A dexterity maximizing continuation method for the inverse kinematics of redundant manipulators

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Abstract— **A continuation method for the inverse kinematics of redundant serial robotic manipulators is presented that maximizes the manipulators kinematic dexterity. The algorithm consists in three steps. A predictor step achieves geometric tracking of the target end-effector configuration. An intermediate perturbation step reconfigures the predictor value in self motion direction that increase dexterity. In a final corrector step an admissible configuration, with increased dexterity, in the neighborhood of the perturbed posture is determined. These three steps are applied iteratively in each continuation step. Dexterity is justified by $\det JJ^T$ and the inverse condition number of JJ^T, J is the manipulator Jacobian. An algebraic expression for the gradient of these measures is given, which allows a straight forward implementation of the algorithm. Except at singularities the iteration procedure converges rapidly and three iterations were sufficient for all considered examples. The target end-effector configuration is provided at discrete sample times and no explicit information about its time dependence is necessary. Thus the approach may be used as a 'plug-in'. Its performance is demonstrated for a 4R wrist, a planar 5R and spatial 10R manipulator.**

Keywords— **Redundant manipulators, inverse kinematics, continuation methods, manipulability, multibody systems**

I. Introduction

Redundancy of robotic manipulators can be exploited to cater for various goals. It is exploited to minimize joint motion or joint torques [4], to maximize the manipulators manipulability [1,14] possibly avoiding singularities [7] and to circumvent obstacles [13]. Common to these strategies is the use of a pseudoinverse solution for the inverse kinematic problem (IKP). In order to achieve certain further goals a potential function is minimized on variety of IKP solutions. For path tracking/planing additional joint increments belonging to the kernel of the manipulator Jacobian and pointing along the gradient of this potential field are added to a particular IKP solution [6]. A proper minimization was, however, not attempted.

In this paper a dexterity maximizing perturbation method for solving the IKP of redundant serial manipulators (SM) is given that accomplishes this minimization. Before deriving the proposed path tracking method the kinematics of redundant SM is described in section 2 using the product-of-exponentials (POE) formulation on the matrix Lie group $SE(3)$. Section 3 recalls kinematic dexterity measures and their frame invariance and independence from some joint variables. Also the key for an efficient implementation of the proposed scheme is provided: the algebraic expression of the gradient of dexterity measures. The article concludes with two examples: a planar 5R and spatial 10R manipulator.

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II. Robot kinematics

Robotic manipulators are rigid multibody system (MBS). As such a SM is constituted by a single branch with n rigid bodies $B_k, k = 1,\ldots,n$ of a tree structured MBS. All bodies numbered in increasing order starting from the ground B_0. Rigid body configurations are described by homogenous matrices forming the Lie group $SE(3)$ with generating Lie algebra $se(3)$ [10]. The configuration of B_k of is expressed by the homogenous matrix $\underset{k}{C}(q) \in SE(3)$ in terms of a rotation matrix $\underset{k}{R} \in SO(3)$ and position vector $\underset{k}{p} \in \mathbb{R}^3$ w.r.t. an inertial frame (IFR)

$$\underset{k}{C}(q) = \begin{pmatrix} \underset{k}{R}(q) & \underset{k}{p}(q) \\ 0 & 1 \end{pmatrix}. \tag{1}$$

With the assumption that all technical joints can be assembled with one-DOF joints the MBS contains n_R revolute and n_P prismatic/screw joints. Their joint variables $q^i, k = 1,\ldots,n$ are generalized coordinates of the configuration space $\mathbb{V}^n = \mathbb{T}^{n_R} \times \mathbb{R}^{n_P}$. With the POE mapping $\underset{k}{C}\colon \mathbb{V}^n \to SE(3)$ [8,11] the configuration of B_k is

$$\underset{k}{C}(q) = \underset{1}{M}\, e^{\underset{1}{\mathsf{X}} q^1} \cdots \underset{k}{M}\, e^{\underset{k}{\mathsf{X}} q^k},\ \underset{i}{M} \in SE(3),\ \underset{i}{\mathsf{X}} \in se(3), \tag{2}$$

where $\underset{i}{M}$ is the constant transformation and $e^{\underset{i}{\mathsf{X}} q^i}$ is the variable transformation of a B_i-fixed to a B_{i-1}-fixed reference frame. q^i is the joint variable of the joint linking B_i to B_{i-1} and the matrix $\underset{i}{\mathsf{X}}$ is given in terms of the screw vector $\underset{i}{X} \in \mathbb{R}^3 \times \mathbb{R}^3$ associated to joint i via[1] $\underset{i}{\mathsf{X}} = \underset{i}{X}^j\, \mathsf{E}_j$ with $\mathsf{E}_1,\ldots,\mathsf{E}_6$ being a basis for the Lie algebra $se(3)$ deduced from the B_{i-1}-fixed joint frame. $\underset{i}{X}^j$ are the Plücker screw coordinates. The respective conversion is denoted by $\underset{i}{X} = \underset{i}{\mathsf{X}}^{\vee}$ and $\underset{i}{\mathsf{X}} = \underset{i}{\widehat{X}}$. The body velocity of B_k is found from $\underset{k}{\mathsf{V}} = \underset{k}{C}^{-1} \underset{k}{\dot{C}}$ as the push forward mapping $T_q\mathbb{V}^n \to se(3)$

$$\underset{k}{\mathsf{V}} = \underset{k}{\mathsf{K}}_i \dot{q}^i,\ \text{with}\ \underset{k}{\mathsf{K}}_i = \mathrm{Ad}_{\underset{k}{C}^{-1}\underset{i}{C}}(\underset{i}{\mathsf{X}}), \tag{3}$$

[1] The Einstein summation convention $X^i\xi_i \equiv \sum_{i=1}^n X^i\xi_i$ is used.

with the Adjoint operator Ad : $SE(3) \times se(3) \to se(3)$. The kinematic basic functions (KBF) $\underset{k}{\mathsf{K}}_i$ (also termed the geometric Jacobian) play a fundamental part in mechanism analysis [9]. In vector form the KBF of B_n is assembled in $K = \left(\underset{n}{K}_i\right) \in \mathbb{R}^{6,n}$. The end-effector (EE) is represented by an EE frame (EFR) mounted on B_n. Let $\underset{E}{M} \in SE(3)$ be the EFR configuration w.r.t. the B_n frame. The EE configuration and velocity is

$$C(q) = \underset{n}{C}(q)\,\underset{E}{M},\ \mathsf{V} = \mathrm{Ad}^{-1}_{\underset{E}{M}}(\underset{n}{\mathsf{K}}_i \dot{q}^i). \tag{4}$$

The $se(3)$ matrix $\mathsf{V} \in se(3)$ contains the angular velocity tensor $\omega \in so(3)$ and the linear velocity vector $v \in \mathbb{R}^3$:

$$\mathsf{V} = \begin{pmatrix} \omega & v \\ 0 & 0 \end{pmatrix} \tag{5}$$

The left invariant metric on $SE(3)$, i.e. a metric for which $\langle X, X\rangle = X^T G X, \mathsf{X} \in se(3)$, is independent from the particular IFR, is

$$G(\alpha,\beta) = \begin{pmatrix} \alpha I & 0 \\ 0 & \beta I \end{pmatrix},\ \alpha,\beta \in \mathbb{R}. \tag{6}$$

This metric, however, depends on a scaling factor α/β that scales rotation vs. translations [10]. The EE accessibility distribution

$$D : q \in \mathbb{V}^n \mapsto D(q) = \mathrm{span}(\mathrm{Ad}^{-1}_{\underset{E}{M}}(\underset{n}{\mathsf{K}}_i(q))) \subset se(3)$$

assigns to each configuration $q \in \mathbb{V}$ the vector space of possible EE velocities. Its involutive closure $\overline{D}$ is equivalent to a subalgebra of $se(3)$ w.r.t. a conjugation. It is always $\overline{D} = \mathrm{span}(\underset{n}{\mathsf{K}}_i, [\underset{n}{\mathsf{K}}_i, \underset{n}{\mathsf{K}}_j], [\underset{n}{\mathsf{K}}_i, [\underset{n}{\mathsf{K}}_j, \underset{n}{\mathsf{K}}_k]], [\underset{n}{\mathsf{K}}_i, [\underset{n}{\mathsf{K}}_j, [\underset{n}{\mathsf{K}}_k, \underset{n}{\mathsf{K}}_l]]])$ but in general less then three Lie brackets are necessary [8]. Here $[\mathsf{X},\mathsf{Y}] = \mathsf{XY} - \mathsf{YX}$ is the Lie bracket on $se(3)$. The accessibility algebra $\overline{D}$ is the vector space containing all possible EE velocities, the corresponding group $\exp \overline{D}$ containes all possible EE configurations, and is thus a main characteristic of the SM.

With P being a projector to $\overline{D}$ the EE Jacobian in

$$V = J\dot{q} \tag{7}$$

is $J := PAd^{-1}_{\underset{E}{M}} K$, where Ad is the matrix of the adjoint map [10] and $\overset{\vee}{\mathsf{V}} \equiv V = (\omega, v)^T$ is the EE twist vector. E.g. in case of planar manipulators $\overline{D} \triangleq se(2)$ and P removes all non-planar components. Denote the instantaneous EE-DOF by $d(q) = \dim D(q) = \mathrm{rank}(J_i(q))$. The global EE-DOF of a SM is $\overline{d} = \max_{q \in \mathbb{V}^n} d(q)$. If the mechanisms DOF n is larger than the EE-DOF the SM is redundant with a redundancy $n - \overline{d}$ and the Jacobian matrix $J \in \mathbb{R}^{n,\overline{d}}$ is non-square.

Since for redundant SM (4) is not injective at regular $q \in V^n$ there is an $n - \overline{d}$ dimensional self motion submanifold $M_{C(q)} \subset \mathbb{V}^n$ such that $C(p) = C(q), \forall p \in M_{C(q)}$. Moreover, there are in general several such self motion manifolds intersecting at EE singularities. The number of self motion manifolds was addressed in [2] and is closely related to the number of inverse kinematics solutions.

III. Kinematic dexterity

Several local kinematic and dynamic dexterity measures (or manipulability measures) were introduced to characterize manipulator dexterity. Two such kinematic dexterity measures are [14], [3]

$$\mu_2 = 1/\sqrt{\kappa(JJ^T)} \text{ and } \mu_3 = \sqrt{\det JJ^T}, \tag{8}$$

where $\kappa(JJ^T)$ is the condition number of JJ^T. Each measure accounts for a particular goal, μ_3 is proportional to the EE velocity ellipsoid in $se(3)$ and μ_2 determines its deformation. It shall be noted that both measures are based on the left invariant metric (6) on $SE(3)$ which depends on the scaling of translations and rotations. However, nevertheless they constitute proper manipulability measures in the sense that both penalize EE singularities, i.e. configurations with $d_E(q) < \overline{d}_E$. Hence regardless of the used scaling path planing strategies may ovoid EE singularities and nearby points by incorporating either measure. A useful result is the following lemma.

Theorem 1: Both measures are invariant from the IFR and μ_3 is invariant from the EFR. It holds that $\mu_2 = \mu_2\left(q^1, \ldots, q^{n-1}\right)$ and $\mu_3 = \mu_3\left(q^{l+1}, \ldots, q^{m-1}\right)$, where $l = \max(i | [\underset{i}{\mathsf{Y}}, \underset{1}{\mathsf{Y}}], 1 \le i = 0)$ and $m = \min(i | [\underset{i}{\mathsf{Y}}, \underset{n}{\mathsf{Y}}] = 0, \forall i \le n)$. Hence μ_3 is at least independent from q^1 and q^n.

For a redundant SM, with prescribed EE motion $C(t)$, the inverse kinematics may select those points from the self motion manifold for a fixed $C(t)$ that maximize dexterity. Aiming the solution of the IKP while maximizing dexterity it will be necessary to compute gradients of the manipulability measures. Crucial for the computational efficiency of the path tracking method is the algebraic determination of these gradients. The derivatives of the KBF are algebraically expressed as [8]

$$\partial_{q^j} \underset{n}{\mathsf{K}}_i = [\underset{n}{\mathsf{K}}_i, \underset{n}{\mathsf{K}}_j],\ i < j < n. \tag{9}$$

The determinant of $A = JJ^T$ is $\det A = \sum_i A^i{}_j A^{*i}{}_j, \forall j$, where $A^{*i}{}_j$ is the cofactor of $A^i{}_j$, and its derivative is $\partial_{q^k} \det A = \sum_i \sum_j \partial_{q^k} A^i{}_j A^{*i}{}_j$. With (9) the terms $\partial_{q^j} A = \partial_{q^j} J J^T + J \partial_{q^j} J^T$ and thus $\nabla_q \mu_3$ can be given algebraically. Though the gradient of μ_2 can also be derived algebraically in what follows the path tracking will only incorporate μ_3.

IV. Path tracking via continuation

The potential advantage of redundant manipulators is that an optimal posture can be chosen from the self motion manifolds that maximizes dexterity and thus possibly avoids EE singularities. Given a desired EE trajectory $\left(\overline{C}(t), \overline{V}(t)\right), t \in [0,1]$ the IKP is to find $q(t)$ such that $C(q(t)) = \overline{C}(t)$. A solution is, with given initial values, obtained by integrating the under-determined DEQ system $J(q)\dot{q} = \overline{V}(t)$ using the particular solution

$$\dot{q} = J^+ \overline{V}(t) \tag{10}$$

in each integration step, where J^+ is the pseudoinverse of J. Except at singularities it can be obtained as

$J^{+} = J^{T}\left(JJ^{T}\right)^{-1}$. Continuation methods for path tracking solve the IKP using the particular pseudoinverse solution $\dot{q} = J^{+}\overline{V}(t) + (I - J^{+}J)\dot{q}_0$ to predict joint velocities. A variety of approaches choose this null-space velocity to achieve a particular goal [1,4,6] but the obtained solution is not optimal and depends on a damping parameter. The algorithm proposed here uses an iterative procedure to avoid these dependence. In order to maximize dexterity while tracking $C(q)$ along $\overline{C}(t)$, the arbitrary tangent vector $(I - J^{+}J)\dot{q}_0 \in \ker J$ to the self motion manifold passing through q is used to serve a gradient search.

Continuation methods basically solve the IKP by adjusting q at discrete sample times $t_{i+1} = t_i + \Delta t_i$ so that $C(q(t_{i+1})) = \overline{C}(t_{i+1})$. Denote with $q_i := q(t_i)$ and $\Delta q_i := q_{i+1} - q_i$. If q_i is a IKP solution at t_i, i.e. $C(q_i) = \overline{C}(t_i)$, then the IKP at time step $i+1$ is to find joint increments Δq_i such that $C(q_i + \Delta q_i) = \overline{C}(t_i + \Delta t_i)$. For infinitesimal joint increments it holds $C(q + dq) = C(q) + \partial_q C(q)\,dq$. The corresponding configuration change $\Delta C(q)(dq) = C^{-1}(q)\,C(q + dq) \in se(3)$ is found as $\Delta C(q)(dq) = I + \widehat{J(q)\,dq}$. For sufficiently small but finite Δq_i it holds

$$\Delta C(q_i)(\Delta q_i) \approx I + \widehat{J(q_i)\,\Delta q_i} \tag{11}$$

though $\Delta C(q_i)(\Delta q_i) \notin se(3)$.

The target increment for the transition $t_i \rightarrow t_{i+1}$ is $\overline{C}^{-1}(t_i)\,\overline{C}(t_i + \Delta t_i)$. If q_i is a solution at time t_i, i.e. $C(q_i) = \overline{C}(t_i)$, then the joint increment steering from $\overline{C}(t_i)$ to $\overline{C}(t_{i+1})$ is thus implicitly determined by $\Delta C(q_i)(\Delta q_i) \stackrel{!}{=} \Delta\overline{C}(q_i, \Delta t_i)$ with

$$\Delta\overline{C}(q_i, \Delta t_i) := \overline{C}^{-1}(q_i)\,C(t_i + \Delta t_i). \tag{12}$$

This increment is, with $(A - I)^{\vee} \equiv A^{\vee}$, predicted by

$$\Delta q_i = J^{+}(q_i)\,\Delta\overline{C}(q_i, \Delta t_i)^{\vee}. \tag{13}$$

The prediction error is the distance of $C(q_i + \Delta q_i)$ and $C(t_{i+1})$, compatible with the left invariant, i.e. IFR independent, metric $G(\alpha, \beta)$ on $SE(3)$ [10]

$$d(A, B) = \alpha\left\|\log R_A^{-1} R_B\right\|_{so(3)} + \beta\left\|p_A - p_B\right\|. \tag{14}$$

Here $\log R = \frac{\varphi}{2\sin\varphi}\left(R - R^{T}\right)$ is the log function on $SO(3)$ and $\|\omega\|^2_{so(3)} = -\frac{1}{2}\mathrm{tr}\left(\omega^2\right)$ is the norm on $so(3)$ such that $\|\log R\|_{so(3)} = |\varphi|$ is the angle of the rotation determined by R. For small rotations the log function is approximated by $\log R \approx \frac{1}{2}\left(R - R^{T}\right)$. Hence the path tracking problem is to find $q(t)$ fulfilling

$$d\left(\overline{C}(t), C(q(t))\right) = 0, t \in [0, 1] \tag{15}$$

for a given curve $\overline{C}(t)$. For redundant SM the solution is not unique and at fixed time a $q \in M_{\overline{C}(t)}$ can be chosen that maximizes dexterity. Presumed $\overline{C}(t)$ is a smooth curve in $SE(3)$ and because C, μ_2 and μ_3 are analytic functions the solution of (15) will be a smooth curve in $\mathbb{V}^n$, except at singularities, where $\dim\ker J > n - \overline{d}$. Let q_i be an optimal solution at t_i such that $d\left(C(q), \overline{C}(t)\right) = 0$ and $\mu_3(q_i) = \max(\mu_3(q), q \in M_{\overline{C}(t_i)})$. The solution $q_{i+1} = q_i + \Delta q_i$ of

$$\left\{\begin{array}{l} \mu_3(q_{i-1} + \Delta q_{i-1}) \rightarrow \max \\ d\left(C(q_{i-1} + \Delta q_{i-1}), \overline{C}(t_i)\right) = 0 \end{array}\right\}. \tag{PT$_i$}$$

will be an optimal posture at time $t_{i+1} = t_i + \Delta t_i$.

A perturbation approach is proposed here to solve (PT$_i$). It consist in a predictor, perturbation and corrector step. The idea is to perturb the predictor value (13) in direction with increasing dexterity, so that Δq_i is closed to the gradient $\nabla_q \mu_3|_{q_i}$. A solution of

$$\begin{array}{rl} & \left\|\Delta q_i - \lambda\nabla_q\mu_3\right\|^2 \rightarrow \min \\ \text{with} & \Delta\overline{C}(q_i, \Delta t_i) = I + \widehat{J(q_i)\,\Delta q_i} \end{array} \tag{16}$$

and $\lambda > 0$ is a combination of the predictor term (13) and a dexterity increasing perturbation vector projected to the null-space of $J(q_i)$

$$\begin{aligned} \Delta q_i \;=\; & J^{+}(q_i)\,(\Delta\overline{C}(q_i, t_i) - I)^{\vee} + \\ & + \left(I - J^{+}(q_i)\,J(q_i)\right)\nabla_q\mu_3|_{q=q_i}. \end{aligned} \tag{17}$$

This solution yields a posture closer to a dexterity maximizing solution of (PT$_i$) but will not fulfill the geometric condition (15) for finite $\Delta\overline{C}$. In a third step the obtained joint increment is corrected with (13) in order to fulfill $d\left(C(q_i + \Delta q_i), \overline{C}(t_{i+1})\right) = 0$. Since the pseudoinverse solution is a minimum norm solution ($\lambda = 0$ in (16)) this correction will remain closed to the perturbed posture with increased dexterity. The prediction, perturbation and correction steps are applied iteratively until the component of $\nabla_q\mu_3$ tangential to the self motion manifold $M_{\overline{C}(t_{i+1})}$ is below a specified threshold ε. This accomplishes a gradient search on $M_{\overline{C}(t_{i+1})}$. The perturbation algorithm applied in each continuation step is summarized as follows:

Predictor-perturbation-corrector algorithm at time t_{i+1}

- ∘ Initialization: $\Delta p_i^0 := 0, \Delta q_i^0 := 0, j := 1$
- Input: actual time $t_{i+1} \equiv t_i + \Delta t_i$, solution q_i of PT$_i$
- Do
  - · predictor step j
    $\rho := J^{+}(q_i + \Delta q_i^{j-1})(\Delta\overline{C}(q_i + q_i^{j-1}, t_i) - I)^{\vee}$
  - · perturbation step j
    $\delta := (I - J^{+}(q_i + \Delta q_i^{j})(J(q_i + \Delta q_i^{j}))\nabla_q\mu_3|_{q=q_i+\Delta q_i^{j-1}}$
    $\Delta p_i^{j} := \rho + \lambda_i\delta$
  - · corrector step j
    $\gamma := J^{+}(q_i + \Delta p_i^{j-1})((\Delta C(q_i + \Delta p_i^{j-1}, t_i) - I)^{\vee}$
    $\Delta q_i^{j} := \arg\min(\Delta t_i\, v_{\max}, \gamma)$
  - · $j := j + 1$

  While $\|\delta\| > \varepsilon_\mu \;\vee\; d\left(\overline{C}(t_{i+1}), C\left(q_i + \Delta q_i^{j}\right)\right) > \varepsilon_d$
- Result: $\Delta q_i := \Delta q_i^{j}$ solving PT$_i$

If t is time then $v_{\max} = \max_{\forall t} \|\dot{q}\|$ is the maximally admissible joint velocity determined by the drive capability and thus $\Delta t_i v_{\max}$ is the maximal in Δt_i achievable joint increment and a maximum for Δq_i. The precision goal for the geometric tracking problem is determined by ε_d and that of the dexterity maximization problem by ε_μ.

As for all gradient based tracking schemes the choice of λ_i is crucial. The step size λ_i must at least ensure that the perturbation $\lambda_i \delta$ is at the same scale as the increment Δq_i and does not exceed the maximally achievable joint increment. An estimate for the maximal norm of $\nabla_q \mu_3$ is found by considering non-redundant SM for which $\mu_3 = |\det J|$. The partial derivative is $\partial_{q^j} \det J = \sum_i^n \det\left(K_1 \cdots \partial_{q^j} K_i \cdots K_n\right)$ and because $\operatorname{span}\left(K_1 \cdots \partial_{q^j} K_i \cdots K_n\right) = \operatorname{span}\left(K_1 \cdots [K_i, K_j] \cdots K_n\right) \subseteq \operatorname{Ad}_{\substack{M\\E}} \overline{D}_E$ there is a $q \in \mathbb{V}^n$ for which $\mu_3(q) = \det\left(K_1 \cdots \partial_{q^j} K_i \cdots K_n\right)(q)$ (not considering joint limits). Since at least $\partial_{q^1} \mu_3 \equiv \partial_{q^n} \mu_3 \equiv 0$ it holds that $\partial_{q^j} \mu_3 \leq (n-2)\, \mu_{3\max}$. This bound proved to be valid also for redundant SM. The step size that limits Δq_i to this bound is

$$\lambda_i = \frac{\Delta t_i\, v_{\max}}{(n-2)\ \mu_{3\max}}. \tag{18}$$

For constant time steps $\lambda_i \equiv \lambda$.

V. Examples

The proposed algorithm was applied to a variety of manipulator structures and proved to be very efficient and robust. Three examples are presented here. The first is a 4R wrist mechanism proposed in [14]. This mechanism, used as orientation device, has a EE DOF $\overline{d} = 3$ and thus a redundancy of $n - \overline{d} = 1$. The initial posture of the 4R wrist is shown in figure 1 corresponding to $q = \left(q^1, q^2, q^3, q^4\right) = (0, \pi/2, -\pi/6, 0)$. The desired EE motion is a rotation about the vertical axis starting from this position with a velocity of $\pi/6$ rad/s. Figure 2 shows results for the standard pseudoinverse and for the proposed perturbation method. In both solutions q^2 and q^4 remained constant. The step size is $\Delta t_i = 0.01$ s and the distance metric with $\alpha = \beta = 1$ was used. While the pseudoinverse solution controls the manipulator through the singularity the predictor-corrector method maximizes its dexterity and thus avoids this singularity. At all sample points one iteration was sufficient to achieve a precision in the range of the working precission.

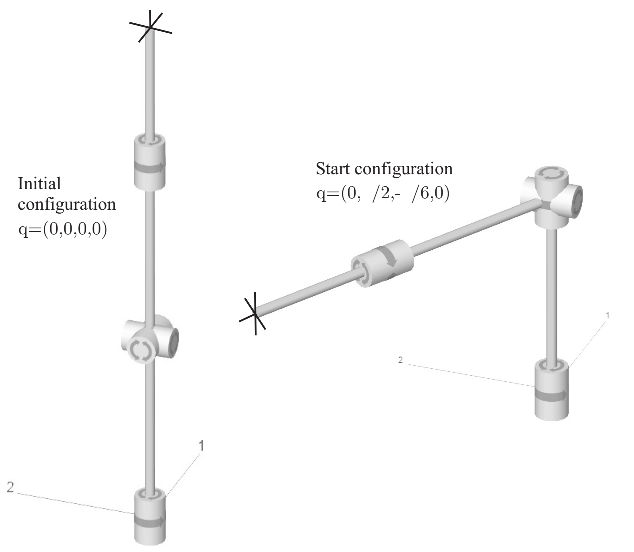

Fig. 1. Redundant 4R wrist in its initial and start configuration.

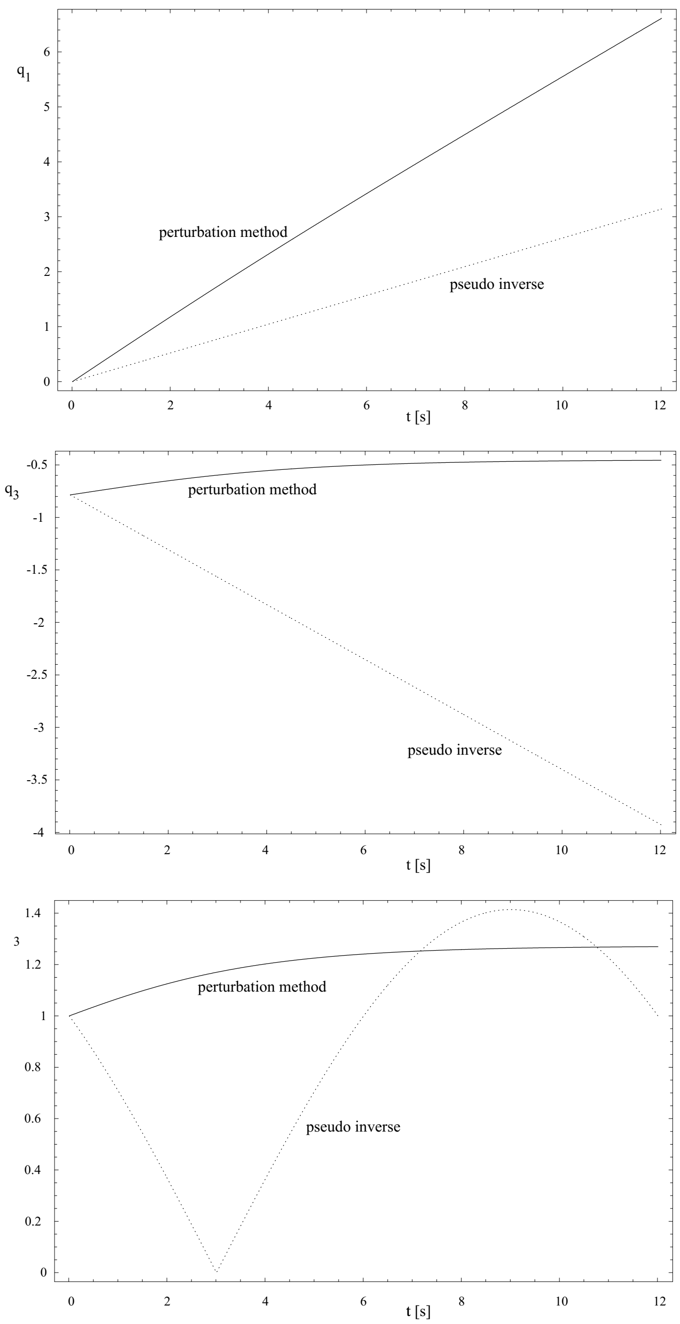

Fig. 2. IKP solutions for the 4R wrist rotating the EE about the vertical axis with constant velocity $\pi/6$ rad/s.

As second example the planar 5R manipulator in figure 3 is considered. It has a EE DOF $\overline{d} = 3$, degree of redundancy $n - \overline{d} = 2$ and unit limb lengths. The EE is tracked along a straight line in $SE(3)$ w.r.t. to the metric (6), with scaling $\alpha = \beta = 1$. That is the EE has to move along a straight line in $\mathbb{R}^3$ connecting the start point $(-4.99, 0.11, 0)$ and target point $(0, 1, 0)$ while rotating with constant speed to the target configuration. The task duration is only 1 second and thus the linear velocity is almost $\sqrt{26}$ m/s. Further a relatively large step size of $\Delta t_i = 0.01$s was chosen. Even though this coarse sampling and fairly high velocity the accuracy goal of $\varepsilon = \varepsilon_d = \varepsilon_\mu = 10^{-6}$ three iterations were sufficient in each time step (figure 4). Obviously at the begin where the manipulator is very closed to a singularity, where J becomes ill conditioned and the convergence is poor and would furthermore be lost in this singularity. At one side this is at due to the predictor (13). On the other side the condition $\|\delta\| < \varepsilon_\mu$ is not a proper convergence criterion because in a singularity there are always increment belonging to $\ker J$ that are not tangent vectors to one of the self motion manifolds. These increment vectors would possibly leave the set of addmissible configurations which in turn violates the geometric convergence criterion. So closed to singularities the algorithm will not converge.

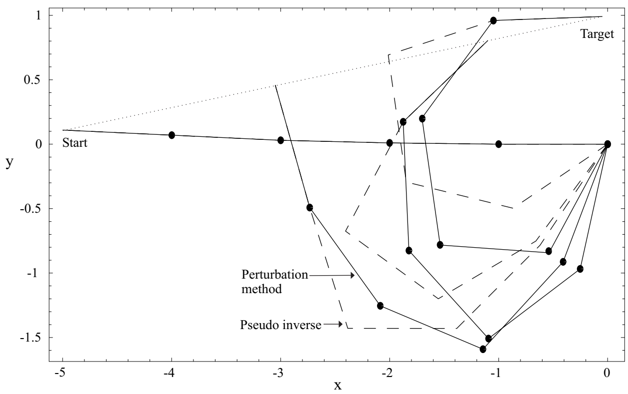

Fig. 3. Motion of the planar 5R mechanism tracking the EE along a straight line in $SE(3)$ from Start to Target EE configuration.

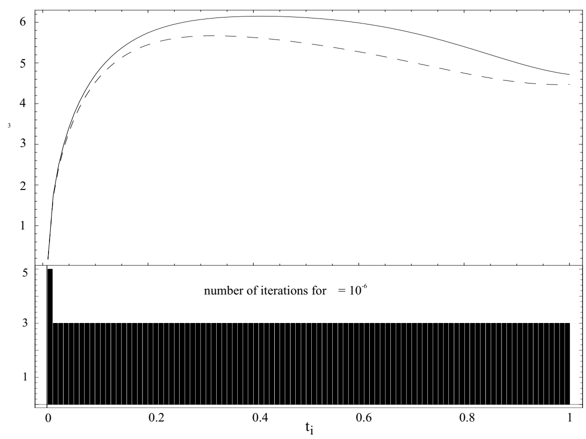

Fig. 4. Manipulability measure μ_3 during motion of the planar 5R mechanism. The shown number of iterations where necessary to achieve the acuracy $\varepsilon = 10^{-6}$

Figure 5 shows results for a more complex spatial 10R manipulator with EE DOF $\overline{d} = 6$ and redundancy $n - \overline{d} = 4$. Starting from the same initial configuration the pseudoinverse and the proposed disturbance method yield completely different motions. Again the task duration is 1 s only and the sampling time is $\Delta t_i = 0.01$ s. The scheme converges within three iterations per time step (Figure 6). Clearly the manipulability measure is increased during the motion for the 5R and 10R example compared to the respective pseudoinverse solution. With more realistic, i.e. lower velocities and smaller step size the method only needs one iteration.

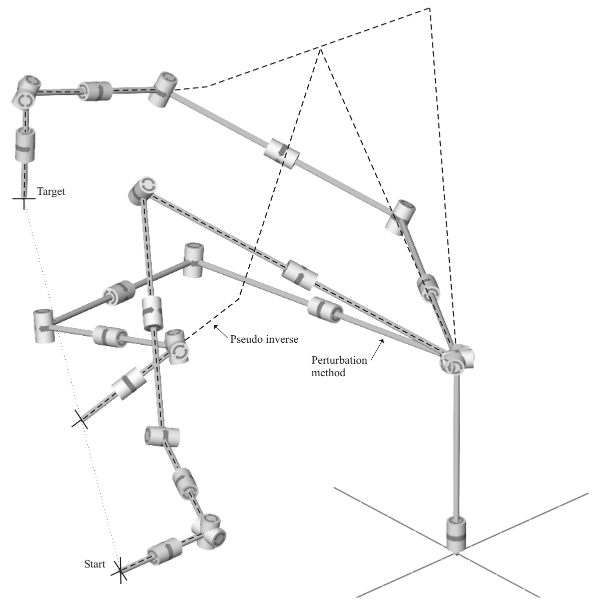

Fig. 5. Motion of the 10R mechanism tracking the EE along a straight line in $SE(3)$ from Start to Target EE configuration.

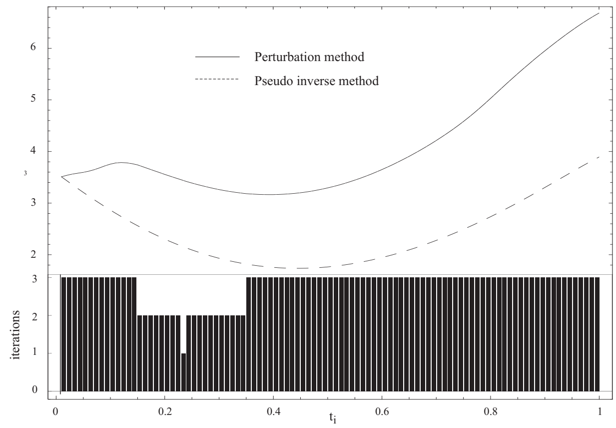

Fig. 6. Manipulability measure μ_3 during motion of the 10R mechanism. The shown number of iterations where necessary to achieve the acuracy $\varepsilon = 10^{-6}$

VI. Conclusions

The introduced perturbation method for the IKP solution for redundant manipulators yields configurations with increased kinematic dexterity. The necessary number of iterations in each continuation step is low and tends to one in regular points. The proposed method can also be applied for joint torque minimizing IKP solution, where instead the inverse of $\mu_3^2 = \det JJ^T$ is maximized, i.e. μ_3 is minimized (Note, however, that EE singularities are attractors for this problem [4]). Also using the inverse condition number μ_2 instead of μ_3 would yield the positive side effect that the error propagation, which is related to the condition number, is kept at minimum. If (13) is replaced by a singularity consistent IKP solution [5,12] it may also contribute to path tracking through singularities, where in singularities the optimal value of μ_2 and μ_3 is zero, however. As further improvement a singularity consistent gradient method for the IKP solution can be constructed based on the distance measure (14), which has the advantage that all gradients are given algebraically in closed form. The approach also extended to parallel manipulators, where the geometric loop constrants and the path tracing constraints are treated in a uniform way.

References

[1] Bayle, B.; Fourquet, J.Y.; Renaud, N.: Using manipulability with nonholonomic mobile manipulators, 3rd International Conference on Field and Service Robotics, Espoo, June 11-13, 2001

[2] J. W. Burdick, On the inverse kinematics of redundant manipulators: Characterization of the self–motion manifold, Proc. IEEE Int. Conf. on Robotics and Automation, May 15–19, Scottsdale, AZ, 1989, pp. 264–270

[3] Gosselin, C.; Angeles, J.: A global performance index for the kinematic optimization of robotic manipulators, Trans. ASME, Vol. 113, Sep. 1991, pp. 220–226

[4] Hollerbach, J.M.; Suh, K.C.: Redundancy resolution of manipulators through torque optimization, Proc. IEEE Int. Conf. Robot. Automation (1985), pp. 1016–1021

[5] Kieffer, J.: Manipulator inverse kinematics for end-effector trajectories with ordinary singularities, Int. J. Robotics Research, 11(3), June 1992, pp. 225–237

[6] Klein, C.A.; Huang, C.H.: Review of psoudoinverse control for use with kinematically redundant manipulators, IEEE Trans. Systems, Man and Cybernetics, Vol. SMC-13, no. 3, 1983, pp. 245–250

[7] Mayorga, R.V; Milano, N; Wong, A.K.C: A fast procedure for manipulator inverse kinematics computation and singularity prevention, J. Robotic Systema, 10(1), 1993, pp. 45-72

[8] Müller, A.; Maisser P.: A Lie–group formulation of kinematics and dynamics of constrained MBS and its application to analytical mechanics, Multibody Systems Dynamics, in press 2003

[9] Müller, A, Local analysis of singular configurations of open and closed loop manipulators, Multibody Systems Dynamics, 8, 2002, pp. 299-328

[10] Murray, R.M., Li, Z.; Sastry, S. S.: A mathematical Introduction to robotic Manipulation, CRC Press, 1993

[11] Park, F.C.; Bobrow, J.E.; Ploen, S.R.: A Lie Group Formulation of Robot Dynamics, Int. J. Robotics Research, Vol. 14, No. 6, 1995, pp.: 609–618

[12] Singh, V.K.; Gupta, K.C.: A manipulator Jacobian based modified Newton-Raphson algorithm for robot inverse kinematics, Adv. Design Automat. ASME DE, 1989, Vol. 93 (3), pp. 327–332

[13] Woernle, C.: Nonlinear control of constrained redundant manipulators, in Computational Kinematics, J. Ageles (ed.), Kluwer, 1993, pp. 119–128

[14] Yoshikawa, T.: Analysis and control of robot manipulators with redundancy, in Brady, M.; Paul. R: Robotic Research: The first international symposium, MIT Press, 1984, pp. 735-747