VISION-BASED PATH FOLLOWING CONTROL OF A UNICYCLE ROBOT ⋆

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Abstract: This paper addresses the problem of driving a wheeled robot of unicycle type along a given path, using vision. The methodology adopted for path following control deals explicitly with vehicle dynamics. Furthermore, by introducing the concept of a "virtual target" to be tracked along the path, it avoids the singularities that are present in a number of path following strategies described in the literature. The paper starts by describing a control law in cartesian space. The relationship between features in the cartesian and image planes are then exploited to derive a nonlinear, vision-based, path following control law for the robot. Simulation results illustrate the performance of the control system proposed.

Keywords: Nonlinear Control, Vision-based Control, Path Following, Mobile Robots.

1. INTRODUCTION

From a control point of view, a camera can be seen as a non-linear sensor that provides measurements of the relative position of the robot carrier with respect to a feature that is captured in the image plane. Considering that this feature corresponds to a path to which the robot must converge while keeping a desired velocity profile, the path following control problem can be formulated in the image plane. Steering the robot along the path is then equivalent to driving asymptotically to zero a generalized error function that includes appropriately defined image variables.

Common methods for path following described in the literature display two major shortcomings :

- The first occurs in the case where a "virtual target" to be tracked along the path is simply defined as the projection of the actual robot on that path (that is, the closest point on the path). In order to avoid the singularities that arise when that distance is not well defined, stringent condition are imposed on the robot's initial condition, thus making it impossible to develop globally convergent path following control laws (MIC 93, SAM 91, LAP 02). This limitation was lifted in (SOE 03) by introducing a new methodology whereby the rate of progression of the "virtual target" along the path is controlled explicitly. This methodology is one of the key building blocks of the strategy for vision-based control proposed in this paper.
- The second limitation is due to the use of a camera as a primary sensor. Generally, vision based control algorithms rely on the inversion of an interaction matrix that relates image and cartesian-based kinematic properties (HUT 96, AND 01, ESP 92). The inversion process introduces new singularities, and this in turn may again reduce the domain of applicability of the control law derived.

In this paper, a visual servoing control law is proposed that combines the non singular path-following strategy exposed in [SOE 03] with a

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geometrical analysis of the image information. The resulting non-linear vision-based control law deals explicitly with vehicle dynamics and yields a non singular path following control strategy.

The paper is organized as follow. Section 2 summarizes the solution to the path following problem in cartesian space, reformulates this problem in terms of camera image variables, and states the resulting visual-servoing control problem rigorously. A solution is provided in Section 3, which also contains illustrative simulation results. Finally, Section 4 contains the main conclusions and discusses issues that warrant further research.

2. PROBLEM FORMULATION

2.1 Notation

Throughout this paper, the following notation will be used.

- The symbol $\{A\} := \{x_A\ y_A\ z_A\}$ denotes a generic reference frame. We let $\{0\}$, $\{F\}$, and $\{B\}$ be inertial, Serret-Frenet, and body axis frames, respectively
- $p^C_{A/B} = [x^C_{A/B} y^C_{A/B} z^C_{A/B}]^T$ denotes the position of frame {A} with respect to {B}, expressed in {C}. Note that $\dot{p}^C_{A/B} = [\ \dot{x}^C_{A/B}\ \dot{y}^C_{A/B}\ \dot{z}^C_{A/B}]^T$ represents the velocity of the frame {A} with respect to {B} expressed in {C}.
- $\Psi_{A/B}$ denotes the orientation of {A} with respect to {B}, and $\dot{\Psi}_{A/B}$ the angular velocity of {A} with respect to {B}.
- R^B_A is the rotation matrix from {A} to {B}.

Note that since the study is made in the horizontal plane, $z^C_{A/B} = cst$ and $\dot{z}^C_{A/B} = 0, \forall (A, B, C)$.

2.2 Parameterization and control strategies

2.2.1. Non-singular Path following

In order to design a path following controller for a robot, the equations of the vehicle must be derived relative to a given path; it is the goal of the controller to drive the robot so as to reach and follow the path, without any time constraints. The parameterization of the problem is illustrated in figure 1. Given a point F that moves continously along that path, a controller that drives θ (angle between the vehicle's main axis and the tangent to the path) to 0 as $x^F_{B/F}$ and $y^F_{B/F}$ go to 0 solves the path following problem. The difference in the strategies that can be adopted concerns the definition of F. In [MIC 93], F is defined as the closest point on the path with respect with the current position of the robot. In that case, the line (BF) is always perpendicular to the path tangent at F. Then the parameterization of the problem is simpler and convergence is guaranteed when $\|BF\|$ and θ are driven to 0. Nevertheless, this method implies a drastic singularity when the point B is located at the center of the path curvature at the point F (θ is undefined). Let's locate the point F by its curvilinear abscissa s_F and name $c_c(s_F)$ the path curvature at this point. Then the singularity occurs when $\|BF\| = (c_c(s_F))^{-1}$. In addition, the analysis [SAM 91] shows that the global convergence is guaranteed only if $\|BF\| < (c^{max}_c)^{-1})$ for all F, and for c^{max}_c defining the maximum curvature encountered on the path. This is a very restrictive hypothesis that implies a drastic limitation on the initial condition $\|BF\|_{t=0} < (c^{max}_c)^{-1}$ and a poor disturbance rejection capability. Another solution consists of defining the point F as a virtual running target that describes the path. The movement control of this target induces supplementary virtual states to the system and transforms the previous constraint in $\|BF\|_{t=0} \neq (c_c(s_F))^{-1}$. The behavior of F (captured in the expression of $\dot{s}_F$) is chosen according to the derivation of some Lyapunov function in the backstepping process, and results in a very cooperative target that quickly converges to the closest point on the path. This method is described in detail in [SOE 03].

2.2.2. Non-holonomic vehicle control

A unicycle type wheeled robot is a non-holonomic system with two inputs, u and r, the forward and heading velocities respectively. We are using the method described in [MIC 93], [SOE 03], which consists in controlling independently the forward velocity to reach a desired value, while the heading control is steering the vehicle toward the path and maintain it to follow the path curvature when the distance to the path is null. This is achieved in solving the heading guidance problem using an approach angle δ as described in (1).

$$\delta(y^F_{B/F}) = -sign(v_t)\theta_a \frac{e^{2k_\delta y^F_{B/F}} - 1}{e^{2k_\delta y^F_{B/F}} + 1} \quad (1)$$

2.2.3. Robot Models

The vehicle has two identical parallel, non-deformable rear wheels that are controlled by two independent motors, and a steering front wheel. It is assumed that the plane of each wheel is perpendicular to the ground and that the contact between the wheels and the ground is pure rolling and non-slipping, i.e., the velocity of the center of mass of the robot is orthogonal to the rear wheel axis. By assuming that the wheels do not slide, a non-holonomic constraint on the motion of the mobile robot of the form $\dot{x}\sin\alpha - \dot{y}\cos\alpha = 0$ is imposed. It is further assumed that the masses and inertias of the wheels are negligible and that the center of mass of the mobile robot is located in the middle of the axis connecting the rear wheels. Each rear wheels is powered by a motor which generates a control torque τ_i, i=1,2. We assume that the center of mass of the vehicle coincides with the origin of {B}. $[x_0 y_0 0]^T$ specifies the position of O_B

in {0} and $\Psi_{B/0}$ is the parameter that specifies the orientation of {B} with respect to {0}(cf. fig 2). u and r denote the robot forward and angular velocities of B with respect to 0, respectively. The kinematic model relates the inertial velocity expressed in the body frame with the one expressed in the universal frame, through the equation:

$$\dot{p}^0_{B/0} = R^0_B.\dot{p}^B_{B/0}$$

Extracting the meaningful relations yields

$$\begin{aligned} \dot{x}^0_{B/0} &= u \cos \Psi_{B/0} \\ \dot{y}^0_{B/0} &= u \sin \Psi_{B/0} \\ \dot{\Psi}_{B/0} &= r \end{aligned} \tag{2}$$

Note that $[u\,r]^T$ defines the input of the kinematic system. The classic unicycle dynamic model is given in the equation (3), considering m as the mass of the vehicle, I its moment of inertia, F and Γ the forward force and moment torque applied by the wheel motors.

$$\begin{aligned} F &= m_u \dot{u} \\ \Gamma &= m_r \dot{r} \end{aligned} \tag{3}$$

Note that the validity of the dynamic model (3) implies that there are no external disturbance, or that we have the perfect knowledge of them. Meanwhile, section 3 deals a controller with guaranteed asymptotic convergence, without or with known disturbances. In the presence of unknown external disturbances the problem consists of proving the boundness of the system output.

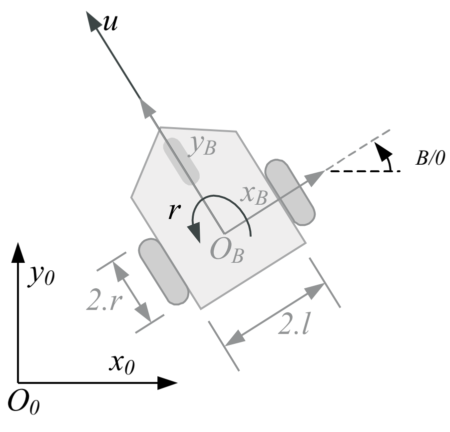

Fig. 1. A wheeled unicycle type robot

Another limitation of the model (3) consists in the knowledge of the value of dynamic parameters of the robot. This robustness requirement can be addressed using backstepping techniques (KRS 95). Note that $[F\ \Gamma]^T$ defines the input of the dynamic system.

2.2.4. Cartesian path following control

In [SOE 03] a dynamic path following controller for a unicycle type robot was derived for the case

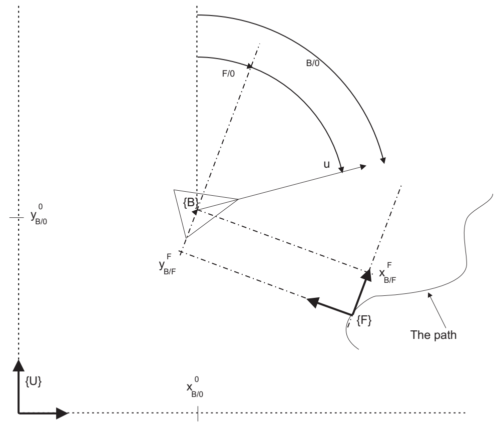

Fig. 2. Frame definition

where the path is expressed in the cartesian frame. We recall the following theorem:

Consider:

- *a wheeled robot described by its kinematic and dynamic models (2) and (3),*
- *a path parameterized by its curvature $c_c(s)$ and curvilinear derivative $g_c(s)$, related to a point located on the path with its curvilinear abscissa s,*
- *a set of available velocity measurements u and r, and $s(0)$ the initial path-position of the virtual target,*
- *u_d and $\dot{u}_d$ the desired forward velocity and acceleration,*
- *the approach angle defined in 1 ,*

Then, the control law

$$\begin{aligned} F &= m_u(\dot{u}_d - K_1(u - u_d)) \\ \Gamma &= m_r(\dot{r}_d - K_2(r - r_d) - (\theta - \delta)) \\ r_d &= \dot{\delta} - K_3(\theta - \delta) + c_c(s)\dot{s} \\ \dot{s} &= u \cos \theta + K_1 x^F_{B/F} \end{aligned} \tag{4}$$

drives asymptotically the robot to the path, considering that the initial position of the robot $x^F_{B/F}(0)$, $y^F_{B/F}(0)$, $\Psi_{B/F}(0)$ is known and such that $y^F_{B/F}(0) \neq c_c(s\,|_{t=0})^{-1}$. Moreover, $\lim_{t\to\infty} u(t) = u_d$.

The parameters K_1, K_2 and K_3 are the positive control gains. The formal proof of the previous theorem can be found in [SOE 03]. Note that the motion of the virtual target along the path (as given by $\dot{s}$) is explicitly controlled. This is in contrast with common approaches described in the literature where the position of the virtual target is defined by the projection of the robot on the path.

2.3 Integration of the vision sensor

For the sake of simplicity, and without loss of generality, we assume the vehicle is equipped with a camera that is rigidly attached to it and points downwards. The image plan {Im} is parallel to the ground and the camera is mounted at the center of mass of the vehicle. Assume the simple pinhole model for the camera, see figure 3, where {B} and {Im} denote the camera and image frame, respectively.

2.3.1. *the projective model*

Let define $\gamma = f/h$, where f is the camera focal length and h the distance of the camera above ground. Let be a projection operator defined as $\Pi([x\ y\ z]^T) = [x\ y]^T$.

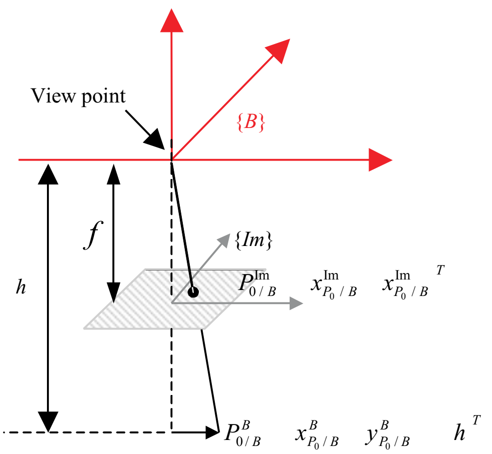

Fig. 3. The projective model of the vision system

Let $p^B_{0/B} = [x^B_{p0/B}\ y^B_{p0/B}\ -h]^T$ be a point on the ground. Its projection on the image plane is the end point of vector:

$$p^{Im}_{0/B} = [x^B_{p0/B}\ y^B_{p0/B}]^T = -\Pi(\gamma p^B_{0/B})$$

2.3.2. *Interaction relation*

Due to the non-holonomic nature of the unicycle system, the interaction matrix, which relates the intertial velocities with the ones visible in the image, includes some singularities, as shown in the following. Since

$$\begin{bmatrix} \gamma \dot{x}^{Im}_{B/F} \\ \gamma \dot{y}^{Im}_{B/F} \\ \dot{\theta}^{Im} \end{bmatrix} = A + B \begin{bmatrix} u \\ r \end{bmatrix}$$

where

$$A = \begin{bmatrix} K_2 x^{Im}_{B/F}(c_c y^{Im}_{B/F} - 1) \\ -c_c K_2 (x^{Im}_{B/F})^2 \\ -c_c K_2 x^{Im}_{B/F} \end{bmatrix}$$

$$B = \begin{bmatrix} c_c y^B_{B/F} \cos\theta & 0 \\ \sin\theta - c_c x^B_{B/F} \cos\theta & 0 \\ -c_c \cos\theta & 1 \end{bmatrix}$$

a visual control relying on an inversion of the previous relation is necessarily singular. Nevertheless, if the robot carries velocity sensors, the necessary complementary information, in order to derive a controller, are contained in the geometrical properties of the image.

2.3.3. *Geometrical relation*

The geometrical information useful to complete the control inputs in cartesian space is

$$\Theta^B = [s\ x^B_{B/F}\ y^B_{B/F}\ \theta\ c_c\ g_c]^T$$

The image contains $x^{Im}_{B/F}$, $y^{Im}_{B/F}$, θ^{Im}, c^{Im}_c, g^{Im}_c. The relation $s = \gamma s^{Im}$ yields

$$\Theta^B = P(\gamma)\Theta^{Im} \tag{5}$$

where

$$P(\gamma) = \begin{bmatrix} \gamma Id_3 & 0_3 \\ 0_3 & G(\gamma) \end{bmatrix},\ G(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\gamma} & 0 \\ 0 & 0 & \frac{1}{\gamma^2} \end{bmatrix}$$

$$\Theta^{Im} = [s^{Im}\ x^{Im}_{B/F}\ y^{Im}_{B/F}\ \theta^{Im}\ c^{Im}_c\ g^{Im}_c]^T$$

Notice that the previous relation is not singular since γ is known, constant and different from 0.

2.4 *Mathematical Formulation*

Equipped with this formalism, we now state the control problem $\mathbf{C_1}$ that is addressed in section 3.

$\mathbf{C_1}$: *given the robot models (2) and (3), a set of measurements available from robot sensors and an available parameterization of the path in the image, compute*

$$U_{dyn} = [F(.)\ \Gamma(.)\ \dot{s}_F]^T$$

so that θ^{Im}, $x^{Im}_{B/F}$, $y^{Im}_{B/F}$ *converge to 0 as t goes to* ∞.

3. THE CONTROLLER

The link between the Cartesian and the visual control can be expressed with the diagram of the figure 4.

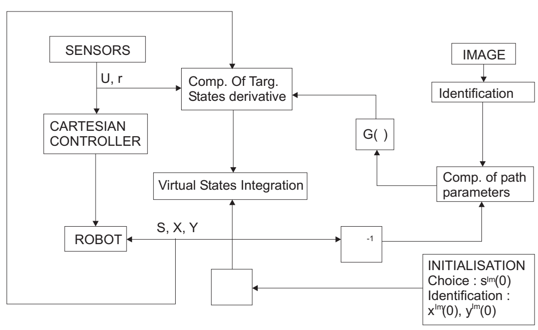

Fig. 4. Vision based controller structure

Inspired by the Cartesian controller developed in [SOE 03] and the visual geometrical relation previously described, we state the following theorem that solves the $\mathbf{C_1}$ problem.

Consider:

- *a wheeled robot described by its kinematic and dynamic models (2) and (3),*
- *a set of available velocity measurements u et r,*
- *u_d and $\dot{u}_d$ the desired forward velocity and acceleration,*
- *the approach angle defined in (1),*
- *a path, visible in the image, and parameterizable by its curvature c_c^{Im} and the curvilinear derivative of the curvature g_c^{Im}, relative to a point located on the path by its curvilinear abscissa s^{Im},*
- *and, $s^{Im}(0)$ the initial path-position of the virtual target in the image,*

Then, considering the visual geometrical relation expressed in (5), the control law described at the equation (4) drives asymptotically the robot to the path, considering that the initial position of the robot is such that $y^F_{B/F}(0) \neq c_c(s\mid_t = 0)^{-1}$. Moreover, $\lim_{t\to\infty} u(t) = u_d$.

The proof is omitted (see [LAP 02b] for details).

4. SIMULATION

4.1 Path parameterization

The path parameterization is acquired in the image plane. Since the robot, and the camera, are subject to arbitrary rotations and translations, the path parameterization should remain non singular (i.e. the image parameterization of the path has to be robust with any displacement and rotation of the robot and so the image). We have chosen to consider a non singular polynomial parameterization of the form

$$\Phi : x^{Im}_{F/Im}(\mu) = \sum_{i=0}^{n} a_i \mu^i \ ; \ y^{Im}_{F/Im}(\mu) = \sum_{i=0}^{n} b_i \mu^i$$

A necessary hypothesis concerns the estimation of the distance travelled on the path by the virtual target and concerns the knowledge of the function $\mu(s)$. This function is not analytically computable with the chosen parameterization, nevertheless we assume to have perfect knowledge of it. The set of the path parameters a_i and b_i are identified in each image.

Assuming we have a precise estimation of the function $\mu(s)$, and given s, we compute

$$\begin{aligned}
\Psi_{F/Im}(s) &= \arctan \frac{(y^{Im}_{F/Im})'}{(x^{Im}_{F/Im})'} \\
c_c^{Im}(s) &= \frac{\partial \Psi_{F/Im}(s)}{\partial \mu}\frac{d\mu}{ds} \\
g_c^{Im} = \frac{\partial c_c^{Im}(s)}{\partial s} &= \frac{\partial c_c(s)}{\partial \mu}\frac{d\mu}{ds} \\
x^{Im}_{F/Im}(\mu(s)) \quad &; \quad y^{Im}_{F/Im}(\mu(s)) \\
(x^{Im}_{F/Im})' = \frac{dx^{Im}_{F/Im}}{d\mu} \quad &; \quad (y^{Im}_{F/Im})' = \frac{dy^{Im}_{F/Im}}{d\mu}
\end{aligned}$$

The estimation of the function $\mu(s)$ is done by integration of

$$\frac{d\mu}{ds} = \frac{1}{\sqrt{[(x^{Im}_{F/Im})']^2 + [(y^{Im}_{F/Im})']^2}}$$

The previous variables constitute a sufficient set of image information to realize the control. The other control inputs are u and r the forward and rotational velocities of the robot, measured by robot sensors.

4.2 Simulation results

Without loss of generality, the parameter γ has been taken equal to 1. The Cartesian parameterization of the path is described in the table 1.

$a_0 = 0$	$a_1 = 0.866$	$a_2 = -0.02$	$a_3 = 10^{-5}$	$a_4 = 1.5.10^{-6}$
$b_0 = 0$	$b_1 = 0.5$	$b_2 = -5.10^{-4}$	$b_3 = 10^{-5}$	$b_4 = 10^{-7}$

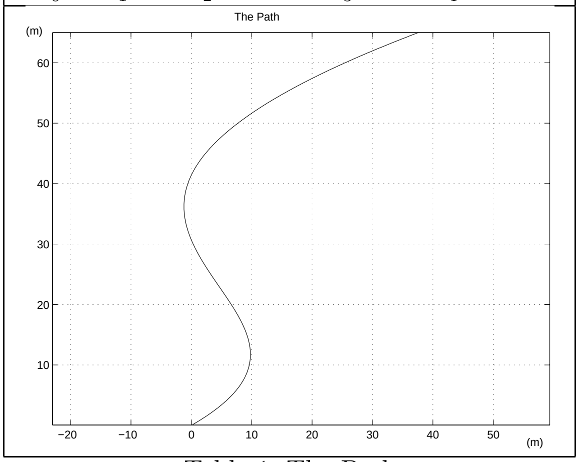

Table 1. The Path

The path parameterization is acquired in the image. The evolution of the image polynomial coefficients appears in the figure 5.

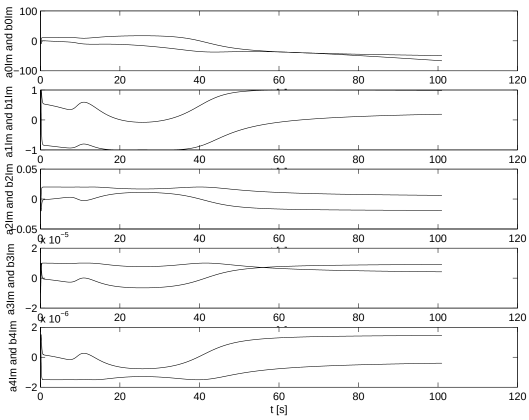

Fig. 5. Evolution of the polynomial coefficient of the image parameterization

The evolution of the distance to the virtual target is displayed in the figure 6. The trajectory of the robot is shown in the figure 7.

5. CONCLUSION

We have developed a non singular non linear vision based control for any parameterizable path

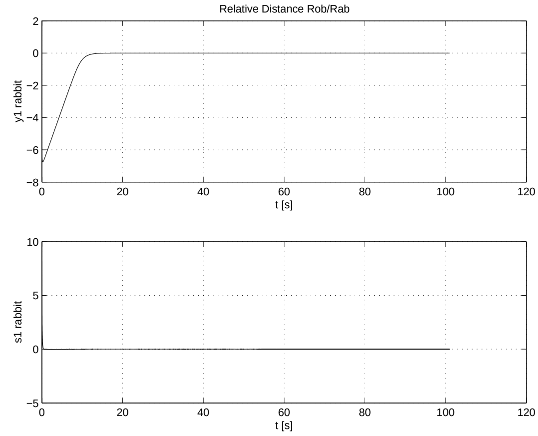

Fig. 6. Evolution of the distance to the target

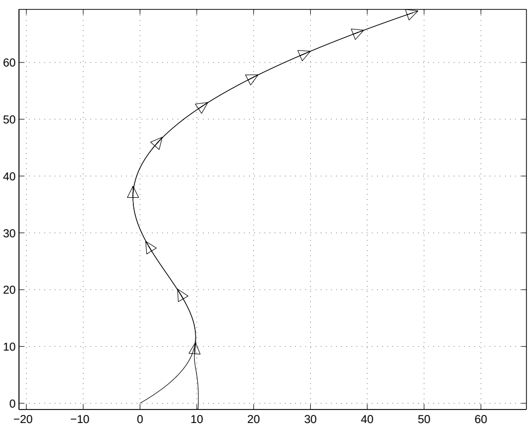

Fig. 7. The trajectory of the robot

visible in the image. One should notice that the proposed structure does not explicitly fuse any information between the image and other sensors. However, the controller uses additional information such that forward and rotational velocities. The natural continuation of this work will be to deal with filtering structures that explicitly complement vision data with that acquired with other sensor suite. Notice that the non-linearity of the problem forbids the use of Extended Kalman Filters since stability guarantees are imposed. The next extension of this work will deal with robustness requirements concerning the misestimation of the dynamic robot and the camera intrinsic parameters.

6. ACKNOWLEDGEMENTS

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