Discrete Time Reaching Law Speed Control of DC Motor Drives

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***Abstract*-- Reaching Law Control (RLC) is an approach to Sliding Mode Control (SMC) design. In this paper, the RLC method is applied to the speed control of a DC motor drive system. Since the discrete time analysis and design is more appropriate for the real applications, the design equations are derived in the discrete time. The implementation problems and the robustness of the control approach are discussed in the paper. Simulation and experimental results supporting the arguments are presented.**

***Index terms*-- sliding mode control, electrical drives, control, DC motors.**

I. INTRODUCTION

Sliding Mode Control (SMC) is a powerful technique to control the non-linear and uncertain (non-deterministic) systems [1,2]. It is a robust control method and can be applied in the presence of model uncertainties, parameter fluctuations and external disturbances provided that the bounds of these uncertainties and disturbances are known. The main disadvantage of the method is the assumption that the control signal can be switched from one value to another at infinite rate. In practical systems, however, it is impossible to manage this since the microprocessor implementation of the control strategy requires a finite sampling time. Direct microprocessor application of the SMC method results in a high frequency oscillation (chattering) about the desired equilibrium point [2]. Although there may exist some applications in which this chattering may be utilised [1], it is generally undesirable since chattering excites the unmodelled high frequency dynamics of the systems.

A new SMC design technique, which is called Reaching Law Control (RLC), was introduced by Gao and Hung in [3]. This approach not only establishes a reaching condition to the sliding line (or surface) directly but also specifies the dynamic characteristics of the system during the reaching phase. Additional merits of the RLC approach include simplification of the solution for SMC and providing a measure for the reduction of chattering. Since the RLC approach is quite new and the classical SMC is a well-known technique, there are only a few practical applications of the RLC approach to motor drive control systems [4,5].

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In this paper, the RLC approach is summarized and then the discrete time implementation of the RLC approach is described for the speed control of a DC motor drive system. Design equations are derived in discrete time and the simulations results validating the mathematical derivations are presented. It is shown that the sampling time of the speed loop is one of the main constraints for the robustness of the RLC approach. The experimental implementation of the RLC approach is realized in a vector controlled induction motor system that is equivalent to a DC motor system due to the vector control [6]. It is found that the noise of torque ripples due to the encoder resolution also restricts the robustness of the RLC approach.

II. REACHING LAW CONTROL (RLC)

Gao and Hung have introduced a new method called Reaching Law Control (RLC) for the design of SMC systems [3]. In their approach, a *reaching law* which is a differential equation specifying the dynamics of the switching function S is first chosen. The control input is then synthesized from the reaching law in conjunction with a known model of the plant and the known bounds of perturbations. It should be noted that the differential equation of an asymptotically stable S is actually a reaching condition. In addition, the dynamic quality of the SMC system in the reaching mode can be controlled by choosing the parameters in the differential equation.

Let us consider a single input second order linear uncertain system:

$$\begin{aligned}\dot{\underline{x}}(t) &= A\underline{x}(t) + \underline{b}u(t) + \underline{d}\,f(t)\\ &= \left(A_n + \Delta A\right)\underline{x}(t) + \left(\underline{b}_n + \Delta\underline{b}\right)u(t) + \left(\underline{d}_n + \Delta\underline{d}\right)f(t) \quad (1)\end{aligned}$$

where $\underline{x}(t)$ is the state vector, u(t) is the control input, A_n, $\underline{b}_n$ and $\underline{d}_n$ are composed of nominal system parameters, ΔA, $\Delta\underline{b}$ and $\Delta\underline{d}$ are the uncertainties introduced by unknown system parameters and f(t) is the external disturbance. ΔA, $\Delta\underline{b}$, $\Delta\underline{d}$

and f(t) are not known exactly but they are bounded. Equation (1) can also be written as

$$\dot{\underline{x}}(t) = A_n \underline{x}(t) + \underline{b}_n \left(u(t) + L(\underline{x}, t)\right) \quad (2)$$

$L(\underline{x}, t)$ is called *lumped uncertainty* given by $L(\underline{x}, t) = B_p \left(\Delta A \underline{x}(t) + \Delta \underline{b} u(t) + \underline{d}\, f(t)\right)$ and bounded as $\left|L(\underline{x}, t)\right| \le L_{\max}$. Note that B_p is the pseudo inverse of $\underline{b}_n$ and given as $B_p = \left(\underline{b}_n^T \underline{b}_n\right)^{-1} \underline{b}_n^T$.

The control problem is to find a control input u such that the state vector $\underline{x}$ tracks a desired trajectory $\underline{x}^d$ in the presence of model uncertainties and external disturbance. The tracking error is defined as (the argument *t* is omitted in the following for simplicity of notation)

$$\underline{e} = \underline{x} - \underline{x}^d = [e, \dot{e}]^T \quad (3)$$

which implies that the states are chosen as $[x, \dot{x}]^T$ (the control canonical form). All controllable systems can be converted to this form and there is no loss of generality in assuming the form (3).

The switching function is

$$S = l\, e + \dot{e} = C \underline{e} \quad (4)$$

If an asymptotically stable reaching law [3] is chosen as

$$\dot{S} = -q\, \mathrm{sgn}(S) - \alpha S \quad (5)$$

where q and α are positive constants, the control input u is derived by using (2)-(4) and (5) as

$$u = \left(C \underline{b}_n\right)^{-1} \left(-C A_n \underline{x} + C \dot{\underline{x}}^d - C \underline{b}_n L - q\, \mathrm{sgn}(S) - \alpha S\right) \quad (6)$$

All the quantities above the right hand side of (6) are known except the lumped uncertainty L. If L in (6) is replaced by a conservative known quantity L_c, then u becomes

$$u = \left(C \underline{b}_n\right)^{-1} \left(-C A_n \underline{x} + C \dot{\underline{x}}^d - C \underline{b}_n L_c - q\, \mathrm{sgn}(S) - \alpha S\right) \quad (7)$$

The dynamics of S is obtained by using (2)-(4) and (7) as

$$\dot{S} = -q\, \mathrm{sgn}(S) - \alpha S + C \underline{b}_n \left(L - L_c\right) \quad (8)$$

By comparing (8) with (5), it can be easily seen that an additional term $C \underline{b}_n \left(L - L_c\right)$ appears in the reaching dynamics of the perturbed system. L_c will be so chosen that it dominates the unknown lumped uncertainty L and thus ensures the reaching law (5). Since L is bounded as $\left|L(\underline{x}, t)\right| \le L_{\max}$ and assuming $C\underline{b}_n$ is a positive constant, a practical choice of L_c is

$$L_c = \begin{cases} L_{\max} & S > 0 \\ -L_{\max} & S < 0 \end{cases} \quad (9)$$

If $C\underline{b}_n$ is negative then the sign of $L_{\max}$ will be opposite in (9) which implies that

$$L_c = L_{\max}\, \mathrm{sgn}(S) \quad (10)$$

and the dynamics of S becomes

$$\dot{S} = -Q\, \mathrm{sgn}(S) - \alpha S + C \underline{b}_n L \quad (11)$$

where $Q = q + C \underline{b}_n L_{\max}$. It should be remembered that the term αS is added in the reaching law (5) to increase the reaching rate [3].

As seen in (7), the control input u contains a sgn(.) function (the ideal relay characteristic) to deal with the uncertainties and disturbances. In continuous time RLC systems, it is assumed that this function switches between +1 and -1 at infinite rate about the S = 0 line. Because of this infinitely fast switching of the control input, an ideal sliding mode exists on the line S = 0, meaning there is no chattering [2]. However, in practical systems, it is impossible to achieve the ideal infinite switching of the control input due to the microprocessor implementation of the control law which requires a finite computation time. Since it is impossible to switch the control input at infinite rate, chattering always occurs in the sliding and steady state modes of a practical RLC system.

Chattering appears as a high frequency oscillation about the desired equilibrium point in the steady state and can excite the unmodelled high frequency dynamics of the system. Since chattering is almost always undesirable for most practical applications, many researchers have directed their work to this problem as reported in [2].

There are several common methods which are used to eliminate or reduce the chattering : The most popular is to replace the discontinuous term sgn(S) by

$$sat(S) = \begin{cases} S / \phi & |S| \le \phi \\ \mathrm{sgn}(S) & |S| > \phi \end{cases} \quad (12)$$

where ϕ is an positive constant and usually called Boundary Layer thickness since using (12) means that a Boundary Layer (BL) around the switching line (or surface) is introduced to eliminate the chattering.

In the following sections, a speed control structure with an anti-windup integrator [7] will be described. Due to the limited integrator, there is no point in employing another limit in the control law of RLC with BL. Thus, the gain multiplied by S, becomes the quantity of interest. The RLC method implies that a control law without the sat(.) function can be directly obtained by simply setting the parameter q equal to zero (see (7)).

III. DISCRETE TIME SPEED CONTROL USING THE RLC APPROACH

In this section, the discrete time speed control system is considered since it is more convenient for experimental implementation. As seen in the previous section, direct implementation of sgn(.) function results in a chattering problem in the discrete time systems. In a speed control system, chattering of the torque demand is usually unacceptable since it may excite the unmodelled mechanical dynamics [1,2].

Let us consider *a discrete time reaching law* without the sgn(.) function :

$$\Delta S(k+1) = -\alpha S(k) \tag{13}$$

where k is the sampling instant (i.e. k = 0,1,2,…), α is a positive constant and Δ operator is defined as

$$\Delta g(k+1) = \frac{g(k+1) - g(k)}{T_s} \tag{14}$$

(T_s is the sampling time) which is supplemented with the condition $\{\Delta g(0) = 0\}$ and the switching function is given by

$$S(k) = \lambda e(k) + \Delta e(k) \tag{15}$$

where

$$e(k) = \omega_{ref}(k) - \omega(k) \tag{16}$$

ω_{ref} and ω are the reference and actual speeds respectively. Note that if the states are defined as the speed error and its derivative, i.e., $\underline{x} = [e, \dot{e}]^T$ and $\underline{x}^d = [0,0]^T$ then the error given by (3) becomes equivalent to the speed error defined by (16). The reaching law (13) basically implies that the switching function S exponentially reduces to zero with a desired dynamics defined by α.

Now let us consider the closed loop speed control system shown in Fig.1, where

$$G_i(z) = \frac{T_s z}{z-1} \tag{17}$$

$$G_p(s) = \frac{1}{Js + B} \tag{18}$$

and K_T is the torque constant, T_e is the electrical torque demand (current control loop delay is ignored) and $G_h(s)$ represents the zero order hold (zoh). The pulse transfer function of the plant is

$$G_p(z) = Z\{G_h(s)G_p(s)\} = \frac{C_p}{z - P_p} \tag{19}$$

where

$$P_p = \exp(-BT_s / J) \quad \text{and} \quad C_p = (1 - P_p)/B .$$

From Fig.1 and using (14)-(16), we obtain

$$\Delta S(k+1) = \frac{(1+\lambda T_s)}{T_s}((P_p - \frac{1}{(1+\lambda T_s)})\Delta e(k) - K_T C_p u(k) + \Delta\omega_{ref}(k+1) - P_p \Delta\omega_{ref}(k)) \tag{20}$$

Setting (20) equal to $-\alpha S(k)$ from (13), and solving for u(k) gives the control law

$$u(k) = \left(\frac{T_s \alpha}{(1+\lambda T_s)K_T C_p}\right)S(k) + \left(\frac{(1+\lambda T_s)P_p - 1}{(1+\lambda T_s)K_T C_p}\right)\Delta e(k) + \frac{1}{K_T C_p}\left(\Delta\omega_{ref}(k+1) - P_p \Delta\omega_{ref}(k)\right) \tag{21}$$

An advance term $\Delta\omega_{ref}(k+1)$ is seen on the right hand side of (21), but this is not a problem since $\omega_{ref}(k)$ is a known reference input. For simplicity, let us assume that ω_{ref} is a step demand (i.e. $\Delta\omega_{ref}(k+1) = \Delta\omega_{ref}(k) = 0$). The control law then becomes

$$u(k) = KS(k) + K_{eq}\Delta e(k) \tag{22}$$

where

$$K = \frac{T_s \alpha}{(1+\lambda T_s)K_T C_p} \tag{23}$$

$$K_{eq} = \frac{(1+\lambda T_s)P_p - 1}{(1+\lambda T_s)K_T C_p} \tag{24}$$

The second term of (22), $K_{eq}\Delta e(k)$, actually corresponds to u_{eq} , equivalent control, which can be interpreted as the control law that would maintain $\Delta S(k) = 0$ if the dynamics were exactly known [1,2].

By using (13) and (14), an expression for S can be obtained as

$$S(k+1) = (1 - \alpha T_s)S(k) \tag{25}$$

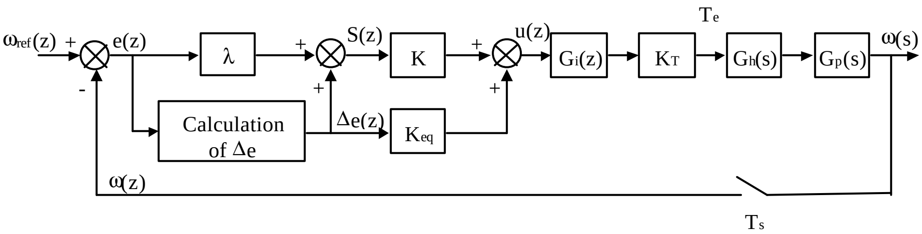

Fig. 1.The closed loop speed control system

In order to have a stable system, α should satisfy

$$0 < a < \frac{2}{T_s} \qquad (26)$$

since a sufficient condition for the stability [8] is

$$|S(k+1)| < |S(k)| \qquad (27)$$

which requires

$$|1 - aT_s| < 1 \qquad (28)$$

It should be noted that if

$$\frac{1}{T_s} < a < \frac{2}{T_s} \qquad (29)$$

then $1-\alpha T_s$ becomes a negative number and S will have damped chattering.

S will exponentially reduce to zero if

$$0 < a \le \frac{1}{T_s} \qquad (30)$$

which requires

$$0 < K \le K_m \qquad (31)$$

where

$$K_m = \frac{1}{(1 + l T_s) K_T C_p} \qquad (32)$$

The system shown in Fig.1 is simulated and Fig.2, 3 and 4 show the phase planes (e(k) & Δe(k)), variations of S, and the speed and torque responses to a step input demand (100 rad/s) for K = K_m, $0.25K_m$ and $1.75K_m$ respectively. The controller is designed for the nominal system parameters which are J = 0.0035kgm^2, B = 0.0007Nms, K_T = 4.1788Nm/A. λ is chosen as 25s^{-1} and the sampling time T_s is 2.5ms.

In Fig.2, a perfect sliding occurs since K = K_m and $\alpha = 1/T_s$ which means S becomes zero after one sampling period (i.e. S(k+1) = 0*S(k)).

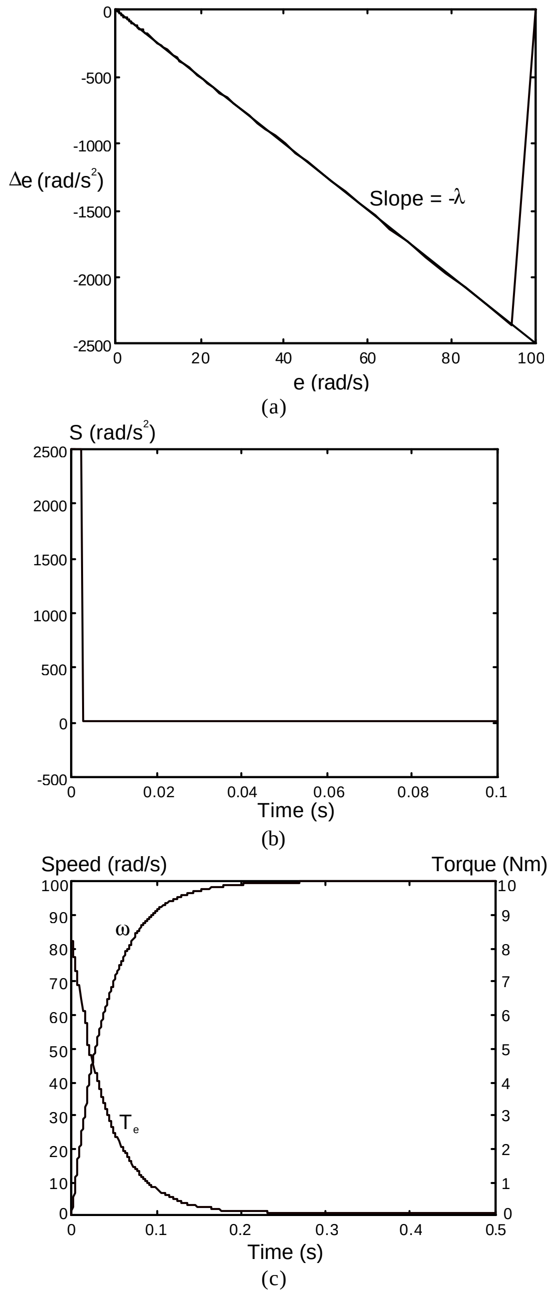

Fig. 2. Simulation results for K = K_m (a) Phase plane (Δe(k) versus e(k)) (b) Variation in S, (c) Speed and torque responses

Fig.3 shows that S exponentially reduces to zero since K = $0.25K_m$ and $\alpha = 0.25/T_s$ (S(k+1) = 0.75*S(k)). On the other hand, a damped chattering is seen in Fig.4 because S(k+1) = -0.75*S(k) due to $K = 1.75K_m$ and $\alpha = 1.75/T_s$. Note that for $K > 2K_m$, the system becomes unstable.

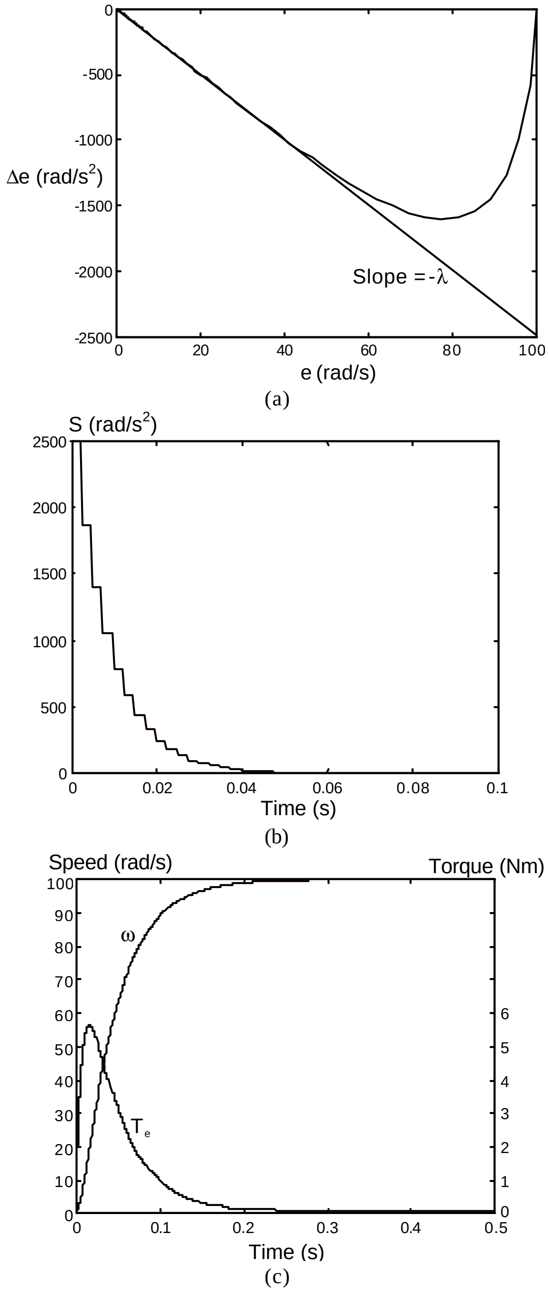

Fig. 3. Simulation results for $K = 0.25K_m$ (a) Phase plane ($\Delta e(k)$ versus e(k)) (b) Variation in S, (c) Speed and torque responses

Finally, Fig.5 shows the experimental and simulation results for K = 0.05 ($\cong 0.16K_m$) where the other parameters (e.g. λ, T_s, J, etc.) are exactly same with the ones used in the simulations above. The value of $K = 0.16K_m$ arises from noise considerations since the encoder resolution noise restricts the controller gain [9]. The details of the experimental system can be found in [9]. As seen in Fig.5c, the torque demand is limited (with an anti-windup mechanism) in order to protect the inverter and other practical circuits in the experimental implementation (the torque demand limitation is also implemented in the simulation). In the experimental results shown in Fig.5, because of the encoder resolution, the signals and the phase plane are not smooth as expected.

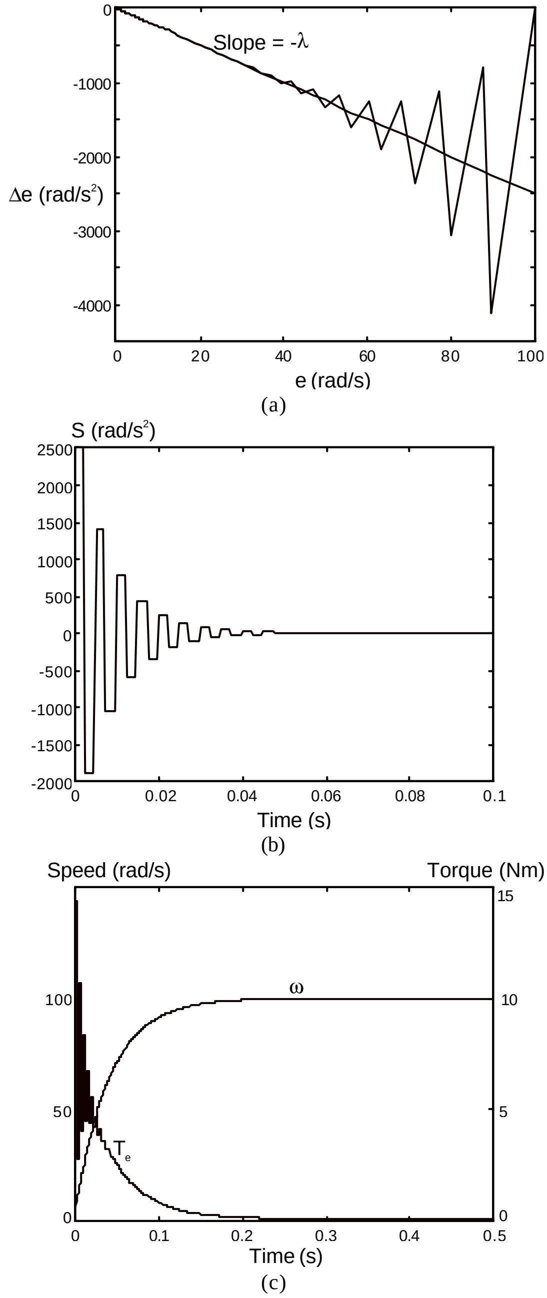

Fig. 4. Simulation results for $K = 1.75K_m$ (a) Phase plane ($\Delta e(k)$ versus e(k)) (b) Variation in S, (c) Speed and torque responses

In this section, the RLC approach has been applied to the discrete time speed control system. The plant was in the nominal conditions (i.e. no parameter variations and no external load torque). It should be noted that the control law similar to (21) can be obtained for the SMC with BL design approach as

$$u(k)=U_{\max}\,sat\big(S(k)\big)+\left(\frac{(1+l\,T_s)P_p-1}{(1+l\,T_s)K_T C_p}\right)\Delta e(k)$$
$$+\frac{1}{K_T C_p}\big(\Delta w_{ref}(k+1)-P_p\Delta w_{ref}(k)\big) \quad (33)$$

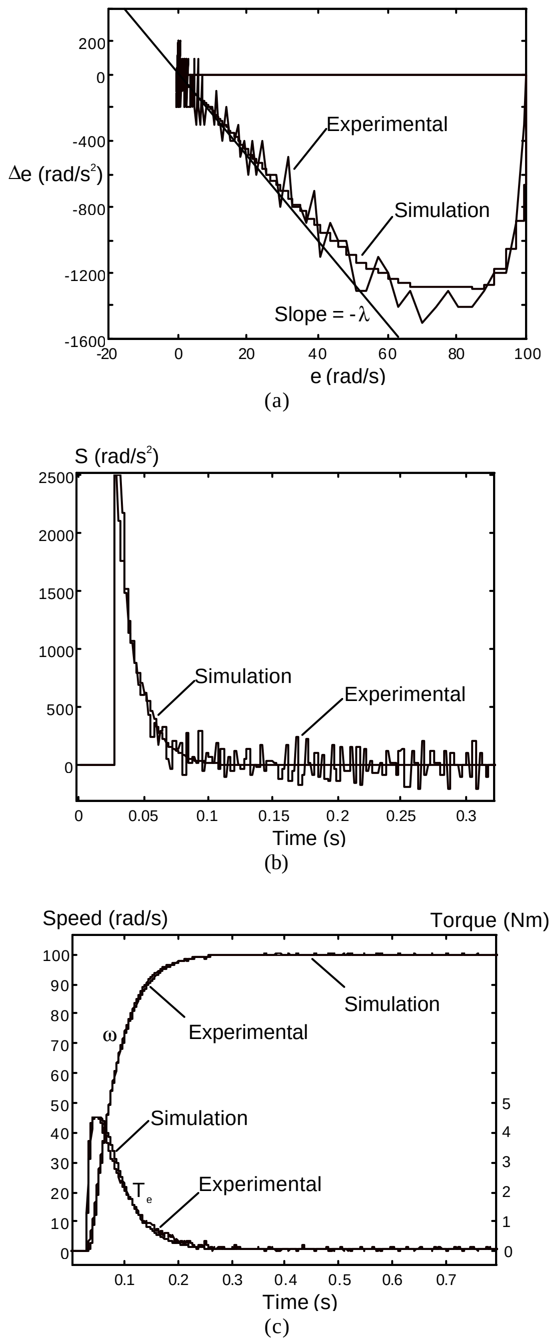

Fig. 5. Experimental and simulation results for K = 0.05 (a) Phase planes (Δe(k) versus e(k)) (b) Variations of S, (c) Speed responses and electrical torque demands

In the BL, $sat\big(S(k)\big)=\frac{S(k)}{\phi}$, thus $\frac{U_{\max}}{\phi}$ directly corresponds to the gain K of the RLC design approach if (21) and (33) are compared. The main difference between these two control law is the limitation due to the sat(.) function as seen in (33). Because of the limited integrator, the U_{max} limit becomes redundant and thus the gain U_{max}/ϕ becomes the quantity of interest. Therefore, the RLC approach has been found more appropriate for the practical implementation of the control structure shown in Fig.1.

IV. CONCLUSIONS

In this paper, RLC method, which is an approach to SMC design, is applied to the speed control of a DC motor drive system. Since the discrete time analysis and design are more appropriate for the practical implementations and in order to see the effects of the sampling time on the robustness and implementation of the RLC approach, the design equations are derived in discrete time. Simulation and experimental results validating the derived equations are presented. It is shown that the robustness of the RLC approach is restricted by the sampling time. However, in practice, the noise emerges as another limiter on the robustness of the method. The studies on the improvement of the robustness of the discrete time RLC approach will be the subject of the further publications.

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