ROBOT FORCE CONTROL: COMMENTS ON SYSTEM CONVERGENCE

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Abstract-**Using two or more different models to adequately represent a system during its operation, is a frequent case in system modeling. On the other hand, successful control of such systems is a quite involving task: we usually employ two or more controllers and specify a switching procedure between them. However, even if each controller may lead to a stable closed-loop system, the overall system convergence is by no means assured when switching is involved. An illustrative example is the case of robot force control. In this paper we carry out a case study of a simplified model of robot arm performing vertical motion. We study a simple linear model of the system and derive a necessary condition to avoid loss of convergence of the system variables.**

Index terms-- **Force control, switching between control laws, convergence analysis.**

I. Introduction

Establishing contact between a robot arm and a specific point of its environment has always been a challenging problem. The main issue is that the system model is different according to whether the contact has been established or not. A direct consequence is the need to use at least two control laws, one for every stage of the procedure and to switch between them. There are even authors (Volpe 1994, Brogliato 1999) who consider that a third regulator is necessary for the transient impact phase in order to avoid dangerous peaks of force and/ or contact loss. For example, when the robot link moves towards a surface we control its speed (and/or position). On the other hand, when we know that contact has been established, we switch to a force control law. Therefore, we have two stages and two (or three) different control laws, which leads to quite a number of possible combinations. Naturally, all controllers are designed to be stable during the time they operate. Nevertheless, careless switching between them can lead to some surprising results. For instance, convergence of the complete system can slow down or (in extreme cases) be lost.

Consequently, it would be useful to avoid switching between controllers. Indeed, (Indri, Tornambe 2001) used the "one family of compensators" approach, while considering a smooth impact between an object and a surface. A different but not dissimilar technique is the so-called implicit force control or mechanical impedance control (Hogan 1985). In this case the same regulator is used for all stages avoiding switching between controllers. However, tuning the regulator to perform well during all stages is a tedious task. Moreover, reaching a reference force value (at the end of the arm motion) implies the knowledge of physical and mechanical characteristics of the environment. For more details on history, background and methods used in impact control the reader is referred to (Wu et al, 1997) and (Volpe et al 1993).

In our case, we study a simple linear model of a robot arm performing vertical motion. Although the influence of nonlinearities is not considered, such models provide an adequate representation of the actual system, especially in 1-D cases. We suppose that knowledge of the environment characteristics is limited. That leads us to use two controllers and a switching procedure. The aim of this study is to define some rules in order to ensure that switching between stable controllers will not affect the system convergence. The paper[1] is organized as follows: In the next section we present the system model during the various stages and choose a controller for each stage. Then, in section 3 we study the system behaviour in the phase plot, with respect to the switching point. Finally we present some concluding remarks.

II. Problem Statement

In this section, the general problem of a robot arm approaching a surface will be examined. We consider a rigid robot arm and we model the surface reaction as a spring - damper system. This approach is rather simple, yet true for a number of industrial processes. As stated above, the most challenging issue of this system is the fact that the system model changes when contact between the arm and

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the surface is established. Hence, we distinguish two principal stages:

a) Stage Σ1 where the tool hurts the surface without exceeding a given maximum force F_{swup}. First of all, we consider a coordinate system in which the tool displacement x_1: =0 when a given reference force F_{ref} is reached. Note that $|F_{swup}| < |F_{ref}|$. Moreover, x_1: >0 when the tool is moving further down. The robot arm can be considered as an object performing a vertical motion under the application of an external force. We consider a normalized model with respect to mass and inertia. Gravity can also be compensated by a specific part of the controller. The surface reaction force during this stage is:

$$F = -k_e(x_1 - x_e) - b_e\dot{x}_1 \tag{1}$$

The reference force, which should ideally be obtained at the end of the tool motion, is given by:

$$F_{ref} = k_e x_e \tag{2}$$

Hence, x_e corresponds to the surface deformation from the static load F_{ref}. According to the choice of coordinate system, F_{ref}<0. In this relatively simple (1-D) case, there are two external forces acting on the robot arm: the gravitational force and the surface reaction (1). It follows that the arm (or tool) displacement is described by the equation:

$$\ddot{x}_1 = -k_e(x_1 - x_e) - b_e\dot{x}_1 + v_g + v \tag{3}$$

where
k_e is the surface stiffness
b_e is the surface damping factor
x_1 is the tool displacement,
v_g is the gravity factor and
v is the user-defined controller.

Define $u = v_g + v$. A variety of control laws u can be used during this stage:

$$\begin{aligned} u &= K_p(x_{ref} - x_1) - K_v\dot{x}_1 \\ u &= K_p(x_{ref} - x_1) - K_v(\dot{x}_{ref} - \dot{x}_1) \text{ etc} \end{aligned} \tag{4a}$$

For the purpose of our analysis we choose to use a control law of the kind:

$$u = -k_v(\dot{x}_1 - \dot{x}_{des}) \tag{4}$$

with k_v a positive constant and $\dot{x}_{des}$ the desired descent speed of the tool.

b) Stage Σ2 during which the tool is in contact with the surface and the measured force already exceeds F_{swup}. As mentioned in stage 1, the tool displacement depends on the surface reaction force and the user-defined control. Thus, the system model during this stage is:

$$\ddot{x}_1 = -k_e(x_1 - x_e) - b_e\dot{x}_1 + u \tag{5}$$

Typical control laws used in that case include:

$$\begin{aligned} u &= K_p(F_{ref} - F) + K_v\frac{d}{dt}(F_{ref} - F) \\ u &= K_p(F_{ref} - F) - K_v\dot{x}_1 \text{ etc} \end{aligned} \tag{6a}$$

Hence F_{swup} is the switching point between the speed/position and force controllers. The (force) control law used, is similar to the second one of (6a):

$$u = \frac{1}{b_e}\left[-b_e k_e x_e - k_e\dot{x}_1 + (k_F - b_e)(F - F_{ref})\right] \tag{6}$$

and k_F another positive constant. Differentiating (1) and using (2), (5) and (6) yields:

$$\frac{d}{dt}(F - F_{ref}) = -k_F(F - F_{ref})$$

Thus, in theory, the force settles down exponentially to the desired value. In practice, errors in the dynamic model may lead to oscillations around the origin.

Remark 1: The choice of control laws (4) and (6) was motivated by their simplicity. In fact, our aim is to propose an analysis framework regardless of the control laws used. We will expand this idea in the following section.

III. Convergence Issues: A Case Study

The system behaviour is more complicated than the above approach suggests. In theory, a "safe" operating mode would be:
a) to approach the surface with the lowest possible speed (stage 1 controller),
b) to establish contact and continue until F_{swup} is reached (switch to stage 2 controller) and
c) to keep moving until the reference force F_{ref} is reached.

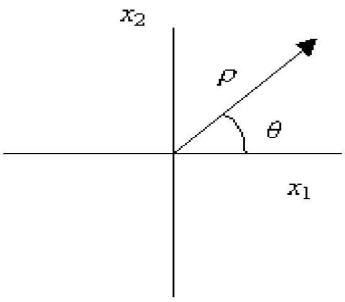
In practice the tool speed during free vertical motion should be as high as possible, even if a high impact speed should clearly be avoided for obvious practical reasons. Due to the non-zero impact speed, the measured force very often exceeds F_{ref}, whereas the tool keeps on moving downwards until it stops. Then, it moves upwards under the surface reaction force. Two things may happen:
- Either the tool moves slowly upwards, reaching the point where $F = F_{ref}$,
- Or the tool bounces back, passing through the F_{swup} force point and continuing upwards. In this case the system switches again to stage 1 controller and the whole procedure

restarts. Due to this possibility, we often introduce a second threshold force value $|F_{swdown}| < |F_{swup}| < |F_{ref}|$. This design helps to prevent an immediate switch to stage 1 controller when the tool moves upwards and past the F_{swup} force value point. Then, convergence to F_{ref} is assured.

Nevertheless, sometimes even this hysteresis is not sufficient to prevent the tool going past the F_{swdown} point. In the remaining part of this section, we will study this particular case. The analysis presented is based on the evolution of the radius ρ as a function of the angle θ and is also motivated by the work of (Xu, Antsaklis, 1999). First, recall that from the phase plot $(\dot{x}_1, x_1)$ (see below), the radius ρ is defined as:

$$\rho = \sqrt{x_1^2 + \dot{x}^2} = \sqrt{x_1^2 + x_2^2} \tag{7}$$

where x_1 is the tool displacement and x_2 the tool velocity. Hence, from the phase plot we obtain:



$$\begin{aligned} x_1 &= \rho\cos(\theta) = \rho c_\theta \\ x_2 &= \rho\sin(\theta) = \rho s_\theta \\ \dot{x}_1 &= x_2 \end{aligned} \tag{8}$$

Then $tan(\theta)$ is defined as:

$$\tan(\theta) = \frac{x_2}{x_1} \tag{9}$$

Differentiating (9) and using (8), (3) or (5), we obtain:

$$\dot{\theta} = \frac{-k_e x_1^2 + k_e x_e x_1 - b_e x_1 x_2 + u x_1 - x_2^2}{\rho^2} \tag{10}$$

Thus from (4) and (6) we obtain two different $\dot{\theta}$, one for each case (Σ1 or Σ2):

$$\dot{\theta}_{\Sigma 1} = -k_e c_\theta^2 - (k_v + b_e)c_\theta s_\theta - s_\theta^2 + \frac{\tau_1}{\rho}c_\theta \tag{11}$$

with $\tau_1 = k_v \dot{x}_{des} + k_e x_e$ and

$$\dot{\theta}_{\Sigma 2} = -(k_F + \frac{k_e}{b_e})c_\theta s_\theta - \frac{k_F k_e}{b_e}c_\theta^2 - s_\theta^2 \tag{12}$$

Following these two results, consider the first derivative of radius ρ:

$$\frac{d\rho}{dt} = \frac{d\rho}{d\theta}\dot{\theta} \Rightarrow \frac{d\rho}{d\theta} = \frac{1}{\dot{\theta}}\frac{d\rho}{dt} \tag{13}$$

Differentiating (7) and using (8), we obtain $d\rho/dt$ for the two cases Σ1 and Σ2:

$$\left.\frac{d\rho}{dt}\right|_{\Sigma 1} = (1 - k_e)\rho s_\theta c_\theta - (b_e + k_v)\rho s_\theta^2 + \tau_1 s_\theta \tag{14}$$

and

$$\left.\frac{d\rho}{dt}\right|_{\Sigma 2} = (1 - \frac{k_e k_F}{b_e})\rho s_\theta c_\theta - (k_F + \frac{k_e}{b_e})\rho s_\theta^2 \tag{15}$$

Consequently, using (13)-(15) we have:

$$\left.\frac{d\rho}{d\theta}\right|_{\Sigma 1} = \rho \frac{(k_e - 1)s_\theta c_\theta + (b_e + k_v)s_\theta^2 - \frac{\tau_1}{\rho}s_\theta}{(k_e - 1)c_\theta^2 + (b_e + k_v)s_\theta c_\theta + 1 - \frac{\tau_1}{\rho}c_\theta} \tag{16}$$

and

$$\left.\frac{d\rho}{d\theta}\right|_{\Sigma 2} = \rho \frac{(\frac{k_F k_e}{b_e} - 1)s_\theta c_\theta + (k_F + \frac{k_e}{b_e})s_\theta^2}{(\frac{k_F k_e}{b_e} - 1)c_\theta^2 + (k_F + \frac{k_e}{b_e})s_\theta c_\theta + 1} \tag{17}$$

Define the following constants:

$$\begin{aligned} K_{t1} &= (k_e - 1), \quad \mathrm{b}_{t1} = (b_e + k_v) \\ K_{t2} &= (\frac{k_F k_e}{b_e} - 1), \quad \mathrm{b}_{t2} = (k_F + \frac{k_e}{b_e}) \end{aligned} \tag{18}$$

Then (16) and (17) can be rewritten as:

$$\left.\frac{d\rho}{d\theta}\right|_{\Sigma 1} = \rho \frac{K_{t1}s_\theta c_\theta + b_{t1}s_\theta^2 - \frac{\tau_1}{\rho}s_\theta}{K_{t1}c_\theta^2 + b_{t1}s_\theta c_\theta + 1 - \frac{\tau_1}{\rho}c_\theta} \tag{19}$$

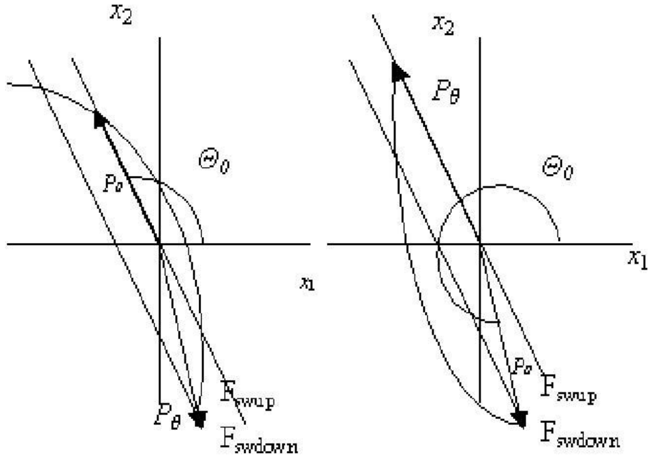
and

$$\left.\frac{d\rho}{d\theta}\right|_{\Sigma 2} = \rho \frac{K_{t2}s_\theta c_\theta + b_{t2}s_\theta^2}{K_{t2}c_\theta^2 + b_{t2}s_\theta c_\theta + 1} \tag{20}$$

In fact, the derivative of ρ with respect to θ has the form of (19) and (20) regardless of the choice of control law in (4a) , (6a). The main change lies on the coefficients τ_1, K_t and b_t. These are different for every control u in (4a) and (6a). For instance, had we chosen u equal to the first eq. of (4a) then we would have obtained $K_{t1}=(k_e-K_p-1)$, $b_{t1}=(b_e+k_v)$ and $\tau_1=(K_p x_{ref}+k_e x_e)$. From (19), (20) it follows that a general expression for the radius ρ_θ would be:

$$\rho_\theta = \rho_0 \exp[\int_{\theta_0}^{\theta} f(w)dw] \tag{21}$$

With $f(w)$ equal to the right hand part of (16) and (17) (w instead of θ) for Σ1 and Σ2 respectively. Note that w is the integration variable. Moreover, the initial radius (for an entry angle θ_0) is ρ_0 whereas the radius at an angle θ is ρ_θ. The integration of the above formula is a tedious task, not least because of the $(\tau_1/\rho)s_\theta$ term on the rhs part of (16). Thus, a different analysis should be performed, in order to investigate the possibility of ρ_θ becoming greater than ρ_0 over a complete cycle in the phase plot. The following drawing illustrates the evolution of the radius as a function of the angle and the switching instants:



Σ1->Σ2 Σ2 -> Σ1

A. *Case Σ2 -> Σ1*

We will first examine the second case, which is the most complicated one. Integrating (19) yields:

$$\int_{\theta_0}^{\theta} \frac{K_{t1}s_w c_w + b_{t1}s_w^2 - \frac{\tau_1}{\rho}s_w}{K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w} dw = \int_{\theta_0}^{\theta} \frac{K_{t1}s_w c_w}{K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w} dw +$$
$$\int_{\theta_0}^{\theta} \frac{b_{t1}s_w^2}{K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w} dw + \tag{22}$$
$$\int_{\theta_0}^{\theta} \frac{-\frac{\tau_1}{\rho}s_w}{K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w} dw = \mathrm{A} + \mathrm{B} + \Gamma$$

Consider the term A on the rhs of (22):

$$\mathrm{A} = \int_{\theta_0}^{\theta} \frac{K_{t1}s_w c_w}{\left[K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w\right]} dw$$
$$= -\frac{1}{4}\int_{\theta_0}^{\theta} \frac{K_{t1}}{\left[K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w\right]} d(c_{2w}) \tag{23}$$

The tool's trajectory (from left to right and around the origin of the phase plot) means that $\dot{\theta}<0$ according to the given convention of positive θ. In view of (9), the denominator in (23) is equal to $-\dot{\theta}$ and thus is greater than zero. That means that A has the same sign as (-1/4), if $cos(2\theta)>cos(2\theta_0)$. Identically, B$>0$ if $\theta>\theta_0$ whereas for Γ we have:

$$\Gamma = \int_{\theta_0}^{\theta} \frac{-\frac{\tau_1}{\rho}s_w}{K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w} dw$$
$$= \int_{\cos(\theta_0)}^{\cos(\theta)} \frac{\frac{\tau_1}{\rho}}{K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w} d(c_w) \tag{24}$$

Hence, Γ has the same sign as τ_1 if $cos(\theta)>cos(\theta_0)$.

In general θ_0 lies in the interval (0,(-π/2)+ε), ε being a small positive number. Thus, $2\theta_0$ lies in the interval (0,-π+2ε). Moreover, θ lies in the interval (-π,(-3π/2)+ε) and consequently 2θ lies in (-2π,-3π+2ε).

Remark 2: We must verify if the condition $\dot{\theta}<0$ is valid, by checking the sign of the denominator of (22). Let us check if it is possible to have:

$$K_{t1}c_w^2 + b_{t1}s_w c_w + 1 - \frac{\tau_1}{\rho}c_w > 0 \quad \forall w \tag{25}$$

A necessary condition for (25) to hold is:

$$(b_{t1}s_w - \frac{\tau_1}{\rho})^2 < 4K_{t1} \quad \forall w \tag{26}$$

Since ρ is a function of w, this condition can be verified for small values of $|\tau_1|$ and large values of ρ. If (26) is valid then $\dot{\theta}<0$ and the system trajectory in the phase plot will surround the origin. Consequently, the system trajectory will not immediately converge to the origin. This, in turn, means that both the tool speed magnitude during the impact and the choice of constant k_v are key factors for the system behaviour. For small values of $|\tau_1|$, (26) becomes:

$$(b_{t1}s_w + \varepsilon)^2 < 4K_{t1} \quad \forall w \qquad (27)$$

with ε a 'small' quantity. Then for $b_{t1}{}^2 < 4K_{t1} \quad \forall w$, (27) is verified, which, in turn, means that the denominator in (22) will be positive regardless of an eventual change of the integration variable.

We provide an example to illustrate the risk of ρ_θ becoming greater than ρ_0

Example: Consider switching points in θ_0 := -π/3 and θ :=-7π/5. Hence, $cos(2\theta_0)> cos(2\theta)$ => A>0, $cos(\theta_0)> cos(\theta)$ => Γ>0 for τ_1 <0 and B<0. Then, depending on the choice of K_t, b_t and τ_1, we could have (A+B+Γ)>0 and as a consequence ρ_θ would not be smaller than ρ_0.

B. *Case Σ1 -> Σ2*

This case can be treated using the same arguments, as above, but as τ_1 =0 the analysis is simpler. As a result, integrating (20) yields:

$$\int_{\theta_0}^{\theta} \frac{K_{t2}s_wc_w + b_{t2}s_w^2}{K_{t2}c_w^2 + b_{t2}s_wc_w + 1}dw = \int_{\theta_0}^{\theta} \frac{K_{t2}s_wc_w}{K_{t2}c_w^2 + b_{t2}s_wc_w + 1}dw +$$

$$+\int_{\theta_0}^{\theta} \frac{b_{t2}s_w^2}{K_{t2}c_w^2 + b_{t2}s_wc_w + 1}dw = \mathrm{A} + \mathrm{B} \qquad (28)$$

or

$$\mathrm{A} = \int_{\theta_0}^{\theta} \frac{K_{t2}s_wc_w}{K_{t2}c_w^2 + b_{t2}s_wc_w + 1}dw = -\frac{1}{4}\int_{\cos(2\theta_0)}^{\cos(2\theta)} \frac{K_{t2}}{[\ldots\ldots]}d(c_{2w}) \quad (29)$$

This time θ_0 > θ, hence B<0 and in the case of our example, for θ_0 =8π/10 > 3π/5 and θ=-π/3 we have cos(2θ_0)> $cos(2\theta)$ => A>0. Thus, depending on the choice of K_t,, b_t and τ_1 we could have (A+B)>0 and ρ_θ would not be smaller than ρ_0.

Consequently, the risks of ρ increasing (instead of decreasing) over a cycle in the phase plot can be eliminated if factors A, B and/or Γ are negative for both processes Σ1 and Σ2. Note, however, that this is only a *necessary* condition. In other words, if one of the factors A, B and/or Γ is positive, we should check their sum for both processes Σ1 and Σ2, for the given initial conditions.

IV. Conclusion

We have chosen to work on the phase plot as it best illustrates the evolution of both the tool displacement and the tool speed. We derived a relationship between the radius ρ and the angle θ and performed a case study on the possibility of ρ increasing on a full cycle. Finally, we formulated a simple rule in order to check how the switching points could affect convergence of the system over a cycle on the phase plot.

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