

Application of Internal Models in the Design of Digitally Controlled Electrical Drives

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Abstract-- The design of a controlling structure with internal models for digitally controlled electrical drives is given. In the control portion of the structure, the nominal model of the plant and model of immeasurable external disturbances are included to improve the system robustness and to extract the expected class of disturbances. Particular attention is paid to the design of IMPACT (Internal Model Principle and Control Together) structure for digitally controlled drives and a suitable modification of the structure is proposed in order to improve the system performance and to facilitate system synthesis.

Index Terms-- Principle of absorption, Immeasurable external disturbance, IMPACT structure, Controlled electrical drives, Newton's predictor.

I. INTRODUCTION

In many applications of controlled electrical drives the high dynamical performance and capability of the system to reject the influence of external disturbances on the steady-state value of the controlled variable are required. In the tracking regime, the tracking error is to be reduced to the level of sensor resolution, in the presence of the generalized disturbance that comprises the external disturbance and uncertainties of plant parameters. These requirements can be achieved by the design of IMPACT controlling structure [1-4] suited for the design of speed- and position-controlled electrical drives.

The IMPACT structure has the merits of both the structures based upon the IMP (Internal Model Principle) and IMC (Internal Model Control) [1,5]. As it is known, IMP means the inclusion of disturbance model into the controlling structure in order to compensate effects of expected class of external disturbances on the system output or the system controlled variable (angular speed or position of the motor shaft). The IMC structure is not suitable for disturbance rejection but it enables the achievement of the robust system stability and high dynamic performance.

In this paper, the conventional IMPACT structure is described and then its modification is proposed for application to the structural design of digitally controlled electrical drives. It will be shown that the application of

predictive filters instead of disturbance observer, including the model of disturbance, gives the same or even better system performance. Unlike the structure with disturbance observer, the alternative control structure, proposed in this paper, is simpler and with a smaller number of tuning parameters within the internal models by which the robustness, filtering properties, and high dynamic performance of the system can be easily adjusted.

II. PRINCIPLE OF ABSORPTION

Suppose that k th sample of an external disturbance $f(t)$ may be determined by a finite number m_0 of previous samples. Then, the disturbance is regular and may be described by extrapolation equation [4]

$$f(kT) = D_f(z^{-1})f((k-1)T) \quad (1)$$

where $D_f(z^{-1})$ is the prediction polynomial of order $m_0 - 1$. Relation (1) is called the equation of extrapolation or prediction [4] and it may be rewritten as

$$(1 - z^{-1}D_f(z^{-1}))f(z^{-1}) = 0 \quad (2)$$

where $f(z^{-1})$ denotes the z -transform of disturbance. Relation (2) is called compensation equation and FIR filter having the pulse transfer function $1 - z^{-1}D_f(z^{-1})$ is the absorption filter or the compensation polynomial [4].

Absorption filter $\Phi_f(z^{-1}) = 1 - z^{-1}D_f(z^{-1})$ is designed for a known class of disturbances and its impulse response becomes identically equal to zero after n sampling instants, where $n \geq m_0$. Hence, the compensation equation (2) may be considered as the absorption condition of a given class of disturbances. The condition can be expressed as

$$\Phi_f(z^{-1})f(z^{-1}) = 0, \text{ for } t = kT \geq (\deg \Phi_f)T. \quad (3)$$

The extrapolation polynomial $D_f(z^{-1})$ is determined by an apriori information about disturbance $f(t)$ [4, 6], nevertheless, it is simply resolved as

$$\Phi_f(z^{-1}) = D_w(z^{-1}), \text{ from } f(z^{-1}) = \frac{N_w(z^{-1})}{D_w(z^{-1})}. \quad (4)$$

In the case of a stochastic disturbance $s(t)$, absorption filter (4) should suppress as much as possible effects of disturbance on the system output. Thus, for a low frequency disturbance $s(t)$, which can be generated by double integration of the white noise, an appropriate choice of

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absorption filter is $\Phi_s(z^{-1}) = (1 - z^{-1})^2$ that corresponds to absorption of linear (ramp) disturbance [1, 7]. In majority of practical applications an appropriate choice might be $D(z^{-1}) = 2 - z^{-1}$. According to (4), prediction polynomial $D(z^{-1}) = 2 - z^{-1}$ rejects ramp disturbances; but, it enables also the extraction of slow varying disturbances and even suppression of the effects of low frequency stochastic disturbances.

III. IMPACT STRUCTURE

In the IMPACT structure shown in Fig.1, the controlling process is given by its pulse transfer function or by polynomials $P_u(z^{-1})$ and $Q(z^{-1})$, and the process dead-time given by integer k . Within the control portion of the structure in Fig.1 (shaded part) two internal models are included: the two-input nominal plant model

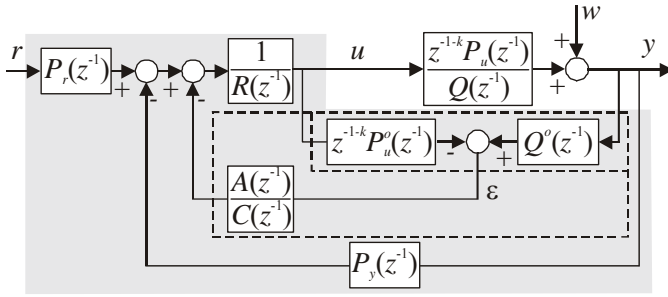


Fig.1. IMPACT controlling structure

$$W^0(z^{-1}) = \frac{z^{-1-k}P_u^o(z^{-1})}{Q^0(z^{-1})} \quad (5)$$

explicitly and the disturbance model embedded into the discrete filter $A(z^{-1})/C(z^{-1})$. Both the internal nominal plant model and disturbance model is treated as the disturbance estimator. The control portion has two control loops that can be designed independently. The minor local control loop is designed by the proper choice of polynomials $A(z^{-1})$ and $C(z^{-1})$, while polynomials $P_r(z^{-1})$ and $P_y(z^{-1})$ in the main control loop are determined to achieve the desired system set point response. For a minimal phase plant, the proper choice of polynomial $P_y(z^{-1})$ is $R(z^{-1}) = P_u^o(z^{-1})$ [1].

Under the nominal conditions ($P_u(z^{-1}) \equiv P_u^o(z^{-1})$, $Q(z^{-1}) \equiv Q^0(z^{-1})$) and for $R(z^{-1}) = P_u^o(z^{-1})$, the closed-loop transfer functions $y(z^{-1})/r(z^{-1})$ and $y(z^{-1})/w(z^{-1})$ are easily derived from Fig.1 as

$$\frac{y(z^{-1})}{w(z^{-1})} = \frac{Q^0(z^{-1})[C(z^{-1}) - z^{-1-k}A(z^{-1})]}{C(z^{-1})[Q^0(z^{-1}) + z^{-1-k}P_y(z^{-1})]} \quad (6)$$

and

$$\frac{y(z^{-1})}{r(z^{-1})} = \frac{z^{-1-k}P_r(z^{-1})}{Q^0(z^{-1}) + z^{-1-k}P_y(z^{-1})}. \quad (7)$$

In virtue of (7), the system set-point response can be adjusted by determining appropriate polynomials $P_r(z^{-1})$ and $P_y(z^{-1})$ according to the desired system closed loop transfer function $y(z^{-1})/r(z^{-1}) = G_{de}(z^{-1})$. Then, the absorption of an external disturbance and the speed of disturbance transient response are adjusted by choosing the structure and parameters of the disturbance estimator.

III.1. ELIMINATION DISTURBANCE

From (6), the steady-state error in the presence of a known class of external disturbance $w(t)$ will become zero if

$$\lim_{z \rightarrow 1} (1 - z^{-1}) \frac{Q^0(z^{-1})[C(z^{-1}) - z^{-1-k}A(z^{-1})]}{C(z^{-1})[Q^0(z^{-1}) + z^{-1-k}P_y(z^{-1})]} w(z^{-1}) = 0 \quad (8)$$

In the case of stable polynomial $C(z^{-1})$ and the plant of nonminimal phase,

$$\lim_{z \rightarrow 1} \frac{Q^0(z^{-1})}{C(z^{-1})[Q^0(z^{-1}) + z^{-1-k}P_y(z^{-1})]} \neq 0 \quad (9)$$

and then the relation (8) is reduced to

$$\lim_{z \rightarrow 1} (1 - z^{-1}) [C(z^{-1}) - z^{-1-k}A(z^{-1})] w(z^{-1}) = 0 \quad (10)$$

As shown later, the stable polynomial $C(z^{-1})$ is to be chosen according to the desired speed of disturbance rejection and the required degree of system robustness and then polynomial $A(z^{-1})$ is determined to satisfy relation (10).

According to the principle of absorption, it is possible to design the observer estimator that rejects any kind of expected disturbances. To this end, consider the class of disturbances having the z -transform $w(z^{-1}) = N_w(z^{-1})/D_w(z^{-1})$. Then, relation (10) is satisfied if the following Diophantine equation holds

$$z^{-1-k}A(z^{-1}) + B_1(z^{-1})\Phi(z^{-1}) = C(z^{-1}) \quad (11)$$

where $\Phi(z^{-1})$ represents the absorption polynomial determined by $\Phi(z^{-1}) \equiv D_w(z^{-1})$. For example, to the polynomial and sinusoidal disturbances ($w(t) = \sum_{i=1}^m d_i t^{i-1}$ and $w(t) = \sin \omega t$) correspond respectively $\Phi(z^{-1}) = (1 - z^{-1})^{m+1}$ and $\Phi(z^{-1}) = 1 - 2z^{-1} \cos \omega T_s + z^{-2}$, where T_s is the sampling period.

A unique solution of the Diophantine equation, which plays a crucial role in the design procedure of the observer estimator, proposed in this paper, does not exist [8]. Relation (11) is a linear equation in polynomials $A(z^{-1})$ and $B_1(z^{-1})$. Generally, the existence of the solution of Diophantine's equation is given in [9]. According to [9], there always exists the solution of (11) for $A(z^{-1})$ and

$B_1(z^{-1})$ if the greatest common factor of polynomials z^{-1-k} and $\Phi(z^{-1})$ divides polynomial $C(z^{-1})$; then, the equation has many solutions. The particular solution of (11) is constrained by the fact that the control law must be causal, i.e., $\deg A(z^{-1}) \leq \deg C(z^{-1})$. Hence, after choosing a stable polynomial $C(z^{-1})$ and degrees of polynomials $A(z^{-1})$ and $B_1(z^{-1})$, and inserting the absorption polynomial $\Phi(z^{-1})$ that corresponds to an expected external disturbance, polynomials $A(z^{-1})$ and $B_1(z^{-1})$ are calculated by equating coefficients of equal order from the left- and right-hand sides of equation (11). Polynomial $A(z^{-1})$ obtained by solving (11) guarantees the absorption of the expected class of disturbances, while the choice of $C(z^{-1})$ affects the speed of disturbance rejection, system robustness, and sensitivity with respect to measuring noise. Good filtering properties and the system efficiency in disturbance rejection are the mutually conflicting requirements. Therefore, to reduce the noise contamination, the low-pass digital filter may be introduced to modify the internal model of the disturbance into

$$\frac{A(z^{-1})}{C(z^{-1})} = \frac{A_f(z^{-1})A_1(z^{-1})}{C(z^{-1})} \quad (12)$$

where $A_f(z^{-1})/C(z^{-1})$ represents the pulse transfer function of the low-pass filter and $A_1(z^{-1})$ is a polynomial that satisfies (11) and thus includes implicitly the internal model of disturbance. The lower bandwidth of the low-pass filter corresponds to a higher degree of system robustness and vice versa [10]. According to [10], complex disturbances require higher order of polynomial $A(z^{-1})$ and it will further reduce system robustness with respect to mismatches of plant parameters.

III.2. PARAMETER SETTING

The main control loop of the system of Fig. 1 is designed to achieve the desired set-point response determined by the system closed-loop transfer function

$$G_{de}(z^{-1}) = \frac{z^{-1-k} H_{de}(z^{-1})}{K_{de}(z^{-1})}. \quad (13)$$

According to (7), the desired closed-loop transfer function is achieved if the following identity holds

$$\frac{z^{-1-k} P_r(z^{-1})}{Q^0(z^{-1}) + z^{-1-k} P_y(z^{-1})} \equiv \frac{z^{-1-k} H_{de}(z^{-1})}{K_{de}(z^{-1})}. \quad (14)$$

To satisfy (14), it is first necessary to solve the Diophantine equation

$$Q^0(z^{-1}) + z^{-1-k} P_y(z^{-1}) = T(z^{-1}) K_{de}(z^{-1}) \quad (15)$$

for polynomials $P_y(z^{-1})$ and $T(z^{-1})$ and then to determine the second polynomial of the main control loop of the system of Fig. 1 as

$$P_r(z^{-1}) = T(z^{-1}) H_{de}(z^{-1}). \quad (16)$$

where $T(z^{-1})$ in (15) is chosen as a stable polynomial.

Recall that, for a minimal phase plant, $R(z^{-1}) = P_u^o(z^{-1})$.

The characteristic polynomial $K_{de}(z^{-1})$ is read from (13) or it may be determined by the desired closed-loop system pole spectrum. To improve the system robustness with respect to uncertainties of plant parameters, polynomial $K_{de}(z^{-1})$ may be extended by factors

$$\prod_{i=1}^n (1 - b_i z^{-1})^i, \quad 0 \leq b_i \leq 0.9. \quad (17)$$

At the beginning, the values of b_i and integer n are to be chosen as small as possible and then they can be increased gradually until the required criterion of robust stability is satisfied. At the same time, polynomial $P_r(z^{-1})$ should be modified into

$$P_r(z^{-1}) = \frac{\prod_{i=1}^n (1 - b_i z^{-1})^i}{\prod_{i=1}^n (1 - b_i)^i} \quad (18)$$

to save the achieved set-point response and to keep unchanged the steady-state value of the system output.

IV. MODIFIED IMPACT STRUCTURE

Fig. 2 shows the modified IMPACT structure for the control plants without transport lags, which may be applied for structural design of digitally controlled electrical drives [1,2]. Signal w_M modeled the influence of load torque on system output y (angular speed or position of the motor shaft).

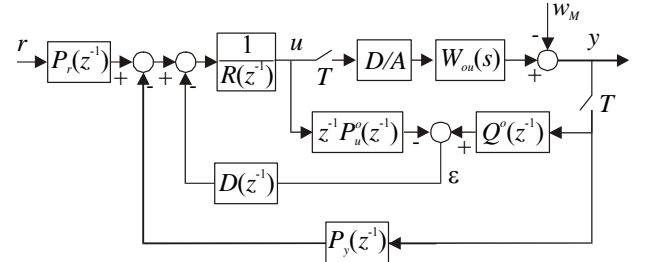


Fig. 2. Modified IMPACT controlling structure

The control plant of the structure in Fig. 2 is given by its nominal pulse transfer function

$$W^o(z^{-1}) = \frac{z^{-1-k} P_u^o(z^{-1})}{Q^o(z^{-1})} \quad (19)$$

which is used as a two-input internal plant model within the control portion of the structure. Signal ε estimates the influence of generalized disturbance on the system output. Uncertainties of plant modeling may be adequately described by the multiplicative bound of uncertainties $\alpha(\omega)$ [11]

$$W(z^{-1}) = W^o(z^{-1})(1 + \delta W(z^{-1})) \quad (20a)$$

$$|\delta W(e^{-j\omega T})| \leq \alpha(\omega), \quad \omega \in [0, \pi/T]. \quad (20b)$$

Fig. 4. Frequency characteristics of Newton's predictive filter for $p = 1$ and $M = 1, 2$ and 3

For further simplification, the low-pass digital filter $S(z^{-1})$ may be adopted as a digital equivalent of the simplest low-pass analogue filter

$$S(z^{-1}) = \frac{1}{T_f s + 1} \bigg|_{s = \frac{2(1-z^{-1})}{T(1+z^{-1})}} = \frac{T(1+z^{-1})}{2T_f + T + (T - 2T_f)z^{-1}} \quad (28)$$

having only one tuning parameter T_f of a clear physical meaning. By increasing the value of T_f the better filtering properties and higher system robustness are achieved, especially within the low frequency band. On the other hand, with greater T_f the speed of disturbance absorption is reduced and vice versa.

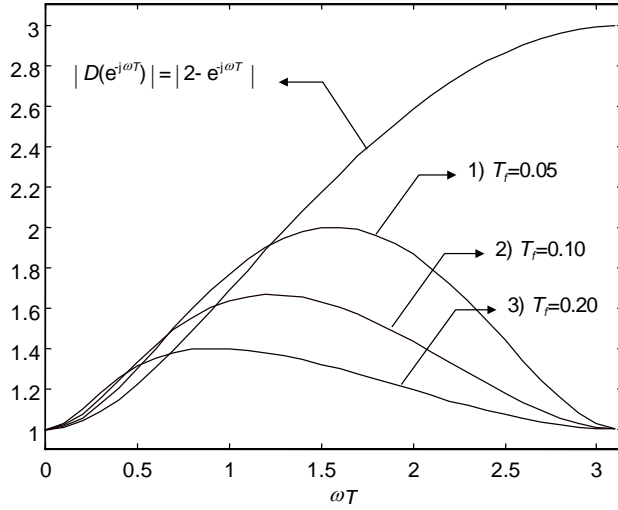


Fig. 5. Frequency characteristics of LSN predictor and prediction polynomial

For comparison, the frequency characteristics of the first order prediction polynomial and LSN predictor are illustrated in Fig. 5.

V. ILLUSTRATIVE EXAMPLE

The efficiency of LSN predictor with $T_f = 0.2s$, when compared with the application of prediction polynomial $D(z^{-1}) = 2 - z^{-1}$, in the case of IMPACT structure of positioning servomechanism is illustrated by Figs. 6 and 7. In the servomechanism, the 16-bits D/A converter and 12-bits R/D converter are applied. The results of simulation runs given in Fig. 6 (a) and (b) and Fig. 7 (a) and (b) are obtained by LSN predictor while the results in Fig. 6 (c) and (d) and Fig. 7 (c) and (d) are accomplished by the prediction polynomial. Notice that LSN filter suppresses the effects of quantization noise on the control variable and slightly slows down the speed of disturbance rejection. Sampling period $T = 0.1s$ is assumed. The control plant is DC motor U12M4T having the electromagnetic gain factor $K = 4.38$ and mechanical time constant $T_m = 0.32s$. The desired close-loop system transfer function is specified by two conjugate complex poles with undamped natural frequency $\omega_n = 2.5$ rad/s and relative damping coefficient $\zeta = 1$.

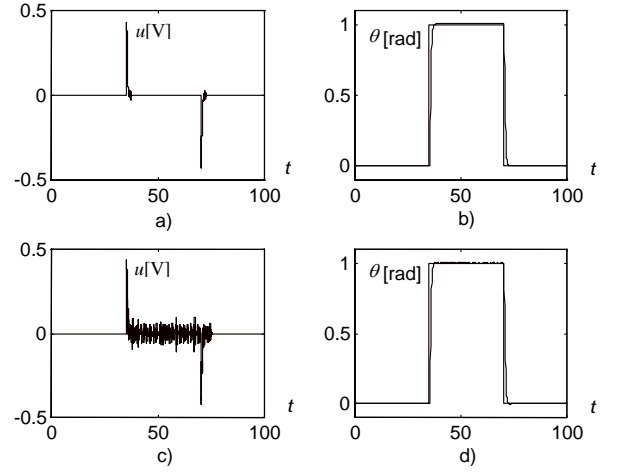


Fig. 6. Operation of IMPACT structure in the absence of torque disturbance

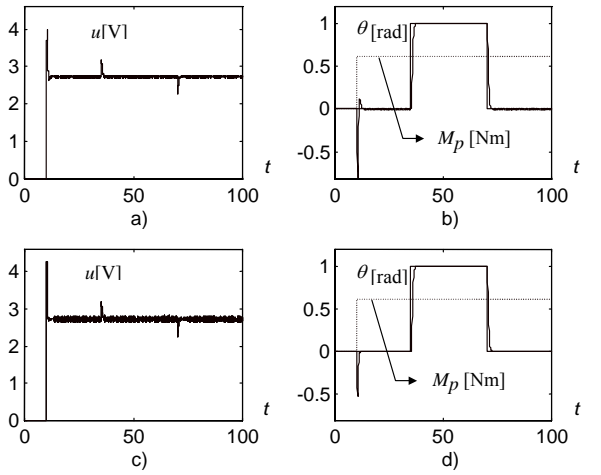


Fig. 7. Operation of IMPACT structure in the presence of torque disturbance

The efficiency of LSN predictor application as a prediction within the IMPACT structure of the speed-controlled electrical drive is illustrated in Figs. 8 and 9. The same DC motor U12M4T and $T = 0.1s$, as in the case of the positioning servomechanism, is applied. According to the proposed procedure, the desired closed-loop system transfer function

$$G_{dc}(z^{-1}) = \frac{0.312898z^{-1} - 0.259182z^{-2}}{1 - 1.687103z^{-1} + 0.740818z^{-2}} \quad (29)$$

is specified and then the following polynomials of the control structure are calculated

$$\begin{aligned} z^{-1}P_u^o(z^{-1}) &= 1.1765z^{-1}, \quad Q^o(z^{-1}) = 1 - 0.73146z^{-1}, \\ R(z^{-1}) &= 1.1765, \quad P_y(z^{-1}) = -0.955642 + 0.740818z^{-1} \quad (30) \\ \text{and } P_r(z^{-1}) &= 0.312898 - 0.259182z^{-1}. \end{aligned}$$

The system simulation is performed when 16-bits D/A and 12-bits R/D converters are applied (Fig. 8). The

standard deviation of the difference between output signals generated with and without quantization noise, in the case of IMPACT structure with LSN predictor (Fig. 8), is 12% less than in the case of the observer based structure. By increasing time constant T_f , quantization noise is more suppressed but, at the same time, the speed of disturbance rejection is slowed down. Hence, the implementation of LSN predictor instead of prediction polynomial gave approximately the same results as in the case of observer implementation in IMPACT structure, but the structure with LSN predictor is significantly simpler. Furthermore, by tuning parameter T_f it is possible, in a simple way, to adjust system dynamic properties, suppression of quantization noise, and to improve robust stability of the system (Fig. 9).

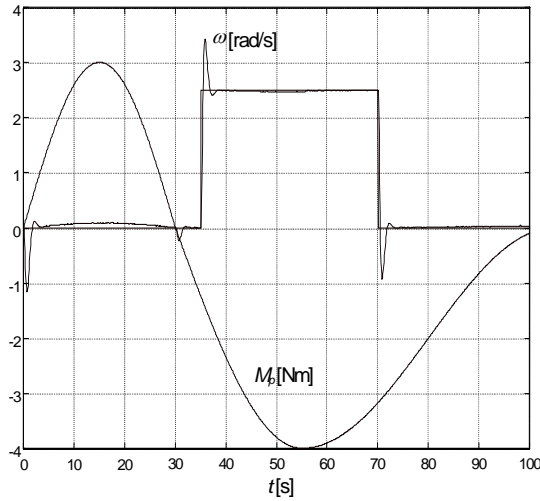


Fig. 8. Responses of IMPACT structure of speed-controlled servomechanism with LSN predictor ($T_f = 0.2s$)

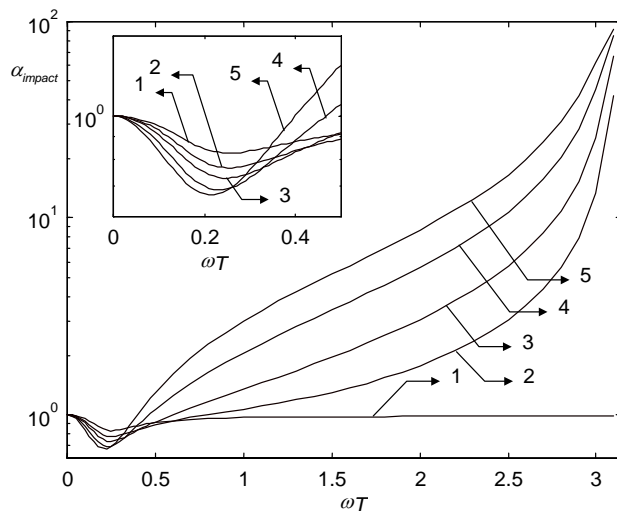


Fig. 9. Frequency characteristics of complementary sensitivity function of IMPACT structure with (1) prediction polynomial and (2) with LSN predictor having: (2) $T_f = 0.05$, (3) $T_f = 0.1$, (4) $T_f = 0.2$, and (5) $T_f = 0.33$.

VI. CONCLUSION

The design procedure of IMPACT structure for digitally-controlled speed and position servomechanisms has been given. It was shown that the set-point response of the structure and speed of disturbance rejection could be adjusted independently. Instead of the design of disturbance estimator within the local loop of the structure, as in the case of basic IMPACT structure, different predictors are employed, for the purpose of disturbance extraction. This alternative approach has several advantages: the relatively easy setting of controller parameters, adjustable speed of disturbance rejection, and control of system robust stability.

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