Design of a Flight Control System for a Non-minimum Phase 5 DOF Aircraft Model

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Abstract

Flight control system designs are complicated if the aircraft dynamics are nonlinear and non-minimum phase. The non-minimum phase property can result from either the choice of output vector or as a consequence of coupling between the moment generating devices and the translational equations of motion. In the latter case the control design is normally carried out by ignoring such coupling . In this paper we consider the former case in which the non-minimum phase property is due to the choice of outputs. This problem arises when the angle of attack is an output . In such case, commanding the angle of attack to a constant value often leads to unstable pitch dynamics. We propose a solution to the stable output tracking problem based on decomposing the aircraft dynamics into a minimum phase part and a non-minimum phase part. For the minimum phase part we use inversion to obtain stable output tracking so that the side slip angle asymptotically tends to zero and the roll angle asymptotically tends to a constant command value. For the non-minimum phase part we use a classical linearization approach to obtain stable output so that the angle of attack asymptotically tends to a constant command value. The ideas are applied to obtain stable output tracking controllers for a five degree of freedom aircraft where the outputs are angle of attack, roll angle and side slip angle.

Keywords

Nonlinear Control, Non-minimum phase, Zerodynamics.

Nomenclature

b	= reference length
$\bar{c}$	= mean aerodynamic chord
C_{i_j}	= dimensionless aerodynamic
D	= drag force
g	= gravitational acceleration
I_i	= moment of inertia
I_{ij}	= product of inertia
L	= lift force
L	= aerodynamic rolling moment
M	= aerodynamic pitching moment
m	= mass of aircraft
N	= aerodynamic yawing moment
p	= roll angle rate
p_c	= roll angle rate command
Q	= dynamic pressure
q	= pitch angle rate
q_c	= pitch angle rate command
r	= yaw angle rate
r_c	= yaw angle rate command
S	= reference wing area
T	= thrust
V	= aircraft velocity, assumed constant
Y	= side force
Z	= total force along z-body axis

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α	= angle of attack
β	= sideslip angle
β_c	= sideslip angle command
ϕ	= bank angle (roll angle)
ϕ_c	= bank angle command
θ	= pitch angle (flight path angle)
θ_c	= pitch angle command
ρ	= air density
δ_a	= aileron deflection
δ_e	= elevator deflection
δ_r	= rudder deflection

I. Introduction

HIGHLY maneuverable jet aircraft provide examples of nonlinear non-minimum phase systems. The non-minimum phase property is a result of aerodynamic body forces that are directly produced by aerodynamic control surfaces. This property may also arise from choice of outputs. Such a case occurs if the angle of attack α, sideslip angle β and roll angle ϕ are chosen as outputs for the aircraft control system.

In recent years nonlinear decoupling theory and dynamic inversion approaches have been applied to design flight control systems. However, it has been shown that straightforward application of inversion approaches to nonlinear non-minimum phase flight control models may result in a system with a linear input/output response but unstable zero dynamics. In [12],[15] and [16] flight controls are designed using nonlinear inversion but no attempt is made to investigate the stability of the resulting zero dynamics.

Progress has been made in [2] and [3] in which a weakly non-minimum phase system is approximated by a minimum phase one. Recent progress in output tracking of nonlinear systems has been achieved based on development of an output regulation theory [4]. Output regulation ensures internal stability with asymptotic output tracking for a class of nonlinear systems but requires solving a set of PDE's. Moreover, this method can lead to large transient errors for non-minimum phase systems [5] and it is limited to reference trajectories generated by an exosystem. Noncausal stable inversion [8] provides internal stability and output tracking but leads to a non-causal solution that requires solving a two point boundary value problem.

In this paper we consider the case in which the non-minimum phase property is due to the choice of outputs. We consider the angle of attack, the side slip angle and roll angle as outputs. In such case, commanding the angle of attack to a constant value often leads to unstable pitch dynamics. We propose a solution to the problem based on decomposing the aircraft dynamics into a minimum phase part and a non-minimum phase part. For the minimum phase part we use inversion to obtain asymptotic output tracking. For the non-minimum phase part we can use any classical linearization approach to obtain the desired output tracking with stability. This work is related to trimmed flight trajectories tracking. In [14] detailed analysis of trimmed flight control are presented. However, in [14] linear control is used to design tracking laws for the set of trimming trajectories about which the aircraft is expected to operate. The contribution of this paper lies in the use of nonlinear control for trimmed flight. The approach presented here has two advantages over the traditional ones: 1) the output tracks the desired command asymptotically and 2) the zero dynamics remains stable for all time. Simulation results of a five degree of freedom aircraft model indicate that this approach is effective for trimmed flight control problems.

II. Mathematical Model of Aircraft

The equations of motion of a conventional aircraft over flat, nonrotating earth with zero ambient winds and constant velocity are given by

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_p \\ f_q \\ f_r \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & 0 & L_{\delta_r} \\ 0 & M_{\delta_e} & 0 \\ N_{\delta_a} & 0 & N_{\delta_r} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\phi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} f_\alpha \\ f_\beta \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\tan\beta\cos\alpha & 1 & -\tan\beta\sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \\ 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2}$$

where,

$$\begin{aligned} f_p &= \frac{1}{I}[L_0 I_y I_z + N_0 I_y I_{xz} + (I_{xz} I_y I_z - (I_y - I_x) I_y I_{xz}) pq - ((I_z - I_y) I_y I_z + I_{xz}^2 I_y) qr] \\ f_q &= \frac{1}{I}[M_0 (I_x I_z - I_{xz}^2) + (r^2 - p^2)(I_x I_z - I_{xz}^2) I_{xz} - (I_x - I_z)(I_x I_z - I_{xz}^2) pr] \\ f_r &= \frac{1}{I}[L_0 I_y I_{xz} + N_0 I_x I_y + (I_{xz}^2 I_y - (I_y - I_x) I_x I_y) pq - ((I_z - I_y) I_y I_{xz} + I_{xz} I_x I_y) qr] \end{aligned}$$

$$\begin{aligned} f_\alpha &= \frac{-QS(C_{L0} + \alpha C_{L\alpha} + (C_{D0} + \alpha C_{D\alpha})\sin\alpha) + mg(\cos\theta\cos\phi\cos\alpha + \sin\theta\sin\alpha)}{\mathrm{m}V\cos\beta} \\ f_\beta &= \frac{QS(\beta C_{Y\beta} - (C_{D0} + \alpha C_{D\alpha})\cos\alpha\sin\beta)}{\mathrm{m}V} \\ &\quad + \frac{g}{V}(\sin\theta\cos\alpha\sin\beta + \cos\theta\sin\phi\cos\beta - \cos\theta\cos\phi\sin\alpha\sin\beta) \end{aligned}$$

$$\begin{aligned} L_0 &= QSb[\beta C_{l\beta} + (pC_{lp} + rC_{lr})b/(2V)] \\ M_0 &= QS\bar{c}[C_{mo} + (qC_{mq} + C_{m\dot{\alpha}}(f_\alpha + q - (p\cos\alpha + r\sin\alpha)\tan\beta))\bar{c}/(2V) + C_{m\alpha}\alpha] \\ N_0 &= QSb[\beta C_{n\beta} + (pC_{np} + rC_{nr})b/(2V)] \end{aligned}$$

$$\begin{aligned} L_{\delta_a} &= \frac{1}{I}[I_y I_z QSbC_{l\delta_a} + I_y I_{xz} QSbC_{n\delta_a}] \\ L_{\delta_r} &= \frac{1}{I}[I_y I_z QSbC_{l\delta_r} + I_y I_{xz} QSbC_{n\delta_r}] \end{aligned}$$

$$\begin{aligned} M_{\delta_e} &= \frac{QS\bar{c}}{I}[(I_x I_z - I_{xz}^2) C_{m\delta_e}] \\ N_{\delta_a} &= \frac{1}{I}[I_y I_{xz} QSbC_{l\delta_a} + I_x I_y QSbC_{n\delta_a}] \\ N_{\delta_r} &= \frac{1}{I}[I_y I_{xz} QSbC_{l\delta_r} + I_x I_y QSbC_{n\delta_r}] \end{aligned}$$

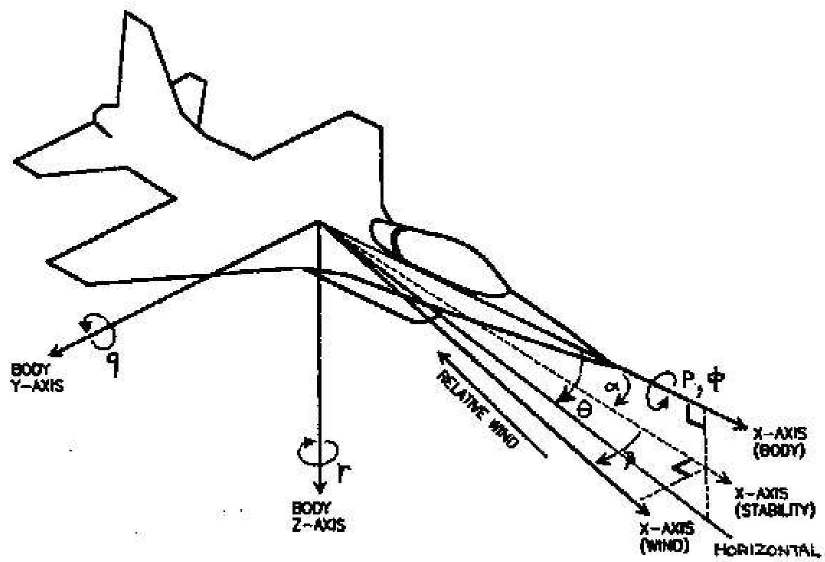

Fig. 1. Model Aircraft and Notations

III. Flight Control Design

In this paper we consider the angle of attack α, the sideslip angle β and the bank angle ϕ as outputs to be tracked. In most manned aircraft, the sideslip angle is required to be zero or small. Tracking of angle of attack is directly related to tracking of normal acceleration [17], which plays an important role in many practical maneuvers. Consequently, we assume that the command angle of attack α_c and command roll angle ϕ_c are constant, while the command sideslip angle $\beta_c = 0$. In this section we consider two different strategies to design tracking control laws. First, nonlinear inversion [18] is considered and then a decomposition approach is introduced in [13].

A. Nonlinear Inversion

Here the objective is to design a control law such that given reference commands $(\phi_c, \alpha_c, \beta_c)^T$ are tracked. A candidate method to design such a control law is the dynamic inversion technique. However, this method can ordinarily not be used if the resulting zero dynamics is unstable. In [15], a simplified aircraft model is introduced and inversion is used to design a control law for this output vector. The resulting controller is input output stable but no attempt is made in [15] to investigate the stability of the resulting zero dynamics.

In this paper we examine the stability of the resulting zero dynamics. The relative degree vector is $[2, 2, 2]$ and the resulting zero dynamics is of order one. To obtain the expression for the zero dynamics we set the outputs and there derivatives to zero and solve for the remaining dynamics. It can be shown that the zero dynamics in this case is given by

$$\dot{\theta} + \frac{g}{V}\cos\theta = \frac{QSC_{L0}}{mV}$$

The above equation has an unstable equilibrium at

$$\theta^* = \cos^{-1}\frac{\rho V^2 SC_{L0}}{2mg}$$

The system with unstable zero dynamics is called non-minimum phase. Simulation results in [15] indicates linear growth in the pitch angle as time proceeds. Clearly such a control law can not be used in practice at least for a significant time period. It is important to mention that the resulting non-minimum phase behavior in this problem is due to the choice of outputs. This differs from the flight control problems studied in [5] and [11] where the non-minimum phase effect is due to the small aerodynamic forces generated by the control surfaces deflections.

B. Decomposition of Aircraft Dynamics

Failure of inversion techniques to provide satisfactory tracking control laws motivates us to seek a different control strategy. In [13,20] we found that multi-input multi-output nonlinear non-minimum phase systems may be decomposed into a minimum phase part and a non-minimum phase part.

For this flight control problem we note that the zero dynamics is linearly controllable through q and hence through δ_e. Further, the angle of attack is also linearly controllable through q. This motivate us to use the elevator to simultaneously track a constant angle of attack and to stabilize the zero dynamics. The remaining inputs the aileron and rudder, are used to track the remaining outputs, namely, roll angle and sideslip angle. In this case the aircraft dynamics can be decomposed into a minimum phase dynamics:

$$\begin{bmatrix} \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_p \\ f_r \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} f_\beta \\ f_\phi \end{bmatrix} + \begin{bmatrix} 1 & \tan\theta\cos\phi \\ \sin\alpha & -\cos\alpha \end{bmatrix} \begin{bmatrix} p \\ r \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \phi \\ \beta \end{bmatrix} \tag{5}$$

and a non-minimum phase dynamics:

$$\begin{aligned} \dot{q} &= f_q + M_{\delta_e}\delta_e && (6)\\ \dot{\alpha} &= \hat{f}_\alpha + q && (7)\\ \dot{\theta} &= q\cos\phi - r\sin\phi && (8)\\ y_3 &= \alpha && (9) \end{aligned}$$

where

$$\begin{aligned} f_\phi &= q\tan\theta\sin\phi \\ \hat{f}_\alpha &= f_\alpha - (p\cos\alpha + r\sin\alpha)\tan\beta \end{aligned}$$

C. Controller Design for the Minimum Phase Dynamics

We can use inversion technique to design tracking control law for the minimum phase dynamics. Using a time scale separation approach, we invert equation (3) for commands p_c and r_c. The control inputs are given by

$$\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix}^{-1} \begin{bmatrix} \dot{p_c} - k_p(p - p_c) - f_p \\ \dot{r_c} - k_r(r - r_c) - f_r \end{bmatrix} \tag{10}$$

where k_p and k_r are positive constants. Define $\widetilde{p} = p - p_c$ and $\widetilde{r} = r - r_c$ to obtain

$$\dot{\widetilde{p}} = -k_p \widetilde{p} \tag{11}$$
$$\dot{\widetilde{r}} = -k_r \widetilde{r} \tag{12}$$

Note that equation (3) can be written as

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} f_\beta \\ f_\phi \end{bmatrix} + \begin{bmatrix} 1 & \tan\theta\cos\phi \\ \sin\alpha & -\cos\alpha \end{bmatrix} \begin{bmatrix} p_c + \widetilde{p} \\ r_c + \widetilde{r} \end{bmatrix} \tag{13}$$

Now in view of equations (11), (12), $\widetilde{p} \to 0$ and $\widetilde{r} \to 0$ as $t \to \infty$. Assuming a time scale separation, equation (13) can be approximated by

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} f_\beta \\ f_\phi \end{bmatrix} + \begin{bmatrix} 1 & \tan\theta\cos\phi \\ \sin\alpha & -\cos\alpha \end{bmatrix} \begin{bmatrix} p_c \\ r_c \end{bmatrix} \tag{14}$$

The commands p_c and r_c can be determined by inverting equation (14) to obtain desired output commands ϕ_c and β_c as follows

$$\begin{bmatrix} p_c \\ r_c \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta\cos\phi \\ \sin\alpha & -\cos\alpha \end{bmatrix}^{-1} \begin{bmatrix} \dot{\phi}_c - k_{\phi_1}(\phi - \phi_c) - k_{\phi_2} w_\phi - f_\phi \\ \dot{\beta}_c - k_{\beta_1}(\beta - \beta_c) - k_{\beta_2} w_\beta - f_\beta \end{bmatrix} \tag{15}$$

where k_{ϕ_i} and k_{β_i}, $(i = 1, 2)$ are constants and w_ϕ and w_β are integrals of outputs tracking errors, i.e

$$\dot{w}_\phi = \phi - \phi_c, \tag{16}$$
$$\dot{w}_\beta = \beta - \beta_c \quad . \tag{17}$$

Define $\widetilde{\phi} = \phi - \phi_c$ and $\widetilde{\beta} = \beta - \beta_c$, to obtain

$$\ddot{\widetilde{\phi}} + k_{\phi_1}\dot{\widetilde{\phi}} + k_{\phi_2}\widetilde{\phi} = 0 \tag{18}$$
$$\ddot{\widetilde{\beta}} + k_{\beta_1}\dot{\widetilde{\beta}} + k_{\beta_2}\widetilde{\beta} = 0 \tag{19}$$

It is clear that if $k_{\phi_i} > 0$ and $k_{\beta_i} > 0$, $(i = 1, 2)$, then the tracking errors tend to zero asymptotically.

D. Controller Design for the Non-minimum Phase Dynamics

In this section we indicate how to design a linear control law for the non-minimum phase dynamics using a classical linearization approach. We assume $\phi \to \phi_c$, $\beta \to (\beta_c = 0)$ and $p \to p_{c_{ss}}$ and $r \to r_{c_{ss}}$ where the subscript ss indicates steady state value. This assumption is based on the fact that output commands for the minimum phase part can be tracked arbitrary fast. Consequently the non-minimum phase dynamics can be approximated as

$$\dot{q} = f_q(q, \alpha, \theta, \phi_c, p_{c_{ss}}, r_{c_{ss}}) + M_{\delta_e}\delta_e \tag{20}$$
$$\dot{\alpha} = f_\alpha(\alpha, \theta, \phi_c) + q \tag{21}$$
$$\dot{\theta} = q\cos\phi_c - r_{c_{ss}}\sin\phi_c \tag{22}$$
$$y_3 = \alpha \tag{23}$$

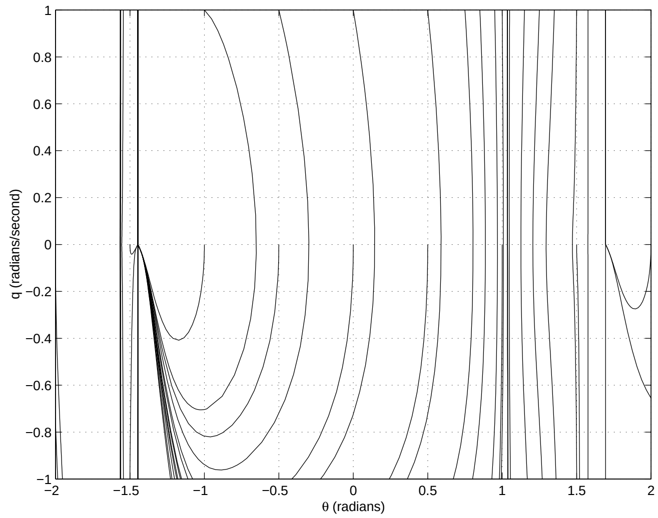

Fig. 2. Phase portrait of the A-4D Fighter/Attacker pitch dynamics for Mach number=0.8, $\phi = 45$, $\beta = 0$, $\alpha = 10$ degrees

Note that $p_{c_{ss}}$ and $r_{c_{ss}}$ are not constants and it can be computed using equation (15) as follows,

$$
\begin{aligned}
p_{c_{ss}} &= -\frac{(g/V)\cos\theta\sin\phi_c\cos\alpha + q\tan^2\theta\sin\phi_c\cos\phi_c}{\cos\alpha + \tan\theta\sin\alpha\cos\phi_c} \\
r_{c_{ss}} &= -\frac{(g/V)\cos\theta\sin\phi_c\sin\alpha - q\sin\phi_c\tan\theta}{\cos\alpha + \tan\theta\sin\alpha\cos\phi_c}
\end{aligned}
$$

For a given angle of attack command α_c, the equilibrium point of the non-minimum phase dynamics can be obtained by solving equations (20), (21) and (22) for $(\theta^*, q^*, \delta_e^*)$. Note that for a given angle of attack α_c, solution for the equilibrium pitch angle θ^* is not guaranteed to exist and even if it does exist it might not be suitable for practical aircraft maneuvers. This obstacle is not due to the design approach but it is due to the physical limitations of the aircraft. Note that θ^* not only depends on α_c but it depends also on ϕ_c and the aircraft speed V.

For any equilibrium of the non-minimum phase dynamics, equations (20), (21) and (22) can be linearized at the equilibrium. Then, an LQR design or any robust linear control approach can be used to obtain a feedback control law to stabilize the system at the desired equilibrium point.

IV. Numerical Results

In this section, simulation results are presented for A-4D Fighter/Attacker aircraft studied in [19]. The complete set of aerodynamic parameters for Mach number=0.8 and altitude of 35,000 feet is given in [19]. The terminal values of ϕ, β and α where chosen as $(\phi_c, \beta_c, \alpha_c) = (45, 0, 10)$ degrees. To solve the tracking problem for this output commands we need to find an equilibrium for the pitch dynamics. For this specific output command the phase plane of q and θ is shown in figure 2. It is clear that there exist multi equilibrium for the pitch dynamics. Only the unstable equilibrium at 1.025 (rad.) (58.74 degrees) is of practical interest to the maneuver considered here. Next we linearize equations (20,21,22) at the equilibrium $(\theta^* = 1.025, q^* = 0.1187, \alpha_c = 0.1745, \delta_e^* = -0.1261)$ to obtain

$$
\begin{bmatrix} \dot{q} \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -1.01 & -9.38 & -1.18 \\ 1 & -0.54 & -0.02 \\ 1.40 & -0.65 & -15.62 \end{bmatrix} \begin{bmatrix} q - q^* \\ \alpha - \alpha_c \\ \theta - \theta^* \end{bmatrix} + \begin{bmatrix} -14.55 \\ 0 \\ 0 \end{bmatrix} (\delta_e - \delta_e^*) \tag{24}
$$

Following the same approach in [13], we use an LQR approach to design a stabilizing feedback law for the nominal system (24):

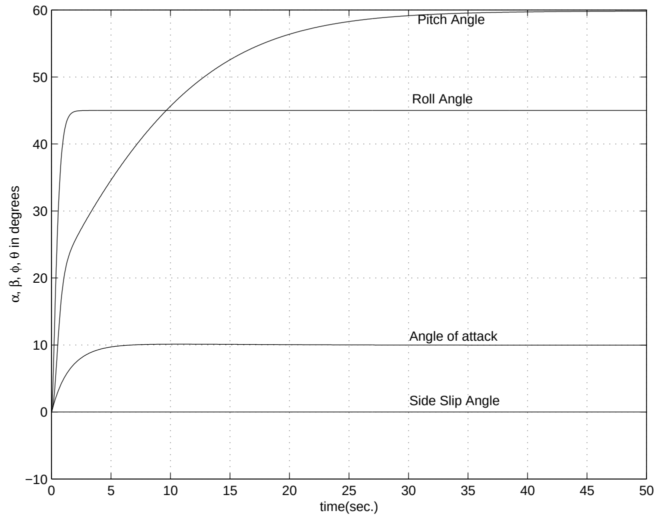

Fig. 3. Aircraft response to commands $\alpha_c = 10(deg.), \beta_c = 0(deg.), \phi_c = 45(deg.)$ for zero initial conditions and 0.8 Mach number speed

$$\delta_e = -k_1(q - q^*) - k_2(\alpha - \alpha_c) - k_3(\theta - \theta^*) + \delta_e^* \tag{25}$$

where $[k_1, k_2, k_3] = R^{-1}B^TP$ and P is the solution of algebraic Riccati equation

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{26}$$

where A and B are the state and control matrices in equation (24), $Q = \text{diag}(20, 1, 1)$ and $R = 1$. To ensure time scale separation between the minimum phase dynamics and the non-minimum phase dynamics the gains of the nonlinear control law (10) are chosen to be ($k_p = k_r = 20, k_{\phi 1} = k_{\beta 1} = 8, k_{\phi 2} = k_{\beta 2} = 16$). The result of the simulation is calculated using MATLAB and it is shown in figure 3. It is clear that all outputs converge to the desired values and the zero dynamics tends to a constant value.

V. Conclusion

In this paper we have shown that if the output of a flight control system is the angle of attack, side slip angle and roll angle, then dynamics inversion can not be used to design an output tracking controller for the system. The reason is that the resulting zero dynamics namely the pitch dynamics is unstable.

We have solved this problem by decomposing the aircraft dynamics into a minimum phase part and a non-minimum phase part. For the minimum phase part we have used dynamics inversion to track constant command roll angle and zero command side slip angle. For the non-minimum phase part we have used LQR approach to track constant command angle of attack while maintaining the zero dynamics stable. This idea is applied to obtain a stable output tracking controllers for a five degree of freedom aircraft where the outputs are angle of attack, roll angle and side slip angle.

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