FUZZY LINEAR MATRIX INEQUALITIES (LMI) CONTROL OF A RECTANGULAR THERMAL CONVECTION LOOP

Giovanni Muscato*, Giacomo Spampinato*, Maria Gabriella Xibilia#

*Dipartimento di Ingegneria Elettrica Elettronica e dei Sistemi, gmuscato@dees.unict.it
Università degli Studi di Catania,Viale A. Doria, 6, 95125 Catania, Italy
#Dipartimento di Matematica, Università di Messina,Salita Sperone 31, 98166 Messina, Italy*

Abstract. In this paper a fuzzy LMI control strategy for a rectangular thermosyphon is proposed. These thermo-fluid-dynamic systems are used to refrigerate heat sources by means of natural circulation of a fluid without mechanical pumping.
Two different fuzzy LMI controllers have been designed in order to guarantee closed loop stability, on the basis of a suitable nonlinear T-S fuzzy model of the system. .In particular the controllers are designed in order to prevent temperature oscillations, associated with the inversion of the flow direction, which compromise the heat removal from the thermal source. Two different strategies have been adopted to introduce suitable constraints on the control action.

1. INTRODUCTION

Thermal convection loops, often named closed loop thermosyphon, are thermo-fluid-dynamical systems mainly devoted to the refrigeration of a heat source by means of natural circulation of a fluid in a closed loop, without mechanical pumping.
The absence of moving components drastically reduces the probability of a failure in the heat removal from the heat source. In fact, this is the main reason for which natural is preferred to force convection in those energy plants in which safety is a primary requirement. Therefore, refrigeration of reactors in nuclear power plants and electrical machine rotor cooling [1], [2], represent the main applications of closed loop thermosyphons.
Other important applications in which closed loop thermosyphons are preferred to forced circulation loops, are those in which the absence of pumping elements allows a considerable costs reduction, e.g. geothermal plants or solar heaters, that have low temperature thermal sources but relatively high circulating flow rate [3], [4], or, finally, where the pumping system cannot be conveniently positioned, such as cooling systems for internal combustion engines, turbine blade cooling or computer cooling [5], [6].
In their basic scheme natural circulation loops lye on a vertical plane, are symmetrical with respect to the vertical axis and consist of a heat source (placed in the bottom and cooled by the circulating flow), a heat sink (placed on top of the loop) in which the circulating flow is cooled, and may have or not two thermally isolated vertical legs connecting the heat exchanging sections.
The boundary conditions usually examined in experimental studies are those in which the bottom section is heated using an imposed heat power [7], [8], [9], whereas the topmost section is generally refrigerated imposing a constant wall temperature at its wall. Whatever the boundary conditions are, the buoyancy caused by the density gradients existing in the loop represents the driving force for the fluid motion. Moreover, the flow direction is determined by the temperature difference existing between different points of the loop, on which the buoyancy depends.
The stability of natural circulation loops mainly depends on the entity of the buoyancy, which is proportional to the vertical temperature difference and therefore is determined by the boundary conditions imposed at the heating section. In particular, for increasing values of the vertical temperature difference, ΔT, the flow will be accelerated due to the growth of the forcing term. On the other hand, the growth both of ΔT and of velocity causes a destabilising effect, which can be schematised as follows:

1. for low ΔT the buoyancy is too weak to cause the fluid motion: heat removal is ensured by conductive heat transfer;
2. when ΔT increases the fluid starts moving either in the clockwise or in the counter-clockwise direction (ideally with the same probability); once the motion has started in one direction it keeps moving in this direction;
3. for higher ΔT the velocity of the flow becomes too high and the flow cannot be sufficiently cooled in its passage through the cooling section. The temperature at the outlet of the cooling section increases and its velocity decreases until the mass of fluid coming out from the cooling section has grown so much to exert a sort of sudden impulse, which makes the fluid move rapidly again.
4. when the temperature at the outlet of the cooling section is higher than that at the inlet, the buoyancy inverts its direction, causing therefore the flow inversion. The velocity in the opposite direction, which is initially very low, gradually increases so that steps 2) and 3) are repeated for the current flow direction.

In the last two cases, the dynamics is often characterised by non-periodical oscillations, which have been shown to be chaotic. Hence, the process leads to temperature oscillations and is associated to inversions of the flow direction. The flow inversion compromises the heat removal from the thermal source and should therefore be avoided.

Stabilising the dynamics of the process therefore represents the main task in the field of natural circulation loops [8], [9], [11]. In particular suitable control action may consist in varying the refrigerant flow rate to the “cold” sink or the heat power to the “hot” source.

A set of PID controllers has been designed by the authors as described in [12] on the basis of the linearised analytic model of the system. Even if they gave good results, no global stability conditions are analytically guaranteed for the non-linear system.

In order to design a controller which guarantees stability, in this paper a different controller design strategy, based on a Fuzzy LMI controller is proposed.

The proposed controller has been designed on the basis of a fuzzy T-S model, obtained by using the approach proposed in [16], starting from an accurate mathematical model.

An efficient modelling approach of the considered system is based on an approximation of its non linear partial derivative equations by using a truncated Fourier series expansion of the variables [13], even if different modelling strategy, based on neural NARMAX models are reported in [14], [15], directly obtained from experimental data.

In Section 2 the experimental system on which the analytical model has been tested is briefly described, while in Section 3 the analytical model and the corresponding fuzzy model are reported. The proposed control strategy and some results are given in section 4.

2. THE EXPERIMENTAL SYSTEM

The experimental natural circulation loop is depicted in Fig. 1.It consists of two copper horizontal tubes (heat transfer sections), two vertical phirex tubes, four horizontal phirex tubes and four 90° phirex bends. The lower heating section consists of twelve independent electrical heating wires, able to provide 0.5 kW each, winding on the outside of the copper tube, so that the system is able to provide up to 6kW. The upper heat extraction system is a coaxial heat exchanger with tap water flowing in the annulus created by an external iron case. In this way it is possible to impose desired values both of the heat flux in the lower heating section and of the temperature of the coolant. The latter condition can be obtained by adopting high values of the water flow rate so to minimise the temperature difference between the inlet and the outlet of the cooling water. An expansion tank open to the atmosphere is installed on the topmost elevation of the loop allowing the fluid volumetric expansion.

The whole system is equipped with eight calibrated (± 0.1 K) *T*-thermocouples (diameter 1.6 mm) located (see Fig. 1): T2 and T4 on the left vertical tube; T5 and T6 on the right vertical tube; T1 and T3 on the lower horizontal tubes; T7 and T8 on the input and output of the cooling water.

An inductive flowmeter is inserted in the main loop while another inductive flowmeter and a electromechanical servo-valve are inserted to measure cooling flow rate.

All the sensors are connected to a data acquisition board in the external computer. The sampling frequency adopted was 1 Hz.

The described experimental system has been used to validate the performance of the mathematical models.

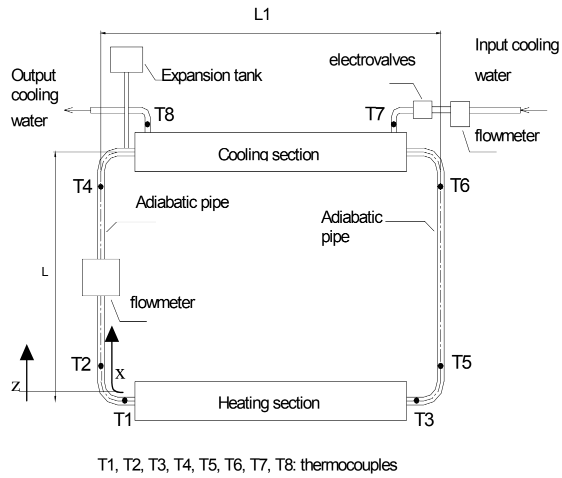

Fig.1 The Experimental system.

3. THE MATHEMATICAL MODEL

The aim of this section is to give a mathematical description of the behaviour of the natural circulation represented in Fig. 1, having generic height and width, indicated respectively with L and L_1, and constant tubes inner diameter equal to r.

The reported model will be converted, as described in the following in a fuzzy model suitable to apply the fuzzy LMI control strategies described in Section 4.

In the following, x represents an abscissa parallel to the loop pipes, with positive direction corresponding to the clockwise path on the loop and with an arbitrarily chosen origin in the left down corner of the loop, as indicated in Fig. 1.

The analytic model, obtained applying the thermo-fluid-dynamic equations to the rectangular geometry of the loop is given as:

$$\frac{dv(t)}{dt} + \frac{1}{r} b \left(\frac{\nu}{2r} \right)^{d} v^{2-d} = \frac{g\beta}{2(L+L_1)} \oint (T - T_0) f(x) dx \quad (1)$$

$$v(0) = v_0$$

$$\frac{\partial T}{\partial t} + v(t) \frac{\partial T}{\partial x} = h(x) + a \frac{\partial^2 T}{\partial x^2} \quad (2)$$

$$T(x,0) = T_0(x)$$

where: v is the fluid velocity, d and b are parameters to be determined experimentally, ν is the cinematic viscosity, g is the gravitational acceleration, β is the

volumetric expansion coefficient, T is the fluid temperature, T_0 is a reference temperature, a is the fluid thermal diffusivity, $f(x)=dz/dx$ and $h(x)$ describes the boundary conditions.
By means of a suitable Fourier series expansion of the known function $f(x)$ and $h(x)$ and of the variable $T(x,t)$, arresting the Fourier series expansion to the third mode and applying the method of residuals in order to separate the terms of the same order, it is possible to rewrite the model as [12]:

$$\dot{v}(t) = -\frac{1}{r} b\left(\frac{\nu}{2r}\right)^{d} v^{2-d} + \frac{2g\beta}{\pi}\cdot \left[\alpha_1(t)\sin\gamma - \beta_1(t)(1-\cos\gamma) + \alpha_3(t)\frac{1}{3}\sin 3\gamma - \beta_3(t)\frac{1}{3}(1-\cos 3\gamma)\right] \tag{3}$$

$$\dot{\alpha}_1(t) = -a\frac{\pi^2}{(L+L_1)^2}\alpha_1(t) + \frac{\pi}{L+L_1}v(t)\beta_1(t) + \frac{\Gamma}{2\pi}sin\gamma \tag{4}$$

$$\dot{\beta}_1(t) = -a\frac{\pi^2}{(L+L_1)^2}\beta_1(t) - \frac{\pi}{L+L_1}v(t)\alpha_1(t) + \frac{\Gamma}{2\pi}(1+\cos\gamma) \tag{5}$$

$$\dot{\alpha}_3(t) = -a\frac{9\pi^2}{(L+L_1)^2}\alpha_3(t) + \frac{3\pi}{L+L_1}v(t)\beta_3(t) + \frac{\Gamma}{6\pi}\sin 3\gamma \tag{6}$$

$$\dot{\beta}_3(t) = -a\frac{9\pi^2}{(L+L_1)^2}\beta_3(t) - \frac{3\pi}{L+L_1}v(t)\alpha_3(t) + \frac{\Gamma}{6\pi}(1+\cos 3\gamma) \tag{7}$$

where α_i and β_i (i=$1,3$) are the real and imaginary part of the coefficients a_k of the expansion $T(x,t)$, $\gamma = \frac{\pi L}{L+L_1}$, $\Gamma = \frac{2}{\rho_0 cr}\left(\frac{\dot{m}c_p\Delta T}{2\pi r L_1}+q\right)$, $\Gamma_1 = \frac{2}{\rho_0 cr}\left(\frac{\dot{m}c_p\Delta T}{2\pi r L_1}-q\right)$
q is the heat flux, ΔT is the inlet-outlet temperature difference of the cooling section, $\dot{m}$ and c_p are mass flow rate and specific heat of the cooling fluid.
In order to validate the model it is necessary to compare its simulation with measurements detected on an experimental loop. To this purpose, it is necessary to reconstruct the temperature function $T(x,t)$ from the variables of the model $\alpha_k(t)$ and $\beta_k(t)$,this is performed with the relation:

$$\begin{aligned} T(x,t) - T_0 &= 2\alpha_1(t)\cos\frac{\pi}{L+L_1}x - 2\beta_1(t)sin\frac{\pi}{L+L_1}x + \\ &+ 2\alpha_2(t)\cos\frac{\pi}{L+L_1}2x - 2\beta_2(t)sin\frac{\pi}{L+L_1}2x + \\ &+ 2\alpha_3(t)\cos\frac{\pi}{L+L_1}3x - 2\beta_3(t)sin\frac{\pi}{L+L_1}3x \end{aligned} \tag{8}$$

In order validate the model several simulations have been compared with experimental data.

As an example, a comparison between a simulation and experimental measurement for the temperature T4 with a constant heating power of 1800 W is reported in Fig. 2 and Fig. 3

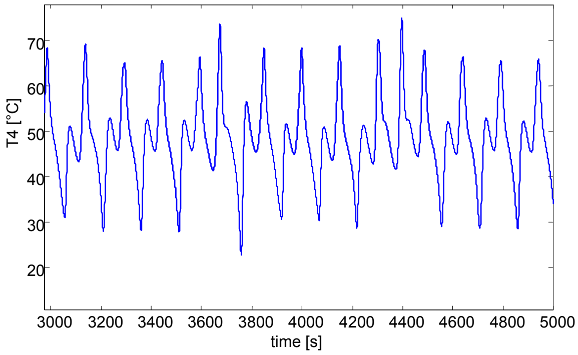

Fig. 2. Simulated temperature T4 for an heating power of 1800 W.

Many other simulations have been compared with experimental data for a set of different power, confirming in all cases the good accuracy of the model.

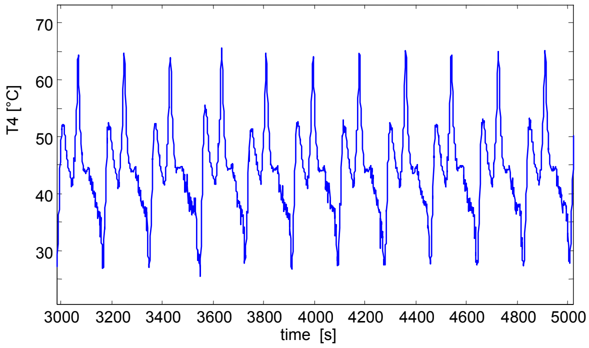

Fig. 3. Experimental temperature T4 for an heating power of 1800 W.

The corresponding fuzzy T-S model of the system has been obtained using the approach reported in [16].

The method is based on a local description of systems of the type:

$$\dot{x}(t) = f(x(t)) \in [a_1, a_2]x(t)$$

The model is constituted by a set of r IF-THEN rules representing local linear I/0 relations in the form:

IF $z_1(t)$ *is* M_{i1} *and ... and* $z_p(t)$ *is* M_{ip}*, THEN*

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), \\ y(t) = C_i x(t), \end{cases} \qquad i=1,2,..,r \tag{9}$$

where M_{ij} are fuzzy set, and $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^q$, are the state, input and output respectively, $A_i \in R^{nxn}$, $B_i \in R^{nxm}$, $C_i \in R^{qxn}$ and the variables $z_1(t)$, ..., $z_p(t)$ depend on the the state variables.

The global system is described by:

$$\dot{x}(t)=\frac{\sum_{i=1}^{r} w_i(z(t))\{A_i x(t)+B_i u(t)\}}{\sum_{i=1}^{r} w_i(z(t))}=\sum_{i=1}^{r} h_i(z(t))\{A_i x(t)+B_i u(t)\}$$

$$y(t)=\frac{\sum_{i=1}^{r} w_i(z(t))C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))}=\sum_{i=1}^{r} h_i(z(t))C_i x(t)) \quad (10)$$

where:

$$w_i(z(t))=\prod_{j=1}^{p} M_{ij}(z_j(t)),\; h_i(z(t))=\frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$$

In our application the state vector is given by:

$$x(t)=\begin{bmatrix} v(t) & \alpha_1(t) & \beta_1(t) & \alpha_3(t) & \beta_3(t)\end{bmatrix}^T$$

The suitable fuzzy sets are built on the basis of the maximum and minimum values of each variable (m_i and n_i), obtained by the simulation of the system mathematical model. The z_i variables have been defined as follows:

$$\begin{aligned}
z_1(t)&=v(t)^{1-d} &&= M_1(z_1(t))\cdot(m_2)+M_2(z_1(t))\cdot(-m_1)\\
z_2(t)&=\alpha_1(t) &&= N_1(z_2(t))\cdot(n_2)+N_2(z_2(t))\cdot(-n_1)\\
z_3(t)&=\beta_1(t) &&= P_1(z_3(t))\cdot(p_2)+P_2(z_3(t))\cdot(-p_1)\\
z_4(t)&=\alpha_3(t) &&= Q_1(z_4(t))\cdot(q_2)+Q_2(z_4(t))\cdot(-q_1)\\
z_5(t)&=\beta_3(t) &&= R_1(z_5(t))\cdot(r_2)+R_2(z_5(t))\cdot(-r_1)
\end{aligned}$$

where the fuzzy set are built satisfying the constraints:

$$\begin{aligned}
M_1(z_1(t))+M_2(z_1(t))&=1\\
N_1(z_2(t))+N_2(z_2(t))&=1\\
P_1(z_3(t))+P_2(z_3(t))&=1\\
Q_1(z_4(t))+Q_2(z_4(t))&=1\\
R_1(z_5(t))+R_2(z_5(t))&=1
\end{aligned}$$

A set of 32 fuzzy rules have then been built by using all the possible combinations of the two membership functions for each variable, in the form (9) and the system output using relation (10).
The fuzzy LMI controller design strategy, based on the above fuzzy model is described in the next Section.

4. THE LMI FUZZY CONTROLLER

The fuzzy LMI control strategy, proposed in [16], is based on partial distributed compensation (PDC), i.e. a compensator fuzzy rule is designed for each system rule.
Each rule of the controller is in the form:

IF *$z_1(t)$ is M_{i1} and ... and $z_p(t)$ is* M_{ip}, THEN
$u(t)=-F_i x(t)$ i=1,2,..,32 (11)

and the controller output is given by:

$$u(t)=-\frac{\sum_{i=1}^{r} w_i(z(t))F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))}=-\sum_{i=1}^{r} h_i(z(t))F_i x(t)$$

The design strategy consists therefore in designing the local gains F_i of the consequents by using suitable global stability conditions expressed as LMI problems, as proved in [16]:

The equilibrium of a closed Fuzzy control system is asymptotically stable in the large if there exist a common positive definite matrix P such that for:
$h_i(z(t))\cdot h_j(z(t))\neq 0$, :

$\{A_i - B_iF_j\}^T P + P\{A_i - B_iF_j\}<0$ $\forall t$, i,j=1,2,…,r

In our application the input matrix is the same in all the 32 subsystems, so that the above conditions reduce to:

$\{A_i - BF_i\}^T P + P\{A_i - BF_i\}<0$ i=1,2,..,r

This can be written as an LMI in the form:

$-XA_i^T + M_i^T B^T - A_iX + BM_i>0$ i=1,2,..,r

where $X=P^{-1}$ and $M=FX$.

The resulting control system is then described by 32 rules in the form:

IF *$z_1(t)$ is M_1 and $z_2(t)$ is N_1 and $z_3(t)$ is P_1 and $z_4(t)$ is Q_1 and $z_5(t)$ is R_1*, THEN $u(t)=-F_1 x(t)$

However stability is not the only requirement to be satisfied in our application. In particular it is fundamental to impose suitable constraints on the control power.
To this aim two different strategies have been pursued. The first strategy is based on directly imposing a constraint on the control input in the form:

$$\|u(t)\|_2<\mu$$

The corresponding LMI problem is then:

$$\begin{cases}
-XA_i^T+Y_i^TB^T-A_iX+BY_i>0\\
\phi^2 I\le X\\
\begin{bmatrix} X & Y_i^T\\ Y_i & \mu^2 I\end{bmatrix}\ge 0\\
X>0
\end{cases}$$

i =1,2,...,32

where ϕ is chosen such that:

$$||x(0)|| < \phi$$

In fig. 4 the trend of the variable $\Delta T_{25}=T_2-T_5$ is reported with the control inserted for $t=1000$ *s*.

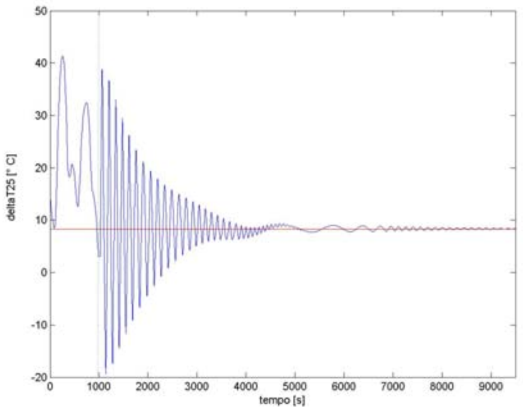

Fig. 4. Trend of the controlled variable $\Delta T_{25}=T_2-T_5$

As it can be observed the oscillations are damped out and the designed controller stabilises the system. The long transient response is due to the imposed constraint on the control input. A previous design performed without considering power input constraint lead to a very fast transient with unacceptable values of power.

A second strategy was adopted based on the minimisation of the quadratic performance index:

$$J = \int_0^{\infty} \left\{ x(t)^T Qx(t) + u(t)^T Ru(t) \right\} dt$$

where $Q \geq 0$ and $R > 0$.

This condition reduces to the following inequality [17] :

:

$$-XA_i^T + Y_i^T B - A_i X + BY_i - XQX + Y_i^T RY_i > 0$$

$i = 1,2,..,32$

where $X=P^{-1}$ and $Y_i=F_iX$., that can be written as an LMI problem:

$$\begin{bmatrix} A_1X + XA_1^T - B\,M_i + M_i^T B^T & XQ^{\frac{1}{2}} & Y_1R^{\frac{1}{2}} \\ Q^{\frac{1}{2}}X & -I & \underline{0} \\ R^{\frac{1}{2}}Y_1 & \underline{0} & -1 \end{bmatrix} < 0$$

$i = 1,2,..,32$

The weighting matrix Q has been selected in order to minimize the fluid velocity, while guaranteeing, by an high value of the R parameter, small power oscillations.

Figure 5 reports the trend of the variable $\Delta T_{25}=T_2-T_5$ with the previous controller inserted at $t=1000$ *s*.

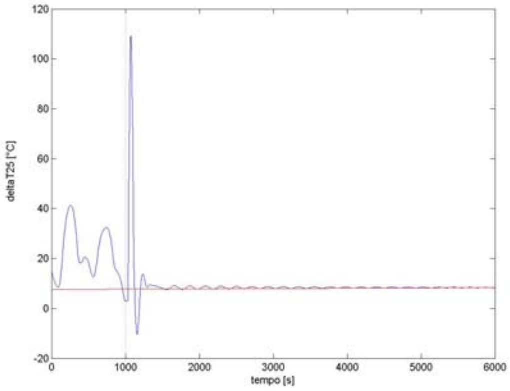

Fig. 5. Trend of the controlled variable $\Delta T_{25}=T_2-T_5$

From this figure it can be observed that this second strategy leads to a faster transient with an increment of the peak value of the temperature oscillations.

5. CONCLUSION

In this paper a Fuzzy LMI control strategy has been applied to the control of a natural circulation loop. The controller has been designed in order to avoid fluid inversion, which compromises efficient heat removal from hot source. In a previous work [12] a set of PID controllers were designed on the basis of the linearised analytical model of the circuit. However no global stability conditions were guaranteed.

The strategy proposed in this paper has been chosen in order to analytically guarantee global stability. This approach required to write the non-linear system in the fuzzy T-S form and then solving a suitable LMI problem to derive the controller gain.

The two designed controllers stabilise the system imposing constraints on the control power. The first approach resulted in a slow transient with low peak temperature values, whereas the second design resulted in a very fast transient with greater temperature peaks. Work is in progress in order to implement both controllers in the experimental circuit, in order to obtain an experimental evaluation of the results and to compare their performance.

REFERENCES

[1] Vijayan P.K., A.K. Nayak, D.S. Pilkhwal, D. Saha, V. Venkat Raj, (1992). Effect of Loop Diameter on the Stability of Single-Phase Natural Circulation in Rectangular Loops., Proc. 5th NURETH-5, Salt Lake City, USA, vol. 1, pp. 261-267.

[2] Greif, R., Y. Zvirin, A. Mertol, (1979). The transient and stability behavior of a natural convention loop. Trans. of the ASME, vol. 101, pp. 684-688.

[3] Kreitlow, D.B., G.M. Reistad, C.R. Miles, G.G. Culver, (1978). Thermosyphon Models for Downhole Heat Exchanger Applications in Shallow Geothermal Systems, ASME J. Of Heat Transfer, vol. 100, pp. 713-719.

[4] Zvirin, Y., A. Shitzer, A. Bartal-Bornstein, (1978). On the Stability of the Natural Circulation Solar Heater, Proc. 6th Int. Heat Transfer Conf., Toronto, Canada, vol. 20, pp. 997-999.
[5] Zvirin, Y.,(1981).A Review of N. C. Loops in PWR and Other Systems, Nuclear Egineering Design, vol. 67, pp.203-225.
[6] Japikse, D. (1973), Advances in Thermosyphon Technology, Advances in Heat Transfer, eds. T.F. Irvine and J.P. Harnett, vol. 9, pp. 1-111.
[7] Cammarata, G., A. Fichera, M. Froghieri, M. Misale, M.G. Xibilia, (1999), A new modelling methodology of natural circulation loop for stability analysis, Proc. Eurotherm Seminar No. 63, pp.151-159.
[8] Singer, J.,Y.Z. Wang, (1991), Controlling a Chaotic System, Physical Review Letters, vol. 66, n. 9, pp. 1123-1125.
[9] Wang, Y., J. Singer, H. H. Bau, (1992), Controlling chaos in a thermal convection loop, J. Fluid Mech., vol. 37, 479-498.
[10] Rubio, D., (1995). Boundary Control for the Chaotic Flow of a Thermal Convection Loop", Proc. 34th Conf. On Decision and Control, New Orleans, U.S.A., pp. 3734-3737.
[11] Wang, Y., J. Singer, H.H. Bau, (1992),Controlling Chaos in a thermal Convection Loop, J. Fluid Mech., vol. 237, pp. 479-498.
[12] G. Muscato, M.G. Xibilia, Modeling and control of a natural circulation loop, Journal of Process Control,vol. 13/3 pp.239-251, Dec. 2002
[13] Fichera, A. G. Muscato, M.G. Xibilia, A. Pagano. A modeling strategy for a rectangular thermal convenction loop, Proc. of the 15th IFAC Triennal World Congress, Barcellona, Spain, July 2002
[14] Fichera,A., G. Muscato, M.G. Xibilia, A. Pagano, (2000). Modelling Unstable Behavior of a Natural Circulation Loop with a Neural Network, Proceedings of the IJCNN 2000, pp.I-75-I-80, Como, Italy.
[15] Fichera, A., G. Muscato, A. Pagano, M.G. Xibilia, (2001),A neural model for natural convection control, Proceedings of the 9th IEEE Mediterranean Control Conference on Control and Automation, Dubrovnik (Croatia).
[16] K.Tanaka and H.O. Wang, Fuzzy Control Systems Desing and Analysis: A Linear Matrix Inequality Approach, John Wiley & Sons, 2001.
[17] Boyd, S. and C. Barratt, "Linear Controller Design – Limits of Performance", Prentice-Hall, 1991.