Control of an Underactuated Underwater Robotic Manipulator

Paolo Di Giamberardino and Furio Sabbadini

Abstract— **The paper deals with the design of a control law for a robotic manipulator externally mounted on an underwater vehicle for performing underwater operations like pick and place, motion of submersed objects, submarine recovers and rescues and so on. The behaviour of the mechanical structure has been restricted to lie on a vertical plane. The control design approach here followed is based on the composition of a continuous time state feedback and a discrete time one, according to a technique that makes use of exact multirate sampling of nonlinear systems. Simulation results show the behaviour of the proposed controllers.**

Keywords— **Underwater vehicle, underwater robotics, multirate sampling, nonlinear control, piecewise continuous control.**

I. Introduction

UNDERWATER Robotic Vehicles (URV), usually less investigated than the analogous space ones, have recently received an increasing interest from many researchers, both for the modeling and the control point of view. One of the most relevant problems arising when dealing with such structures is related to the difficulty of computing precise models for URVs. The difficulty is due to the high density, complex, unstructured underwater environment. The modeling and the control of a robotic manipulator fixed to the ground are well understood problems and can be easily addressed starting from either the Euler-Lagrange or Newton-Euler method ([1]). However, the development of models for underwater robotic vehicles presents some difficulties, mainly due to the presence of uncertainties in the description of the hydrodynamic forces.

Various underwater vehicles have been developed and experimentally tested. In the category of tethered vehicles one can find TROJAN and RUM III. In the untethered group, EPAULARD, ARCS, ROVER and JASON ROV. Informations about them can be easily obtained from many web sites.

If the configuration of an underwater vehicle is given by the six-dimensional Special Euclidian Group and the velocity is constrained so that the only forward velocity component is different from zero, the vehicle has four degrees of freedom (one translational and three rotational). This type of underwater vehicle was studied in [2], where it was show that the nonholonomic underwater vehicle is controllable but not stabilizable with a smooth static feedback control law. A feedback controller for solving a tracking problem was proposed under the assumption of a nonzero forward velocity. The solution is applied to the kinematic motion control of the vehicle only, without including the dynamical aspects. In [3] an autopilot for underwater vehicles is proposed. However, the lack of precision on the model and the uncertainties presented by the unstructured hydrodynamic environment make also this approach quite difficult. In [4] a sliding mode control for a single input is proposed for robust trajectory control.

Once one deals with an underwater manipulator, the main tasks are to move the manipulator to the prescribed location and to guarantee its operative performances while working. When a robotic manipulator is connected to an underwater vehicle, a multibody hydrodynamic problem arises. Such a problem has been addressed by many researchers and some results are present in literature ([5], [6], [7], [8], [9]). In [5] an underwater vehicle equipped with a manipulator is described. A coordinated control scheme is developed to control the vehicle and manipulator simultaneously and to compensate the end-effector error resulting from motion of the vehicle. The control system is based on a discrete-time approximation of the dynamics. The model of the robot used in the paper is planar. In [6] the author expands the classic Newton-Euler mechanics to formulate the dynamic model of an underwater manipulator. In [7] an algorithm for dynamical simulation, based on the articulated body dynamics for an unmanned underwater vehicle with a robotic manipulator, is proposed. These dynamic models, developed following the Newton-Euler formulation, result in a set of equations presented in recursive form, which can be used for simulations but not for control purpose. In [9] a dynamical model for an underwater vehicle with an n-axis robot arm is developed on the basis of Kane's method. This technique provides a direct method for incorporating external environmental forces into the mathematical description. The model developed in the paper includes four major hydrodynamic forces: added mass, profile drag, fluid acceleration and buoyancy. The model derived is a closed form solution which can be used in modern model-based control schemes.

The present paper proposes a different approach to the control problem of the underwater manipulator. The mathematical model used is computed following the idea of the simplified formulation for the hydrodynamic forces and using the Kane's formulation ([10]) for the interaction with the external forces, as described in next section. The control scheme proposed is based on a design technique, successfully used in different applicative contexts, which makes use of a digital control computed from the multirate sampling of the nonlinear dynamics preliminarily modified by a continuous state feedback and a change of coordinates. Such technique is shortly recalled in section 3, after that, in section 2, the description of the mechanical system under

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study, the computation of the mathematical model and the description of the particular operative conditions are addressed. The results of a simulation are reported in section 4. Some conclusions and acknowledgments end the paper.

II. The underwater manipulator

A. Description of the mechanical structure

The mechanical structure here addressed is composed by an underwater vehicle provided with a three degrees of freedom robotic manipulator. The vehicle is supposed to be actuated by there orthogonal forces and three torques, while the manipulator is composed by three links connected by rotational joints: the first one with a vertical axis w.r.t the vehicle, the second and the third ones with axis orthogonal to the previous one, parallel to the horizontal plane of the ship and transversal to the axis of the associated links. Figure 1 shows the manipulator scheme and the definition of the variables describing it. In order to compute the mathematical model of our full structure, in this section the definition of the inertial reference frame together with the reference frame for the submarine vehicle that indicates its position and its attitude in the space are introduced. Subsequently, the relations between the two chosen reference systems is given.

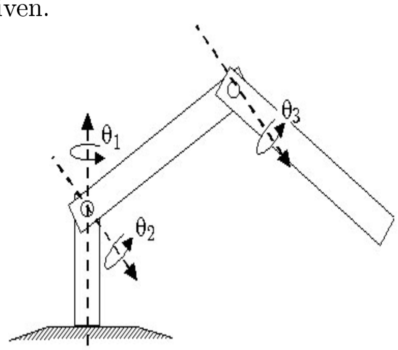

Figure 1: scheme of the three links manipulator

B. The mathematical model

For the computation of the mathematical model an inertial reference frame $O-xyz$ is considered, together with a $C-x_0y_0z_0$ frame, fixed to the center of mass of the submariner, with x_0 axis coincident with the main longitudinal axis of the vehicle, the y_0 axis coincident with the transversal one, so that the x_0y_0 plane coincides with the horizontal one w.r.t. the vehicle. Finally, the z_0 axis coincides with the vertical one of the ship. Figure 2 depicts such choices for the reference frames.

The relative orientation between $O-xyz$ and $C-x_0y_0z_0$ frames is described by the roll pitch and jaw angles φ, θ and ψ respectively. With such a choice, $\Phi = (\varphi, \theta\psi)^T$ denotes the relative angular position, i.e. the rotation vector, while

$$R(\Phi) = R_z(\psi)R_y(\theta)R_x(\varphi) =$$

$$\begin{pmatrix} c_\theta c_\psi & s_\varphi s_\theta c_\psi - c_\varphi s_\psi & c_\varphi s_\theta c_\psi + s_\varphi s_\psi \\ s_\theta s_\psi & s_\varphi s_\theta s_\psi + c_\varphi c_\psi & c_\varphi s_\theta s_\psi - s_\varphi c_\psi \\ -s_\theta & s_\varphi c_\theta & c_\varphi c_\theta \end{pmatrix} \qquad (1)$$

defines the rotation matrix that transforms vector representation from $C-x_0y_0z_0$ frame into the inertial one. In (1) c_x and s_x denote $\cos(x)$ and $\sin(x)$ respectively, with x one of φ, θ and ψ.

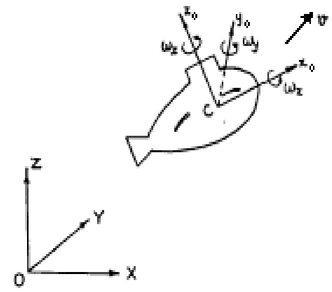

Figure 2

With this notation, it is well known that the relationship between the angular velocity ω in the body fixed frame and the rate of change of Φ in the inertial frame is given by

$$\omega = \begin{pmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & s_\phi c_\theta \\ 0 & -s_\phi & c_\phi c_\theta \end{pmatrix} \dot{\Phi} = T(\Phi)\dot{\Phi} \qquad (2)$$

where $T(\Phi)$ is singular for $\theta = \frac{\pi}{2} + K\pi$. However, since these values correspond to the submariner in vertical position, the singularities do not affect the description during the normal operative conditions.

The mechanical system is completed with the manipulator previously described. It is external and fixed to the hull of the vehicle, at a distance $R = (R_x, R_y, R_z)$ from its center of mass.

For the computation of the mathematical model, the vector of generalized variables q has to be defined. In this case, q has dimension nine and it is given by

$$q = (\, x_c \quad y_c \quad z_c \quad \varphi \quad \theta \quad \psi \quad \theta_1 \quad \theta_2 \quad \theta_3 \,)^T \qquad (3)$$

where the first six components describe the position (center of mass) and the orientation of the ship, while the last three ones are the angles of the arms of the manipulator. Then, the generalized velocities are given by $\dot{q}$ but it must be recalled that if ω is preferred to be present in the mathematical description instead of $\dot{\Phi}$, (2) must be used.

In order to simplify the computation of the model, the approach here used follows the technique based on Kane'e equation ([10]), since it eliminates the interaction forces, between links, which do not produce work and considers the entire system like one single entity. This approach requires the calculation of the generalized active forces and the generalized forces of inertia for each link of the system.

In a few words, given a system $\mathcal{S}$ with N degrees of freedom, its behaviour satisfies the Kane's equation ([10])

$$F_i + F_i^* = 0 \qquad i = 1, \ldots, N$$

where the F_is are the generalized inertia forces, while the F_i^*s are the generalized active ones. Then, it is required to find all the active forces involved with the structure under study.

The active forces acting on the system are given by the sum of the gravity and the hydrodynamics forces.

The hydrodynamics forces induced by the motion of a rigid body in the underwater environment are highly not linear. A general discussion of the hydrodynamics forces and their interaction with a submerged body can be found for example in [6].

For sake of simplicity, some approximations must be introduced. For example, in [9] four separated forces have been proposed. They are the *added mass*, the *buoyancy*, the *fluid acceleration* and the *profile drag*.
Added mass: when a body accelerates in a fluid, the particles of the fluid next to the body tend to assume the same acceleration of the body. Such a fluid layer produces a force of the same amplitude, but with opposite sign, of the force of reaction produced by the body.
Buoyancy: it is well known that the buoyancy acting on a generic body is proportional to the mass of the fluid occupied by the body itself and it is applied to its center of mass.
Fluid acceleration: the force due to the acceleration of the fluid is similar to the Archimedes' force since it is proportional to the moved fluid and its acceleration.
Viscous friction: the force due to the acceleration of the fluid is similar to the Archimedes' force since it is proportional to the moved fluid and its acceleration.
Profile drag: the force due to the viscous friction on a body moving in a fluid depends on the square of the relative speed between body and fluid. The shape of the body and the density of the fluid characterize the value of the viscosity coefficient.

In order to complete the model, input forces and torques are considered. For a fully actuated structure, three forces acting on the submarine and six torques, three for the vehicle and one for each revolution joint of the manipulator, are introduced.

The result of the computation brings to the dynamical model of the form

$$B(q)\ddot{q} + C(q, \dot{q}) + G(q) + F(q) = D(q)\tau \qquad (4)$$

C. *The planar underactuated model*

The long expression of (4) is not reported for sake of simplicity and, also, because in the present paper the interest is posed on a simplified version of (4), that is a planar version, with its behaviour laying on the vertical plane of the submarine constrained to be parallel to the z axis of the inertial frame. Then, in (4) y_c, θ_1, φ and ψ must be set equal to zero together with their derivatives. At the same time, any force or torque producing motion outside the considered plane are set equal to zero. In the so reduced dynamics, a further simplification is performed in this paper, i.e. no translational motion is considered and then both x_c and y_c dynamics are ignored and the forces along these directions are set equal to zero. The presence of x_c and z_c and then a more complete and realistic solution to the problem are contained in a quite longer paper whose preparation is in progress. However, it is important to underline that the only significative difference is the length of the mathematical expressions involved and not in the methodologies as well as in the effectiveness of the results.

After all these simplifications, the mathematical model obtained has the form

$$\tilde{B}(\tilde{q})\ddot{\tilde{q}} + \tilde{C}(\tilde{q}, \dot{\tilde{q}}) + \tilde{G}(\tilde{q}) + \tilde{F}(\tilde{q}) = \tilde{D}\tilde{\tau} \qquad (5)$$

where $\tilde{q} = (\theta \quad \theta_2 \quad \theta_3)^T$, $\tilde{\tau} = (\tau_b \quad \tau_2 \quad \tau_3)^T$, with τ_b the torque acting on the submarine, τ_2 and τ_3 the torques generated by the actuators of the manipulator moving the second and the third link respectively, and

$$\tilde{D} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

Moreover, the control problem here addressed involves the possibility of achieving the end effector positioning making use of manipulator controls only, i.e. with no actuation by the submarine.

The problem arises from the fact that usually it is very difficult for an underwater vehicle to change its orientation while moving slowly. This means that it is not reasonable to imagine that the vehicle can contribute actively to the angular positioning of the manipulator. Then, a practical solution needs to avoid the use of the pitch control torque of the submarine. This fact has two effects: a simplification of the model, since it brings to neglect the first component τ_b of the input $\tilde{\tau}$ in (5) and to consider the matrix

$$P = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}$$

instead of $\tilde{D}$, and a complication in the control design, since one has to deal with an underactuated dynamics subject to a nonintegrable kinematic constraint given by the angular momentum conservation and expressed by

$$(1 \quad 1 \quad 1)\,\tilde{B}(\tilde{q})\dot{\tilde{q}} = 1^T\tilde{B}(\tilde{q})\dot{\tilde{q}} = 0 \qquad (6)$$

Since no traditional control design technique, like the ones based on linear approximation, exact feedback linearization and so on can be applied due to the presence of the nonholonomic constraint (6), in next section a technique already used in different fields, from mobile robotics ([11]) to aeronautic applications ([12]) and to space applications ([13]), will be shortly recalled and then used to get an effective solution to the control problem here posed. The mathematical model used is then (5) with the further positions just described, which bring to

$$\tilde{B}(\tilde{q})\ddot{\tilde{q}} + \tilde{C}(\tilde{q}, \dot{\tilde{q}}) + \tilde{G}(\tilde{q}) + \tilde{F}(\tilde{q}) = P\hat{\tau} \qquad (7)$$

with

$$\hat{\tau} = \begin{pmatrix} \tau_2 \\ \tau_3 \end{pmatrix}$$

III. The control strategy

A. The multirate digital approach

The control design approach here adopted makes use of a discrete time controller based upon the sampled equivalent dynamics of the given system according to the technique described firstly in [14] and exposed in [15].

Such approach requires that the dynamics under study admit a *finite sampled equivalent* or an *exact sampled equivalent* ([15]). These concepts are equivalent to the exact computability of the nonlinear sampled equivalent model in the sense that it is possible to give the analytic expression of the discrete time dynamics whose behaviour, at the sampling instants, is coincident with the continuous time one once fed with piecewise constant inputs.

More precisely, given a nonlinear dynamics of the form

$$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) \tag{8}$$

with $x \in M \subset \mathcal{R}^n$, $u \in U \subset \mathcal{R}^m$, $f, g_1, \ldots, g_m$ real analytic vector fields on M, and assumed piecewise constant inputs over the time intervals $[k\Delta, (k+1)\Delta[$, the solution $x[k+1]$ at time $t = (k+1)\Delta$, starting from $x[k]$ at time $t = k\Delta$, at least for small values of Δ, is an analytic function $F_\Delta : M \times U \to M$ defining the equivalent sampled model of (8) ([10])

$$x[k+1] = F_\Delta(x[k], u[k]) = e^{\Delta L_{f+\sum_{i=1}^{m} u_i[k] g_i}}(Id)\Big|_{x=x[k]} \tag{9}$$

If the analytic expression of F_Δ is computable, then it is called the exact sampled equivalent of (8). Moreover, if F_Δ is a polynomial in Δ of finite order $\bar{k}$, then it is called the finite sampled equivalent. Such properties can be achieved also by a preliminary state feedback and coordinates change.

For the solution of many control problems, it can be useful, or sometimes necessary, to use a multirate sampled model ([16]). This means that each control input u_i in (8) is assumed constant over a time interval δ, with $\delta = \frac{\Delta}{r}$, that is $u_i(t) = u_i^h$ for $t \in [k\Delta + (h-1)\delta, k\Delta + h\delta[$, $h = 1, \ldots, r$. The computation of the solution $x[k+1]$ at time $t = (k+1)\Delta$, starting from $x[k]$ at time $t = k\Delta$ under the previous hypothesis gives the expression

$$\begin{aligned} x_{k+1} &= F_\delta(x_k, u_k^1, \ldots, u_k^r) = \\ &= e^{\delta L_{f+\sum_{i=1}^{m} u_i^1 g_i}} \circ \cdots \circ e^{\delta L_{f+\sum_{i=1}^{m} u_i^r g_i}}(Id)\Big|_{x=x_k} \end{aligned} \tag{10}$$

The properties of finite or exact computability of the sampled equivalent dynamics clearly allow the design of an exact solution of a given control problem and, at the same time, make the procedure more simple from a mathematical point of view.

Then, the design of the control law for the system under study, performed in the next subsections, is organized in the following way: firstly, a state feedback and a coordinates change are computed in order to put the dynamics in a finitely discretizable form. This part constitutes the continuous part of the control. Then, a multirate sampling and a digital control design are performed, so getting the discrete time part of the control. The overall control law is then a piecewise continuous one.

B. Computation of the continuous feedback

The computation of the control law, according to the technique previously recalled, begins with the determination of a static state feedback and a diffeomorphism whose goal it to transform the dynamics (7) into a finitely discretizable one. Under the diffeomorphism

$$\begin{pmatrix} x \\ u \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \tilde{q} \\ H(\tilde{q})\dot{\tilde{q}} \end{pmatrix} = \begin{pmatrix} \tilde{q} \\ P^T \dot{\tilde{q}} \\ 1^T \tilde{B}(\tilde{q})\dot{\tilde{q}} \end{pmatrix} = \begin{pmatrix} \tilde{q} \\ P^T \dot{\tilde{q}} \\ 0 \end{pmatrix} \tag{11}$$

and the state feedback

$$\hat{\tau} = c^{-1}(x)\left(a - b(x,u)\right) \tag{12}$$

it is easy to verify that (7) is transformed into

$$\begin{aligned} \dot{x} &= K(x)u \\ \dot{u} &= a \end{aligned} \tag{13}$$

where

$$K(x) = \left(\, \gamma_1(x_2,x_3) \quad \gamma_2(x_2,x_3) \,\right) = \\ = \begin{pmatrix} s_1(x_2,x_3) & s_2(x_2,x_3) \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

is composed by the first two columns of $H(\tilde{q})^{-1}$. In 12

$$c(x) = P^T \tilde{B}^{-1}(x) P$$

and

$$b(x,u) = -P^T \tilde{B}^{-1}(x)\left(\tilde{C}(x, K(x)u) + \tilde{G}(x) + \tilde{F}(x)\right)$$

Form (13) has an interesting property, proved in ([14]): if the first part, i.e.

$$\dot{x} = K(x)u$$

admits a state feedback

$$u = \beta(x)v$$

and a coordinates change

$$z = Z(x)$$

which make it finitely discretizable, then the state feedback

$$a = \dot{\beta}(x)\beta^{-1}(x)u + \beta(x)w \tag{14}$$

and the diffeomorphism

$$\begin{pmatrix} z \\ v \end{pmatrix} = \begin{pmatrix} Z(x) \\ \beta^{-1}(x)u \end{pmatrix} \tag{15}$$

transform (7) into a system admitting a finite sampled equivalent. In this case, if one supposes the existence of a function $\lambda(x)$ such that

$$L_{\gamma_2(x)}\lambda(x) = 0 \tag{16}$$

and

$$L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x) \neq 0 \tag{17}$$

for any $x \in \mathcal{R}^3$ or, at least, in a suitable manifold in $\mathcal{R}^3$, then it is easy to verify that choosing

$$Z(x) = \begin{pmatrix} x_2 \\ L_{\gamma_1(x)}\lambda(x) \\ \lambda(x) \end{pmatrix}$$

one can write

$$\begin{aligned} \dot z_1 &= \dot x_2 = u_1 = v_1 \\ \dot z_2 &= L^2_{\gamma_1(x)}\lambda(x)u_1 + L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)u_2 = v_2 \\ \dot z_3 &= L_{\gamma_1(x)}\lambda(x)u_1 + L_{\gamma_2(x)}\lambda(x)u_2 = z_2 v_1 \end{aligned} \tag{18}$$

once that the state feedback

$$u = \beta(x)v = \begin{pmatrix} 1 & 0 \\ -\frac{L^2_{\gamma_1(x)}\lambda(x)}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} & \frac{1}{L_{\gamma_2(x)}L_{\gamma_1(x)}\lambda(x)} \end{pmatrix} v \tag{19}$$

has been used. Clearly, (18) admits a finite sampled equivalent, given by

$$\begin{aligned} z_1[k+1] &= z_1[k] + \Delta v_1[k] \\ z_2[k+1] &= z_2[k] + \Delta v_2[k] \\ z_3[k+1] &= z_3[k] + \Delta z_2[k]v_1[k] + \frac{\Delta^2}{2}v_1[k]v_2[k] \end{aligned} \tag{20}$$

Then, the transformations for (13) are given by (14) and (15). The last question is: does such a function $\lambda(x)$ exist? The answer is positive. Denoting by $\sigma(x_2)$ the first element of $ad_{\gamma_1(x)}\gamma_2(x)$ computed for $x_3 = 0$, the function $\lambda(x)$ is given by

$$\lambda(x) = \frac{x_1 - \int_0^{x_3} s_2(x_2,\eta)d\eta}{\sigma(x_2)} \tag{21}$$

It is easy to verify, by direct computation, the fulfillment of (16) and (17).

Under the feedback obtained by the cascade of (12) and (14) and with the coordinates change given by the composition of (11) and (15), (7) assumes the form

$$\begin{aligned} \dot z_1 &= v_1 \\ \dot z_2 &= v_2 \\ \dot z_3 &= z_2 v_1 \\ \dot v_1 &= w_1 \\ \dot v_2 &= w_2 \end{aligned} \tag{22}$$

C. Computation of the discrete time control

The discrete time part of the controller is designed by computing a multirate sampling of (22), choosing the multirate order so that the control can be computed by inversion. In the present case the choice

$$w_1 = \begin{cases} w_{1,1} & \text{if} \quad t \in [k\Delta, k\Delta + 2\delta) \\ w_{1,2} & \text{if} \quad t \in [k\Delta + 2\delta, (k+1)\Delta) \end{cases}$$

$$w_2 = \begin{cases} w_{2,1} & \text{if} \quad t \in [k\Delta, k\Delta + \delta) \\ w_{2,2} & \text{if} \quad t \in [k\Delta + \delta, k\Delta + 3\delta) \\ w_{2,2} & \text{if} \quad t \in [k\Delta + 3\delta, (k+1)\Delta) \end{cases} \tag{23}$$

brings to the following multirate finite sampled dynamics, equivalent to (22) and feedback equivalent to (7):

$$\begin{aligned} z_1[k+1] = \; & z_1[k] + 4\delta v_1[k] + 2\delta^2 (3w_{1,1} + w_{1,2}) \\ z_2[k+1] = \; & z_2[k] + 4\delta v_2[k] + \tfrac{\delta^2}{2}(7w_{2,1} + 8w_{2,2} + w_{2,3}) \\ z_3[k+1] = \; & z_3[k] + 4\delta z_2[k]v_1[k] + \\ & +2\delta^2 (4v_1[k]v_2[k] + 3z_2[k]w_{1,1} + z_2[k]w_{1,2}) + \\ & +\tfrac{\delta^3}{6}(88v_2[k]w_{1,1} + 40v_2[k]w_{1,2} + \\ & +37v_1[k]w_{2,1} + 26v_1[k]w_{2,2} + v_1[k]w_{2,3}) + \\ & +\tfrac{\delta^4}{24}(281w_{1,1}w_{2,1} + 136w_{1,2}w_{2,1} + \\ & +207w_{1,1}w_{2,2} + 129w_{1,2}w_{2,2} + 8w_{1,1}w_{2,3} + \\ & +7w_{1,2}w_{2,3}) \\ v_1[k+1] = \; & v_1[k] + 2\delta(w_{1,1} + w_{1,2}) \\ v_2[k+1] = \; & v_2[k] + \delta w_{2,1} + 2\delta w_{2,2} + \delta w_{2,3} \end{aligned} \tag{24}$$

From (24) it is possible to compute the controls $w_{i,j}$ which steer the system form any configuration at time $k\Delta$ to any other one at time $(k+1)\Delta$.

D. The full control scheme

The full control scheme is obtained by the cascade of the continuous and discrete state feedback previously computed. The final expression is a piecewise continuous function over the time interval $[k\Delta, (k+1)\Delta[$, obtained from (12), where a is given by (14) with w piecewise constant according to (23).

IV. Simulation results

Simulations have been performed in order to validate the control scheme proposed and to show its effectiveness. The results of a manoeuvre from the initial condition

$$\tilde q = (\,\theta \quad \theta_2 \quad \theta_3\,)^T = (\,0 \quad 0 \quad -\tfrac{\pi}{2}\,)^T$$

to the final one

$$\tilde q = (\,\theta \quad \theta_2 \quad \theta_3\,)^T = (\,0 \quad \tfrac{1.8}{\pi} \quad -\tfrac{\pi}{2}\,)^T$$

are reported in the following figures, where the time histories of the three angles $\theta(t)$, $\theta_2(t)$ and $\theta_3(t)$ are depicted in figure 3,4 and 5 respectively. Figure 6 shows the piecewise constant control $w(t)$ and figure 7 reports the effective control input $\hat\tau(t)$.

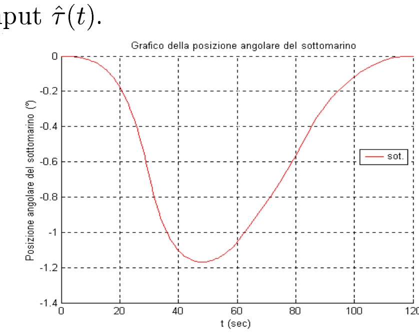

Figure 3: time history of $\theta(t)$

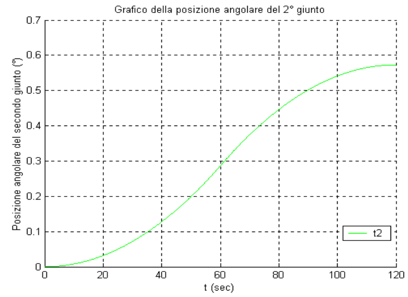

Figure 4: time history of $\theta_2(t)$

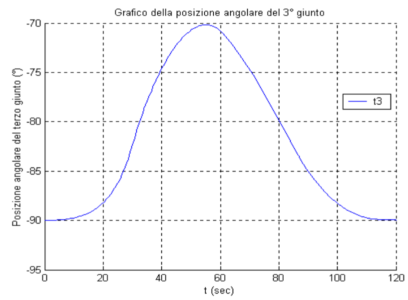

Figure 5: time history of $\theta_3(t)$

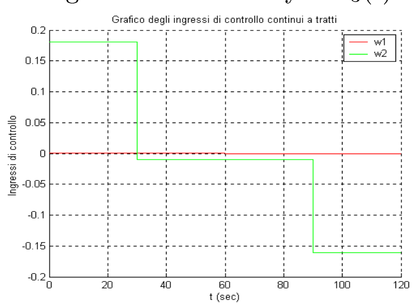

Figure 6: the piecewise constant controls $w_1(t)$ and $w_2(t)$

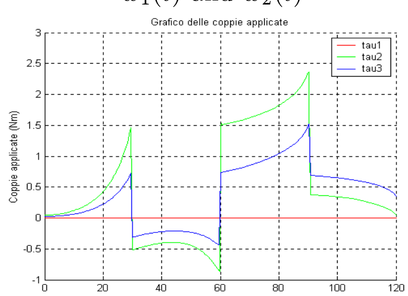

Figure 7: the control torques $\tau_1(t)$ and $\tau_2(t)$

The performances of the controller are well evidenced by such results.

V. Conclusion

In the paper a control strategy for an underwater robotic manipulator mounted on the hull of a submarine is proposed, under the hypothesis of a vertical planar motion and using only the robot actuators. In this configuration, the system is characterized by the presence of a nonholonomic constraint which makes the solution of the control problem quite difficult. The technique adopted is based upon a discrete time state feedback performed after a preliminary continuous time one: the first for achieving the control at prefixed time instants while the latter for technical reasons related to the exact solvability of the discrete time problem.

The solution is strongly dependent on the model, since it contains an inversion of the discrete time dynamics. This fact could reduce the robustness of the proposed approach if no additional actions were performed. However, it has been shown, for example in [17], how robustness can be achieved within this control technique, just adding a further digital state feedback.

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References

[1] Craig J.J., Introduction to robotics, Addison-Wesley Publishing Company, 1989.

[2] Nakamura Y. and Savant S., Nonlinear tracking control of autonomous underwater vehicles, IEEE International Conference on Robotics and Automation, A4-A9, 1992.

[3] Goheen K.R. and Jeffrey E.R., Multivariable self-tuning autopilots for autonomous and remotely operated underwater vehicles, IEEE Journal of Oceanic Engineering, Vol. 15 No. 3 pp. 144-151, 1990.

[4] Yoerger D.R. and Slotine J.J.E., Robust trajectory control of underwater vehicles, IEEE Journal of Oceanic Engeneering, Vol. 10, No. 4, pp. 462-470, 1985.

[5] Mahesh H., Yuh J. and Lakshmi R., A coordinated control of an underwater vehicle and robotic manipulator, Journal of Robotic System, Vol. 8, No. 3 pp. 339-370, 1991.

[6] Ioi K. and Itoh K., Modeling and simulation of an underwater manipulator, Advanced Robotics, No 4, pp 303-317, 1990.

[7] McMillan S., Orio D.E. and McGhee R.B. Efficient dynamic simulation of an underwater vehicle with a robotic manipulator, IEEE Transaction on System Man and Cybernetics, Vol 25, No. 8, 1995.

[8] Kato N. and Lane D.M., Coordinated control of multiple manipulators in underwater robots, 9th Int. Symposium on Unmanned Untethered Submersible Technology, pp. 34-50, 1995.

[9] Tarn T.J., Shoults G.A. and Yang S.P., A dynamic model of an underwater vehicle with a robotic manipulator unsing Kane's method, Autonomous Robots, Vol.3 pp. 269-283, 1996.

[10] Kane T.R., Likens P.W. and Levinson D.A., Spacecraft Dynamics, McGraw-Hill, 1983.

[11] Di Giamberardino P., Monaco S. and Normand-Cyrot D., Why multirate sampling is instrumental for control design purpose: the example of the one-leg hopping robot, Proc. 41st IEEE Control and Decision Conference, 2002.

[12] P. Di Giamberardino, M. Djemai, S. Monaco and D. Normand–Cyrot, Exact Steering and Stabilization of a PVTOL Aircraft, Proc. 36th IEEE Control and Decision Conference, pp. 2049–2054, 1997.

[13] Di Giamberardino P., Monaco S. and Normand-Cyrot D., Hybrid control scheme for maneuverig space multibody structures, Journal of Guidance, Control and Dynamics, Vol.23, No 2, 2000.

[14] Di Giamberardino P., Monaco S. and Normand–Cyrot D., Digital Control through Finite Feedback Discretizability, Proc. Int. Conf. Rob. and Aut., 1996.

[15] Monaco S. and Normand-Cyrot D., Issues on nonlinear digital control, European Journal of Control, Vol. 7, N. 2-3, 2001.

[16] Monaco S. and Normand–Cyrot D., On the Sampling of Linear Analytic Control Systems, Proc. 24th IEEE Control and Decision Conference, pp. 1467–1472, 1985

[17] Di Giamberardino P., Grassini F., Monaco S. and Normand–Cyrot D., Piecewise Continuous Control for a Car–Like Robot: Implementation and Experimental Results, Proc. 35th IEEE Control and Decision Conference, pp. 3564–3569, 1996.