Adaptive Control Scheme Based on Hybrid Adaptation of Lead-lag Compensator Parameters

Zdenko Kovačić, Stjepan Bogdan, Mario Punčec

University of Zagreb, Faculty of Electrical Engineering and Computing
Unska 3, 10000 Zagreb, CROATIA
E-mail: zdenko.kovacic@fer.hr, stjepan.bogdan@fer.hr , mario.puncec@fer.hr,
URL: http://flrcg.rasip.fer.hr

Abstract. This paper presents a method of adaptation achieved by adding a lead-lag compensator to a principal controller and by adapting parameters of the lead-lag compensator rather than the parameters of the principal controller. The parameters, gain and lead time constant, are adapted by using an integral criterion and a sensitivity model derived at first for the second order reference model and then applied to the high-order systems of similar dynamic characteristics. A proposed hybrid adaptation algorithm has been tested in the angular speed control loop of a permanent magnet synchronous motor (PMSM) drive, and the simulation results confirmed its effectiveness in spite of its simplicity.

Key words: adaptive control, integral criterion, sensitivity functions, reference model, angular speed control, servo system

1. Introduction

Many control solutions still use controllers with fixed parameter values. These values guarantee a desired system performance only in a certain range around the operating point. If a process (control object) is characterized by large dead-times, stochastic process disturbances, continuous parameter variations, varying production levels and other sources of unbalanced control response, commissioning may often be difficult.

Providing that process parameters are first identified from collected process inputs and outputs, controller computation and adjustment can be automated in many ways [1, 2, 3, 4]. For example, auto-tuning controllers are adaptive controllers whose functions involve an on-line process parameter identification and controller parameter computation. Self-tuning controllers also include controller adjustment. These controllers adapt their parameters in an automatic way by means of a process parameter estimator, and thus may control even the most difficult control loops.

The other way of adapting principal controllers is to use a reference model as a model for a desired dynamic behavior. This philosophy is present in various model reference-based adaptive control (MRAC) schemes [5, 6]. A full-order reference model can provide the best impact of the adaptation mechanism, but reduced-order reference models are usually preferred because of a simpler design and implementation [7]. Very often a second-order reference model is used to determine the desired dynamic characteristics of a high-order system with similar dynamic characteristics. Adaptation mechanisms in MRAC schemes are dominantly non-linear devices, which may generate an additional signal (signal adaptation) or change principal controller parameters (parameter adaptation).

In this paper we present an adaptive control scheme containing a principal controller and a lead-lag compensator whose parameters, gain coefficient and lead time constant, are adapted by using an integral criterion and a sensitivity model derived for a second order reference model. This control scheme represents further improvement of control schemes described in [8, 9, 10], where only integral criterion-based adaptation was discussed. Adjustments of both gain and lead time constant are adapted on-line by using functions that relate their changes to the changes of the integral criterion and the sensitivity functions. The integral criterion is calculated as the ratio of integrals (areas) determined by a model reference response and a system response. The sensitivity functions describe the influence of lead-lag compensator parameters on the system response.

The proposed control scheme has been applied to the angular speed control loop of a PMSM drive. The simulation results show its effectiveness in case of very large changes of system parameters. Due to its apparent simplicity, the proposed adaptive control scheme can be implemented in any contemporary control equipment.

2. The structure of an adaptive system

Let us assume for an unknown, time-varying, probably non-linear and stable process that its dynamics in a selected operating point could be approximated well enough with the second-order transfer function:

$$G_P(s) = \frac{Y(s)}{U(s)} \approx \frac{K_A}{T_A{}^2 \cdot s^2 + 2 \cdot \xi \cdot T_A \cdot s + 1} = G_A(s) \quad (1)$$

where $G_P(s)$ is referring to the process and $G_A(s)$ to its approximation.

By changing the operating point, system parameters may change. This would impose the need for extensive controller commissioning in various operating points, unless adaptive control is applied.

For the purpose of adaptation, let us use a second-order reference model to define a desired behavior of the adaptive closed-loop control system:

$$G_M(s) = \frac{Y_M(s)}{U_R(s)} = \frac{1}{\frac{T_M}{K_M} \cdot s^2 + \frac{1}{K_M} \cdot s + 1} \quad (2)$$

where $U_R(s)$ and $Y_M(s)$ are the reference model input and output, respectively, while T_M and K_M are the model time constant and the gain (Fig. 1.).

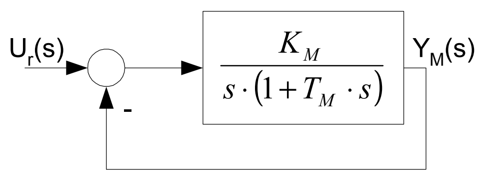

Fig. 1. The structure of a 2nd-order reference model.

The reference model response can be assumed as "nominal" dynamics. A decision which adaptation algorithm would be appropriate for the studied system depends on many factors; desired system precision and dynamics, on-line computational power, noise level, non-linearities encountered in the system, accuracy of the linearized model, etc.

Here we propose an adaptation algorithm suitable for systems approximated with the transfer function (1). The adaptation mechanism is based on the reference model (2) and it tunes parameters of the added lead-lag compensator leaving the principal controller as it is. The principal controller can be any standard type of controller or any of advanced control algorithms such as neural network, fuzzy logic or hybrid controllers [10]. The structure of the adaptive closed-loop control system is shown in Fig. 2.

The transfer function of the lead-lag compensator

$$G_f(s) = K_f \cdot \frac{T_f \cdot s + 1}{T_M \cdot s + 1} \quad . \quad (3)$$

has two adaptive parameters, gain coefficient K_f and lead time constant T_f that are adjusted on-line by means of the adaptation algorithm.

All changes in system parameters will be compensated by changes of these two parameters. Initial values of these parameters are one for gain K_f, and T_M for the lead time constant T_f, so that the lead-lag compensator has no influence on the control loop dynamics at the beginning of adaptation.

3. The adaptation mechanism

The aim of the proposed method is to determine on-line the lead-lag compensator parameters, which would enforce the closed-loop system to follow the reduced order reference model (2) as closely as possible. An adaptive gain coefficient K_f should reach the value which would provide convergence of the open-loop system gain to the value of the reference model gain K_M (see Fig. 1). Similarly, an adaptive lead time constant T_f must compensate for a dominant process time constant, so that the lag time constant T_M (that is equal to the reference model time constant) becomes a dominant one.

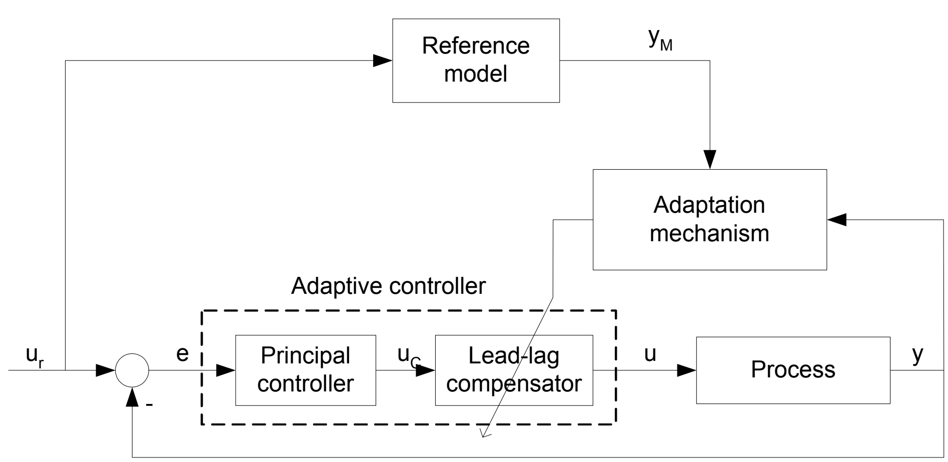

Fig. 2. The structure of the adaptive control system.

A recursive law for tuning the lead-lag compensator parameters has the following form:

$$
\begin{aligned}
K_f^{\,new} &= K_f^{\,old} \cdot \left(1 + \gamma_K \cdot \Delta K_f\right) \\
T_f^{\,new} &= T_f^{\,old} \cdot \left(1 + \gamma_T \cdot \Delta T_f\right)
\end{aligned}
\tag{4}
$$

where γ_K and γ_T are tuning coefficients, and ΔK_f and ΔT_f are changes of the gain coefficient and the lead time constant, respectively.

As mentioned before, initially $K_f=1$ and $T_f=T_M$, so that the lead-lag compensator does not influence the system behavior at the beginning of tuning.

There are two problems related to the viability of the tuning law (4). The first one is to find out how ΔK_f and ΔT_f should vary with the system response changes. The second problem is how to find such tuning coefficients γ_K and γ_T in (4), which would guarantee the tuning stability and consequently, the overall stability of the closed loop system.

To resolve these two problems, we need some well defined measure of system dynamics, and for this purpose, a reference model (2) can be used. Then we need to find such criteria which would link changes of parameters ΔK_f and ΔT_f with a changed system behavior taking a reference model behavior as the etalon.

3.1. Sensitivity model-based adaptation

One possible way to assess the influence of parameter variations on the system response is to build a sensitivity model and use so obtained sensitivity functions [11, 12]. When only the output of the system is considered, then the Kokotovic method of sensitivity points is preferable to the canonical system sensitivity model [12, 13]. With the aid of the Kokotovic method of sensitivity points, sensitivity model of reference model (2) shown in Fig. 1 can be derived. For the given model and its parameters semi relative sensitivity functions have the following form:

$$
\begin{aligned}
\eta_{Mi}(s) &= \lambda_{Mi} \frac{\delta Y_M(s, \underline{\lambda}_M)}{\delta \lambda_{Mi}} = \\
&= \frac{1}{1 + G_o(s, \underline{\lambda}_M)} S_{Mi}(s) Y_M(s, \underline{\lambda}_M)
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
S_{Mi}(s) &= \frac{\lambda_{Mi}}{G_{oM}(s, \underline{\lambda}_M)} \frac{\delta G_{oM}(s, \underline{\lambda}_M)}{\delta \lambda_{Mi}} \\
G_{oM} &= \frac{K_M}{s(1 + T_M s)} \\
\underline{\lambda}_M &= \begin{bmatrix} K_M & T_M \end{bmatrix}^T
\end{aligned}
\tag{6}
$$

Sensitivity functions are obtained in the sensitivity points of the sensitivity model by adding a transfer functions block $S_{Mi}(s)$. For the reference model (2), $S_{KM}(s) = 1$, $S_{TM}(s) = -\dfrac{T_M s}{1 + T_M s}$, and the block diagram of the corresponding sensitivity model is shown in Fig. 3.

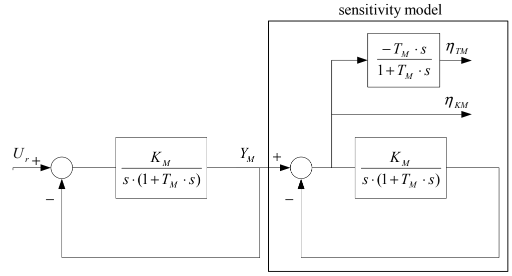

Fig.3. A sensitivity model of the second-order reference model (2).

In case of a stepwise unity change of the reference input, the resulting sensitivity functions $\eta_{KM}(t)$ and $\eta_{TM}(t)$ shown in Fig. 4 indicate that during the rise time the open-loop gain K_M is more influential than a dominant time constant T_M. When overshoot and peak time is considered, the influence of both parameters is almost leveled.

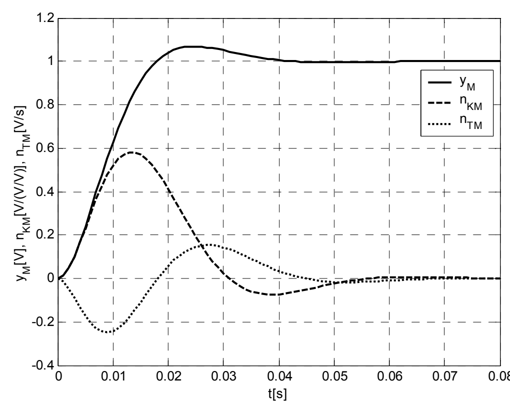

Fig.4. A second-order reference model response and sensitivity functions with respect to open-loop gain K_M and time constant T_M with nominal performance indices $\sigma_m = 5\%, t_m = 0.025$ [s].

In order to get sensitivity functions with respect to lead-lag compensator parameters, $\eta_{Kf}(t)$ and $\eta_{Tf}(t)$, the idea is to replace the sensitivity model of a closed-loop high-order system with the sensitivity model of the reference model (see Fig. 3). Then the system response y(t) becomes the input of the sensitivity model.

Changes of the system output y(t) due to small variations of the system parameters are given in the time domain by the following:

$$\Delta y(t,\underline{\lambda}) = \sum_i \eta_{\lambda i}(t)\frac{\Delta\lambda_i}{\lambda_{i0}(t)} = \\ = \eta_{Kf}(t)\frac{\Delta K_f}{K_{f0}(t)} + \eta_{Tf}(t)\frac{\Delta T_f}{T_{f0}(t)} \tag{7}$$

where $K_{f0}(0) = 1$ and $T_{f0}(0) = T_M$.

Lead-lag compensator parameter variations ΔK_f and ΔT_f, which provide desired changes of the system response $\Delta y(t)$ may be computed directly from (7) if sensitivity functions $\eta_{Kf}(t)$ and $\eta_{Tf}(t)$ are known. The following strategy was adopted: gain K_f is adjusted in the moment when $\eta_{Kf}(t)$ reaches its maximum, while T_f is adjusted when the system response reaches the peak value $y(t_m)$:

$$\Delta K_f = \frac{\Delta y(t)}{\max\left[\eta_{Kf}(t)\right]} K_{f0}(t) \\ \Delta T_f = \frac{\Delta y(t_m)}{\eta_{Tf}(t_m)} T_{f0}(t_m) \tag{8}$$

Now, tuning coefficients γ_K and γ_T must be defined. In general, larger values of γ_K and γ_T cause larger changes of K_f and T_f. Smoother adaptation may be expected if tuning coefficients are in the range $0 < \gamma_K, \gamma_T \leq 1$. A care must be taken to choose a value of γ_T which would not cause a negative value for T_f. This would change the structure of the lead-lag compensator and make the closed-loop system unstable.

3.2. Integral criterion-based adaptation

For tuning K_f and T_f, in [9] and [10] an integral criterion has been used (it is assumed that system noise has characteristics of white noise):

$$I(t_i) = \int_0^{t_i} y(t)\cdot dt \tag{9}$$

where y(t) is the unity step system response, and t_i is an integration time.

Integration time t_i is treated as a parameter that satisfies condition $y(t_i) = a$ (Fig. 5.). In the studied case a corresponds to 63% of the imposed stepwise change of the reference input $u_r(t)$.

If integral (9) is intended to be used as a measure of adaptation, the question arises how nominal process dynamics could be put in relation with the changes of K_f and T_f if the process is treated as a black box. Under such conditions, this is apparently a non-feasible task, and the answer to this question may be found in the reference model counterpart.

The integral (9) of the reference model (2) has a form:

$$I_M(t_i) = \int_0^{t_i} y_M(t)\cdot dt = \\ = t_i - \frac{1}{K_M} + \frac{e^{-\alpha}}{K_M}\cdot\left(\cos(\alpha\beta) + \frac{\frac{t_i}{2}\sin(\alpha\beta)\cdot(1-2\cdot K_M\cdot T_M)}{\alpha\beta}\right) \tag{10}$$

where $\alpha = \dfrac{t_i}{2\cdot T_M}$, $\beta = \sqrt{4\cdot T_M\cdot K_M - 1}$.

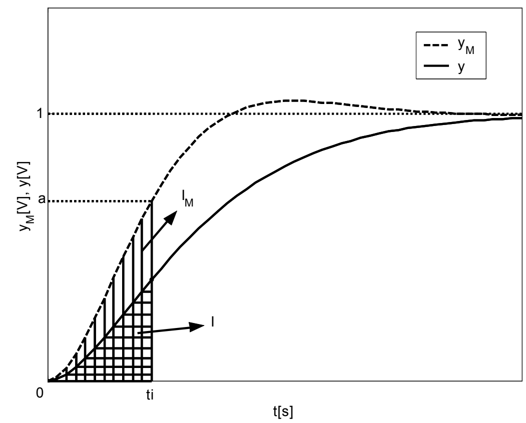

Fig. 5. Illustration of a reference model integral (I_M) and a closed-loop system integral (I).

Variations of K_M and T_M cause variations of $I_M(t_i)$. Let us denote $I_M(t_i)$ obtained for nominal reference model parameter values K_{M0} and T_{M0} as I_{M0}. By varying K_M and T_M in a certain range around nominal values, it is possible to establish relations between $\{K_M, T_M, I_M(t_i)\}$ and $\{K_{M0}, T_{M0}, I_{M0}\}$. These relations obtained for different values of gain K_M and time constant T_M, are shown in Figs. 6 and 7, respectively. It must be pointed out that these relations can be calculated either prior (off-line) or during start-up (on-line) of the controller.

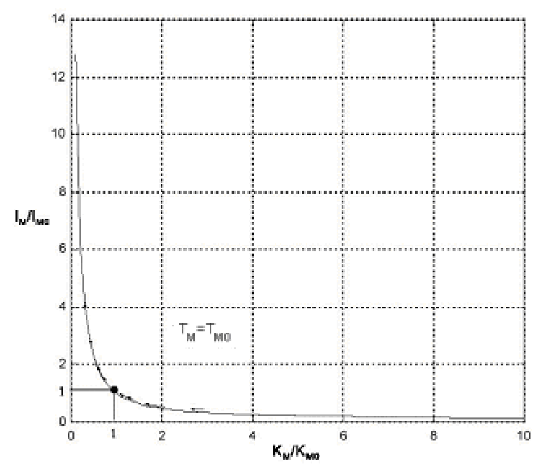

Fig. 6. A relation between $I_M(t_i)$ and a model gain K_M.

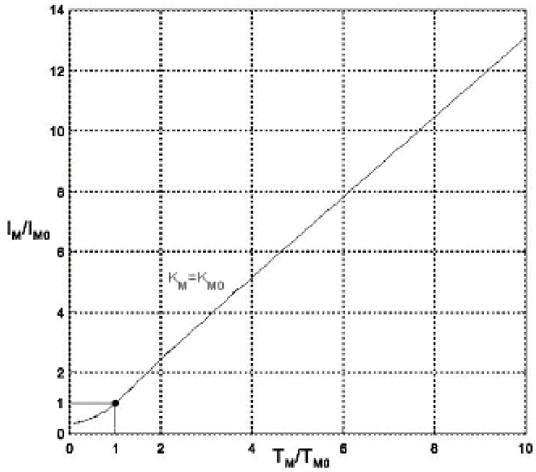

Fig.7. A relation between $I_M(t_i)$ and a model time constant T_M.

Once having exact relations $K_M/K_{M0}=f(I_M/I_{M0})$ and $T_M/T_{M0}=f(I_M/I_{M0})$, and having an integral I_M measured, we exactly know how much we must change K_M and T_M to enforce the integral I_M to be equal to the nominal integral I_{M0}. There is just one step more to obtaining an adaptation mechanism for the closed-loop system. Putting in relation integrals that are shown in Figs. 6 and 7, i.e. by substituting integral I_M with the closed-loop system integral $I(t_i)$, aforementioned relations obtain the form $K_f/K_{M0}=f(I(t_i)/I_{M0})$ and $T_f/T_{M0}=f(I(t_i)/I_{M0})$. They directly determine a sign and a magnitude of ΔK_f and ΔT_f, which should enforce integral (9) to get as close as possible to the desired integral value (10).

Since both relations have the same purpose, parameters K_f and T_f are adjusted alternatively, one parameter in each consecutive tuning iteration. This causes restrictions on the range of successfully compensated parameter variations. Accordingly, tuning coefficients γ_K and γ_T should be chosen to provide stable and smooth tuning.

3.3. A hybrid adaptation algorithm

In order to overcome noticed constraints of integral criterion-based adaptation of both parameters, two methods for tuning K_f and T_f have been combined, as shown in Fig. 8.

Here K_f is tuned according to the integral-based criterion (Fig. 6 and relation (4)), while T_f is simultaneously tuned by using the sensitivity function η_{TM} and relation (8).

4. Simulation results

The proposed hybrid adaptation algorithm has been tested in the angular speed control loop of a permanent magnet synchronous motor (PMSM) drive with a digital PI controller. The structure of the controlled process is shown in Fig. 9.

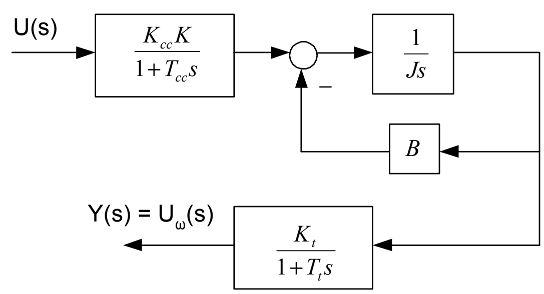

Fig.9. A block scheme of the controlled process.

The nominal values of controlled process parameters are as follows: K_{cc}=1 A/V - current closed-loop gain, T_{cc}=0.05 ms – current closed-loop time constant, K=0.9837 Vs – motor constant, J=0.00176 kg·m^2 – moment of inertia, B=0.000388 Nms – viscous friction coefficient, K_t=0.063 V·s – tachometer gain and T_t=2.5 ms – tachometer time constant.

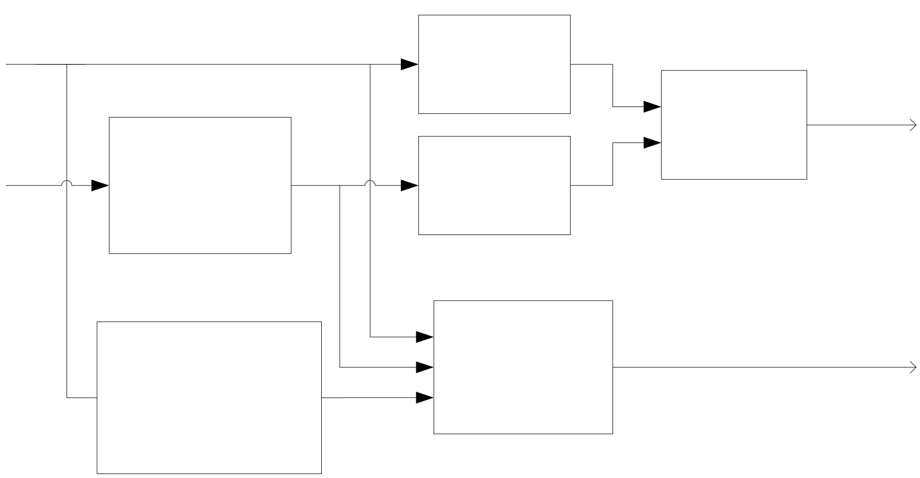

Fig. 8. A block diagram of a hybrid adaptation algorithm.

The second order reference model (2) dynamics is defined with the maximum overshoot σ_m=5.5% and the peak time t_m=0.025s. The PI and lead-lag compensator parameters were determined for nominal process parameters. Simulation experiments were concerned with adaptation to large variations of the moment of inertia. In the first experiment J has been set 5 times larger than the nominal value, $J=5\cdot J_n$. Fig. 10 shows development of process and reference model responses. A very large initial tracking error $e_M=y_M-y$ is reduced several times after only two iterations (Fig. 11).

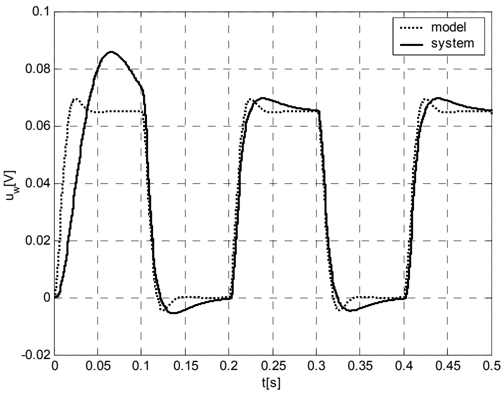

Fig.10. Process and reference model responses from the start of adaptation, $J=5\cdot J_n$.

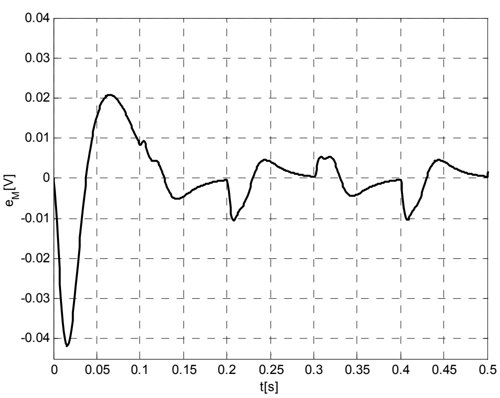

Fig.11. Tracking error responses, $J=5\cdot J_n$.

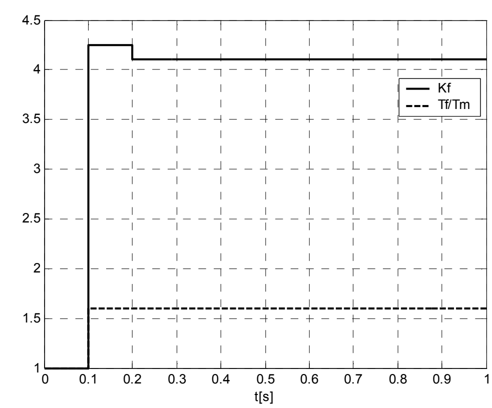

Fig.12. Convergence of the lead-lag compensator gain coefficient K_f and lead time constant T_f, $J=5\cdot J_n$.

One may see in Fig. 12 that values of lead-lag compensator parameters converge to their steady-state values in only two iterations.

In the next experiment an open loop gain K_o has been increased 5 times. A rapid improvement of process responses is shown in Fig. 13 resulting in the swift reduction of the tracking error (Fig. 14). Also, initial oscillations in the system response have been completely eliminated. Convergence of parameter values is very fast and completed in only four iterations (Fig. 15).

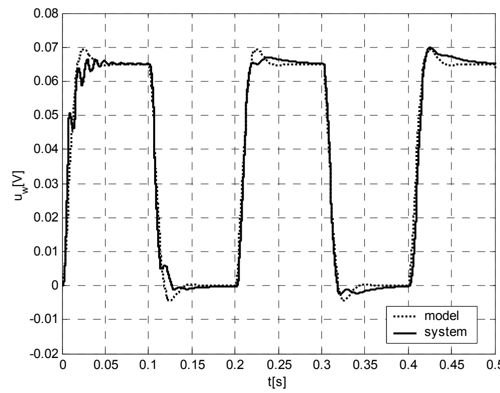

Fig.13. Process and reference model responses from the start of adaptation, $5\cdot K_{on}$.

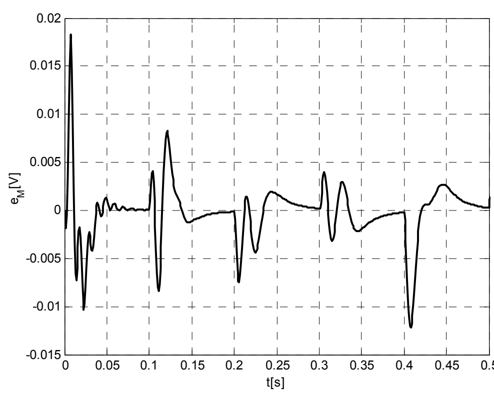

Fig.14. Tracking error responses, $5\cdot K_{on}$.

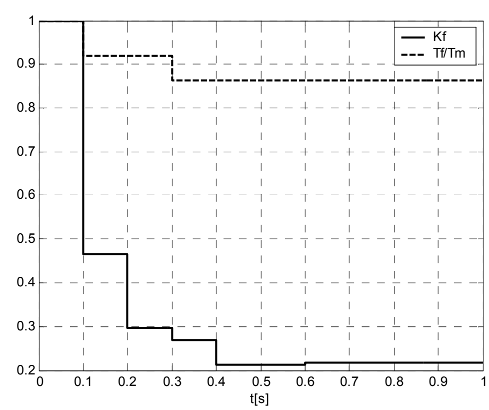

Fig.15. Convergence of the lead-lag compensator gain coefficient K_f and lead time constant T_f, $5\cdot K_{on}$.

5. Conclusions

Besides well established model reference-based and process identification-based adaptive control schemes, there is still a room for development of other adaptive control structures which would be simple enough and effective enough to raise the level of principal controller performance.

This paper presents a method of adaptation achieved by adding a lead-lag compensator to a principal controller and by adapting parameters of the lead-lag compensator rather than the parameters of the principal controller. The parameters, gain and lead time constant, are adapted by using an integral criterion and a sensitivity model derived at first for the second order reference model and then applied to the high-order systems of similar dynamic characteristics.

Sensitivity functions used for adaptation of a lead time constant are obtained from the sensitivity model based on the Kokotovic method of sensitivity points. The integral criterion-based adaptation algorithm is calculated as the ratio of integrals (areas) determined by a model reference response and a system response.

The proposed hybrid adaptation algorithm has been applied to the linearized model of the angular speed control loop of a permanent magnet synchronous motor (PMSM) drive controlled primarily with a digital PI controller. The simulation results have proved effectiveness of the controller in case of very large changes of system parameters (moment of inertia and open-loop gain).

The proposed adaptive control method can be applied not only to linear, but also to non-linear principal controllers, providing that the controlled process dynamics can be described well enough with the reduced second order reference model.

Regarding future work on the adaptive control scheme presented in this paper, an experimental verification on the laboratory setup of a servo drive should be made. Also, some other integral criteria such as ITAE and IAE should be taken into consideration for the purpose of convergence speed assessment. Stability issues should also be worked out in terms of finding such tuning coefficients which would guarantee the overall stability of the adaptive control system.

6. Acknowledgements

The work in this paper was performed within the project 036042 "Integrated Control of Robotized Plants" that was supported by a grant from the Ministry of Science and Technology, Republic of Croatia.

7. References

[1] Astrom K.J., Theory and applications of adaptive control - A Survey, *Automatica*, Vol. 19, No. 5(1983).

[2] Chalam V.V., *Adaptive control systems-Techniques and applications*, Marcel Dekker, New York (1987).

[3] Clarke D.W. and Gawthorp P.J., Self-Tuning Controller, *Proc. IEE,* Vol. 122, pp. 929-934(1975).

[4] Eykoff P., *System Identification*, Wiley, New York (1974).

[5] Landau Y.D., *Adaptive control: the model reference approach*, Marcel Dekker, New York (1987).

[6] Butler H., *Model reference adaptive control*, Prentice Hall, New York, London (1992).

[7] Ioannou P.A. and Kokotovic P.V. *Adaptive Systems with Reduced Models*, Springer, Berlin (1983).

[8] Kovačić Z., Bogdan S. and Vikalo H. Design and Parameter Adaptation of a Fuzzy Servo Controller, *Proceedings of 6th International Fuzzy Systems Association World Congress*, Sao-Paulo, Brazil, pp. 389-392(1995).

[9] Kovačić Z., Bogdan S. and Punčec M. Integral-Criterion-based Adaptive PID Angular Speed Control", CD-ROM Proceedings of the 10th International Power Electronics and Motion Control Conference, Dubrovnik, Croatia, (2002).

[10] Kovačić Z., Bogdan S. and Punčec M. Adaptive Fuzzy Logic Control Based on Integral Criterion, *Proceedings of the 15th IEEE International Symposium on Intelligent Control ISIC'00*, Patras, Greece, pp. 55-60(2000).

[11] Kokotović P.V., Method of sensitivity points in the investigation of linear control systems, *Aut. Remote Control* (U.S.S.R.), Vol. 12, pp. 1512-1518, 1964.

[12] Cruz J.B., *Feedback Systems*, McGraw-Hill, New York, 1972.

[13] Frank P.M., *Introduction to System Sensitivity Theory*, Academic Press, New York, 1978.