Sliding Mode Control with Decreased Chattering for Nonminimum Phase Plants

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***Abstract*—The idea of the system with sliding mode control and adaptation of the switched β^+ and β^- amplitudes of the relay output [1] for nonminimum phase plants is applied in the paper. This is possible owing to proposal of original parallel compensator which from the point of view of control removes the nonminimum phase property of the plant. The adaptation causes that the difference $(\beta^+ - \beta^-)$ is decreased and both the values tend to be placed symmetrically with respect to the needed value of the control. Owing to this the chattering effect appearing in the sliding control is decreased, significantly and the system remains robust. The proposed idea is illustrated in simulations.**

***Index Terms*—Sliding mode control; chattering decrease; nonminimum phase plants; parallel compensator; relay control.**

I. Introduction

The control systems which uses sliding mode technique have now good theoretical elaborations (e.g [4, 5]) as well as successful practical applications (e.g. commonly used voltage stabilization of car alternators). This kind of systems works well both with linear and non-linear plants. Sliding mode technique can be also used for decoupling multivariable systems both with linear and non-linear [3] multivariable plants.

It is well known, that the systems with sliding mode control are very robust so they work well even in the case of large and rapid parameter changes. However with the switching action of the relay there is connected the so called chattering effect which sometimes is not accepted by users and/or by actuators. Therefore chattering decrease is interesting from application point of view.

It is known that nonminimum phase plants are difficult for control. One can notice that usual sliding mode control can not be applied to the nonminimum phase plants successfully. This is caused by the fact that the step response of these plants is non monotonic i.e. at the beginning it is negative and later on –positive. Therefore the usual sliding mode control applied to these plants gives non stable waveforms.

In the present paper the idea of the sliding mode control with decreased chattering elaborated first in [1] is used to nonminimum phase plants. The independent adaptation of both the switched amplitudes β^+ and β^- of the relay output causes that the difference $(\beta^+ - \beta^-)$ is decreased and both the amplitudes are symmetrically placed with respect to the demanded value of the plant input signal. Owing to this the chattering effect is decreased, significantly. The successive application of this sliding mode control to nonminimum phase plants is possible owing to the original proposal of the parallel compensator similar to the Smith predictor. Owing to this proposal the control may be designed similarly as for minimum phase plants.

The contribution of the paper is in proposal of the parallel compensator owing to which the sliding mode control with decreased chattering may work well also for nonminimum phase plants.

II. A parallel compensator

Let a nonminimum phase plant is described by the following transfer function

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + ... + b_m}{s^n + a_1 s^{n-1} + ... + a_n} \tag{1}$$

where $m < n$ and $a_i, i = 1, 2, ..., n, b_j, j = 1, 2, ..., m$ are real coefficients. Let $s_1, s_2, ...s_n$ and $z_1, z_2, ..., z_m$ be the poles and zeros of the plant, respectively. Assume that the plant is stable, i.e. $Res_i < 0, i = 1, 2, ..., n$ while $Rez_j > 0, j = 1, 2, ..., k < m$, and $Rez_j < 0, j = k + 1, k + 2, ..., m$

Let

$$\bar{L}(s)) = (s - z_1)(s - z_2)...(s - z_k) \tag{2}$$

be the polynomial composed of nonminimum phase zeros. Denote also by

$$\bar{M}(s) = (s - s_1)(s - s_2)...(s - s_k) \tag{3}$$

Assume that the polynomial $\bar{M}(s)$ has real coefficients i.e. it contains or real poles or conjugate pair of complex poles.

Let

$$r^* = Min_i|Res_i|, \quad i = k + 1, k + 2, ..., n \tag{4}$$

and assume that

$$| \, Res_i \, | \geq f r^*, i = 1, 2, ..., k, \tag{5}$$

and that the nonminimum phase zeros are not to close to the imaginary axis (e.g. $|Rez_i| > r^*, \quad i = 1, 2, ...k$). Here f is an integer equal to at least several. The assumption (4) means that the modes corresponding to the poles $p_i, i = 1, 2, ..., k$ decay several times faster than the slowest mode among those corresponding to $i = k + 1, k + 2, ..., n$.

We may write:

$$b_0 s^m + b_1^{m-1} + ... + b_m = \bar{L}(s)(\tilde{b}_0 s^p + \tilde{b}_1 s^{p-1} + ... + \tilde{b}_p) \tag{6}$$

$$s^n + a_1 s^{n-1} + ... + a_n = \bar{M}(s)(s^q + \bar{a}_1 s^{q-1} + ... + \bar{a}_q) \quad (7)$$

where $p = m - k, \quad q = n - k$ and $\tilde{b}_0 = b_0$.

Let us notice that the transfer function

$$\bar{G}(s) = \frac{\tilde{b}_0 s^p + \tilde{b}_1 s^{p-1} + ... + \tilde{b}_p}{s^q + \bar{a}_1 s^{q-1} + ... + \bar{a}_q} \; \frac{b_m}{\tilde{b}_p} \; \frac{\bar{a}_q}{a_n} \quad (8)$$

(which results from (1) by neglecting in the numerator k modes corresponding to nonminimum phase zeros and in the denominator – k modes corresponding to the fast k poles $s_1, s_2, ..., s_k$) has the same relative degree as $G(s)$, i.e. $n - m = q - p$ and additionally it has the same gain since $\bar{G}(0) = G(0) = b_m/a_n$. Therefore the models (1) and (8) for relatively small (working) frequencies have approximately the same frequency responses, though the model (1) is nonminimum phase while (8) is minimum phase one. This also means that after same time (shorter if f is bigger) the step responses of both the models (1) and (8) are approximately the same.

Therefore we create the compensator with transfer function

$$C(s) = \bar{G}(s) - G(s) \quad (9)$$

and apply it, in parallel, to the plant $G(s)$ as on Fig. 1. Then the resulting replacement plant

$$G_r(s) = G(s) + \bar{G}(s) - G(s) = \bar{G}(s) \quad (10)$$

is minimum phase and stable and for it the sliding mode control with decreased chattering [1, 2] may be used. Note that in this approach the design of the control may be made using the plant model $\bar{G}(s)$.

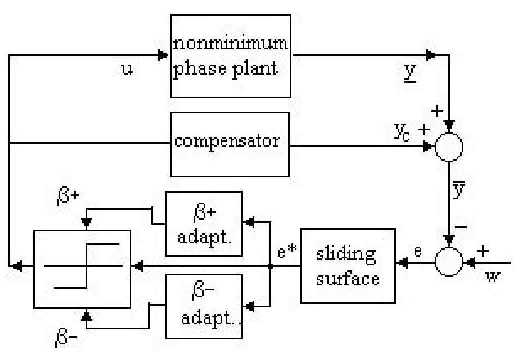

Fig. 1. Sliding mode control with adaptation.

III. Sliding mode control

In the system shown on Fig. 1 the replacement plant is described by

$$\bar{G}(s) = \frac{\bar{b}_0 s^p + \bar{b}_1 s^p + ... + \bar{b}_p}{s^q + \bar{a}_1 s^{q-1} + ... + \bar{a}_q} \quad (11)$$

where

$$\bar{b}_i = \tilde{b}_i \cdot \frac{b_m}{\tilde{b}_q} \frac{\bar{a}_q}{a_n} \quad , i = 0, 1, ..., p. \quad (12)$$

Let $\underline{y}$, y_c and y denote the outputs of the models $G(s)$, $C(s)$ and $\bar{G}(s)$, respectively, excited by the same input u. Then from Fig. 1 it results

$$\bar{y} = \underline{y} + y_c = \underline{y} + y - \underline{y} = y \quad (13)$$

Let w be the set point of the closed Loop (CL) system and $e = w - \bar{y} = w - y$ be the control error. Let $c_0, c_1, ..., c_{d-1}, \quad d = q - p - 1 = n - m - 1$ be the coefficients chosen so that the equation

$$e* = c_0 e^{(d)} + c_1 e^{(d-1)} + ... + c_{d-1} e^{(1)} + e = 0 \quad (14)$$

has stable transients of a good quality. The equation (14) describes the sliding surface. The number d has been chosen so that $\dot{e}^*$ has stepwise changes for stepwise changes of u.

By means of the proper choice of the values β^+ and β^- the following bang-bang relay control law can be created

$$\begin{array}{l} u = \beta^+ \quad if \quad e* > 0 \\ u = \beta^- \quad if \quad e* < 0 \end{array} \quad (15)$$

The values β^+ and β^- should be chosen so that

$$\begin{array}{l} \dot{e}^* = \dot{w}^* - \dot{y}^* < 0 \quad if \quad e^* > 0 \\ \dot{e}^* = \dot{w}^* - \dot{y}^* > 0 \quad if \quad e^* < 0 \end{array} \quad (16)$$

From (15), (16) it results that the law (15) realizes the bang-bang relay control of the variable e^* on the level zero. When the initial value of e^* is non zero then it is moved by means of the control (15) to zero and then it is held in zero by means of successive switchings.

The CL system described by the plant equations (11) sliding surface equation (14) and the control law (15) implements the sliding mode control. From above considerations it results that if the initial states $e, e^{(1)}, ..., e^{(d-1)}$ of the system does not lay on the sliding surface (14) then the control (15) move them to the surface (14) and then by means of successive switchings slide them along the surface. Thus, if the sliding control is realized the equation (14) is approximately fulfilled and the transients of the error e are determined by the solutions of this equation. From assumption concerning the coefficients $c_i, \quad i = 0, 1, ..., d-1$ it results that then, the system is stable and has good transients.

It is worthwhile to stress that the system may work correctly even in the case when the parameters of the plant are not known accurately. Really, in the intervals of constant w, when the sliding mode control is realized it is $e(t) \approx 0$ and $y_c(t) \approx 0$. Since $e(t) = w(t) - \bar{y}(t) = w(t) - \underline{y}(t) - y_c(t) \approx w(t) - \underline{y}(t)$, then $\underline{y}(t) \approx w$. Similar relations are valid for relatively slowly varying $w(t)$ (and eventually slowly varying disturbances and parameters).

IV. The Choice of the Values β^+, β^-

For any bounded set of the states $e, e^{(1)}, ..., e^{(d-1)}$ it is easy to choose some appropriate values of β^+ and β^- such that the inequalities (16) are fulfilled. But in the process of design this set is not given but rather some expected variations of the excitations (set point and/or disturbance) can be assumed. Using this one can determine the values β^+, β^-. Of course, the choice of the values β^+, β^-, assuring the sliding control, is possible for sufficiently smooth variations of the excitations.

One from the possible choices is $\beta^+ = \bar{\beta}$ and $\beta^- = -\bar{\beta}$ where $\bar{\beta}$ is sufficiently large value. Larger value $\bar{\beta}$ faster variations of the output y are possible and bigger variations of the disturbance may be compensated. However with switching appearing in the sliding mode control the effect of chattering is related which is stronger when $\bar{\beta}$ is larger. Some times users or actuators do not accept to strong chattering effects, especially if they are lasting to long time. Therefore, there arises the idea of adaptation of the values of β^+ and β^- to the actual system work conditions (expressed by needed output variations, parameter changes or appearing actually disturbances). This idea makes it possible to adapt the values β^+, β^- so that the difference $(\beta^+ - \beta^-)$ is sufficiently small and it is increased when this is needed. In this system the chattering effect is reduced significantly. On the other hand the sliding mode control appearing in the system causes that the latter tolerates parameter changes even in the case of nonminimum phase plants.

V. Adaptation of β^+ and β^-

In section 3 it was noted that the considered system with sliding mode control works as bang-bang relay control of the variable e^* on the level zero. The successive switchings of the relay causes characteristic oscillations of the variable e^* shown in Fig. 2. These oscillations contain information about actual values of β^+ and β^-. Namely, there is dependence between maximum positive slope $\dot{e}^*_{mx}$ of the curve in the interval (t_1, t_2) and the value of β^- as well as the minimum (negative) slope $\dot{e}^*_{min}$ of the curve in the interval (t_2, t_3) and the value of β^+. Smaller values of β^- causes greater values of the maximum slope $\dot{e}^*_{mx}$ and greater value of β^+ causes smaller value of the minimum slope $\dot{e}^*_{min}$.

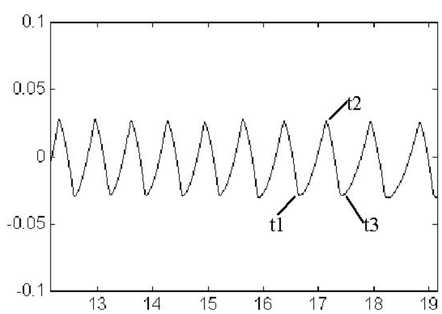

Fig. 2. Oscillation of the variable e^*.

This dependence can be used to control the maximum positive and minimum negative slope of the curve on the prescribed level by means of changing the values of β^- and β^+, respectively. Choosing for instance the prescribed levels (set points) for $\dot{e}^*_{mx} = \dot{e}^*_s$ and $\dot{e}^*_{min} = -\dot{e}^*_s$ (where $\dot{e}^*_s$ is a small positive value chosen from trails) we can cause that the values of β^+ and β^- are mutually close and symmetrically placed with respect to the needed value of the control u.

The block diagram of the system which implements this idea is shown in Fig. 1. The system implements sliding mode control and contains additionally two channels for independent adaptation of the values of β^+ and β^-, as well as the proposed compensator. A more detailed description of these channels will be given in the Example further on. Now, it is worthwhile to add that the sliding mode control in the main channel is realized in the continuous-time, relay system, while the channels for adaptation of β^+ and β^- works in the discrete-time.

In implementation of the sliding mode control the signal e^* is related to the error e by means of the formula (14). Then the successive differentiation of e must be performed. The differentiation gains the noises. Therefore, it is better to use in the place of the ideal differentiation described by the transfer function (TF) s the approximate differentiation described by the TF

$$G_d(s) = \frac{k_d s}{s + k_d} \tag{17}$$

where k_d is sufficiently large gain (e.g. $k_d = 100$). Owing to the inertia appearing in (17) the high frequency noise is not gained.

It is worthwhile to realize that implementation of the sliding mode control is possible in the case of not to high d (e.g d = 1, 2). For higher d some difficulties connected with higher order differentiation may appear.

VI. Example

Consider the nonminimum phase plant described by the transfer function

$$G(s) = \frac{-2s + 3}{s^3 + 4s^2 + 4s + 3} \tag{18}$$

The poles of the plant are: $p_1 = -3, p_2 = -0,5 + j0,8660, p_3 = -0,5 - j0,8660$. The denominator of the model (11) in accordance with (6) results from division

$$(s^3 + 4s^2 + 4s + 3) : (s + 3) = s^2 + s + 1$$

i.e. the faster mode (s+3) is deleted. Similarly for numerator we use the formula (7), (11) and (12) obtaining finally

$$\bar{G}(s) = \frac{1}{s^2 + s + 1} \tag{19}$$

Thus the compensator takes the form

$$C(s) = \frac{1}{s^2 + s + 1} - \frac{-2s + 3}{s^3 + 4s^2 + 4s + 3} \tag{20}$$

The design will be made for the minimum phase plant (19). It is easy to check that for the plant (19) we obtain $d = 1$. Let $c_0 = T$, then the dependence (14) takes the form

$$e^* = T\dot{e} + e \tag{21}$$

where T is the time constant of the transients appearing when the sliding mode control is realized.

The SIMULINK block diagram of the system with sliding mode relay control together with adaptation of the values β^+ and β^- is shown in Fig. 3. The derivative $\dot{e}$ is obtained by means of element (17) with $k_d = 100$. The relay with hysteresis H is realized by means of two elements: ”Backlash” and ”Sign”. Some small hysteresis makes it possible to obtain more expressive oscillations bringing the information about values of β^+ and β^-.

The adaptation of the values β^+ and β^- realizes the block ”adaptation” which works in the discrete-time with sampling period h. The period h should be several times smaller than the minimal switching period of the relay. The main part of this block is the MATLAB function $slidad22$ described by

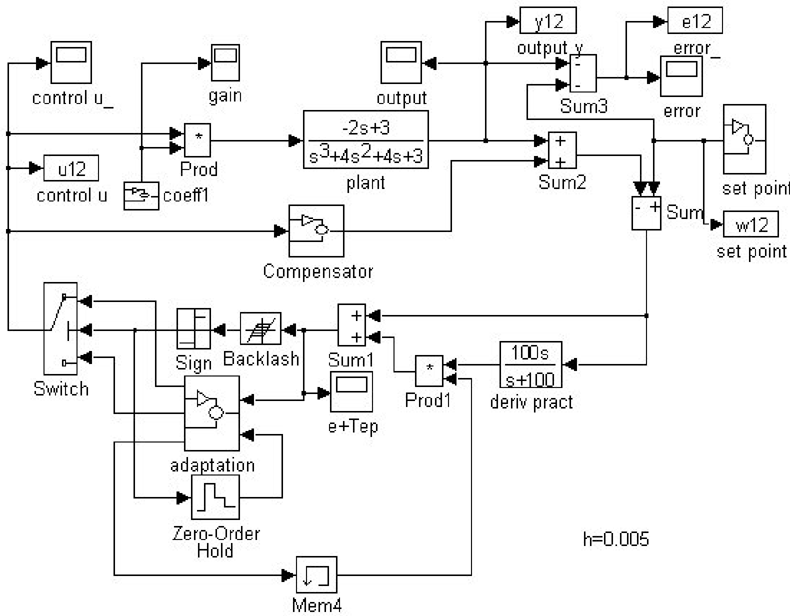

Fig. 3. SIMULINK block diagram of the system with sliding mode control and adaptation of β^+ and β^-.

the program shown in Fig. 4. In this program the notation of some variables or parameters is different from that used in the paper. Namely $ep = \dot{e}$, $dep = \dot{e}_s^*$, $bp1 = \beta^+$, $bm1 = \beta^-$, $p_1(t) = sign(e^*(t-h))$, $ep1p = \dot{e}_{mx}^*$, $ep1m = \dot{e}_{min}^*$, $ep1(t) = \dot{e}(t-h)$, $p = H(e^*)$ where $H(e^*)$ denotes the characteristic of the relay with hysteresis; $t1$ – instant of the last switching; dt – time interval assuring that the correction of β^+, β^- is made when the sliding control appears; bmx – maximum value of β^+ ($-bmx$ is the minimum value of β^-); $e = e^*$ de – the threshold of the error e: for $|e| < de$ the adaptation of β^+, β^- is realized; for $|e| > de$ it is set up $\beta^+ = bmx$ and $\beta^- = -bmx$; the same notation with indexes 1 or 2 e.g. $bp1$ and $bp2$ means that $bp1(t) = bp2(t-h)$; c – denotes a positive coefficient in the dependencies establishing new value of β^+ (β^-) in accordance with

$$\beta_2^+ = \beta_1^+ + c(\dot{e}_{min}^* + \dot{e}_s^*) \tag{22}$$

$$\beta_2^- = \beta_1^- + c(\dot{e}_{mx}^* - \dot{e}_s^*) \tag{23}$$

The formulas (22) and (23) are used in the conditions determined by the program.

VII. Results of simulations

The results of simulations for sinusoidal change of the gain from $k_{min} = 0,8$ to $k_{max} = 1,2$ and stepwise change of the set point from $w_{min} = -2$ to $w_{max} = 2$ are shown in Fig. 5. The parameters of the algorithm, chosen experimentally, were $dt = 0,5, dep = 0,05, c = 2,5, bmx = 12, de = 0,05, T = 0,1, h = 0,005, H = 2 \times 0,005$.

Thus the gain $k(t)$ is varied continuously in the interval (0.8,1.2) i.e. $k_{mx}/k_{min} = 1.5$ while in the compensator it is assumed that $k = 1$. It is seen that there exists adaptation of the values β^+ and β^- to the current needs. The large difference $(\beta^+ - \beta^-)$ appears only when some fast change of the output is needed (after stepwise change of the set point w). Since the plant is non minimum phase then after appearance of the stepwise change of w an undershot appears i.e. at the beginning there is the change of the output in the opposite to the needed direction. Note, that after stepwise change of w there is inevitable undershot, however at the vicinity of $t = 30$ there is no overshot (because of small gain) and for $t = 64$ the overshot is very small.

The results of simulations shown in Fig. 6 were performed for stepwise change of the gain from $k_{min} = 0.8$ to $k_{mx} = 1.2$ and sinusoidal change of he set point $w(t)$. It is seen that the difference $(\beta^+ - \beta^-)$ increases only after stepwise change of the gain $k(t)$. After that the adaptation decreases the difference $(\beta^+ - \beta^-)$ fast and closes both the values β^+ and β^- to the needed value of the control. At the vicinity of $t = 22$ and $t = 44$ the stepwise change of $k(t)$ influences the output $y(t)$. But at the vicinity of $t = 64$ there is negligible influence of stepwise change $w(t)$ on the output $y(t)$. There is one exception in increase of the difference $(\beta^+ - \beta^-)$. Really at the vicinity of $t = 57$ the difference increases without stepwise change of $k(t)$. This is caused by the relatively fast variation of $w(t)$ when the small difference is not sufficient for tracking.

VIII. Conclusions

Usual sliding mode control can not work with the nonminimum phase plants because the system then either becomes non stable or has some non acceptable oscillations of the output variable.

In the present paper it is shown that it is possible to implement sliding mode control, with adaptation of the relay amplitudes, for some nonminimum phase plants, using the proposed parallel compensator. The condition is that in the plant

```
function b=slidad22(u)

e=u(1); ep=u(2); t=u(3); dt=u(4); dep=u(5); c=u(6);
bmx=u(7); bp1=u(8); bm1=u(9); p1=u(10); t1=u(11);
ep1p=u(12); ep1m=u(13); ep1=u(14); p=u(15);
de=u(16); T=u(17);
bp2=bp1; bm2=bm1; t2=t1; ep2p=ep1p; ep2m=ep1m;

if abs(e)>de
 T2=T;
 bp2=bmx; bm2=-bmx;
else
 T2=T;
 if sign(ep)<sign(ep1)
  t3=t;
  if t3-t1<dt
   bp2=bp1+c*(ep1m+dep); ep2m=0;
  end
 end
 if sign(ep)>sign(ep1)
  t3=t;
  if t3-t1<dt
   bm2=bm1+c*(ep1p-dep); ep2p=0;
  end
 end
 if sign(ep)==sign(ep1)
  if sign(ep)==1
   if ep>ep1p
    ep2p=ep;
   end
  end
  if sign(ep)==-1
   if ep<ep1m
    ep2m=ep;
   end
  end
 end
end
if p~=p1
 t2=t;
end
b=[bp2,bm2,t2,ep2p,ep2m,T2];
```

Fig. 4. Algorithm of adaptation of the relay output.

there is at least so many poles corresponding to relatively faster modes as the nonminimum phase zeros and the nonminimum phase zeros are not to close to the imaginary axis. In this case it is possible to design the parallel compensator in which an important role plays the minimum phase model having for small (working) frequencies the same frequency response as the plant. The idea of the compensator is similar to that of the Smith predictor. Owing to the compensator the considered system works as with minimum phase plant.

The system works well also when the gains of the plant and compensator are different, especially for slow variations of parameters or variables. Owing to the adaptive action, the switched outputs β^+ and β^- of the relay are then suited to the needed value of the control u, so that the chattering effect connected with the sliding mode control is decreased, significantly.

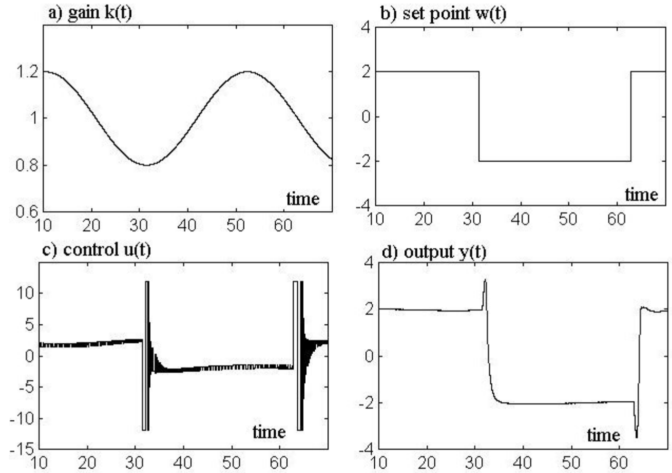

Fig. 5. Results of simulations for sinusoidal gain and stepwise set point.

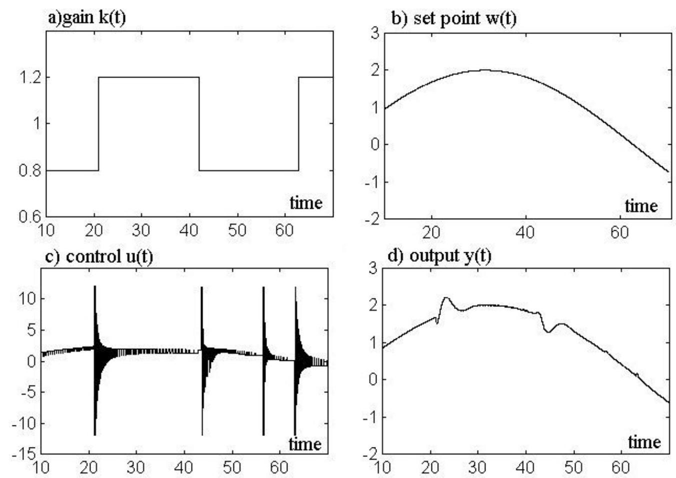

Fig. 6. Results of simulations for stepwise gain and sinusoidal set point

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