Controller Tuning for Integrating Processes with Time Delay Part I: IPDT Processes and the Pseudo-Derivative Feedback Control Configuration

Kostas G.Arvanitis, George Syrkos, Iakovos Z.Stellas, and Nick A.Sigrimis, *Member, IEEE*

Abstract— **In this paper, the use of the Pseudo-Derivative Feedback (PDF) structure in the control of integrating plus dead-time (IPDT) processes is investigated. Simple methods for tuning the PDF feedback controller are presented. The PDF control structure and the proposed tuning methods ensure smooth closed-loop response to set-point changes, fast regulatory control and sufficient robustness against parametric uncertainty. The proposed methods require small computation effort and they are particularly useful for on-line applications, since they require prior information that can easily be obtained using the relay autotuning method. Simulation results show that our methods are favorably compared to the already known PI/PID controller tuning methods for IPDT processes.**

Index Terms— **Controller tuning, dead-time processes, integrating processes, process control, Pseudo-derivative feedback.**

I. Introduction

INTEGRATING plus dead time (IPDT) model was found to be suitable for a number of processes. In [1]-[4], it has been suggested that using the IPDT model for feedback controller tuning has several advantages. This model has the ability to adequately represent the dynamics of a wide variety of systems over the frequency range of interest for conventional three-term controllers (e.g. PID controllers), which still predominate in process control and are sufficient for most needs. Since the IPDT model contains only two parameters (i.e. process gain and time delay), it is very simple for identification. For single-input, single-output (SISO) systems that contain two parameters, only one, simple relay feedback experiment [5] is needed for the estimation of these parameters. For multiple-input, multiple-output (MIMO) systems the parameters of the off-diagonal elements of the transfer function matrix can also be estimated during the relay experiment. Moreover, systems with large time constants can effectively be approximated by the IPDT model, over the critical frequency range, i.e., near the ultimate frequency.

In spite of the appealing usefulness and simplicity of the IPDT model, a limited number of tuning methods is available for such processes, as compared to other process types (e.g. first order plus dead time (FOPDT) processes). The classical Ziegler-Nichols method results in a rather oscillatory response that may become unstable even for small perturbations in the model parameters. The method reported in [1], which is based on the Internal Model Control (IMC) [7] structure, can lead to poor control when the adjustable parameter is not chosen properly. The method reported in [2] is based on the classical frequency response method of the maximum closed loop log modulus [8] and provides quite acceptable results. In [9], tuning rules for PI controllers have been proposed based on the stability analysis. In [10], PI controller settings for IPDT processes have been obtained based on stability analysis and optimization methods. In [4], the method of [2] has been extended to the problem of tuning PID controllers. In [11], a method based on the maximum peak resonance specification is proposed for PI controller tuning of IPDT processes. In [12], [13], a method is proposed for designing PID controllers for IPDT models based on specification in terms of desired control on signal trajectory scaled with respect to the magnitude of the coefficient of Taylor's series applied to stable portion of the transfer function. In [14], PID controller tuning methods have been proposed for IPDT processes based on minimizing integral criteria by the use of genetic algorithms. In [15], a PID controller tuning method for IPDT processes has been proposed, which relies on the approximation of an ideal controller by taking the first three terms of its McLaurin series expansion. Set-point pre-filters are used in [15] to obtain smooth closed-loop response to set-point step changes. A simple PI/PID tuning method for IPDT models based on matching coefficients of like powers of s in the numerator and the denominator of the closed-loop transfer function for a servo-problem has recently been presented in [16], wherein it is proposed to use set-point weighting parameters to avoid overshoot. Alternative tuning rules for IPDT models, based on

K. G. Arvanitis is with the Agricultural University of Athens, Department of Agricultural Engineering, Iera Odos 75, 11855, Athens, GREECE (phone: +302105294034; fax: +302105294039; e-mail: karvan@aua.gr).

G.Syrkos is with the Technological Education Institute of Piraeus, Department of Automation, Petrou Ralli and Thivon 250, 12244, Athens, GREECE (e-mail: gsyrkos@teipir.gr).

I.Z.Stellas is with the the Agricultural University of Athens, Department of Agricultural Engineering, Iera Odos 75, 11855, Athens, GREECE (e-mail: istel@tee.gr).

N.A.Sigrimis is with the Agricultural University of Athens, Department of Agricultural Engineering, Iera Odos 75, 11855, Athens, GREECE (e-mail: n.sigrimis@computer.org).

appropriate modifications of the classical PI/PID control structure (i.e. filtered derivative, two-degree-of-freedom controllers, set-point weighting, etc.) have also been reported in the literature (see [6] and the references therein).

The goal of this three-part paper is to investigate some design methods based on integral control and "pseudo-derivative feedback" (PDF), and to explore some aspects of the control configuration developed by Phelan [17]. These are put forward here as an alternative means of tuning a two or three-term controller for integrating plus dead time processes. In particular, Part I of the paper is devoted to the tuning of PDF controllers for IPDT models. Part II of the paper is devoted to the robust stability analysis of the proposed control scheme under structured perturbations in the model parameters, and to a comparison of the proposed control and tuning methods with known conventional PI/PID controller tuning methods, in terms of robustness. Finally, in Part III of the paper, an extension of the proposed methods to the case of first order lag plus integral plus dead-time (FOLIPDT) processes is presented.

The PDF controller is essentially a variation of both the integral control with derivative-feedback algorithm (IDF) [17] and of the conventional PID control algorithm. The three algorithms are different from each other, mainly in the way that the feedback of the controlled variable is realized. This difference reflects on some fundamental differences related to the performance of the closed-loop system in set-point changes that will be explained, in full detail, in the following section. In Part I of the paper, our attention is focused on the two simpler possible forms of the general PDF control structure. The first contains only proportional action in the feedback path, and it is designated here as the "PD-0F control structure". The second contains both proportional and derivative action in the feedback path, and it is called the "PD-1F control structure". The step-by-step development of the proposed controllers is outlined and two methods for tuning their settings are presented. In particular, the proposed tuning rules rely on approximations of the delay term through first order Taylor and Padé expansions and of the crossover frequency of the Nyquist plot of the loop transfer function. They are expressed in terms of adjustable parameters, which can be appropriately selected, either to achieve a desired damping ratio for the closed-loop system or to ensure the minimization of classical integral criteria, such as the integral of squared error (ISE) criterion, the integral of squared error plus normalized square controller output deviation (ISENSCOD) criterion [18], and the integral of squared error plus the normalized squared derivative of the controller output (ISENDCO) criterion, for either set-point tracking or regulatory control. It is worth mentioning, that explicit and precise rules and formulas, for the selection of the adjustable parameters are proposed in the paper. The proposed tuning methods require small computational effort and they are particularly useful for on-line applications, since it requires prior information that can easily be obtained using the relay autotuning method. A variety of simulation studies have been performed in the paper and the performance of the proposed methods is compared to that of known PI/PID controller tuning methods for integrating processes. The results obtained from the application of the proposed controller tuning methods to the control of IPDT processes are rather encouraging. In contrast to known conventional PI/PID tuning rules that result on large overshoot in the closed-loop response, the proposed controller structures and tuning methods ensure smooth closed loop response to set-point step changes. This enhanced performance is plausible without the need for setpoint weighting or the introduction of set point filters. The comparison also reveals that the proposed methods provide fast attenuation of step load disturbances, in addition to enhanced closed-loop response in set-point changes. Moreover, as it is analytically shown in Part II of the paper, the proposed methods are favorably compared with most of the known PI/PID tuning methods in terms of stability robustness. Finally, as it is shown in Part III of the paper, the proposed control and tuning methods can easily be extended to other classes of integrating processes, like the first order lag plus integral plus dead time (FOLIPDT) process model. Overall, the PDF algorithm can provide a better understanding of the role of the three controller terms than does the conventional PID algorithm and permits the development of simple and effective design procedures for integrating processes with time delay.

II. IPDT Processes and the PDF Controller Structure

IPDT processes are described by the following transfer function model

$$G_P(s)=K\exp(-ds)/s \tag{1}$$

where K and d are the process gain and the time delay, respectively. The magnitude and the argument of the IPDT model are given by

$$\arg\left(G_P(j\omega)\right) = -\pi/2 - d\omega \quad , \quad \left|G_P(j\omega)\right| = K/\omega$$

Solving the equation $\arg(G_p(j\omega_u)) = -\pi$, we take $\omega_u=\pi/(2d)$. Therefore, the critical gain and critical period are given by

$$K_u = \left|G_P(j\omega_u)\right|^{-1} = \pi/(2Kd)\,,\ P_u = 2\pi/\omega_u = 4d \tag{2}$$

In this paper, our aim is to investigate the pseudo-derivative feedback configuration, which was first proposed by Phelan [17] and which is put forward here as a preferred starting point, compared to the "standard" model of a PI/PID controller, for understanding and implementing two or three-term-controllers for IPDT processes. The general PDF control structure is shown in Fig. 1. The transfer function $G_{CL}(s)$ of the closed loop system is

$$G_{CL}(s) = \frac{K_I G_P(s)}{s + \left(K_{D,n-1}s^n + \ldots + K_{D,1}s^2 + K_{D,0}s + K_I\right)G_P(s)} \tag{3}$$

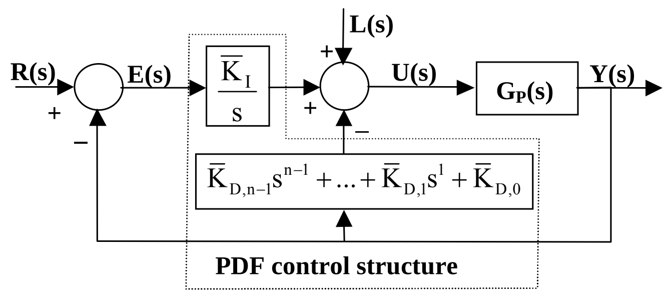

Fig. 1. The general PDF control structure.

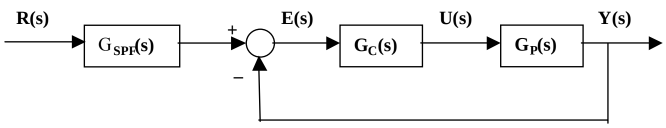

Fig. 2. PI/PID controller with set point filter control structure equivalent to PD-0F and PD-1F control, respectively.

The PDF controller is essentially a variation of the conventional PID controller that differs in the following main points: (a) In contrast to standard PID controllers, in the PDF controller, only the integral term, which is dedicated to steady state error elimination, is located in the forward path, while the remaining terms, which are mainly dedicated in assigning the desired closed-loop performance (stability, responsiveness, disturbance attenuation, etc.), are located in the feedback path. (b) The conventional PID controller acts on the process error in such a way that its elements contribute to both closed-loop poles and zeros. In contrast, the PDF controller does not contribute to closed-loop zeros, and hence it is expected that it will not worsen the overshoot of the closed-loop response. In other words, the two configurations differ in the way they react to set-point changes (as it can be easily checked, they are equivalent for load or disturbance changes). The PID controller often has an abrupt response to a step change because the step is amplified and transmitted directly to the feedback control element and downstream blocks. This can induce a significant overshoot in the response that is unrelated to the closed loop system damping. For this reason, it is a common practice to ramp or filter the set-point. The PDF structure avoids this because naturally ramps the controller effort, since it internalizes the pre-filter that one would apply to cancel any zeros introduced in the PI/PID control configuration.

In the present paper, we focus our attention on the two simpler possible forms of the general PDF control structure. The first contains only proportional action in the feedback path (i.e. $K_{D,i}=0$, for $i=1,\dots,n-1$ and $K_{D,0}\equiv K_P\neq 0$). We call this feedback scheme, the PD-0F control structure. The second contains both proportional and derivative action in the feedback path (i.e. $K_{D,0}\equiv K_P\neq 0$, $K_{D,1}=K_d\neq 0$ and $K_{D,i}=0$, for $i=2,\dots,n-1$). We call this feedback scheme, the PD-1F control structure. We shall next analyze the behavior of both the PD-0F and the PD-1F control structure, in the case where the system under control is an IPDT process with a transfer function model of the form (1).

To this end, observe that, equation (3), in the case of a PD-0F controller and for IPDT models of the form (1), takes the form

$$G_{CL}(s)=\frac{KK_I\,exp(-ds)}{s^2+K(K_Ps+K_I)exp(-ds)} \tag{4}$$

The proposed PD-0F controller is equivalent to a PI controller with a low-pass set-point filter, as shown in Fig. 2, where $G_C(s)=K_P(1+1/\theta s)$, $G_{SPF}(s)=1/(\theta s+1)$ and

$$\theta=K_P/K_I \tag{5}$$

Taking into account this equivalence, the loop transfer function of an IPDT system controlled by a PD-0F controller is given by

$$G_L(s)=\frac{KK_P(\theta s+1)\,exp(-ds)}{\theta s^2}=\frac{K(K_Ps+K_I)exp(-ds)}{s^2}$$

The argument of the loop transfer function h is given by

$$arg(G_L(\omega))=-(\pi/2)-d\omega-tan^{-1}((\theta\omega)^{-1}) \tag{6}$$

On the other hand, equation (3) in the case of a PD-1F controller and for IPDT models of the form (1), takes the form

$$G_{CL}(s)=\frac{KK_I\,exp(-ds)}{s^2+K(K_ds^2+K_Ps+K_I)exp(-ds)} \tag{7}$$

It is easy to check that the proposed PD-1F controller is equivalent to a PID controller with a set-point filter, as shown in Fig. 2, where

$$G_C(s)=K_P(1+1/(\theta s)+\delta s)\ ,\ G_{SPF}(s)=1/(\delta\theta s^2+\theta s+1)$$
$$\delta=K_d/K_p$$

This equivalence suggests that the loop transfer function of an IPDT system controlled by a PD-1F controller is given by

$$G_L(s)=\frac{KK_P(\delta\theta s^2+\theta s+1)\,exp(-ds)}{\theta s^2}=\frac{K(K_ds^2+K_Ps+K_I)exp(-ds)}{s^2}$$

In the following sections, our aim is to propose simple methods for tuning the PD-0F and PD-1F controllers for IPDT processes.

III. Tuning the PD-0F Controller

Using the first order approximation $exp(-ds)\approx 1-ds$, in the denominator of (4) and after some easy algebraic manipulations we obtain

$$G_{CL}(s) \approx \frac{exp(-ds)}{\lambda^2 s^2 + 2\zeta\lambda s + 1} \tag{8a}$$

$$\lambda = \sqrt{\left(K^{-1} - dK_P\right)K_I^{-1}} = \sqrt{\theta\left(K^{-1}K_P^{-1} - 1\right)} \tag{8b}$$

$$\zeta = \frac{K_P K_I^{-1} - d}{2\sqrt{\left(K^{-1} - dK_P\right)K_I^{-1}}} = \frac{\theta - d}{2\sqrt{\theta\left(K^{-1}K_P^{-1} - d\right)}} \tag{8c}$$

The Routh stability conditions about equation (8) yield

$$dK_I < K_P < d^{-1}K^{-1} \tag{9}$$

provided that

$$K_I < 1/(d^2 K) \tag{10}$$

Clearly, inequality (9) provides the admissible range of the PD-0F controller parameter K_P.

We next apply the approximation $tan^{-1}(x) \approx x$ (which is valid for $x<<1$) in (6). For frequencies close to the critical frequency, equation (6) can be approximated by (observe that in these frequencies the argument of the tan^{-1} function is always less than 0.2)

$$arg\left(G_L(\omega)\right) \approx -(\pi/2) - d\omega - (\theta\omega)^{-1}$$

Solving the equation $arg(G_L(\omega_{PC})) = -\pi$ yields

$$\omega_{PC} = \frac{\pi}{4d}\left[1 + \sqrt{1 - \frac{16d}{\pi^2\theta}}\right] \tag{11}$$

We are now able to present a method for tuning the PD-0F controller parameters. The proposed method is as follows.

PD-0F Controller Tuning Method for IPDT models

As for K_P, we choose the middle value of the allowed range given by inequality (9). That is

$$K_P = \left(dK_I + d^{-1}K^{-1}\right)/2 \tag{12}$$

The reason for this choice of K_P is that we want to keep maximum robustness margins for the both sides of the process parameters, when the parameters get either larger or smaller than their estimated values. Using (5), relation (12) can be written as

$$K_P = \theta\left[(2\theta - d)dK\right]^{-1} \tag{13}$$

Observe now that if θ is somehow specified, then, the PD-0F controller settings can be obtained from (13) and (5). As for θ, it is proposed to choose

$$\theta = 2\pi/(\alpha\omega_{PC}) \tag{14}$$

where ω_{PC} is given by (11) and $\alpha>0$ is an adjustable tuning parameter. Substituting (11) in (14) and after some algebraic manipulations we obtain

$$\theta = 4d\pi^2\left[\alpha\left(\pi^2 - \alpha\right)\right]^{-1} \tag{15}$$

provided that $\alpha<\pi^2$. Therefore, from (15), (13) and (5) we finally obtain

$$K_P = 4\pi^2\left[\left((8-\alpha)\pi^2 + \alpha^2\right)dK\right]^{-1} \tag{16a}$$

$$K_I = \alpha\left(\pi^2 - \alpha\right)\left[\left((8-\alpha)\pi^2 + \alpha^2\right)d^2K\right]^{-1} \tag{16b}$$

Note that, with this value of K_I, inequality (10) is always satisfied.

In general, the value of the parameter α can be selected arbitrarily in the range $0<\alpha<\pi^2$, thus permitting on-line tuning. However, it would be useful for the designer to follow certain rules, based on some criteria relative to the closed-loop system performance, in order to choose the adjustable parameter α.

A first criterion for the choice of the parameter α is related to the responsiveness of the closed-loop system. From the previous analysis, it becomes clear that the closed-loop system response may take on any desired form by choosing an appropriate value of the damping ratio ζ (which depends on K_P and θ and hence on the adjustable parameter α) in the second order approximation (8). Let this particular value of ζ to be denoted as ζ_{des}. In order to satisfy $\zeta=\zeta_{des}$, the adjustable parameter α must be selected as

$$\alpha = \frac{\pi^2}{2}\left(1 - \sqrt{1 - \frac{16}{\pi^2\left(4\zeta_{des}^2 + 1\right)}}\right) \tag{17}$$

provided that $\zeta_{des} > \sqrt{4/(\pi^2) - 1/4} \approx 0.3941$. Relation (17) can be easily obtained by substituting (15) and (16a) in (8c) and then solving the resulting equation with respect to α.

An alternative tuning can be obtained from the minimization of integral criteria. Such criteria include the integral of squared error due to unit step set-point changes (ISE-SP) or due to unit step load changes (ISE-L), the integral of squared error plus the normalized squared controller output deviation from its final value u_∞ [18] for either set-point tracking or regulatory control (ISENSCOD_SP and ISENSCOD_L, respectively) and the integral of squared error plus the normalized squared derivative of the controller output for set-point tracking or regulatory control (ISENSDCO_SP and ISENSDCO_L, respectively). These integrals have the forms

$$J_{ISENSCOD} = \int_0^\infty \left\{\left[y(t) - r(t)\right]^2 + K^2\left[u(t) - u_\infty\right]^2\right\}dt \tag{18a}$$

$$J_{ISENSDCO} = \int_0^\infty \left\{\left[y(t) - r(t)\right]^2 + K^2\dot{u}^2(t)\right\}dt \tag{18b}$$

TABLE I
OPTIMAL VALUES OF THE ADJUSTABLE PARAMETER α THAT MINIMIZE SEVERAL INTEGRAL CRITERIA

Optimal value of α	Minimum value of the integral
2.0432	d×ISE_SP = 2.7658
1.6952	d×ISENSCOD_SP = 3.1343
1.8045	d×ISENSDCO_SP = 2.9053
2.1483	d^3×ISE_L = 7.3241
1.7779	d^3×ISENSCOD_L = 10.379
1.8315	d^3×ISENSDCO_L = 8.9623

TABLE II
OPTIMAL VALUES OF THE ADJUSTABLE PARAMETER β THAT MINIMIZE SEVERAL INTEGRAL CRITERIA

Optimal value of β	Minimum value of the integral
1.6202	d×ISE_SP = 2.7658
1.4040	d ×ISENSCOD_SP = 3.1343
1.4746	d ×ISENSDCO_SP = 2.9053
1.6807	d^3×ISE_L = 7.3241
1.4576	d^3×ISENSCOD_L = 10.379
1.4916	d^3×ISENSDCO_L = 8.9623

Note that in the case of IPDT processes $u_\infty=0$. Tuning methods based on the minimization of ISE guarantee small error and very fast response, particularly useful in the case of regulatory control. However, the closed-loop step response is very oscillatory, and the tuning can lead to excessive controller output swings that cause process disturbances in other control loops. In contrast, minimization of criteria (18a) and (18b) leads to smoother closed-loop responses that are less demanding for the process actuators.

Since there is no close form solution for the minimization of the above integrals in the case of time-delay systems, simulation must be used instead. Here, optimization algorithms are used to obtain the optimal values of α that minimize the aforementioned integrals. Table I summarizes the optimal values of the tuning parameter α and the corresponding minimum values of the above integrals.

An alternative way to tune the PD-0F controller parameters that differs from the above method in the way the parameter θ is chosen is next described. The controller gain K_P is once again chosen as suggested by (12) (or equivalently (13)). As for θ, it is now proposed to choose

$$\theta = P_u / \beta = 4d / \beta \tag{19}$$

where P_u is the critical period ($P_u=2\pi/\omega_u$) and β is an adjustable parameter. Note that, $\beta = \alpha(\pi^2 - \alpha)\pi^{-2}$. With this choice, we obtain

$$K_P = 4\left[(8-\beta)dK\right]^{-1}\ ,\ K_I = \beta\left[(8-\beta)d^2K\right]^{-1} \tag{20}$$

provided that $\beta<8$. The adjustable parameter β can be chosen to satisfy a desired damping ratio ζ_{des} in the second order approximation (8). In order to satisfy $\zeta=\zeta_{des}$, the adjustable parameter β must be selected as

$$\beta = 4\left[4\zeta_{des}^2 + 1\right]^{-1} \tag{21}$$

where, in producing (21), use was made of (19) and of the first of (20) in (8c), in order to obtain a quadratic equation with respect to β, whose admissible solution is given by (21).

Alternative controller settings can be obtained from the minimization of integral criteria. Table II summarizes the optimal values of the tuning parameter β and the corresponding minimum values of several integral criteria, which has been obtained using optimization algorithms

The proposed method requires prior information that can easily be obtained using the relay autotuning method. Indeed, from (2)

$$d=P_u/4\ ,\ K=2\pi/(K_uP_u)$$

Therefore, one simple relay feedback experiment can provide us the information needed for tuning the PD-0F controller.

IV. TUNING THE PD-1F CONTROLLER

Using the first order Padé approximation $exp(-ds)\approx (1-0.5ds)/(1+0.5ds)$, in the denominator of (7) yields

$$G_{CL}(s) \approx \frac{KK_I\, exp(-ds)}{s^2 + \dfrac{K\left(K_d s^2 + K_P s + K_I\right)}{1+0.5ds}(1-0.5ds)}$$

$$= \frac{KK_I\, exp(-ds)}{s^2 + \left[2d^{-1}KK_d s + 2d^{-1}\left(KK_P - 2d^{-1}KK_d\right) + Q(s)\right]\left(1-0.5ds\right)}$$

where

$$Q(s) = \left(KK_I - 2d^{-1}KK_P + 4d^{-2}KK_d\right)/\left(1+0.5ds\right)$$

Observe now that if K_d is selected as

$$K_d = \left(dK_P/2\right) - \left(d^2K_I/4\right) \tag{22}$$

then $Q(s)=0$ and $2d^{-1}\left(KK_P - 2d^{-1}KK_d\right) = KK_I$. Therefore,

$$G_{CL}(s) \approx \frac{KK_I\, exp(-ds)}{s^2 + \left(2d^{-1}KK_d s + KK_I\right)\left(1-0.5ds\right)}$$

or, after some easy manipulations,

$$G_{CL}(s) \approx \frac{exp(-ds)}{\rho^2 s^2 + 2\xi\rho s + 1} \tag{23a}$$

$$\rho = \sqrt{\frac{1}{KK_I} - \frac{dK_P}{2K_I} + \frac{d^2}{4}} = \sqrt{\theta\left(\frac{1}{KK_P} - \frac{d}{2}\right) + \frac{d^2}{4}} \tag{23b}$$

TABLE III
OPTIMAL VALUES OF THE ADJUSTABLE PARAMETER γ THAT MINIMIZE SEVERAL INTEGRAL CRITERIA

Optimal value of γ	Minimum value of the integral
1.8972	d ×ISE_SP = 2.7658
1.5393	d ×ISENSCOD_SP = 3.1343
1.3329	d ×ISENSDCO_SP = 2.9053
1.6281	d^3×ISE_L = 7.3241
1.2264	d^3×ISENSCOD_L = 10.379
0.5962	d^3×ISENSDCO_L = 8.9623

$$\xi = \frac{\dfrac{K_P}{K_I} - d}{2\sqrt{\dfrac{1}{KK_I} - \dfrac{dK_P}{2K_I} + \dfrac{d^2}{4}}} = \frac{\theta - d}{2\sqrt{\theta\left(\dfrac{1}{KK_P} - \dfrac{d}{2}\right) + \dfrac{d^2}{4}}} \tag{23c}$$

The Routh stability conditions about equation (23) yield

$$dK_I < K_P < (dK_I / 2) + 2/(dK) \tag{24}$$

provided that $K_I<4/(d^2K)$.

We are now able to present a simple method for tuning the PD-1F controller parameters. The proposed method is as follows.

PD-1F Controller Tuning Method for IPDT models

As for K_P, we choose the middle value of the allowed range given by the inequality (24). That is

$$K_P = \left(\left(3dK_I / 2\right) + 2d^{-1}K^{-1}\right)/ 2 = \left(3dK_I / 4\right) + d^{-1}K^{-1}$$

Using (5), the above relation can equivalently be written as

$$K_P = d^{-1}K^{-1}\left(1 - \frac{3d}{4\theta}\right)^{-1} \tag{25}$$

It is now clear that if θ is somehow specified, then, the PD-1F controller settings can be obtained from (25), (5) and (22). Here, as for θ, it is proposed to choose

$$\theta = P_u / \gamma = 4d / \gamma \tag{26}$$

where γ is an adjustable parameter. Then, we obtain

$$K_P = 16\left[(16 - 3\gamma)dK\right]^{-1} \tag{27a}$$

$$K_I = 4\gamma\left[(16 - 3\gamma)d^2K\right]^{-1} \tag{27b}$$

$$K_d = (8 - \gamma)\left[(16 - 3\gamma)K\right]^{-1} \tag{27c}$$

provided that $\gamma<16/3$.

As in the case of the PD-0F controller, the adjustable parameter γ can be specified to obtain a desired damping ratio ξ_{des} for the second order approximation (23). Substituting (26) and (27a) in (23c) and solving the resulting equation with respect to γ, we obtain

$$\gamma = 4\left(2\xi_{des}^2 + 1\right)^{-1}$$

Alternative controller settings can be obtained from the minimization of integral criteria. Table III summarizes the optimal values of the tuning parameter γ and the corresponding minimum values of several integral criteria, which has been obtained using optimization algorithms.

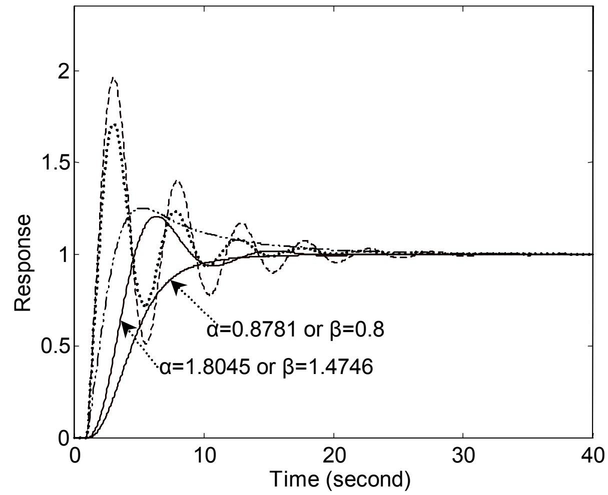

Fig. 3a. Servo response for different methods for PI and PD-0F controller tuning, without the use of setpoint filters. Solid: proposed method (α=0.8781 or β=0.8, and α=1.8045 or β=1.4746); dash: C-PS method (α_I=1.225); dot: C-PS method (α_I=1.1); dash-dot: T-L method.

V. Simulation Studies

In order to demonstrate the effectiveness of the proposed control structure and tuning methods for IPDT processes and to provide a comparison with existing tuning formulas for conventional PI/PID controller tuning, two examples are elaborated in this section

A. SISO Example

We first proceed with a comparison of the proposed method for PD-0F controller tuning with the methods for PI controller tuning reported in [2], [9], [10], [11] and [16]. To this end, the IPDT model with parameter values K=1%/sec and d=1 sec is considered. The PI controller settings given by the T-L method are K_P=0.4863, θ=8.7527. C-method provides the settings K_P=0.6710, θ=3.6547. The settings obtained from the application of the K-L-A method are K_P=0.6042, θ=5, while the P-P method yields K_P=0.5325, θ=4.1600. For α_I=1.225, the C-PS method provides the PI controller settings K_P= 1.1011, θ=4.9444, while for α_I=1.1, we obtain K_P=1.0476, θ=10.5. The servo responses obtained by applying the T-L method and the C-PS method for the above values of the adjustable parameter α_I are shown in Fig. 3a. The performance of the proposed controller, whose settings are given by relation (20) for ζ_{des}=1 (α=0.8781 or β=0.8), and for α=1.8045 or β=1.4746

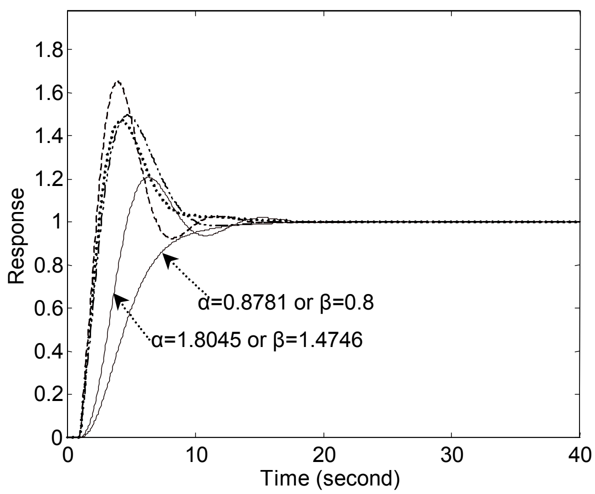

Fig. 3b. Servo response for different methods for PI and PD-0F controller tuning, without the use of setpoint filters. Solid: proposed method (α=0.8781 or β=0.8, and α=1.8045 or β=1.4746); dash: C-method; dot: K-L-A method; dash-dot: P-P method.

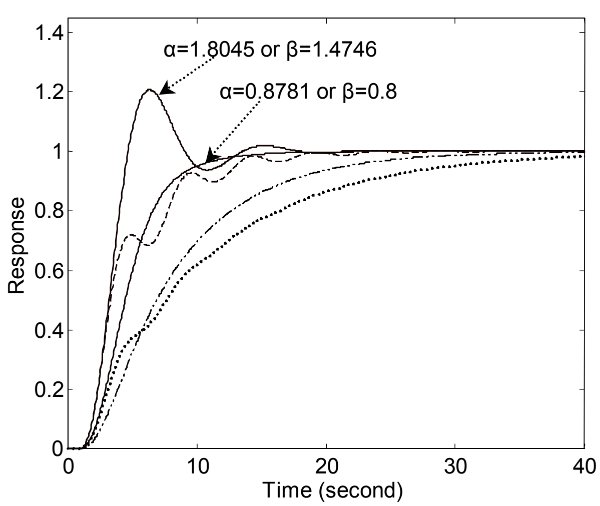

Fig. 3c. Legend as in Fig. 3a, but with a setpoint filter of the form $1/(\theta s+1)$ added in the PI controller configuration.

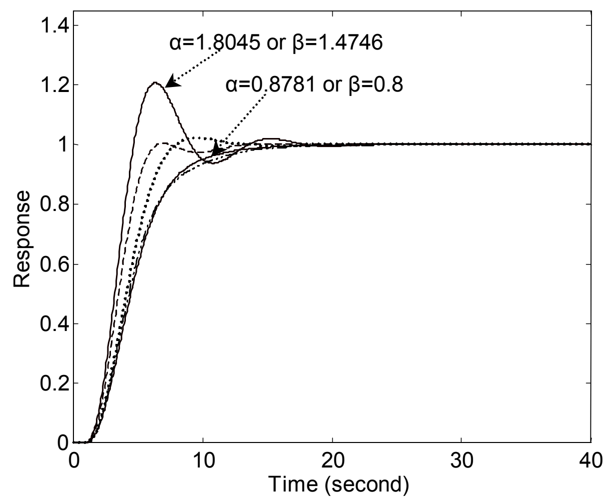

Fig. 3d. Legend as in Fig. 3b, but with a setpoint filter of the form $1/(\theta s+1)$ added in the PI controller configuration.

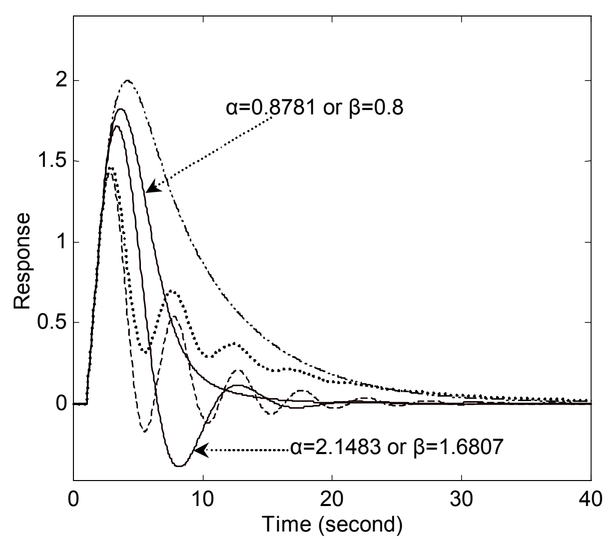

Fig. 3e. Regulatory response for different methods for PI and PD-0F controller tuning and a for a unit step load change. Other legend as in Fig. 3a.

(which corresponds to the minimum of the ISENDCO_SP criterion) is also given in Fig. 3a. Fig. 3b illustrates the servo responses obtained by the application of the C-method, the K-L-A method and the P-P method together with those obtained by the proposed controller structure and tuning method. From the above figures, it becomes clear that both the C-PS and the C-method give excessive overshoots. Moreover, the C-PS method yields an oscillatory response. The overshoot is smaller, when the P-P or the K-L-A method is applied, but still remains near to 50%. The T-L method yields the smallest overshoot among the conventional PI controller tuning methods, but the settling time obtained is quite large. Our method is the best in terms of both overshoot and settling time.

As it has been mentioned above, the proposed PD-0F controller is equivalent to a PI controller with a set-point filter. Therefore, it is fair to perform a comparison of the proposed controller tuning methods, in the case where a set point filter of the form $1/(\theta s+1)$, although not suggested, is added, in order to implement the control. Figures 3c and 3d, illustrate the servo responses obtained by the application of controller and tuning method with the above mentioned PI the methods under comparison. Our method gives a faster response than the T-L method and the C-PS method. The C-PS method for α_I=1.225 gives an oscillatory response. Our method for α=0.8781 (or β=0.8) is comparable to the K-L-A method and the C-method in terms of settling time, while it gives almost identical response to set-point changes with the P-P method. Finally, our method for α=1.8045 (or β=1.4746) provides a faster response, but the settling time is worst than that provided by the C-method, the K-L-A method or the P-P method.

We next perform a comparison of the proposed control and tuning method with the conventional PI controller tuning methods mentioned above, in the case of regulatory control. The regulatory responses obtained by applying the T-L method and

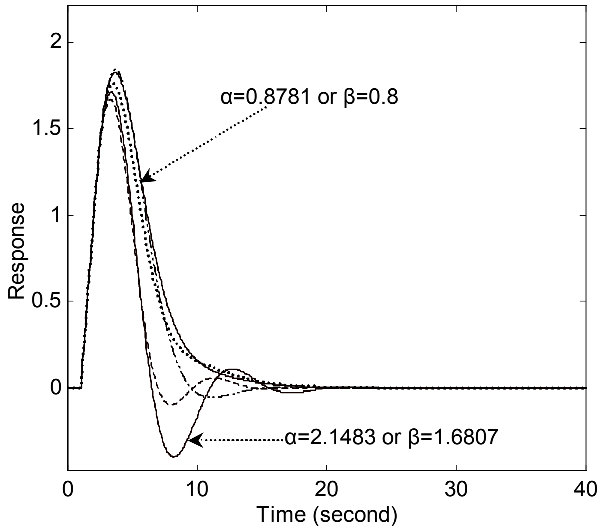

Fig. 3f. Regulatory response for different methods for PI and PD-0F controller tuning and for a unit step load change. Other legend as in Fig. 3b.

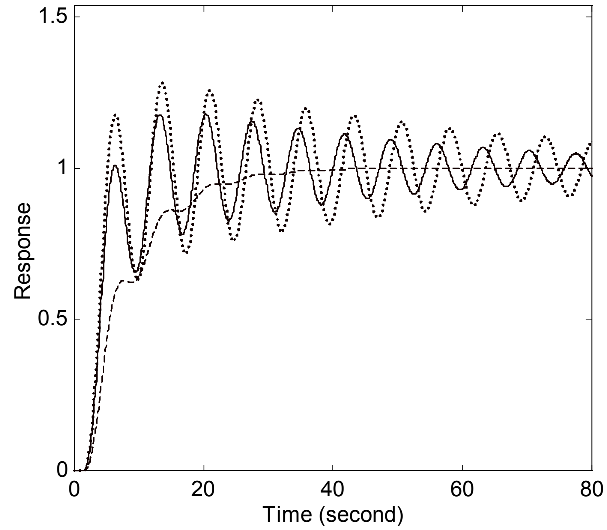

Fig. 3g. Servo response under simultaneous parametric uncertainty. K=1, d=1 for controller design and K=1.5, d=1.5 in the process. Solid: proposed method; dash: T-L method; dot: P-P method.

the C-PS method, for α_I=1.225 and α_I=1.1, are shown in Fig. 3e. Figure 3f illustrates the regulatory responses obtained by applying the C-method, the K-L-A method and the P-P method. A unit step load change is assumed. The performance of the proposed controller for ζ_{des}=1 (α=0.8781 or β=0.8), and for α=2.1483 or β=1.6807 (which corresponds to the minimum of the ISE_L criterion) is also given in Figures 3e and 3f. From these figures, it becomes clear that the smallest error is provided by the CP-S method. However, this method provides a very oscillatory response, and for α_I=1.1 the settling time is large. The T-L method provides poor regulatory control, since its response presents large error and settling time. For regulatory control, our method is comparable to the C-method, K-L-A method and P-P method in terms of maximum error and settling time. In particular, the proposed control and tuning method gives results almost identical to those obtained from the application of the P-P method for PI controller tuning.

The robustness of the proposed control and tuning method is studied by using 50% simultaneous perturbation in K and d from their nominal values in the simulation (K=1.5, d=1.5), whereas the controller settings are those calculated for the process with nominal parameters (K=1, d=1). Fig. 3g shows the servo response. A set-point filter of the form $1/(\theta s+1)$ is used in the PI controller configuration. The responses are obtained for the regulatory problem as shown in Fig. 3h. Note that, with this simultaneous uncertainty, the C-PS method, the C-method and the K-L-A method give unstable responses both for servo and regulatory control. For the servo problem, the proposed control and tuning method gives also an unstable response, in the case where α=1.8045 or β=1.4746, while for the regulator problem, the proposed method gives an unstable response when α=2.1483 or β=1.6807. Obviously, the controller settings provided by the T-L method give the best robust performance. The proposed control and tuning method is worst than the T-L method and slightly better than the P-P method in terms of robustness. Of course, this is expected based on the analysis presented in Part II of the paper.

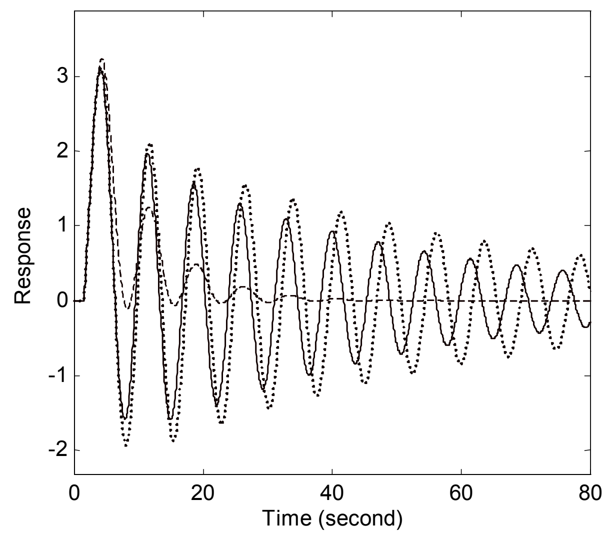

Fig. 3h. Regulatory response under simultaneous parametric uncertainty. Other legend as in Fig. 3g.

We next perform a comparison of the proposed method for PD-1F controller tuning with the methods for PID controller tuning reported in [4], [12] and [13], [14]-[16]. For the IPDT model with parameter values K=1%/sec and d=1 sec, the PID settings given by the method in [4] (L-method) are K_P=0.7726, θ=8.8, δ=0.64, while N=10 (see [4], [6] for details). The W-C method provides the settings K_P=1.1431, θ= 2.2178, δ=0.4307 for ζ_I=1 and β_I =0.5, and K_P= 1.3382, θ= 1.9086, δ=0.4152 for ζ_I= $\sqrt{2}/2$ and β_I= 0.5. The settings obtained from the application of the V-method are K_P=1.37, θ=1.49, δ=0.59. The method in [15] (L-L-P method) provides the PID controller settings K_P=1.2879, θ=2.6533, δ=0.4255, with the parameter λ= 0.6. The C-PS method, for α_I=1.25, provides the PID controller settings K_P=1.2346, θ=4.5, δ=0.45. The servo responses

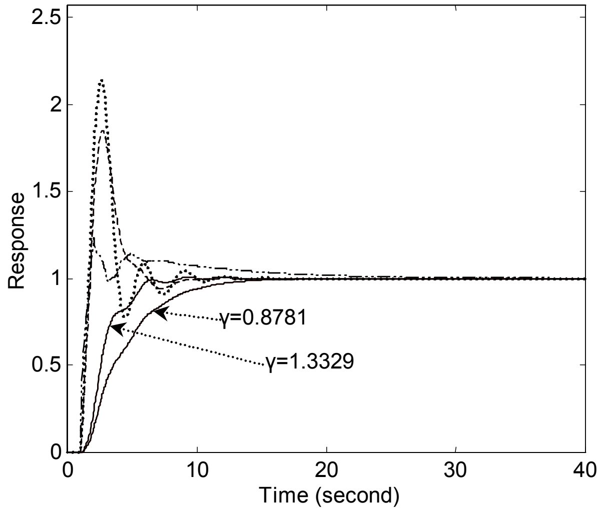

Fig. 4a. Servo response for different methods for PID and PD-1F controller tuning, without the use of setpoint filters. Solid: proposed method (γ=0.8781 and γ=1.3329); dash: W-C method (ζ_l=1); dot: W-C method ($\zeta_l=\sqrt{2}/2$); dash-dot: L-method.

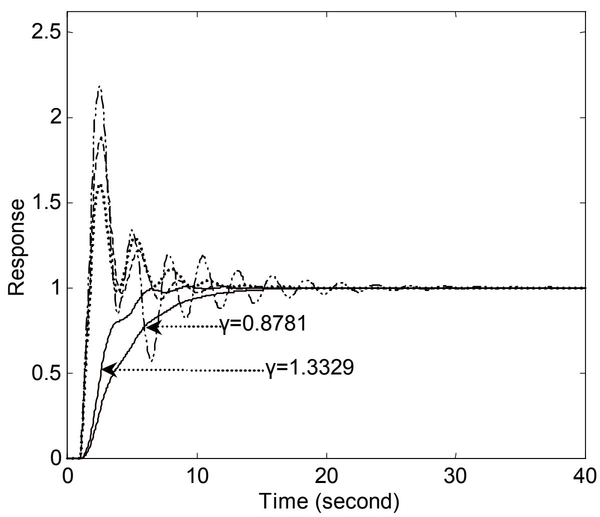

Fig. 4b. Servo response for different methods for PID and PD-1F controller tuning, without the use of setpoint filters. Solid: proposed method (γ=0.8781 and γ=1.3329); dash: L-L-P method; dot: C-PS method; dash-dot: V-method.

obtained by applying the L-method and the W-C method, are shown in Fig. 4a. The performance of the proposed controller, whose settings are given by relations (27a)-(27c) for γ=0.8781 (ξ_{des}=1.3333), and for γ=1.3329 (which corresponds to the minimum of the ISENDCO_SP criterion) is also given in Fig. 4a. Fig. 4b illustrates the servo responses obtained by the application of the V-method, the C-PS method and the L-L-P method together with those obtained by the proposed control and tuning method. It is noted that no set-point filter is used in order to implement the PID controller. Clearly, all known PID tuning methods except the L-method give excessive overshoot. Moreover, the V-method gives an oscillatory response with large settling time. The L-method provides less overshoot (only 20%), but it is the worst in terms of settling time. The proposed control and tuning method provides smooth response and acceptable settling time.

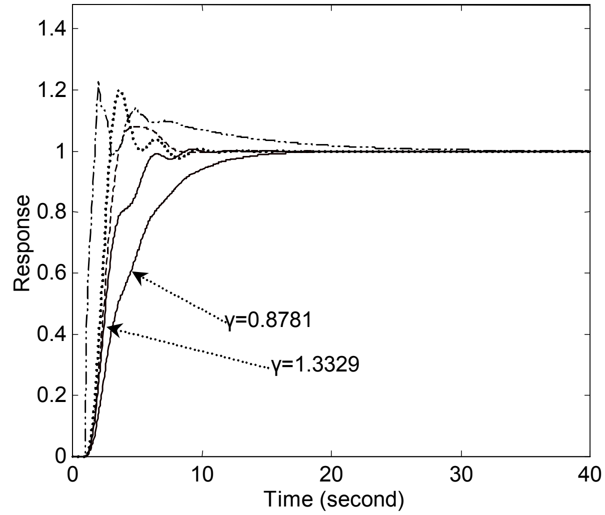

Fig. 4c. Servo response with a setpoint filter of the form $1/(\delta\theta s^2+\theta s+1)$ added in the PID controller configuration when the W-C method is applied. Other legend as in Fig. 4a.

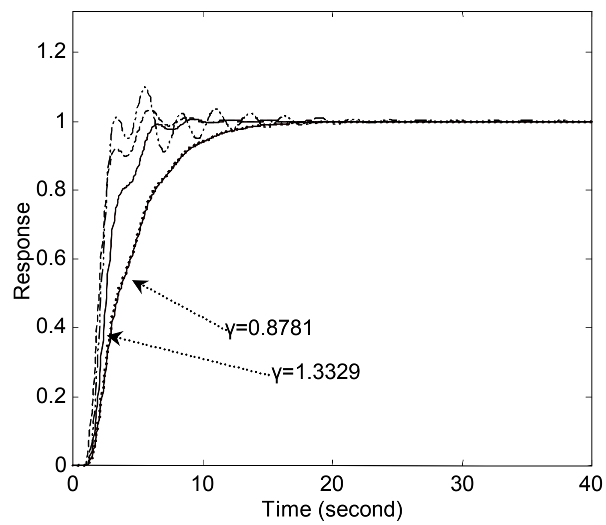

Fig. 4d. Servo response with setpoint filters added in the PID controller configuration. A set point filter of the form $1/(\delta\theta s^2+\theta s+1)$ is used with the V-method and the C-PS method. A first order set point filter [15] is used with the L-L-P method. Other legend as in Fig. 4b.

Figures 4c and 4d illustrate the servo responses obtained by the applying the above PID tuning methods and the proposed controller and tuning method, in the case where a set-point filter is used in the PID control configuration, to reduce the overshoot. The W-C, the C-PS and the V-method is associated with a second order filter of the form $1/(\delta\theta s^2+\theta s+1)$, while the L-L-P method is associated with an appropriate first order filter (see [15] for details). No set-point filter is used in case of the L-method, because the derivative term is already filtered. Even in this case the proposed method gives a smoother response with satisfactory settling time. In particular, the proposed control and tuning method gives a servo response almost identical to that obtained from the application of the C-PS method for PID controller tuning.

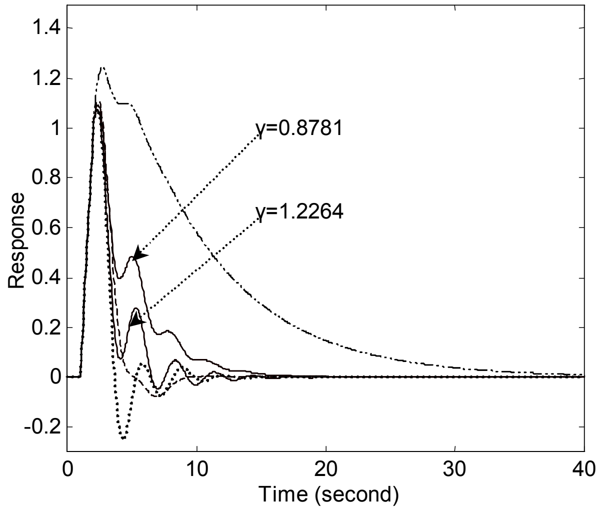

Fig. 4e. Regulatory response for different methods for PID and PD-1F controller tuning and for a unit step load change. Solid: proposed method (γ= 0.8781 and γ=1.2264). Other legend as in Fig. 4a.

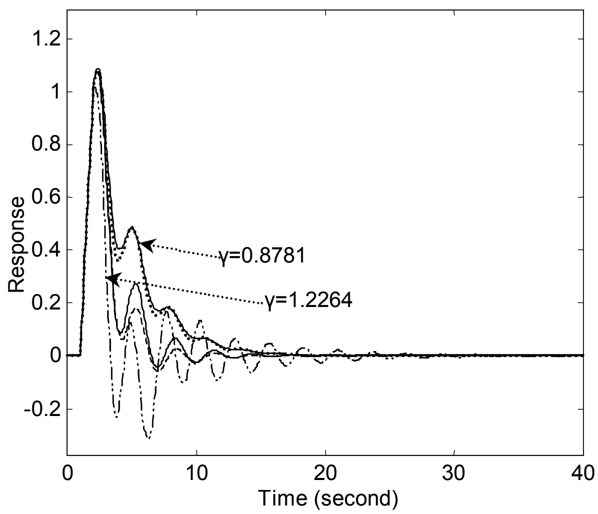

Fig. 4f. Regulatory response for different methods for PID and PD-1F controller tuning and for a unit step load change. Solid: proposed method (γ= 0.8781 and γ=1.2264). Other legend as in Fig. 4b.

The performance of the proposed PD-1F controller tuning method and of the conventional PID controller tuning methods mentioned above, in the case of regulatory control, is illustrated in Figures 4e and 4f. In the comparison, a unit step load change is assumed. The performance of the proposed controller for γ=0.8781 and for γ=1.2264 (which corresponds to the minimum of the ISENSCOD_L criterion) is also given in Figs. 4e and 4f. From these figures, it becomes clear that the L-method provides poor regulatory control, since its response presents large error and settling time. The smallest error is provided by the V- method. However, this method provides a very oscillatory response, and the settling time is large. For regulatory control, our method is comparable to the C-PS method, the L-L-P method and the W-C method in terms of maximum error and settling time. In particular, the proposed control and tuning method gives a regulatory response almost

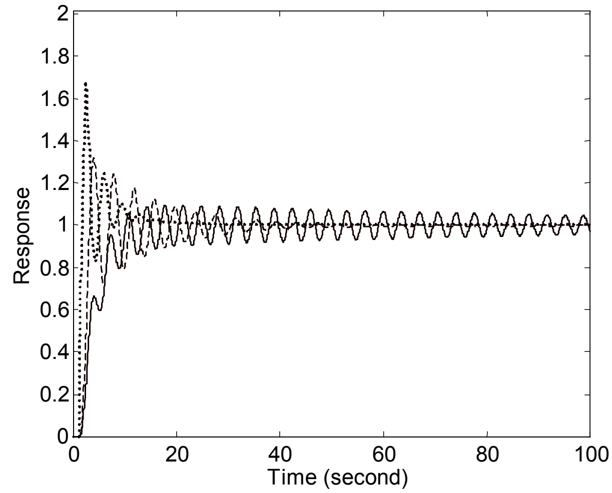

Fig. 4g. Servo response under simultaneous parametric uncertainty. K=1, d=1 for controller design and K=1.2, d=1.2 in the process. Solid: Proposed method (γ=0.8781); dash: W-C method (ζ_l=1); dot: L-method.

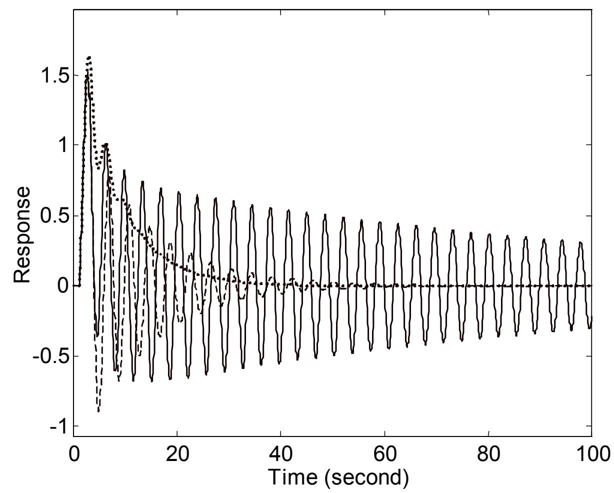

Fig. 4h. Regulatory response under simultaneous parametric uncertainty. K=1, d=1 for controller design and K=1.2, d=1.2 in the process. Other legend as in Fig. 4g.

identical to that obtained from the application of the C-PS method for PID controller tuning.

The robustness of the proposed control and tuning method is studied by using 20% simultaneous perturbation in K and d from their nominal values in the simulation (K=1.2, d=1.2), whereas the controller settings are those calculated for the process with nominal parameters (K=1, d=1). Fig. 4g shows the servo response. A set-point filter of the form $1/(\delta\theta s^2+ \theta s+1)$ is used in the PID controller configuration associated with the W-C method for ζ_l=1. The responses are obtained for the regulatory problem as shown in Fig. 4h. Note that, with this simultaneous uncertainty, the C-PS method, the V-method, the L-L-P method and the W-C method for $\zeta_l=\sqrt{2}/2$ give unstable responses both for servo and regulatory control, since they cannot tolerate 20% simultaneous parametric uncertainty. For the servo problem, the proposed control and tuning method gives also an unstable response, in the case where γ=1.3329, while for the regulator problem, the proposed method gives an unstable response when γ=1.2264. Obviously, the controller settings provided by the L-method give the best robust performance. The proposed control and tuning method is worst than both the L-method and the W-C method in terms of robustness, a fact that is expected based on the analysis reported in Part II.

B. MIMO Example

It is interesting and challenging to investigate the usefulness of the proposed methods in the context of MIMO systems control. To this end, we consider here the well-known and studied Wood and Berry (W&B) column [19]. The W&B column has the following transfer function matrix

$$\begin{bmatrix} x_d(s) \\ x_b(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ Q(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8e^{-8.1s}}{14.9s+1} \\ \dfrac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} F(s)$$

Among the multivariable PI controllers that have been proposed for the W&B column, we consider here the application of the BLT tuning method reported in [8], [20], [21] and the application of the Inverse Nyquist Array (INA) reported in [22].

The critical gain and critical period of the diagonal elements of the transfer function matrix are the following

$K_{u,1}$=2.1 , $P_{u,1}$=3.91 , $K_{u,2}$= - 0.42 , $P_{u,2}$=11.13

Thus, application of the proposed method for PD-0F controller tuning, for α=0.8781 or β=0.8, gives

$K_{P,1}$=0.7427 , $K_{I,1}$=0.1520 , $K_{P,2}$= - 0.1485 , $K_{I,2}$= - 0.0107

Application of the BLT method for tuning multivariable PI controllers of the form

$$G_C(s) = diag\{K_{P,i}(1+1/(\theta_i s))\},\ i=1,2 \tag{28}$$

gives

$K_{P,i}=K_{u,i}/(2.2f)$, $\theta_i=fP_{u,i}/1.2$

and the tuning factor f takes the value f=2.55 for the W&B column [8], [19]. Application of the INA method for the design of multivariable PI controllers has been reported in [22], where the following controller parameters are proposed

$$K_{P,1}=0.2,\ \theta_1=7.2\ \ ,\ \ K_{P,2}=-0.047,\ \theta_2=8.1$$

Using considerably more analysis, in order to achieve increased diagonal dominance at intermediate frequencies, we can take the following set of controller parameters

$K_{P,1}$=0.61 , θ_1=8.1 , $K_{P,2}$= - 0.085 , θ_2=7.6

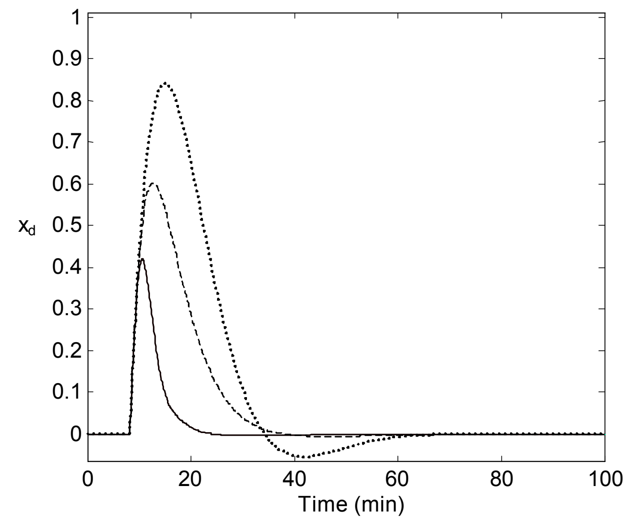

Fig. 5a. Closed-loop distillate composition (x_d) response of the W&B column to load disturbance. Solid: Proposed method; dash: BLT method; dot: INA method with a multivariable controller of the form (28).

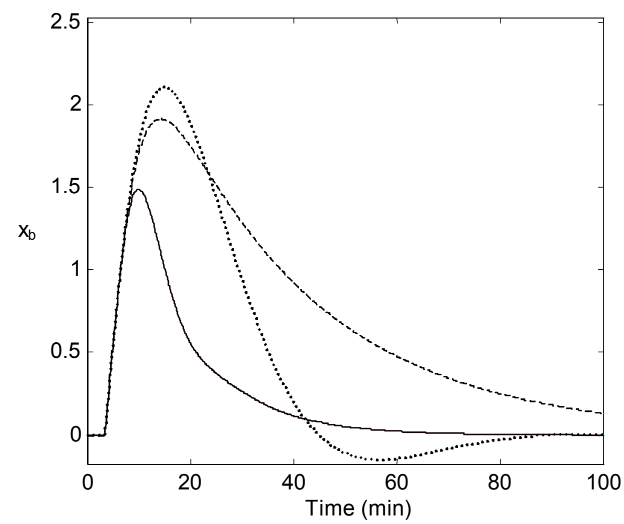

Fig. 5b. Closed-loop bottoms composition (x_b) response of the W&B column to load disturbance. Other legend as in Fig. 5a.

and the following precompensator

$$K_{PC} = \begin{bmatrix} 1 & 0.9 \\ -0.44 & 1 \end{bmatrix}$$

In this case, the controller transfer function is

$$G_{C,INA}(s) = K_{PC} diag\{K_{P,i}(1+1/(\theta_i s))\},\ i=1,2 \tag{29}$$

In Figures 5a and 5b the variation of the outputs of the system for a unit step change in the feed flow rate disturbance is given for the proposed PD-0F controller tuning method (for α=0.8781 or β=0.8), the BLT method and the INA method given by equation (28). In Figures 5c and 5d the same closed loop experiment is considered but the equation (29) is used for the INA method.

It is interesting to note that the proposed PD-0F controller

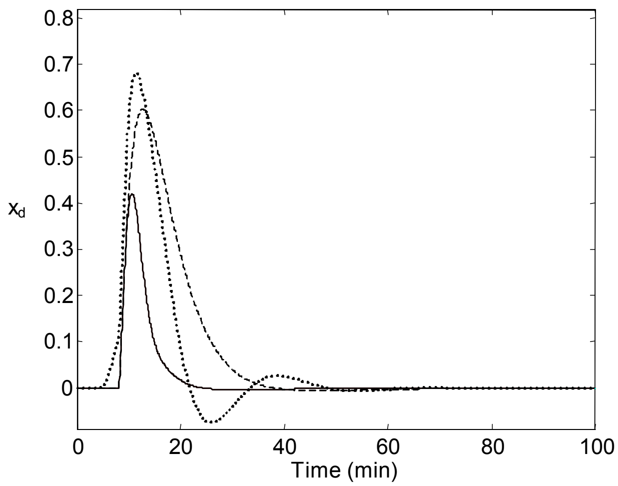

Fig. 5c. Closed-loop distillate composition response of the W&B column to load disturbance. Solid: Proposed method; dash: BLT method; dot: INA method with a multivariable controller of the form (29).

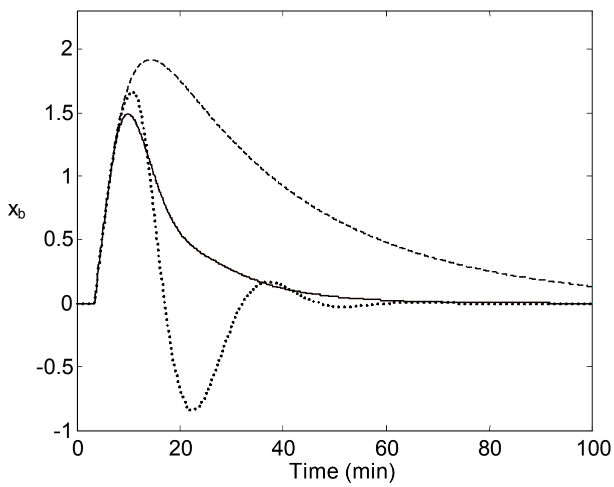

Fig. 5d. Closed-loop bottoms composition response of the W&B column to load disturbance. Other legend as in Fig. 5c.

tuning method gives better results than the BLT or the INA method. This is a rather surprising result, since in contrast to the proposed method both BLT and INA method require considerable off-line computation in order to design the diagonal multivariable PI controllers. When the INA method, with a controller of the form (29), is used, the response of x_b is greatly improved. However, the response of x_d is worse than the one obtained using the proposed method. In conclusion, the closed loop response obtained using the proposed PD-0F controller tuning method is at least comparable with the response obtained with well known multivariable design methods for PI controller tuning that require considerable computational effort.

VI. CONCLUSIONS

Simple methods for tuning PD-0F and PD-1F controllers for IPDT processes have been proposed, in this first part of the paper, and their performance has been compared with that of conventional PI/PID controller tuning methods. The comparison reveals that the proposed control and tuning methods are superior to most of the existing PI/PID tuning methods in both servo and regulatory control problems, while they provide a more robust performance, as it will analytically be shown in Part II of the paper. Taking into account the present analysis and the results reported in Part II, among existing tuning methods, only the method reported in [12], [13] really provides better results, in terms of closed-loop performance and stability robustness.

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