Adaptive control based on plant-parameterization using δ-models

Frantisek Gazdos, Petr Dostal, and Petr Navratil

Abstract—**The paper focuses on adaptive control of nonlinear technological processes. It uses the modified iterative scheme of the closed-loop identification and control design based on plant-parameterization. The modification enables to identify the whole plant using only one coefficient and without the necessity of reducing the order of a new plant-model. Moreover, instead of using a least squares algorithm, only a simple formula for identification is used. In addition, introduction of δ-models helps to cope with numerical instabilities of discrete models occurring when a sampling interval is being shortened. The proposed algorithm is verified using simulation experiments.**

Index Terms—**adaptive control, closed-loop identification, delta models, iterative methods, Youla-Kucera parameter.**

I. Introduction

FAST progress in computer technology during last several decades has enabled to implement sophisticated control strategies (such as adaptive and robust control) that are able to cope with nonlinearity and uncertainty of technological processes. All these algorithms, however, when implemented into PCs, IPCs (industrial PCs) or PLCs (programmable logical controllers) has to be discretized. This fact gives rise to problems with a sampling interval. It is usually chosen according to the dynamics of a controlled process but presence of nonlinearity and uncertainty forces us to shorten it. Unfortunately, as generally known, shortening of a sampling interval causes numerical instabilities of discrete models. One way of avoiding the problems is introducing "δ-models" [9], [10]. The basic property and the great advantage of δ-models is that when shortening a sampling interval, δ-operator converges to the derivative operator. This fact simply means that δ-models approach to continuous-time models when the sampling interval is sufficiently short.

This paper focuses on practical implementation and enhancement of latest achievements in the field of adaptive and robust control. It utilizes the iterative approach to identification in the closed loop and control design [3], [13]. The method is based on plant-parameterization, also called "dual Youla-Kucera parameterization", see e.g. [1], and its modification [8], [2], [7]. It uses fractional description in the ring of stable and proper rational transfer functions (R_{PS}), and, one of the main advantages of the algorithm is that a new model of the process is estimated without the necessity of consequent reduction of its order. Also, the whole plant is identified only by very few parameters, and for the identification, stable-filtered signals are used.

This work "improves" the method from a practical point of view so that it can easily be implemented into PCs, IPC or PLCs. Then, all necessary signals can be measured in such short time-intervals (compared to the dynamics of a controlled process) that they can be considered as continuous, and what is essential, without the necessity of facing numerical problems of discrete models. In addition, as shown in illustrative section of this paper, the method can be "simplified" so that we can avoid using the least squares identification algorithm and what is more, the whole plant can be identified only by one simple parameter representing the difference between the real plant and a model.

The first section of the paper describes the basics of the used iterative identification and control technique, second part focuses on the algorithm itself, and a useful modification follows. Next part introduces δ-models and final section illustrates the procedure of transforming the algorithm into δ-representation when controlling simple nonlinear system. The paper concludes providing some simulation results and summarizing main contributions of the approach.

II. Methodology

A. Basic principles

Consider a feedback control set-up as shown in Fig. 1, where y, u, e, r are signals of controlled output, control input,

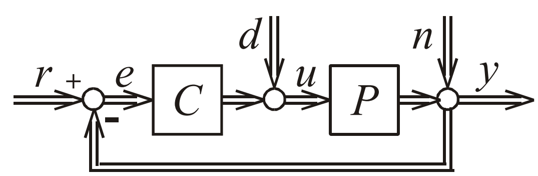

Fig. 1. Classical feedback control set-up

control error and the reference; n represents additive output noise and d is a disturbance signal. Next, let P stands for a transfer function of a plant and C for a controller

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$$P = \frac{N}{D} \tag{1}$$

$$C = \frac{X}{Y} \tag{2}$$

with N, D and X, Y coprime transfer function from φ, where φ denotes the set of stable and proper rational transfer functions (R_{PS}). Then, by coprimeness, a stabilizing controller can be found using so-called Bezout identity [15] as a particular solution of diophantine equation

$$D \cdot Y + N \cdot X = 1. \tag{3}$$

When one stabilizing controller is found, then the set of all controllers that stabilize P can be expressed as

$$C_S = \frac{X + D \cdot S}{Y - N \cdot S}, \tag{4}$$

where S is an *arbitrary parameter* from φ. This approach, called "Youla-Kucera parameterization" greatly facilitates design of controllers.

Dual problem to finding all stabilizing controllers if only one is known is *finding all plants stabilized by the one controller*. Again, consider the control set-up from Fig. 1 with fractional description of a nominal plant and a controller according to (1), (2), and suppose that Bezout equation (3) holds. Then, the *set of all plant models stabilized by the one controller C* is given by

$$P_M = \frac{N + Y \cdot S}{D - X \cdot S}. \tag{5}$$

Again, S is an arbitrary parameter representing a stable and proper transfer function. This approach, when the plant is parameterized, also called "dual Youla-Kucera parameterization", is near to closed-loop identification in the presence of noise and consequently to adaptive control. Let us explain this fact more.

The basics of closed-loop identification with the help of dual Y-K parameterization were independently given by several authors [5], [4], [12], [11]. The *key-idea is to directly identify Y-K parameter "S"* instead of classical identification of model coefficients. The most important fact is that *this is a standard open-loop identification problem*. In addition, estimated models of the process are guaranteed to be stabilized by the currently proposed controller.

The choice of the plant model according to (5) *and substitution into* Fig. 1 yields the alternative set-up of the closed loop as shown in Fig. 2.

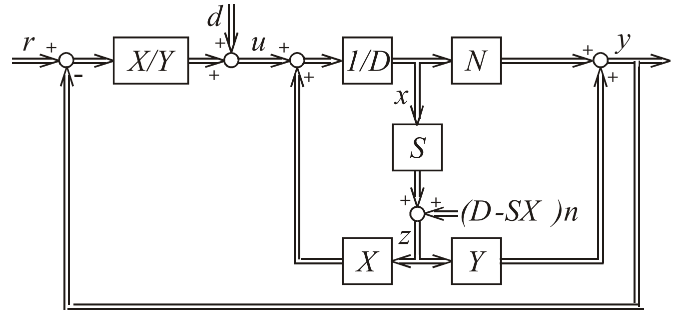

Fig. 2. Alternative set-up of a closed-loop

It can easily be proved that the substitution with input u, n and output y implies

$$y = P_M \cdot u + n. \tag{6}$$

Now, the task is to identify a true nonlinear plant in the closed-loop from noise-contaminated measurements of u and y. As stated above, the key-identification idea is to identify the parameter S rather than the coefficients of P_M. To see that this is a standard open-loop identification problem observe from Fig. 2 following relationships related to signals x and z (used for identification of the parameter S):

$$x = Y \cdot d + X \cdot r = Y \cdot u + X \cdot y \tag{7}$$

$$z = D \cdot y - N \cdot u \tag{8}$$

$$z = S \cdot x + \left(D - S \cdot X\right) \cdot n. \tag{9}$$

The last equation is the most significant for the identification. In this formula, signals x and z are measurable on the closed loop via (7) - (8), and if n is independent of r and d, then x and n are independent processes. Also $S \in \varphi$, i.e. it is stable. Hence, the *identification of S is a standard open-loop identification problem*.

B. *Algorithm of adaptive control*

Here, let us show how the ideas above help us to prepare the algorithm of adaptive control. First, let the time axis be divided into intervals, so that during the *k-th* interval, the control input into the plant is driven by a controller C^k. Next, let there be a nominal model P^k of the plant P stabilized by the controller. Employing $P^k = P$, which leads to $S = 0$ (and shows that coprime factors of P^k approximately equal to those of the real system), the following *algorithm of adaptive control based on iteration* of these steps can be prepared:

1) For the model of the process, P^k, a stabilizing controller is computed using Bezout equation (3):

$$D^k \cdot Y^k + N^k \cdot X^k = 1, \tag{10}$$

2) the auxiliary signals x and z are formed as

$$\begin{aligned} x &= Y^k \cdot u + X^k \cdot y \\ z &= D^k \cdot y - N^k \cdot u \end{aligned}, \tag{11}$$

3) then, these signals are used for identification of S (in a least squares algorithm) with the output error

$$\varepsilon = z - \widehat{S} \cdot x , \tag{12}$$

where x is the input and z the output signal.

4) A new transfer function of the identified model is computed using rel. (5):

$$P_M^{k+1} = \frac{N^k + Y^k \cdot \widehat{S}}{D^k - X^k \cdot \widehat{S}} . \tag{13}$$

5) Employing

$$P^k = P_M^{k+1} , \tag{14}$$

6) the algorithm continues by step 1) with the new transfer function of the model, P^k.

The fact that *parameters of the estimated model are close to the real process* is easy to recognize from the record of S: *If coefficients of this parameter converge to the zero value*, then, the identified model represents a good approximation of the real system. In addition, the parameter S actually "describes" the difference between our estimated model and the true plant.

The presented method, however, suffers from a drawback: computation of a new transfer function using (13) increases the order of the model so that consequent *use of* a *reduction method is advisable*. In order to avoid this complication, the original algorithm was slightly modified.

C. Modification of the algorithm

The modification is based on works [11], [14], and the method is directed towards the identification of coprime factors of $\boldsymbol{P_M}$ employing the auxiliary signal x (measurable on the closed-loop and uncorrelated to the noise signal). From Fig. 2, (supposing $n = 0$), it is also possible to derive these formulas

$$\begin{aligned} y &= \left(N + Y \cdot S\right) = N_M \cdot x \\ u &= \left(D - X \cdot S\right) = D_M \cdot x \end{aligned}, \tag{15}$$

where N_M, D_M are coprime factors of the plant model (5)

$$P_M = \frac{\left(N + Y \cdot S\right)}{\left(D - X \cdot S\right)} = \frac{N_M}{D_M} . \tag{16}$$

Then, the combination of (8) and (9) with (15) yields the following equation

$$S = D \cdot N_M - N \cdot D_M , \tag{17}$$

which is the "key-one" in modification of the original algorithm. *All steps are identical* to the original version *except for step 4)*. Here, the *new transfer function of the identified model is computed using* (17):

$$\begin{aligned} \widehat{S} &= D^k \cdot N_M^{k+1} - N^k \cdot D_M^{k+1} \\ P_M^{k+1} &= \frac{N_M^{k+1}}{D_M^{k+1}} \end{aligned} . \tag{18}$$

Now, what is important, if orders of D and D_M, and also N and N_M are chosen the same, then *it is not necessary to reduce the order of a new plant model* as in the original algorithm. This "smart" approach, which ensures that the order of a new plant model is under control, was introduced in [8], utilized for MIMO systems in [2] and further elaborated in [7], [6].

D. Delta-representation

In order to avoid numerical problems occurring when a sampling interval is shortened (after discretization of continuous-time systems), Middleton and Goodwin in [9] introduced a special type of discrete models originally defined as:

$$\delta = \frac{z-1}{T_0} , \tag{19}$$

where "*z*" represents the complex variable of Z-transform and T_0 is a sampling interval. The main advantage of these models is that *when shortening a sampling interval, δ-operator converges to the derivative operator*, i.e.

$$\lim_{T_0 \to 0} \delta = s . \tag{20}$$

Moreover, it can be proved that all models of the form defined as

$$\gamma = \frac{z-1}{\lambda \cdot T_0 \cdot z + \left(1 - \lambda\right) \cdot T_0} , \quad 0 \le \lambda \le 1 \tag{21}$$

have the property of convergence to the derivative operator [10]. The most frequently used are the following ones

$$\lambda = 0 \qquad \delta = \frac{z-1}{T_0}$$

$$\lambda = 1 \qquad \sigma = \frac{1-z^{-1}}{T_0} \quad . \tag{22}$$

$$\lambda = 0.5 \qquad \gamma = \frac{2}{T_0}\frac{z-1}{z+1}$$

All these models are generally called "δ-models". In this paper, the first definition (19) was utilized to implement and improve the modified algorithm of adaptive control based on plant-parameterization.

III. Illustrative Example

A. Controlled process

Consider the plant to be controlled as depicted in Fig. 3.

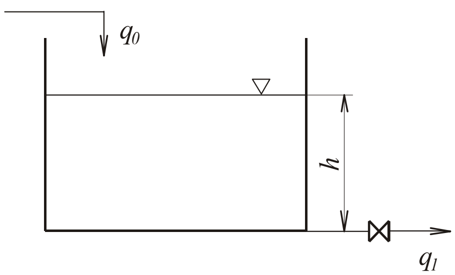

Fig. 3. Controlled process

The process represents a tank for liquid with inflow q_0 as the input variable and a level of liquid, h, as the state variable to be controlled; q_1 is the outflow from the tank. Suppose a mathematical model of the process in the form [16]

$$\begin{aligned} F \cdot \frac{dh}{dt} + q_1 = q_0 \\ q_1 = c_1 \cdot h + c_2 \cdot \sqrt{h} \end{aligned}, \tag{23}$$

where F is the section of the tank and c_1, c_2 are constants obtained from the real process. Next, let us introduce deviation variables defined as

$$\begin{aligned} y = h - h^s \\ u = q_0 - q_0^s \end{aligned}, \tag{24}$$

where h^s is the steady state level of liquid in the tank and q_o^s is the steady state inflow into the tank. Constants corresponding to the real plant are: c_1 = 1.53322.10^{-3} dm^2/s, c_2 = 3.31142.10^{-3} dm$^{5/2}$/s, F = 1.44 dm^2, h_s = 1.5 dm, $q_0{}^s$ = 0.006359 dm^3/s and the inflow rate q_0 can vary in the range from -0.006359 dm^3/s to 0.004161 dm^3/s.

B. Preparation of the algorithm

Model of the process in the continuous-time form was chosen as

$$P_M(s) = \frac{n_0}{s + d_0}, \tag{25}$$

where the initial estimate of the constants n_0, d_0 needed to start the algorithm was obtained from a linearized model of the process as: $n_0(0)=0.6944\ dm^{-2}$, $d_0(0) = 0.0020\ s^{-1}$.

The stabilizing controller (providing also asymptotic tracking of the reference) was proposed using Bezout equation (3) in the form

$$C(s) = \frac{x_1 \cdot s + x_0}{s}, \tag{26}$$

with x_1, x_0 computed using these formulas

$$x_1 = \frac{1}{n_0} \cdot (2 \cdot \alpha - d_0), \quad x_0 = \frac{\alpha^2}{n_0}, \tag{27}$$

where α *is a real positive constant* (used for the conversion into stable and proper rational transfer functions) chosen so that *strong stability* of the controller is ensured. The Youla-Kucera parameter S used for the identification of the process was chosen in the form

$$S(s) = \frac{s_0}{s + \alpha}. \tag{28}$$

This choice enables us to *identify the whole process using only the one coefficient* s_0, and what is more (as shown further), we can directly identify it using simple formula *without the necessity of using a least squares algorithm*.

The signals used for identification of parameter S (after conversion into R_{PS}) take the form:

$$\begin{aligned} x = Y \cdot u + X \cdot y &= \frac{s}{s+\alpha} \cdot u + \frac{x_1 \cdot s + x_0}{s+\alpha} \cdot y \\ z = D \cdot y - N \cdot u &= \frac{s+d_0}{s+\alpha} \cdot y - \frac{n_0}{s+\alpha} \cdot u \end{aligned} \tag{29}$$

(note that these signals represent *stable* filtration of the input and output signal). Further, the equation for calculation of new coefficients of the plant model has the form of (17):

$$\begin{aligned} &\widehat{S} = D^k \cdot N_M^{k+1} - N^k \cdot D_M^{k+1} \quad \Rightarrow \\ &\Rightarrow \quad \frac{\widehat{s}_0}{s+\alpha} = \frac{s+d_0^k}{s+\alpha} \cdot \frac{n_0^{k+1}}{s+\alpha} - \frac{n_0^k}{s+\alpha} \cdot \frac{s+d_0^{k+1}}{s+\alpha} \cdot \end{aligned} \tag{30}$$

C. Implementation of the algorithm

Now, in order to implement the algorithm of adaptive control and improve numerical aspects, let us transform it from continuous-time form into discrete one using δ-operator defined by (19). If we suppose that all signals are measured in such short intervals (compared to the dynamics of the controlled process) that they can be considered as continuous, then, we can utilize the formula (20) introducing δ-operator and simply substitute the derivative operator "s" by δ-operator defined e.g. by (19). Further, realize that the "*z*-variable" in the formula represents the shift operator and its inversion, z^{-1}, *represents one-step signal delay*. Then, the method of adaptive control can be transformed into simple and easily programmable form as follows:

1) For the model of the process given by rel. (25), the stabilizing controller (26) is computed using (27) and is implemented by this equation:

$$u(k) = u(k-1) + x_1 \cdot e(k) + (x_0 \cdot T_0 - x_1) \cdot e(k-1). \quad (31)$$

2) Computation of auxiliary signals *x* and *z* takes the form:

$$\begin{aligned} x(k) = &(1-\alpha \cdot T_0) \cdot x(k-1) + u(k) - u(k-1) + \\ &+ x_1 \cdot y(k) + (x_0 \cdot T_0 - x_1) \cdot y(k-1) \end{aligned} \quad (32)$$

$$\begin{aligned} z(k) = &(1-\alpha \cdot T_0) \cdot z(k-1) + y(k) + \\ &+ (d_0 \cdot T_0 - 1) \cdot y(k-1) - n_0 \cdot T_0 \cdot u(k-1) \end{aligned} \quad (33)$$

3) Now, these signals are used for *simple* identification of parameter *S*:

$$\hat{s}_0 = \frac{z(k) + (\alpha \cdot T_0 - 1) \cdot z(k-1)}{T_0 \cdot x(k-1)}. \quad (34)$$

4) Using the identified parameter s_0 and (30), the new model coefficients are computed as:

$$n_0^{k+1} = \hat{s}_0 + n_0^k, \quad (35)$$

$$d_0^{k+1} = \frac{d_0^k \cdot n_0^{k+1} - \hat{s}_0 \cdot \alpha}{n_0^k} \quad (36)$$

5) These new coefficients are used as the new estimate for the next iteration step.

Recall that the constant T_0 represents the sampling interval and can be chosen as short as possible to "catch" the continuous-time character and nonlinearity of the plant. For the start of the algorithm, see (34), we must ensure that $x(k\text{-}1) \neq 0$.

D. Simulation results

For the simulation experiment, the sampling interval was chosen $T_0 = 6\ s$, and the period of adaptation (i.e. calculation of new model coefficients and consequent controller re-design) was $T_A = 10.T_0 = 60\ s$. The positive constant α was derived from the model coefficient d_0 and was responsible for dynamics of the control and identification process. Let us add that the algorithm was stable even for far shorter sampling intervals.

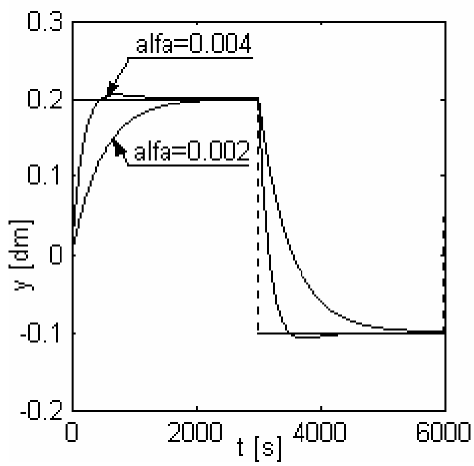

Fig. 4. New setpoint response of the plant output. See the influence of the positive constant α.

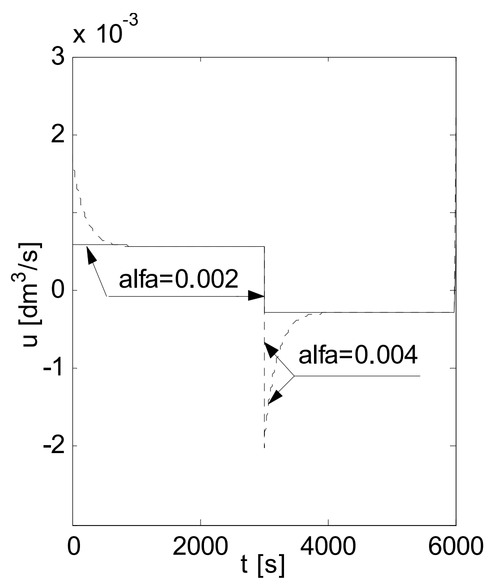

Fig. 5. Control input response. Again note how the positive constant α can influence smoothness of the control input.

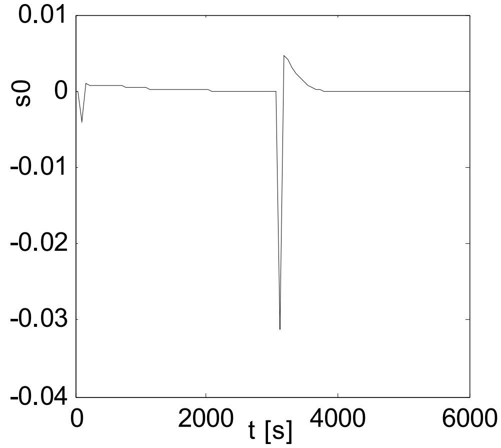

Fig. 6. Identified Youla-Kucera parameter *S* for $\alpha = 0.002$. Observe that the parameter converges to the zero value very soon, which means that very quickly, the identified model coefficients are close to the real plant.

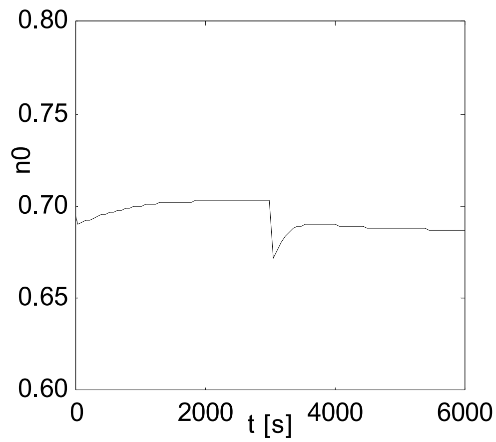

Fig. 7. Adaptation of the model coefficient n_0 for $\alpha = 0.002$.

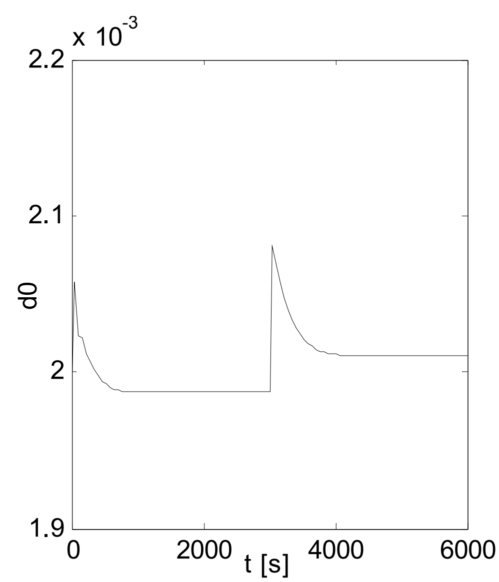

Fig. 8. Adaptation of the model coefficient d_0 for $\alpha = 0.004$.

IV. Conclusion

The proposed adaptive algorithm based on iteration technique provides effective identification method (the whole plant is identified only by one coefficient and using a simple formula and stable filtered signals). In addition, the process of identification and control can be influenced by one tunable constant and discrete implementation enables to use "extremely" short sampling intervals without the danger of numerical instability of the algorithm. Now, the task is to ensure good convergence of the iteration technique even if initial estimates of model coefficients are not known very precisely.

Acknowledgment

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