p\tilde{\theta}\dot{\tilde{\theta}} + \tilde{\omega}\dot{\tilde{\omega}} + W\dot{W} + \frac{1}{\Gamma}\tilde{\beta}\dot{\tilde{\beta}} \\ &= \tilde{\omega}(k_p\tilde{\theta} + \tau - \dot{\omega}_m^*) - W(\tanh(k_s\dot{\theta}_{ld}) - \frac{\sigma}{B_n}W) \\ &\quad + \frac{1}{\Gamma}\tilde{\beta}\dot{\hat{\beta}}.\end{aligned}$$

Substituting for τ above and utilizing the inequality in 12 yields

$$\dot{V} = -k_v\tilde{\omega}^2 - (\frac{\sigma}{B_n} - \frac{\varsigma}{2})W^2 + \frac{1}{2\varsigma}\tanh^2(k_s\dot{\theta}_{ld}) + \frac{1}{\Gamma}\tilde{\beta}\dot{\hat{\beta}}. \tag{31}$$

Replacing the update law for $\hat{\beta}$ results in

$$\begin{aligned}\dot{V} &\leq -k_v\tilde{\omega}^2 - (\frac{\sigma}{B_n} - \frac{\varsigma}{2})W^2 + \frac{1}{2\varsigma}\tanh^2(k_s\dot{\theta}_{ld}) \\ &\quad + \tilde{\beta}(-e(t)\sigma W(t)).\end{aligned}$$

As discussed earlier, the second term is negative as long as $|W| > \frac{B_n}{\sigma}$ where σ can be arbitrarily chosen. As for the last term in 32, one needs to examine what the error parameterizations reduces to. Because of the shape of the backlash hysteresis one of three situations may arise after tracking has been established by the inner loop. They are as follows

1. If θ_{ld} and θ_l lie on the upward line of the backlash hysteresis, then both of $\dot{\theta}_l > 0$ and $\dot{\theta}_m > 0$. In addition $\hat{\beta}\sigma W(t) = \hat{\beta}B_n sgn(\theta_{ld}) = \hat{\beta}B_n$. Let the exact backlash

distance be rewritten as $B = \beta^* B_n$ where β^* is the factor that exactly adjust B_n to match B.

$$\begin{aligned}
\theta_l &= \theta_m^* - B = \theta_{ld} + \sigma W(t) - B \\
e(t) &= \theta_l - \theta_{ld} = \hat{\beta}\sigma W(t) - \beta^* B_n \\
&= \tilde{\beta} B_n. \qquad (32)
\end{aligned}$$

2. If θ_{ld} and θ_l lie on the downward line of the backlash hysteresis, then both of $\dot{\theta}_l < 0$ and $\dot{\theta}_m < 0$.

$$\begin{aligned}
\theta_l &= \theta_m^* + B = \theta_{ld} + \sigma W(t) + B \\
e(t) &= \theta_l - \theta_{ld} = \hat{\beta}\sigma W(t) + \beta^* B_n \\
&= \hat{\beta} B_n \mathrm{sgn}(\dot{\theta}_{ld}) + \beta^* B_n = -\tilde{\beta} B_n \qquad (33)
\end{aligned}$$

3. If $\dot{\theta}_{ld}$ changes sign then $\sigma W(t)$ will reverse its sign appropriately thereby providing the necessary jump needed to traverse the backlash distance. This will result in a transient error due to the time needed for the motor to track the sudden switching time of $W(t)$. Nevertheless, by properly designing the PD-controller, the time needed for the motor to track θ_m^* can be greatly reduced. As a result, there are two cases to be considered here. First if $\dot{\theta}_{ld}$ goes from positive to negative at $t = t_0$ then $\sigma W(t_0)$ will equal to $-\hat{\beta} B_n$ at t_{0+}

$$\begin{aligned}
\theta_l &= \theta_m^*(t_0) + \beta^* B_n = \theta_{ld} + \sigma W(t_0) + \beta^* B_n \\
e(t_0) &= \theta_l - \theta_{ld} = \sigma W(t_0) + \beta^* B_n \\
e(t_{0+}) &= -\hat{\beta} B_n + \beta^* B_n = -\tilde{\beta} B_n \qquad (34)
\end{aligned}$$

For the second case , when $\dot{\theta}_{ld}$ goes from negative to positive, a similar argument can show that $e(t_{0+}) = \tilde{\beta} B_n$ Therefore, because of the designed trajectory the case where by the motor and load angle are on the inner segment will be avoided by proper jumps from one parallel line to the next of the backlash hysteresis.

Therefore, based on the previous statements the tracking error may be written as

$$e(t) = \tilde{\beta} B_n \mathrm{sgn}(\dot{\theta}_{ld}) = \tilde{\beta}\sigma W(t). \qquad (35)$$

This shows that in all three cases $e(t)$ is bounded since $W(t)$ as shown earlier is BIBO stable. In addition, using $e(t) = \tilde{\beta}\sigma W(t)$ allows the derivative of the Lyapunov in 32 to become

$$\dot{V} \leq -k_v \tilde{\omega}^2 - (\frac{\sigma}{B_n} - \frac{\varsigma}{2}) W^2 + \frac{1}{2\varsigma} \tanh^2(k_s \dot{\theta}_{ld}) - (\tilde{\beta}\sigma W(t))^2. \qquad (36)$$

Since the hyperbolic tangent function can be upper bounded by 1 then by properly choosing the values of σ and ς to satisfy the inequality

$$\frac{\sigma}{B_n} > \frac{1}{2}(\varsigma + \frac{1}{\varsigma})$$

ensures that $\dot{V} < 0$. Nevertheless, since $\sigma W(t)$ is a smooth approximation to signum function a bounded error will be unavoidable. Simulation results for the system show significant improvement of tracking error and a clear reduction in the backlash effect.

VI. Simulation Results

A. Exact Backlash Inverse for a Known Backlash Nonlinearity

Simulations for the system with a known backlash spacing equal to $B = 1$ degree under the controller developed here were compared with simulation results obtained for the same system under a PD-controller only. As can be seen form Figure 3 the tracking error is drastically reduced when backlash inverse controller is used over a PD-controller with the same gains. The gains k_p and k_v where equal to 404 and 40 respectively. Meanwhile, the σ-parameter was set equal to 15. The improvement in the output trajectory is further demonstrated in Figure 4 where $\theta_{ld} = 20sin(\omega t)$ drawn in a dashed line, moreover, the solid line shows the actual motor load angle tracking performance. In comparison to the adaptive backlash inverse controller performance, the dash/dot line plotted on the same figure represents the load tracking with only a PD-controller to demonstrate the drastic improvement accomplished by our proposed method. In addition, the inverse backlash signal of θ_{ld} defined as θ_m^* is shown in Figure 5 . It can be seen that an almost vertical displacement occurs at the points where the input velocity $\dot{\theta}_{ld}$ is changing its sign. Finally, the control effort is shown in Figure 6(a), whereas The backlash inverse state $\sigma W(t)$ is shown in Figure 6(b). Figure 6 shows that the backlash inverse dynamics are switching between the values B and $-B$ as expected.

B. Adaptive Control of a System with Unknown Backlash Nonlinearity

Simulations for the system with backlash amount of $B = 1$ degree under the adaptive controller developed here were compared with simulation results obtained for the same system under a PD-controller with backlash parameter unknown. The simulations were performed with the nominal estimate of backlash spacing B_n is assumed to be equal to $0.8B$ degrees thereby underestimating the actual backlash spacing. The tracking error under an adaptive backlash inverse controller is shown in Figure 8. Note that the adaptive backlash inverse controller was applied after five seconds from the start of the simulation. This was done to demonstrate the level of improvement in the tracking performance of our proposed controller. The tracking of the reference signal $\theta_{ld} = 10\sin(\omega t)$ is shown in Figure 9. As can be seen form Figure 5 the tracking error is drastically minimized when the adaptive backlash inverse controller is used as compared with a PD-controller with the same gains. The gains k_p and k_v where equal to 404 and 40 respectively. Meanwhile, the σ-parameter was set equal to 15. Note that $\hat{\beta}$ reaches its final value $\beta^* = 1.2$ which is the required factor needed to adjust B_n. This confirms the fact that the inner closed-loop stability is achieved regardless of what the estimate of $\hat{\beta}$.

VII. Conclusion

In this paper we have presented a new backlash inverse controller for the case of output backlash nonlinearity. The new controller was applied to a D.C. motor system which resulted in a great reduction in the load tracking error. The controller then was modified for the case of unknown backlash spacing. The adaptive backlash inverse controller performed amicably well when applied to D.C. motor suffering from unknown backlash nonlinearity. Stability proofs were presented based on the Lyapunov method. Examples and simulations were presented to show the efficacy of the proposed scheme.

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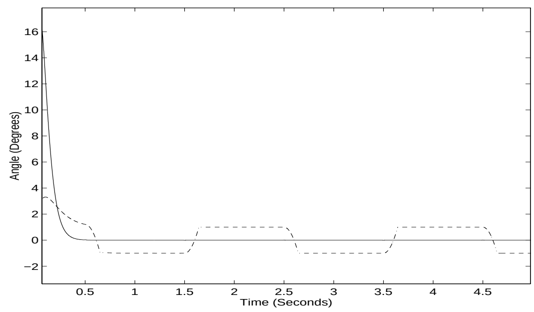

Figure 3: Tracking error $(\theta_l - \theta_{ld})$, under an exact backlash inverse controller is shown in solid, while the system tracking error under a PD controller is shown in dashed line.

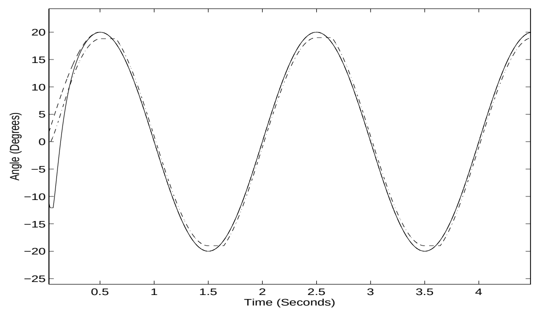

Figure 4: The system under exact backlash inverse (solid) tracking a desired trajectory $20\sin(\omega t)$ shown in (- -) line vs the tracking performance of the system under PD-controller is shown (.-.) line.

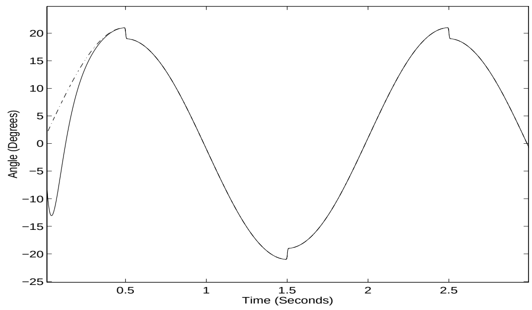

Figure 5: Motor angle tracking (solid) of backlash inverse trajectory θ_m^* (dashed).

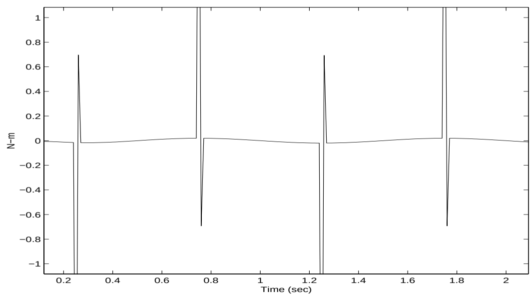

Figure 6: Control effort.

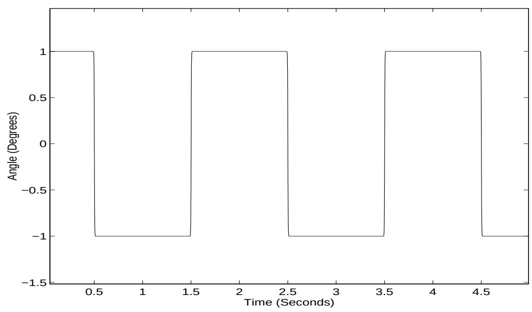

Figure 7: σW Backlash inverse dynamics

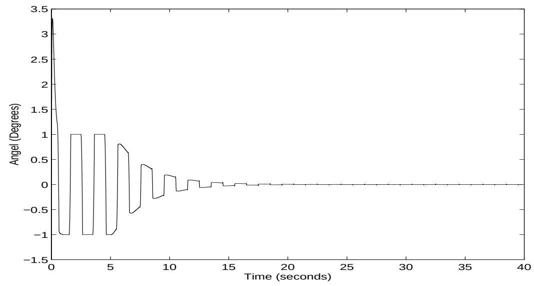

Figure 8: Tracking error $(y(t) - \theta_{ld})$ with adaptive backlash inverse (solid) versus PD-controller (dashed.) The adaptive backlash inverse was applied for $t \geq 5$ seconds.

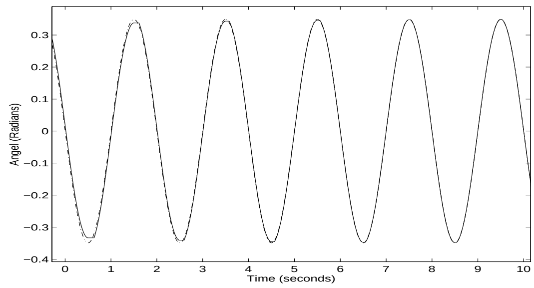

Figure 9: Load angle (solid) tracking of a reference signal $10\sin(2\pi t)$ (dashed) under adaptive backlash inverse controller.

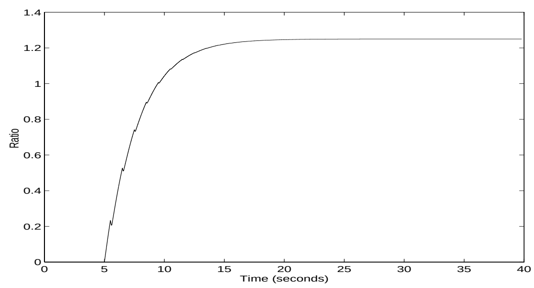

Figure 10: Evolution of $\hat{\beta}$ where $\beta^* = 1.2$.