PARTIAL EIGENSTRUCTURE ASSIGNMENT BY STATE FEEDBACK: AN LMI APPROACH

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Abstract

A new methodology of the partial eigenstructure assignment by state feedback via Linear matrix inequality (LMI) is given. It enables to adopt the Sylvester matrix equation $AX + BY = XH$ and to check stability using parameter dependent Lyapunov functions which are derived from LMI conditions. We show in this work that it is possible to avoid some limiting assumptions needed for the resolution of the Sylvester equation by using a reduced-order system obtained by the projection of the trajectories of original system onto a subspace associated with the undesirable open-loop eigenvalues. **Key-words**: Partial eigenstructure assignment, State feedback, Sylvester equation, LMI technique.

1 Introduction

In the pole assignment problem, it is often useful to modify only some of the open-loop eigenvalues, while leaving the remainder unchanged, this is the so-called partial pole assignment. The problem of eigenstructure assignment consists on the resolution of different type of linear and non linear algebraic equation. One can cite the work of [4] where the resolution of the equation $XA + XBX = HX$ is presented as a technique of partial eigenstructure assignment leading to built robust controllers [5]. For the same purpose, the following generalized Sylvester matrix equation $AX+BY = XH$ is used in the literature [8], [10] [9] and the references therein. In [9], the partial eigenstructure assignment problem, in which both the open-loop and the closed-loop eigenvalues are allowed to possess arbitrary geometric and algebraic multiplicities, is addressed. Its solution is similar to the one obtained by [4]. Under non degeneracy assumptions, the proposed algorithm is based on a series of transformations leading to the solution of a Sylvester equation. A well known result [8] states that a non singular solution X of the Sylvester equation is generically obtained if the pair (A, Y) is observable and $\lambda(A) \bigcap \lambda(H) = \emptyset$. However, in our knowledge, there does not exist any numerical procedure which systematically gives a non singular solution to the general Sylvester equation.

The LMI theory has been successfully used in many areas of automatic control [3], [6], [1], [12],[15], [2]. There exist many efficient algorithms to numerically solve a given LMI problem. In this paper, we address the eigenstructure assignment problem by state feedback in terms of an LMI problem. This formulation allows one to find a non singular solution to the Sylvester equation with possible additional specifications on the eigenvectors of the closed-loop systems and without restrictive assumptions. Also, a solution is provided for the partial stabilization problem, that is find a feedback F which ensures the asymptotic stability of the reduced-order system in the closed-loop and keeps unchanged the stable mode of the open-loop system.

The rest of the paper is organized as follows: The background of the asymptotic stability by means of LMI together with the technique of the reduced order system are recalled in the second section. Section 3 presents the main result of this paper which consists in a LMI allowing a partial eigenstructure assignment. An algorithm and an example illustrating this new technique are also presented in this section.

2 Preliminary results

Consider the continuous-time invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$, with $rank(B) = m \leq n$.
We assume that:

- H1): The pair (A, B) is controllable.
- H2): The open-loop system has $r \leq m$ undesirable or unstable eigenvalues.

Consider the following feedback control:

$$u(t) = Fx(t);\ F \in \mathbb{R}^{m\times n} \tag{2}$$

Matrix F is computed such that the closed-loop system is asymptotically stable;

$$\dot{x} = (A + BF)x(t) \tag{3}$$

It is essential to recall the necessary and sufficient condition of asymptotic stability of system (3) by using the LMI's.
It is question of computing matrices P and F such that:

$$\begin{cases} (A+BF)^T P + P(A+BF) < 0 \\ P = P^T > 0 \end{cases} \tag{4}$$

By using the following variables:

$$P = X^{-1}, F = YX^{-1}, X = X^T > 0; \quad X \in \mathbb{R}^{n\times n}; Y \in \mathbb{R}^{m\times n} \tag{5}$$

Inequality (4) becomes an LMI in X and Y which is rewritten as follows:

$$\begin{cases} AX + XA^T + BY + Y^T B^T < 0 \\ X = X^T > 0 \end{cases} \tag{6}$$

The solution of the LMI (6) leads to a regulator of gain matrix F such that system (3) is asymptotically stable.
Let Λ_o: be the subset formed by the $(n-r)$ open-loop eigenvalues that belong to some desirable stable region.

Λ_1 : be the subset formed by the r eigenvalues that one wants to assign in closed-loop by using the state feedback (2).

The problem of partial pole assignment consists in computing matrix F such that $\Lambda = \lambda(A+BF) = \Lambda_o \cup \Lambda_1$.
In this work, we introduce the linear matrix inequality (LMI) to compute a matrix F which assigns the desired spectrum and stabilizes the system in the closed-loop.

This work is also based on the use of the technique of reduced-order system obtained by the projection of the original system trajectories in the subspace associated with the undesirable eigenvalues [13], [7] and [14]. This can be achieved by a Schur decomposition of the system matrix in two blocks associated respectively with the desirable and undesirable open-loop eigenvalues. Thus, let us recall the main outlines of this technique detailed in [7].

Let us define a subspace S_o associated with the $(n-r)$ stable open-loop eigenvalues and consider S_r a complementary subspace to S_o, i.e $S_o \oplus S_r = \mathbb{R}^n$. Note that S_r can be associated with the unstable or undesirable eigenvalues.

In this way, consider the following change of basis in (1):

$$x = [Q_o \,|Q_r] \left[\begin{array}{c} z_o \\ z_r \end{array} \right]; z_0 \in \mathbb{R}^{n-r}, z_r \in \mathbb{R}^r \tag{7}$$

where the matrix $Q \in \mathbb{R}^{n\times n}$ is orthonormal,

$$Q = [Q_o |\, Q_r]; Q_o \in \mathbb{R}^{n\times(n-r)}; Q_r \in \mathbb{R}^{n\times r}$$
$$Q^TQ = QQ^T = \mathbb{I}_n; Q_r^TQ_r = \mathbb{I}_r \tag{8}$$

such that,

$$\begin{cases} \text{the columns of } Q_o \text{ span } S_o \triangleq Ker(F) \\ \text{the columns of } Q_r \text{ span } S_r \end{cases}$$

Matrix Q can be obtained from a Schur decomposition of matrix A by reordering, if necessary, its Schur blocks [11].

In the orthonormal basis formed by the columns of matrix Q, the open-loop system (1) is represented by :

$$\left[\begin{array}{c} \dot{z}_o \\ \dot{z}_r \end{array} \right] = \left[\begin{array}{cc} R_o & R_2 \\ 0 & R_r \end{array} \right] \left[\begin{array}{c} z_o(t) \\ z_r(t) \end{array} \right] + \left[\begin{array}{c} B_o \\ B_r \end{array} \right] u(t) \tag{9}$$

where,

$$R = Q^TAQ = \left[\begin{array}{cc} R_o & R_2 \\ 0_{(r\times(n-r))} & R_r \end{array} \right];$$
$$B_Q = \left[\begin{array}{c} B_o \\ B_r \end{array} \right] = \left[\begin{array}{c} Q_o^T \\ Q_r^T \end{array} \right] B \tag{10}$$

$$\begin{cases} z_o \text{ is the projection of } x \text{ on } S_o \text{ along } S_r \\ z_r \text{ is the projection of } x \text{ on } S_r \text{ along } S_o \end{cases}$$

Note that the dynamic of z_r associated with the undesirable poles to be modified, is decoupled from z_o. Thus we can isolate the following open-loop reduced-order system:

$$\dot{z}_r = R_r\, z_r(t) + B_r u(t) \tag{11}$$

Recall that it is always possible to have $rank(B_r) = r$ and the reduced pair (R_r, B_r) completely controllable [7].

In the new basis, the feedback matrix F is represented by:

$$F_Q = F[Q_o \,|\, Q_r] = [0_{m\times(n-r)} |\, F_r], \tag{12}$$

with $rank(F_r) = r$. In this way, matrix F_r assigns the desired spectrum of the closed-loop reduced-order system :

$$\dot{z}_r = (R_r + B_rF_r)\, z_r(t) \tag{13}$$

- Notice that if $r = m$, the stabilizing state feedback gain is given by $F = F_rQ_r^T$.
- If $r < m$, the control vector is ordered in such a way that the reduced-order system (11) can be rewritten under the form:

$$\dot{z}_r = R_r\, z_r(t) + [B_{r1} \,|\, B_{r2}] \left[\begin{array}{c} u_1(t) \\ u_2(t) \end{array} \right] \tag{14}$$

where $B_{r1} \in \mathbb{R}^{r\times r}$ is square, full-rank matrix. To achieve the desired control requirements, we impose that matrix F_r has the form:

$$F_r = \left[\begin{array}{c} F_{r1} \\ 0_{(m-r)\times r} \end{array} \right]; F = F_rQ_r^T \tag{15}$$

the fact that $(R_r,\, B_{r1})$ is also controllable, implies that the vector u_2 is inactive, that is, $u_2(t) = 0_{(m-r)}$. In this case, system (14) can be rewritten as follows:

$$\dot{z}_r = (R_r + B_{r1}F_{r1})z_r(t) \tag{16}$$

3 Main result

In this section, we present two results. The first associates the LMI of asymptotic stability of linear continuous-time systems to the technique of order reducing. The second, concerns the problem of partial eigenstructure assignment.

Lemma 1 *The system (3) with assumptions H1) and H2) is asymptotically stable if and only if the reduced-order following LMI holds:*

$$\left\{\begin{array}{l} R_rX_r + X_rR_r^T + B_rY_r + Y_r^TB_r^T < 0 \\ X_r = X_r^T > 0 \end{array}\right. \tag{17}$$

Proof. The system (1) with (2), (5) and assumptions H1) and H2) is asymptotically stable if and only if the LMI (6) is satisfied. Rewrite (6) equivalently as follows,

$$\left\{\begin{array}{l} X^{-1}A + A^TX^{-1} + X^{-1}BYX^{-1} + \\ X^{-1}Y^TB^TX^{-1} < 0 \\ X = X^T > 0 \end{array}\right. \tag{18}$$

By using the following transformations:

$$\begin{array}{rcl} Q_r^TA &=& R_rQ_r^T; Q_r^TB = B_r; F_r = Y_rX_r^{-1} \\ Y &=& Y_rQ_r^T, X^{-1} = Q_rX_r^{-1}Q_r^T \end{array} \tag{19}$$

and multiplying in the left and in the right by matrices Q_r^T and Q_r respectively, inequality (18) becomes,

$$\left\{\begin{array}{l} X_r^{-1}R_r + R_r^TX_r^{-1} + X_r^{-1}B_rY_rX_r^{-1} + \\ X_r^{-1}Y_r^TB_r^TX_r^{-1} < 0 \\ X_r = X_r^T > 0 \end{array}\right. \tag{20}$$

Multiplying in the left and in the right this latter by matrix X_r, one obtains,

$$\left\{\begin{array}{l} R_rX_r + X_rR_r^T + B_rY_r + Y_r^TB_r^T < 0 \\ X_r = X_r^T > 0 \end{array}\right.$$

This leads to the asymptotic stability of the reduced-order system.

The reciprocal is easily established by multiplying in the left and in the right inequality (17) by matrix X_r^{-1} to obtain inequality (20). The use of the transformations (19) leads to inequality (18). ■

Remark 2 *It is worth noting that this result can be used for partial stabilization. In this case, the resolution of the reduced-order LMI (17) is obviously more efficient than the resolution of the full LMI (18). Besides, one can further simplifies the LMI (17) by using Finsler's lemma:*

$$\left\{\begin{array}{l} (B_r)_\perp^T\left(R_rX_r + X_rR_r^T\right)(B_r)_\perp < 0 \\ X_r = X_r^T > 0 \end{array}\right. \tag{21}$$

where $(B_r)_\perp$ is the orthogonal complement of B_r.

The second result of this section concerns the problem of partial eigenstructure assignment which is related to a reduced-order Sylvester equation associated to a given matrix H_r by using the LMI technique.

Theorem 3 *For a matrix $H_r \in \mathbb{R}^{r\times r}$ given such that $\lambda(H_r) = \Lambda_1$, and X_r, Y_r solutions of the following LMI problem:*

$$\left\{\begin{array}{c} X_r^T + X_r > 0 \\ u.\ c.:\ R_r\,X_r + B_rY_r - X_r\,H_r = 0 \end{array}\right. \tag{22}$$

the regulator of gain $F = F_r\,Q_r^T$; with $F_r = Y_rX_r^{-1}$ assigns the spectrum $\Lambda_o \cup \Lambda_1$ for the system (3) with assumptions H1 and H2.

Proof. Consider the following change of variables,

$$\tilde{x}(t) = P\,x$$

where matrix P is given by (5), it follows,

$$\dot{\tilde{x}} = P(A + B\,YP)x(t)$$

If there exists a stable matrix $H \in \mathbb{R}^{n\times n}$ such that:

$$P(A + B\,YP) = HP, \tag{23}$$

then system (3) is transformed to,

$$\dot{\tilde{x}} = H\,\tilde{x}(t) \tag{24}$$

For $P = X^{-1}$, the equation (23) becomes a linear equation in X and Y,

$$A\,X + B\,Y \;= X\,H, \qquad (25)$$

which is the so-called Sylvester matrix equation which is frequently used in the problem of eigenstructure assignment for linear systems [9].

Following the reduced-order system technique quoted in Section 2, we can easily apply the above steps to the system (13) with a similar change of variables:

$$\tilde{z}_r(t) = X_r^{-1}\,z_r$$

Give a stable matrix H_r such that,

$$H_r \in \mathbb{R}^{r\times r};\, \lambda(H_r) = \Lambda_1$$

Using the same transformations (19) and $H = Q_r H_r Q_r^T$, equation (25) becomes,

$$R_r\,X_r + B_r\,Y_r \;= X_r\,H_r, \qquad (26)$$

which is the Sylvester equation associated to the following reduced-order system:

$$\dot{\tilde{z}}_r(t) = H_r\;\tilde{z}_r(t)$$

Thus, to compute matrix F which assigns the spectrum $\Lambda_o \cup \Lambda_1$, one has only to solve the LMI (22). ■

Comments 4

- *It is known [8] that a non singular solution X of the Sylvester equation (25) is generically obtained if the pair (A, Y) is observable and $\lambda(A) \bigcap \lambda(H) = \emptyset$. These two assumptions are not needed in our approach. Further,*
  - *The solution X_r of the LMI (22) is only non singular but not necessary positive definite, hence this LMI is not restrictive.*
  - *The spectrum of matrix H_r can contain some eigenvalues of the remainder spectrum Λ_0 of matrix A.*
- *One has to note that the solution of the LMI (17) is symmetric definite positive as required by the partial stabilization problem while the solution of the LMI (22) is only non singular as required by the pole assignment problem.*
- *The resolution of the LMI (22) can be associated to the results of [5] where matrix H_r has to be chosen according to some conditions to ensure a robust pole assignment for uncertain systems.*

To apply the result of Theorem 3, one has, however, to distinguish two different cases:

- In the case $r = m$, the solution of the problem is given by the resolution of the LMI (22), with $F = F_r Q_r^T = Y_r X_r^{-1} Q_r^T$.
- For the case $r < m$, the solution of the problem is given by the resolution of the LMI (27) where matrix $B_r \in \mathbb{R}^{r\times m}$ is changed to matrix $B_{r1} \in \mathbb{R}^{r\times r}$ and $Y_r \in \mathbb{R}^{m\times r}$ to $Y_{r1} \in \mathbb{R}^{r\times r}$.

$$\left\{ \begin{array}{c} X_r^T + X_{\;r} > 0 \\ \text{u. c.} R_r\,X_r + B_{r1}Y_{r1} - X_r\,H_r = 0 \end{array} \right. \qquad (27)$$

In this context, the feedback matrix F that assigns the desired closed-loop eigenvalues and guarantees the asymptotic stability of the system (3) is obtained as follows:

$$F = \left[\begin{array}{c} Y_{r1}X_r^{-1} \\ 0_{(m-r)\times r} \end{array} \right] Q_r^T$$

It is worth noting that, for the two main cases, we realize $\lambda(R_r + B_r F_r) = \lambda(H_r)$ and $\lambda(A + BF) = \lambda(H_r) \cup \Lambda_o$.

The following algorithm summarizes the steps of calculations followed during the development of this new approach by introducing the LMI. It is worth to recall that the use of a Schur decomposition guarantees numerical robustness in the computation of the open-loop eigenvalues while it determines a new basis for the associated subspaces [11].

Algorithm 5

- *Step 1: Verify that (A, B) is controllable and fix the undesirable eigenvalues of matrix A. Λ_0 is the set of the remainder $(n-r)$ eigenvalues of A.*
- *Step 2: Apply a Schur decomposition of matrix A by reordering, if necessary, its Schur blocks to have matrix $Q_r \in R^{n\times r}$ and the reduced-order system (11) associated with the undesirable eigenvalues of matrix A.*
- *Step 3: Give $H_r \in R^{r\times r}$ such that $\lambda(H_r) = \Lambda_1$.*
- *Step 4:- If $r < m$ compute the LMI (27); $F_{r1} = Y_{r1}X_r^{-1}$ and*

  $$F_r = \begin{bmatrix} F_{r1} \\ 0_{(m-r)\times r} \end{bmatrix}$$

  - If $r = m$ compute the LMI (22); $F_r = Y_r X_r^{-1}$
- *Step 5:Compute the gain matrix $F = F_r Q_r^T$.*
- *Step 6:Verify that $\lambda(A + BF) = \Lambda_0 \bigcup \Lambda_1$*

Example 6 *In order to illustrate the use of the proposed methodology, we consider the same example treated by [9].*

$$A = \begin{bmatrix} -5 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \; B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The open-loop eigenvalues of the system are given by:

$$\lambda(A) = \{-4.9711, \;\; -0.0973, \;\; 2.0684\}$$

The undesirable open-loop eigenvalue is 2.0684. *In this case $r = 1 < m$.*
Applying a Schur decomposition to matrix A, we obtain;

$$Q = \begin{bmatrix} 0.9851 & 0.1427 & 0.0962 \\ 0.0284 & -0.6864 & 0.7267 \\ -0.1697 & 0.7131 & 0.6802 \end{bmatrix};$$

$$Q_r = \begin{bmatrix} 0.0962 \\ 0.7267 \\ 0.6802 \end{bmatrix}$$

$$R = \begin{bmatrix} -4.9711 & -1.4069 & 0.0268 \\ 0 & -0.0973 & 0.1413 \\ 0 & 0.0000 & 2.0684 \end{bmatrix}$$

$$B_Q = \begin{bmatrix} -0.1697 & 0.0284 \\ 0.7131 & -0.6864 \\ 0.6802 & 0.7267 \end{bmatrix}$$

Choose $H_r = -1$, the resolution of the LMI (27) yields the following solution,

$$X_r = 500.5; \; Y_{r1} = -2257.7686$$

$$F_{r1} = Y_{r1}X_r^{-1} = -4.5112; \; F_r = \begin{bmatrix} F_{r1} \\ 0 \end{bmatrix}$$

From the Algorithm, we have the next feedback matrix that assigns the desired closed-loop spectrum,

$$F = F_r Q_r^T = \begin{bmatrix} -0.4341 & -3.2783 & -3.0684 \\ 0 & 0 & 0 \end{bmatrix}$$

The assigned spectrum in closed-loop is then computed,

$$\lambda(A + BF) = \{-4.9711, \, -0.0973, \, -1\} = \Lambda_0 \cup \lambda(H_r)$$

Assume now that the undesirable eigenvalues are $\{-0.0973, 2.0684\}$. In this case $r = m = 2$. For that, the same Schur decomposition of matrix A is still used, where matrix Q_r is now given by:

$$Q_r = \begin{bmatrix} 0.1427 & 0.0962 \\ -0.6864 & 0.7267 \\ 0.7131 & 0.6802 \end{bmatrix}$$

Choose random matrix H_r of spectrum $\lambda(H_r) = \{-1, -2\}$ as follows:

$$H_r = \begin{bmatrix} -2.0811 & 0.7588 \\ -0.1161 & -0.9186 \end{bmatrix}$$

The resolution of the LMI (22) yields the following solutions with X_r non symmetric and non definite positive,

$$\begin{aligned} X_r &= \begin{bmatrix} 2826.329 & -4050.0884 \\ 4050.0884 & 2826.329 \end{bmatrix}; \\ Y_r &= \begin{bmatrix} -16151.8 & 0 \\ -8461.3281 & -7388.2863 \end{bmatrix} \end{aligned}$$

$$F_r = \begin{bmatrix} -1.8716 & -2.6819 \\ 0.2463 & -2.2611 \end{bmatrix}$$

From the Algorithm, we have the next feedback matrix that assigns the desired closed-loop spectrum,

$$F = \begin{bmatrix} -0.5251 & -0.6643 & -3.1588 \\ -0.1824 & -1.8122 & -1.3632 \end{bmatrix}$$

The assigned spectrum in closed-loop is then computed,

$$\lambda(A + BF) = \Lambda_0 \cup \lambda(H_r)$$
$$= \{-4.9711, -1.0000, -2.0000\}$$

Note that one can add a decoupling constraint on the solution X of the LMI (27), like $X(2,1) = 0$. In this case, the obtained solutions are given by:

$$X_r = \begin{bmatrix} 2452.6958 & 0 \\ 0 & 2452.6958 \end{bmatrix};$$
$$Y_r = \begin{bmatrix} -3787.773 & -3987.507 \\ 3153.5506 & -6349.1127 \end{bmatrix}$$

Remark 7 *For a system with an order $n > 3$, the use of the reduced-order system and the linear matrix inequality proposed in this work is very efficient while in the previous works based on the resolution of Sylvester equation, the steps are increasingly complex, owing to the numbers of the elementary transformations and the arbitrary choice of several parameters. Moreover, it is noticed that the number of the conditions to check for this approach is very limited. Indeed, it is not necessary to have $\lambda(A) \bigcap \lambda(H_r) = \emptyset$ nor a particular choice of the matrix H_r (such diagonal or full).*

4 Conclusion

In this paper, a new approach of partial eigenstructure assignment by state feedback is presented. This technique is based on the use of the reduced-order system and the LMI's. The number of assumptions is reduced while a non singular solution is guaranteed. The results are given for the continuous-time systems, but they can also be extended to the discrete-time systems. The paper presents a simple algorithm and an illustrative example.

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