Selection of the initial controllers in design of the critical control systems

Takahiko Ono, Tadashi Ishihara, Hikaru Inooka

Abstract— **This paper proposes a method for selecting an initial controller required in the critical control system design. The initial controller is determined so that the response of the closed-loop system follows that of the pre-given target system with appropriate input-output performance via LMI optimization. The controller, which ensures the specification, is determined by performing a further parameter search from the obtained initial controller. A numerical example is given to show the validity of the proposed method.**

Keywords— **Control system design, Critical system, Model following, Model Matching, Linear matrix inequality, Convex optimization.**

I. Introduction

MOST control systems have constraints on the bounds of the responses. If the violation of these bounds causes unacceptable or fatal operation, the system is said to be critical [1]. In the framework of the principle of matching [2], [3], the methods for designing controllers for critical systems have been studied. In this framework, the controller is designed so that the outputs are maintained within the prespecified bounds for all inputs belonging to the set, called the possible set. Generally, the problem of designing such a controller is formulated as the admissibility problem of solving the inequalities of the form

$$\phi_i(p) \le \varepsilon_i \quad (i = 1, 2, \ldots, n) \tag{1}$$

where p is a decision vector comprising parameters of a controller. As inequality solvers to (1), the moving boundaries process [4], the goal attainment method [5] and the simulated annealing method [6] are available. In design using these inequality solvers, however, the efficiency of search is much affected by the initial point of p since $\phi_i(p)$ is not a convex function and the set of admissible solutions to (1) is not a convex set. Accordingly, a careful choice of an initial point, that is, an initial controller is required for computationally efficient design. However, the method for choosing an initial controller has not been discussed enough.

This article proposes a method for selecting an initial point which is frequently required in designing controllers for critical systems. The initial search point is selected so that one of the inequalities in (1) is ensured. This is realized by applying the idea of the model following. Especially, in order to deal with the problem of designing a low-order controller, the idea of the model matching using coprime factorization [7] is adopted. The actual controller, which ensures the design specification stated by the conjunction of inequalities, is determined by performing a parameter search from the obtained initial point. The validity of the proposed method is examined with a numerical example of multi-objective critical control system design.

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II. Preliminary

A. Notations

This article uses the following notations. Let $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of all real numbers and the set of all nonnegative real numbers, respectively. The p-norm of the signal $f : \mathbb{R}_+ \mapsto \mathbb{R}$ is defined by

$$\|f\|_p := \begin{cases} \int_0^\infty |f(t)|dt & \text{for } \ p = 1 \\ \sup\{|f(t)| : t \in \mathbb{R}_+\} & \text{for } \ p = \infty. \end{cases}$$

To express the transfer function of a linear time-invariant system simply, the following compact notation is used.

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] := D + C(sI - A)^{-1}B$$

The set of all proper and real rational stable transfer functions is denoted by RH_∞. The set of all strictly proper and real rational stable transfer functions is denoted by RH_2.

B. Principle of matching

The principle of matching is the framework for design of control systems proposed by Zakian [2], [3]. The feature of the principle is that it considers two sets of inputs. The first set, denoted by $\mathcal{P}$, is the set of all exogenous inputs that are actually applied or likely to be applied to a system such as reference signals, disturbances or sensor noises. This set is called the possible set, and it characterizes the environment in which the system is operated. The second set, denoted by $\mathcal{T}$, is the set of all inputs that ensure the design specification. This set is called the tolerable set, and it characterizes the control system. When $\mathcal{P}$ is included in $\mathcal{T}$, the control system and its environment are said to be matched. Particularly, if the difference set $\mathcal{T}\backslash\mathcal{P}$ is small, they are said to be well matched. The primary aim of the principle of matching is to match the control system to its environment, that is, to achieve $\mathcal{P} \subseteq \mathcal{T}$. A further aim is to obtain a better match by adjusting $\mathcal{P}$ and $\mathcal{T}$. Adjusting $\mathcal{T}$ means improving the control performance by changing the controller parameters, while the adjusting $\mathcal{P}$ means the improving the environment. In this sense, the principle of matching is the framework for matched environment-system design.

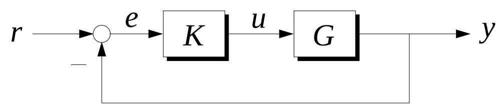

Fig. 1. Unity feedback control system.

C. Problem statement

In design of controllers for critical systems, it is indispensable to consider the feature of exogenous inputs since they are the primary factors which cause the violation of the tolerable bound of the response. For this reason, the principle of matching is suitable for controller design for critical systems. This article considers the following critical system design in accordance with the principle of matching.

1) Control system – The system to be designed is the unity feedback system shown in Fig. 1. The plant is a strictly proper linear time-invariant single-input single-output system with the transfer function

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]. \tag{2}$$

It is assumed that (A, B) is stabilizable, (A, C) is detectable and the initial state of G is zero.

2) Possible set – The exogenous input is the reference command signal r, which is known only to be extent that it has a bound D on the rate of change and zero initial condition. Accordingly, the possible set is modeled as

$$\mathcal{P} = \left\{ f : \mathbb{R}_+ \mapsto \mathbb{R} : \begin{array}{c} f : \text{piecewise smooth} \\ \|\dot{f}\|_\infty \leq D,\ f(0) = 0 \end{array} \right\}. \tag{3}$$

3) Tolerable set – Let $z_i(t, f)$ be the ith response to the input f at time t and let ε_i is the tolerable bound of z_i. The tolerable set is defined by

$$\mathcal{T} = \{ f : \mathbb{R}_+ \mapsto \mathbb{R} : \|z_i(f)\|_\infty \leq \varepsilon_i \ \ (i = 1, 2, \ldots, n) \}, \tag{4}$$

where one of the outputs is a tracking error: $z_i = e$ for a certain i.

4) Matching condition – The controller, denoted by K in Fig. 1, is given as an mth-order transfer function. It is assumed that its initial state is zero. The controller is designed so that the control system is matched to the environment characterized by $\mathcal{P}$, that is, $\mathcal{P} \subseteq \mathcal{T}$ is ensured. Let p denote the vector comprising controller parameters and let $z_i(t, h, p)$ denote the unit step response at time t for the controller characterized by p. It is known that the matching condition, $\mathcal{P} \subseteq \mathcal{T}$, is equivalent to the conjunction of the inequalities

$$D\|z_i(h, p)\|_1 \leq \varepsilon_i \quad (i = 1, 2, \ldots, n). \tag{5}$$

Accordingly, the controller, namely, the parameter vector p is determined so that (5) is met.

III. Selection of an initial controller

Some inequality solvers need an initial point of the parameter, p^0, to obtain the admissible solutions to (5). In this article, p^0 is determined so that it lies in the inside of the set of p's that satisfy

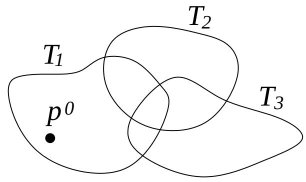

Fig. 2. Initial search point p^0. It is determined so that it lies in the inside of T_1, where T_i is the set of all solutions to $D\|e(h, p)\|_1 \leq \varepsilon_i$.

$$D\|e(h, p)\|_1 \leq \varepsilon_i. \tag{6}$$

The condition (6) is evaluated by the unit step response. Therefore, the model following approach is applicable to the problem of determining p^0: prepare the ideal open-loop system called the target system, T_m, that meets (6), and then determine p^0 so that $e(t, h, p^0)$ follows the unit step response of the target system $e_m(t, h)$. Considering the fact that $e(0, h, p) = 1$, the target system should be chosen so that $e_m(0, h) = 1$. Furthermore, for the left hand of (6) to be finite, it is necessary that $e_m(\infty, h) = 0$. Accordingly, T_m is chosen from the set

$$\mathcal{S} := \left\{ 1 - R(s) : \begin{array}{c} R(s) \in RH_2 \\ R(\infty) = 0,\ R(0) = 1 \\ D\|e_m(h)\|_1 \leq \varepsilon_i \end{array} \right\}. \tag{7}$$

In this article, the transfer function of T_m is expressed by

$$T_m(s) = \left[\begin{array}{c|c} A_m & B_m \\ \hline C_m & 1 \end{array}\right]. \tag{8}$$

Generally, it is desirable that the structure and the order of a controller can be determined flexibly. The model following method proposed by [7] can fulfill this requirement. In this method, with coprime factorization, the error system between the closed-loop system and the target system is expressed in the form of series-interconnection of two systems. Then, by stabilizing one system using the strictly positive real lemma and by minimizing the H_∞-norm of the other system, the model following is achieved. Especially, since the problem is solved in the framework of LMI optimization, the controller can be obtained efficiently. However, if the follow-up performance is not good, a designer must introduce the weighting function and adjust it by try and error. In this article, the same coprime factorization is applied, but the bounded real lemma is used instead of the strictly positive real lemma and the L_1-norm is minimized instead of H_∞-norm to avoid introducing a weighting function. Hereby, two new adjustable scalar parameters are introduced and the follow-up performance can be improved easier by tuning them.

A. Coprime factorization of the error system

Let us review the description of the error system given in the form of series-interconnection of two systems. Let $T_c(s)$

be the transfer function of the closed-loop system from the reference command signal to the tracking error and let $T(s)$ be the error system: $T(s) := T_m(s) - T_c(s)$. Supposed that $G(s)$ has $\tilde{G}_D(s)$ and $\tilde{G}_N(s)$, which are left-coprime over RH_∞:

$$G(s) = \tilde{G}_D(s)^{-1}\tilde{G}_N(s), \quad \tilde{G}_D(s), \tilde{G}_N(s) \in RH_\infty. \tag{9}$$

The state-space models of $\tilde{G}_D(s)$ and $\tilde{G}_N(s)$ can be given by

$$[\ \tilde{G}_D(s)\ \ \tilde{G}_N(s)\ ] = \left[\begin{array}{c|cc} A_H & H & B \\ \hline C & 1 & 0 \end{array}\right] \tag{10}$$

where $A_H := A + HC$ and H is chosen so that A_H is stable. On the other hand, supposed that $K(s)$ has $K_N(s)$ and $K_D(s)$, which are right-coprime over RH_∞:

$$K(s) = K_N(s)K_D(s)^{-1}, \quad K_D(s), K_N(s) \in RH_\infty. \tag{11}$$

The state-space models of $K_N(s)$ and $K_D(s)$ can be expressed in the observable canonical form:

$$[\ K_D(s)\ \ K_N(s)\ ] = \left[\begin{array}{c|cc} A_K & B_{KD} & B_{KN} \\ \hline C_K & 1 & D_{KN} \end{array}\right] \tag{12}$$

where A_K is the m-by-m stable matrix and B_{KD}, B_{KN} and C_K are defined by

$$\begin{aligned} A_K &:= \begin{bmatrix} 0 & \cdots & 0 & -a_1 \\ 1 & & 0 & -a_2 \\ & \ddots & & \vdots \\ 0 & & 1 & -a_m \end{bmatrix}, \\ B_{KD} &:= [\ g_1\ g_2\ \cdots\ g_m\ ]^t, \\ B_{KN} &:= [\ h_1\ h_2\ \cdots\ h_m\ ]^t, \\ C_K &:= [\ 0\ \cdots\ 0\ \ 1\ ],\quad D_{KN} := d. \end{aligned} \tag{13}$$

Then the controller is parametrized by

$$K(s) = \frac{ds^m + (a_m d + h_m)s^{m-1} + \cdots + (a_1 d + h_1)}{s^m + (a_m + g_m)s^{m-1} + \cdots + (a_1 + g_1)}. \tag{14}$$

In this case, $K(s)$ is arbitrarily determined only by B_{KD}, B_{KN} and D_{KN}. Essentially it is independent of A_K. Furthermore, this parametrization has flexibility in determining the type of a controller. For instance, by taking $m = 1$ and $g_1 = -a_1$, $K(s)$ can be a PI controller.

Defining the vector of design parameters as

$$p = [\ B_{KD}^t\ \ B_{KN}^t\ \ D_{KN}\ ]^t \in \mathbb{R}^{2m+1}, \tag{15}$$

the error system can be expressed by

$$T(s,p) = P(s,p)Q(s,p)^{-1}, \tag{16}$$

where $T(s,p)$ means the transfer function determined by p and

$$\begin{aligned} P(s,p) &:= T_m(s)Q(s,p) - \tilde{G}_D(s)K_D(s) \\ Q(s,p) &:= \tilde{G}_D(s)K_D(s) + \tilde{G}_N(s)K_N(s). \end{aligned} \tag{17}$$

Substituting (10) and (12) into (17), $Q(s,p)$ can be given by

$$Q(s,p) = \left[\begin{array}{c|c} A_q & B_q(p) \\ \hline C_q & 1 \end{array}\right], \tag{18}$$

where A_q, $B_q(p)$ and C_q are defined as follows.

$$\begin{aligned} A_q &:= \begin{bmatrix} A_H & HC_K & BC_K \\ 0 & A_K & 0 \\ 0 & 0 & A_K \end{bmatrix} \\ B_q(p) &:= [\ (H + BD_{KN})^t\ \ B_{KD}^t\ \ B_{KN}^t\ ]^t \\ C_q &:= [\ C\ \ C_K\ \ 0\ ] \end{aligned} \tag{19}$$

On the other hand, $P(s,p)$ is given as

$$P(s,p) = \left[\begin{array}{c|c} A_p & B_p(p) \\ \hline C_p & 0 \end{array}\right] \tag{20}$$

where A_p, $B_p(p)$ and C_p are of the following form.

$$\begin{aligned} A_p &:= \begin{bmatrix} A_m & B_m C & 0 & B_m C_K & 0 \\ 0 & A_H & 0 & HC_K & BC_K \\ 0 & 0 & A_H & HC_K & 0 \\ 0 & 0 & 0 & A_K & 0 \\ 0 & 0 & 0 & 0 & A_K \end{bmatrix} \\ B_p(p) &:= [\ B_m^t\ \ (H + BD_{KN})^t\ \ H^t\ \ B_{KD}^t\ \ B_{KN}^t\ ]^t \\ C_p &:= [\ C_m\ \ C\ \ -C\ \ 0\ \ 0\ ] \end{aligned} \tag{21}$$

In particular, when the plant G is stable and $\tilde{G}_D(s)$ is set to 1, that is, $H = 0$, the matrices A_p, $B_p(p)$ and C_p can be given by

$$\begin{aligned} A_p &= \begin{bmatrix} A_m & B_m C & B_m C_K & 0 \\ 0 & A & 0 & BC_K \\ 0 & 0 & A_K & 0 \\ 0 & 0 & 0 & A_K \end{bmatrix} \\ B_p(p) &= [\ B_m^t\ \ (BD_{KN})^t\ \ B_{KD}^t\ \ B_{KN}^t\ ]^t \\ C_p &= [\ C_m\ \ C\ \ 0\ \ 0\ ]. \end{aligned} \tag{22}$$

Design parameters appear affinely in $B_q(p)$ and $B_p(p)$. This feature makes it possible to solve the model following problem via LMI optimization as shown below.

B. Selection of an initial point via LMI optimization

The initial parameter p^0 is chosen so that the unit step response of the error system is nearly 0. The procedure is as follows: 1) The block diagram of $T(s,p)$ can be drawn by Fig. 3. Noting that the stability of $Q^{-1}(s,p)$ is equivalent to that of its strictly proper part

$$Q_0(s,p) := \left[\begin{array}{c|c} A_q - B_q(p)C_q & B_q(p) \\ \hline -C_q & 0 \end{array}\right], \tag{23}$$

make the unit step response of $Q^{-1}(s,p)$ bounded by applying the bounded real lemma to $Q_0(s,p)$. 2) Next, by minimizing the L_1-norm of $P(s,p)$, make the response of

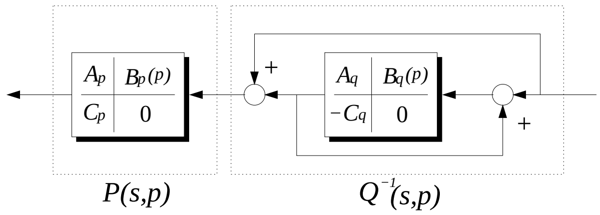

Fig. 3. Block diagram of the error system.

$P(s,p)$ to such a bounded signal nearly 0. Especially, these steps can be performed in the framework of LMI optimization.

Minimizing the maximum amplitude of the output of a linear time-invariant system is equivalent to minimizing the L_1-norm of its unit impulse response. Accordingly, the model following problem of our concern can be stated as follows.

Problem: Let $e_\delta(t,p)$ denote the unit impulse response of $P(s,p)$. For a given positive real number γ, determine p so that $\|e_\delta(p)\|_1$ is minimized subject to $Q_0(s,p) \in RH_\infty$ and $\|Q_0(p)\|_\infty < \gamma$.

According to the bounded real lemma, the sufficient condition for $Q_0(s,p) \in RH_\infty$ and $\|Q_0(p)\|_\infty < \gamma$ can be given as the existential condition of $X = X^t > 0$ such that

$$\begin{bmatrix} A_qX + XA_q^t & XC_q^t - \frac{1}{\gamma}B_q(p) \\ C_pX - \frac{1}{\gamma}B_q^t(p) & -\frac{1}{\gamma} \end{bmatrix} < 0. \tag{24}$$

Meanwhile, minimizing $\|e_\delta(p)\|_1$ is equivalent to the problem of minimizing ξ subject to

$$\|e_\delta(p)\|_1 < \xi. \tag{25}$$

However, this article minimizes ξ under the sufficient condition for (25)

$$|e_\delta(t,p)| < \lambda\xi\exp(-\lambda t), \tag{26}$$

where λ is the real number which satisfies

$$0 < \lambda < \sigma_{\min} \tag{27}$$

and $\sigma_{\min}$ is the minimum absolute value of the real part of the eigenvalues of $P(s,p)$. It is noted that $\sigma_{\min}$ is determined independently of p. It is known that the inequality in (26) holds if there exists $Y = Y^t > 0$ that satisfies the following three matrix inequalities [8], [9].

$$\begin{gathered} A_pY + YA_p^t + 2\lambda Y < 0 \\ \begin{bmatrix} Y & YC_p^t \\ C_pY & \lambda\xi \end{bmatrix} > 0 \\ \begin{bmatrix} P & B_p(p) \\ B_p^t(p) & \lambda\xi \end{bmatrix} > 0 \end{gathered} \tag{28}$$

The matrix inequalities in (24) and (28) are the LMIs to the variables p, X and Y. Accordingly, the parameter p^0 that achieves the model following can be obtained by solving the LMI optimization problem of minimizing ξ subject to (24), (28), $X = X^t > 0$ and $Y = Y^t > 0$ for given γ and λ.

Let us discuss the characteristic of the proposed method. When $\|Q_0(p)\|_\infty < \gamma$, $\|Q^{-1}(p)\|_\infty < 1+\gamma$. Therefore, the maximum amplitude of the unite step response of $Q^{-1}(s,p)$ is estimated as $1+\gamma$. Generally, the maximum amplitude of the response of a linear time-invariant system to persistent input with bound N is calculated as a product of the L_1-norm of its unit impulse response and N. From this fact, it can be seen that the maximum amplitude of the unite step response of the error system is approximated as $(1+\gamma)\xi$. Hence, the difference of the responses of T_m and T_c is estimated as less than $(1+\gamma)\xi$. Although a designer is required to set γ and λ in advance, it is comparatively easy to determine them since they have the clear meanings: the parameter γ determines the bound of the input of $P(s,p)$, while λ determines the decay rate of the unit impulse response of $P(s,p)$. This is contrast to the method in [7], in which the weighting function must be adjusted by try and error.

IV. Design example

Consider designing the unity feedback control system depicted in Fig. 1. The plant P consists of the actual plant and the actuator. Its transfer function is given by

$$P(s) = \frac{27697}{s(s^2+1429s+42653)}. \tag{29}$$

In order to operate the system safely, the control input to P is limited to $|u(t)| \le 10.0$. In this sense, this system is critical. The reference command signal r is restricted in the rate of change and known only to the extent that it belongs to the set $\mathcal{F}(1.5)$:

$$r \in \mathcal{F}(1.5) \tag{30}$$

In terms of the principle of matching, this means that the possible set is not adjustable. The goal of design is to find the linear time-invariant controller which always maintains the tracking error within ± 0.01745 for any reference command signal in $\mathcal{F}(1.5)$ under the restriction $|u(t)| \le 10.0$, namely, for $z := [\,e\ u\,]^t$,

$$1.5\,\|z_1(h)\|_1 \le 0.01745,\ \ 1.5\,\|z_2(h)\|_1 \le 10.0. \tag{31}$$

Note that $P(s)$ has a pole at the origin, $z_1(t,h)$ and $z_2(t,h)$ converge to 0 as t increases if $K(s)$ is a stabilizing controller. Hence, $K(s)$ does not have to have an integrator. For this reason, the controller is parametrized by a real vector $p := [\,p_1,\ldots,p_5\,]^t \in \mathbb{R}^5$ as

$$K(s) = \frac{p_1s^2+p_2s+p_3}{s^2+p_4s+p_5}. \tag{32}$$

Whole procedure to obtain the admissible controller that satisfies (31) is as follows:

Step 1: Choose the target system $T_m(s)$ from the set $\mathcal{S}$ which is defined by (7).

Step 2: Give $\gamma > 0$ and choose λ so that (27) holds.
Step 3: Search for B_{KD}, B_{KN} and D_{KN} that minimize ξ subject to (24), (28), $X = X^t > 0$ and $Y = Y^t > 0$ by using an LMI solver.
Step 4: Determine the initial controller based on (14).
Step 5: Simulate $z_1(t, h, p^0)$ and $z_2(t, h, p^0)$. If they are not satisfactory, adjust γ and λ and repeat Step 3 until they are acceptable. If they are satisfactory, proceed to the next step.
Step 6: Search for controllers that satisfy (31) from p^0 by means of a numerical search.

In this example, the target system is given as

$$T_m(s) = 1 - \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} \tag{33}$$

where $\zeta = 0.9$ and $\omega = 300$. Then

$$1.5\|e_m(h)\|_1 = 0.00904 < 0.01745. \tag{34}$$

For $\gamma = 1.0$ and $\lambda = 112.0\,(= 0.7\,\sigma_{\min})$, the initial point p^0 was searched for by using the function `mincx` in MATLAB LMI Control Toolbox. As a result of the search, the following controller was obtained.

$$K^0(s) = 4999.86\,\frac{(s + 1748.03)(s + 26.10)}{(s + 1675.09)(s + 676.80)} \tag{35}$$

Note that (35) is expressed in the zero-pole-gain form to specify the poles, zeros and gain. The unit step response is illustrated in Fig. 4. The dotted line shows the trajectory of the unit step response of the target system given in (33). The dashed line is the trajectory of $z_1(t, h, p^0)$. The controller $K^0(s)$ realizes a good follow-up performance. Therefore it is adopted as an initial controller.

The controller $K^0(s)$ yields

$$\begin{aligned} 1.5\|z_1(h,p)\|_1 &= 0.01148 < 0.01745 \\ 1.5\|z_2(h,p)\|_1 &= 17.35 > 10.0. \end{aligned} \tag{36}$$

Since the second specification is still not met, the parameter search is performed until (31) is satisfied. The function `fgoalattain` in MATLAB Optimization Toolbox, which is the package of the goal attainment method [5], is used. Consequently, some admissible controllers could be obtained. For instance, one of them is

$$K(s) = 4125.01\,\frac{(s + 1230.32)(s + 37.02)}{(s + 2559.43)(s + 442.95)}. \tag{37}$$

This controller attains

$$\begin{aligned} 1.5\|e(h,p)\|_1 &= 0.01709 < 0.01745 \\ 1.5\|u(h,p)\|_1 &= 9.969 < 10.0, \end{aligned} \tag{38}$$

so the specification in (31) is satisfied. In Fig. 4, the unit step response $z_1(t, h, p)$ is shown with the solid line. Figs. 6 and 7 show the simulation results for the reference command signal shown in Fig. 5. The maximum slope of the reference signal is 1.5 and it belongs to the set $\mathcal{F}(1.5)$. In Figs. 6 and 7, the dotted lines show the tolerable bounds

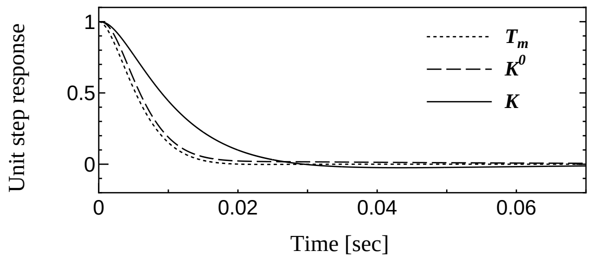

Fig. 4. Unit step responses of the target system (dotted line), the closed-loop systems for $K^0(s)$ in (35) (dashed line) and for $K(s)$ in (37) (solid line).

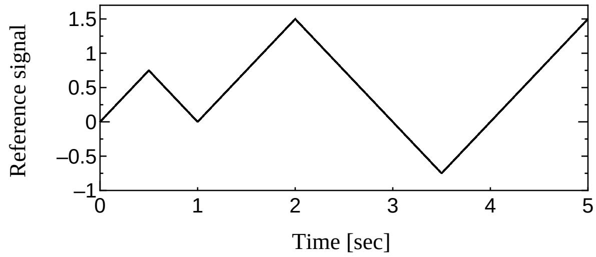

Fig. 5. Reference command signal $r(t)$.

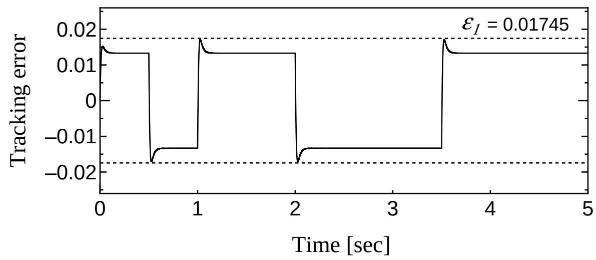

Fig. 6. Tracking error $e(t)$.

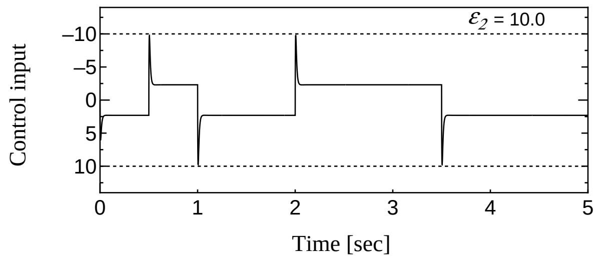

Fig. 7. Control input $u(t)$.

of the responses. As these figures indicate, the tracking error is maintained within the tolerable bounds under the restriction $|u(t)| \le 10.0$.

In this example, the design parameter, which characterizes (37), was obtained after thirteen movements from p^0. From this fact, it could be said that, compared to the case where a designer chooses an initial controller by try and error, the proposed method can realize computationally more efficient design of a critical control system.

V. Conclusions

In this article, by using the idea of the model following control, the method for selecting an initial controller, which is required in matched environment-system design, was proposed. The basic idea comes from the model matching originated by [7]. In the proposed method, it is possible to set the order of the controller arbitrarily. Furthermore, there is flexibility in specifying the structure of a controller. Although it is necessary to give the two parameters (γ and λ) in advance, it is easy to determine them. To improve the follow-up performance, a designer has only to adjust these two parameters. The controller, which ensures the specification, can be designed efficiently by performing a search from the obtained initial parameter. In the numerical example, the validity and the applicability of the proposed method were shown.

This article considered the controller design for critical systems with the rate-limited exogenous inputs. However, the proposed method is also applicable to the different design problems if the design specification can be evaluated by the deterministic input like a unit impulse or a unit step.

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