Control of unsteady Electromagnetic acceleration in parallel plate channels

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Abstract— **In the present work, we consider a simplified "slug" model that describes the flow in an ablative parallel-plate pulsed plasma thruster, a form of electromagnetic propulsion. The complex flow in the PPT can be described by the compressible, viscous, magnetohydrodynamic (MHD) PDEs coupled to an external circuit. The one-dimensional "slug" model used in this study is coupled with an *open loop* hybrid controller. Under this approach, the physical system parameters (resistor, capacitance) are reset to different constant values at discrete time intervals during the PPT pulse. Extensive simulation studies are included to demonstrate the feasibility of the proposed hybrid open loop control scheme.**

Keywords— **Magnetoplasmadynamic accelerator, pulsed plasma microthruster, distributed parameter systems, nonlinear approximation, switched controllers, hybrid system.**

I. Introduction

The pulsed plasma microthruster (PPT) is an electromagnetic onboard propulsion device depicted in Figure 1, [1]. The parallel-plate ablative PPT consists of two electrodes that are connected to a capacitor and use solid Teflon as propellant. The capacitor is discharged in microsecond duration pulses.The arc ablates the propellant and creates a plasma that is accelerated by electromagnetic forces to produce thrust [2]. The result of the Teflon® decomposition and ionization processes in the PPT channel is an unsteady, partially ionized plume that consists of plasmoids emitted at the pulsing frequency of the thruster [3]. The fundamental plasma flow processes inside a PPT are complex and subject to continuing investigations. The PPT flow can be described with various sets of the single and multi-fluid magnetohydrodynamic equations (MHD). The MHD equations are coupled with a heat transfer model that describes the physics of the discharge and Teflon ablation [2], [4], [5].

The PPT flown on several spacecraft is currently considered for onboard propulsion and attitude control maneuvers [6]. Therefore one of the objectives related to enhancing the performance of a PPT is to increase the terminal plasma velocity, which relates to the I_{sp} [1]. This can be achieved by the external circuit, as shown in Figure 1, in which the circuit parameters can be optimized in order to yield the largest possible plasma velocities. Coupling the circuit equation with the MHD equations will produce a faithful albeit complex system with the obvious implementation issues. Instead, simplified models for the plasma mass are assumed and coupled to the external circuit equation. This immediately simplifies the governing equations along with the optimization procedure for optimal circuit design. In the absence of external disturbances, in the form of sinks, the system parameters (circuit parameters) may be optimized to produce the best achievable plasma terminal velocities dictated by the circuit limitations. When disturbances are included, terminal velocities decrease and thus dynamic adjustment of circuit parameters is warranted in order to bring terminal plasma velocities to the desired levels.

Due to the simplified dynamic equations assumed for the PPT, one may not update circuit parameters in a continuous fashion as this would invalidate the very same dynamic equations of the simplified system. The alternative is to intermittently change circuit parameters, which then leads to a *hybrid* system. This direction is undertaken in this manuscript in which switching of circuit parameters takes place in order to improve terminal velocities in the presence of sink terms.

The modelling equations of the system under consideration are summarized in section II. The simplified model is described in section III and the system analysis is presented in section IV. Sensitivity studies on circuit parameters are performed in section V and the proposed hybrid control along with results of our numerical studies are presented in section VI.

II. Modelling Equations

A typical set of equations that has been used to described the PPT flow is the following single-fluid MHD equations:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho uu+p)=\frac{\partial \tau_{xx}}{\partial x}+jB, \tag{2}$$

$$\begin{aligned}\frac{\partial}{\partial t}(\rho e_t)+\frac{\partial}{\partial x}\left[\rho u\left(e_t+\frac{p}{\rho}\right)\right]&=\\ \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)&+\frac{\partial}{\partial x}(u\tau_{xx})+\frac{j^2}{\sigma},\end{aligned} \tag{3}$$

$$\frac{\partial B}{\partial t}+\frac{\partial}{\partial x}(uB)=\frac{\partial}{\partial x}\left(\frac{1}{\mu_0\sigma}\frac{\partial B}{\partial x}\right), \tag{4}$$

where $\rho,u,p,\tau_{xx},j,B,e_t,\lambda,\sigma$ and μ_0 are the density, flow velocity, pressure, viscous stress, current density, magnetic induction field, specific internal energy, heat conduction coefficient conductivity of the plasma and permeability of free space, respectively. The above system (1)-(3) is coupled to an external circuit with equation

$$L_0\frac{d^2Q}{dt^2}+(R_0+R_p)\frac{dQ}{dt}+\frac{Q}{C}=0, \tag{5}$$

where L_0, R_0, R_p, C are the circuit inductance, the external resistance, the plasma resistance and the circuit capacitance. The above set of PDEs is of mixed hyberbolic/parabolic type depending on the range of parameters. The constraint of divergence-free magnetic induction is considered as an initial condition satisfied at all times, but removes the hyperbolicity of

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the ideal MHD equations. These PDEs may be written as an evolution system in an appropriate abstract space(Hilbert/Sobolev) [7], [8]. Existence and well-posedness of the above system may be examined in the framework of the resulting evolution system. Furthermore the associated approximate controllability [9] of (1)-(5), may also be examined but while of importance, will not be considered here but rather will be examined at a later stage.

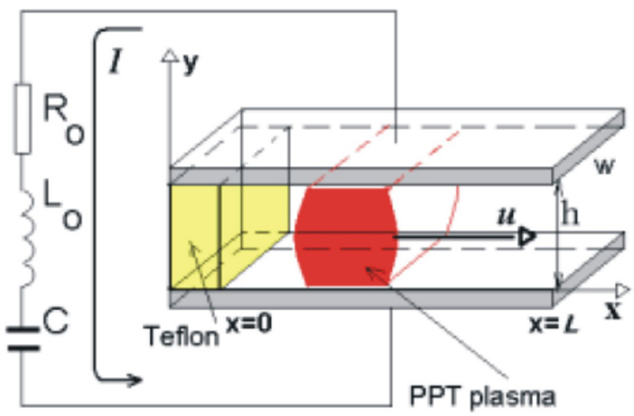

Fig. 1. Pulsed Plasma Thruster.

III. Model Simplification-Slug Model

One-dimensional simplified models derived from the unsteady 1-D MHD equations are often used to describe the PPT (1)-(4). One such model used in literature is the *slug model* [1], where the entire plasma produced in a discharge is represented by a constant mass element of a conventional electric model (2), i.e. no mass is accumulated or lost as the accelerating plasma traverses the channel. The equations capturing the salient dynamics of such a process are given by the coupled nonlinear integro-differential system

$$\Big[1+x(t)\Big]\frac{dJ(t)}{dt}+J(t)\frac{dx(t)}{dt}+\beta J(t)=1-\int_0^t J(\tau),d\tau, \quad (6)$$

$$\frac{d^2x(t)}{dt^2}=\alpha J^2(t), \quad (7)$$

with initial conditions given by

$$x(0)=0,\ \frac{dx(0)}{dt}=0,\ J(0)=0,\ \frac{dJ(0)}{dt}=1. \quad (8)$$

The above equations are derived by nondimensionalizing the circuit equation which includes the capacitor voltage. The nondimensionalized variables, namely current $J(t)$, time t and distance $x(t)$ are related to the current $\bar{J}(t)$, time $\bar{t}$ and position $\bar{x}(t)$ via

$$x=\frac{L_1}{L_0}\bar{x}, \quad ,t=\frac{1}{\sqrt{L_0C}}\bar{t}, \quad J=\frac{1}{V_0}\sqrt{\frac{L_0}{C}}\bar{J},$$

where L_1 is the channel inductance per unit length and V_0 is the initial voltage. The control objective now becomes that of manipulating the circuit variables so that a desired particle velocity $dx(t)/dt$ is achieved while all other state variables remain bounded. An efficient numerical scheme will allow one to perform sensitivity analysis of the above variables on the state quantities thus paving the way for a methodologically rigid control scheme. In addition to the computational model, a control scheme is proposed that takes into account system limitations.

IV. System Analysis

The above set of equations (7),(8) can be placed in state space form via

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \alpha x_3^2(t) \\ (1+x_1(t))\dot{x}_3(t) &= -x_3(t)x_2(t)-\beta x_3(t)+1-x_4(t) \\ \dot{x}_4(t) &= x_3(t), \end{aligned} \quad (9)$$

with initial conditions

$$x_1(0)=0,\ x_2(0)=0,\ x_3(0)=0,\ x_4(0)=0, \quad (10)$$

where

$$x_1(t)=x(t), \qquad x_2(t)=\frac{dx(t)}{dt},$$
$$x_3(t)=J(t), \qquad x_4(t)=\int_0^t J(t)\,dt.$$

Following the analysis in [1], the slug position $x_1(t)$ is always nonnegative and thus one may divide the third equation in (9) by $(1+x_1(t))$. In vector form, the above then becomes

$$\dot{\mathbf{x}}=f(\mathbf{x}) \quad (11)$$

with

$$f(\mathbf{x})=\left[\begin{array}{c} x_2(t) \\ \alpha x_3^2(t) \\ \dfrac{1-x_4(t)-x_3(t)x_2(t)-\beta x_3(t)}{(1+x_1(t))} \\ x_3(t) \end{array}\right],$$

and $\mathbf{x}(t)=[x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)]^T$. The asymptotic behavior of the above system has been studied extensively in [1] for different ranges of the system parameters α and β. For example, when $\alpha\gg 1$, one observes distorted damped current $J(t)$ with monotonically accelerating slug trajectories which produce a given terminal velocity at the current completion, see Fig. 2. This is dictated by the initial energy storage and resistive losses throughout the pulse. On the other hand, if $\alpha\ll 1, \beta\ll 1$, it is observed that $J(t)$ exhibits slowly damped sinusoidal oscillations, see Fig. 3. As in the previous case, the slug trajectories do not oscillate but exhibit minor fluctuations in the time derivatives (slug velocities) in approaching a terminal velocity.

The control objective would be to ensure that the slug velocities reach their terminal values as soon as possible with the largest possible steady-state value. Sensitivity analysis along with solution of an associated inverse problem would certainly allow one to find the set of achievable terminal slug velocities

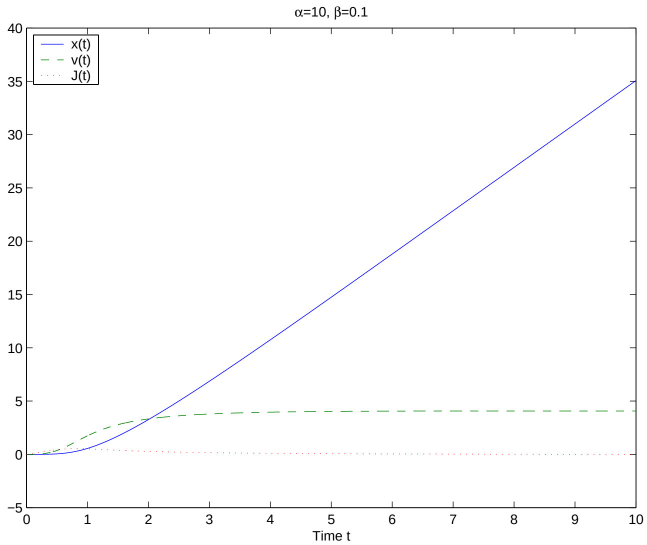

Fig. 2. Case with $\alpha \gg 1$.

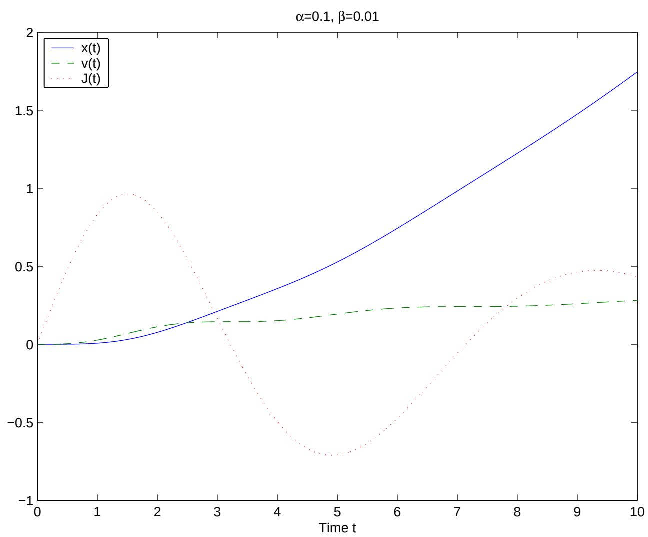

Fig. 3. Case with $\alpha \ll 1$.

subject to circuit constraints, and via some form of optimization, to find these optimal parameter values of α, β. This then may be considered as "static" optimization in the sense that one chooses fixed constant values of these parameters for the duration of current exhaustion. In reality, one may encounter "damping" (or "sink") terms due to disturbances, which, for constant values of the parameters α and β, would yield terminal slug velocities below the achievable ones corresponding to the ideal case with no sinks.

If one were free to choose these two parameters, i.e. consider them as the two control inputs, then a Lyapunov method would provide the dynamic adjustment of both $\alpha(t)$ and $\beta(t)$ in feedback form (i.e. as nonlinear functions of x_1, x_2, x_3, x_4). Toward that end, one redefines the above system (9) or (11) via

$$\tilde{x}_1(t) = x_1(t) - x_{ss}, \qquad \tilde{x}_2(t) = x_2(t) - v_{ss},$$

where x_{ss} and v_{ss} are the desired (or ideally achievable) steady state position and velocity, respectively; e.g. corresponding to the values achieved in the ideal case with no sink terms. This then gives rise to the following *error* system which includes sinks terms

$$\dot{\tilde{\mathbf{x}}} = f_0(\tilde{\mathbf{x}}) + g_1(\tilde{\mathbf{x}})\alpha(t) + g_2(\tilde{\mathbf{x}})\beta(t) + d(\tilde{\mathbf{x}}(t)), \tag{12}$$

where

$$\tilde{\mathbf{x}}(t) = \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \mathbf{x}(t) - \begin{bmatrix} x_{ss} \\ v_{ss} \\ 0 \\ 0 \end{bmatrix},$$

$$f_0(\tilde{\mathbf{x}}(t)) = \begin{bmatrix} \tilde{x}_2(t) + v_{ss} \\ 0 \\ \dfrac{1 - x_4(t) - \tilde{x}_2(t)x_3(t) - v_{ss}x_3(t)}{1 + \tilde{x}_1(t) + x_{ss}} \\ x_3(t) \end{bmatrix},$$

$$g_1(\tilde{\mathbf{x}}(t)) = \begin{bmatrix} 0 \\ x_3^2(t) \\ 0 \\ 0 \end{bmatrix}, \quad g_2(\tilde{\mathbf{x}}(t)) = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{x_3(t)}{1 + \tilde{x}_1(t) + x_{ss}} \\ 0 \end{bmatrix},$$

and $d(\tilde{\mathbf{x}}(t))$ denotes the unknown term due to disturbances. The error system now has the origin as its equilibrium. The control objective now becomes that of choosing the "control" terms $\alpha(t), \beta(t)$ so that regulation of the state $\tilde{\mathbf{x}}(t)$ to zero is achieved in finite time. To do so, a Lyapunov function $V(\tilde{\mathbf{x}}(t))$ is sought that is positive definite. Choosing $\alpha(t), \beta(t)$ so that the time derivative of the Lyapunov function along the trajectories of (12) becomes negative semidefinite for all $d(\tilde{\mathbf{x}}(t))$ belonging to a certain class of functions, would provide closed loop stability and consequently asymptotic convergence of $\tilde{\mathbf{x}}(t)$ to zero [10]. This then would accommodate for the presence of "sink" terms and thus ensure that the terminal slug velocities are close to the nominal ones achievable under nominal conditions. Unfortunately, if one changes the system parameters α, β indiscriminately and continuously, as would be the case of feedback form, then the above model (6)-(8) ceases to be valid. A way to avoid this is to consider the infinite dimensional system (1)-(5) along with a finite dimensional controller associated with it. This would immediately increase the computational cost and the controller complexity with the obvious effects on its real-time control implementability.

An alternative to the above would be to intermittently update the constant values of α, β at the onset of a time subinterval; i.e. consider piecewise constant variations for α, β. This may be placed in an adaptive parameter adjusting scheme wherein hybrid adaptation takes places at discrete time instances and whose objective is to bring the values of the "unknown" parameters α, β to some "ideal" values. These unknown ideal values correspond to the optimal values of the equivalent system that includes the unknown "sinks" and which provide the allowable terminal slug velocities. The drawback to this is twofold since (i) one needs to have functional knowledge of the "sink" term, and parameterize it by available quantities, e.g. known nonlinear functions of the four states, and (ii) one must a priori guarantee the existence of the optimal values of α,β regardless of the class of damping functions ("sinks").

To address this predicament, one may intermittently change the values of α,β as suggested above, but in an "open loop" fashion. This requires parametric studies to be performed a priori for a range of different functions describing "sinks" motivated by physical phenomena; i.e. consider all $d \in \mathcal{D}$, where the class of functions $\mathcal{D}$ provides existence of the solution to (12). This then results in *multiple models* corresponding to the different choices of "sinks". To demonstrate this, we adopt the notation

$$f_i(\mathbf{x}) = f(\mathbf{x},\alpha_i,\beta_i),$$

where $f(\mathbf{x},\alpha_i,\beta_i)$ denotes the vector field in (11) evaluated at the specific values of $\alpha = \alpha_i$ and $\beta = \beta_i$. The decision policy to change the values of α,β would then require one to match current values of the state $\mathbf{x}(t)$ with those from a bank of multiple models generated by computer simulations. Once the system trajectories are matched at a given time instance with the computer values, one may then adjust the values of α,β accordingly. This type of switching may be viewed as *state-dependent* switching [11]. Following [12], one rewrites the above system as

$$\dot{\mathbf{x}}_i(t) = f_i(\mathbf{x}), \quad i \in \mathcal{I}, \tag{13}$$

where the family of functions f_i, $i \in \mathcal{I}$ maps $\mathbb{R}^4$ to $\mathbb{R}^4$ and $\mathcal{I}$ is some index set, which in this manuscript is assumed to be a subset of a finite dimensional linear vector space. Concomitant to the above is the *switching* signal [11], [13]. This is a piecewise constant function $\sigma : [0,\infty) \to \mathcal{I}$. This switching signal serves the role of specifying, at each time instant t, the index $\sigma(t) \in \mathcal{I}$ of the active subsystem $f_i(\mathbf{x})$, in other words which system in (13) is being followed. The size of the index set $\mathcal{I}$ is dictated by physical and hardware considerations, and hence we have for our system under consideration that $\mathcal{I} = \{1,2,\ldots,m\}$ with $m \leq 10$.

This appears to be the most promising alternative, assuming a form of uniqueness and one-to-one correspondence of "sink" expressions with given $\alpha's$ and $\beta's$. In addition to this, one may employ sensitivity analysis by using gradients of the current slug velocity with respect to α,β and adjust accordingly.

Due to practical considerations, a simplified form of the latter will be considered in this study. Nominal values of α,β will be used as starting values, and will be adjusted to other nearby values according to a gradient-like method.

V. Sensitivity Studies

Prior to performing sensitivity analysis on the system in (11), we consider the analysis of the circuit of pulsed accelerators, which is given by

$$L_0\ddot{Q} + R_0\dot{Q} + \frac{Q}{C} = 0,$$

where L_0 is the circuit inductance, R_0 the external resistance and C the capacitance. The charge is denoted by $Q(t)$ and the current by $J(t) = -\dot{Q}(t)$. The above equation is essential in deriving the nondimensionalization of (6)-(7). Using a fixed inductance of $L_0 = 5\times10^{-5}$ henry and enforcing the ratio $4L_0/R_0^2 \approx 10^{-2}$ to get the initial resistance as $R_0 = \sqrt{20}\times10^{-3}$ ohm, we consider the damped case which requires a capacitance $C_0 \approx 10^4\mu$F. For an underdamped circuit, we chose $C = 0.05C_0 = 500\mu$F. To investigate the effects of intermittently changing the external circuit resistance on the circuit current, we changed the resistance via $R_{k+1} = R_k + (k-1)4R_0$, with R_k denoting the resistance at the time instance $k\Delta T$ where $\Delta T = T_{final}/10$ and $T_{final} = 10^{-4}$ seconds; i.e. change(increase) the resistance every 10μsec by a fixed value of $4R_0$. Fig. 4 depicts the evolution of both the circuit charge $Q(t)$ and circuit current $J(t)$ for the case where the resistance is kept constant (solid line) and the case of variable resistance. The fact that the current $J(t)$ can converge to zero faster than the case of fixed resistance, it allows us to further investigate the effects of varying the resistance in the PPT in Fig. 1. The circuit values also allow us to find the nondimensionalized time which is given by

$$t = \frac{1}{\sqrt{L_0C}}\,\bar{t} = \frac{\bar{t}}{\sqrt{5\times10^{-10}}},$$

and hence for a time duration of 100μsec produces a simulation time of 10 (nondimensionalized) time units, see Fig. 7. We first

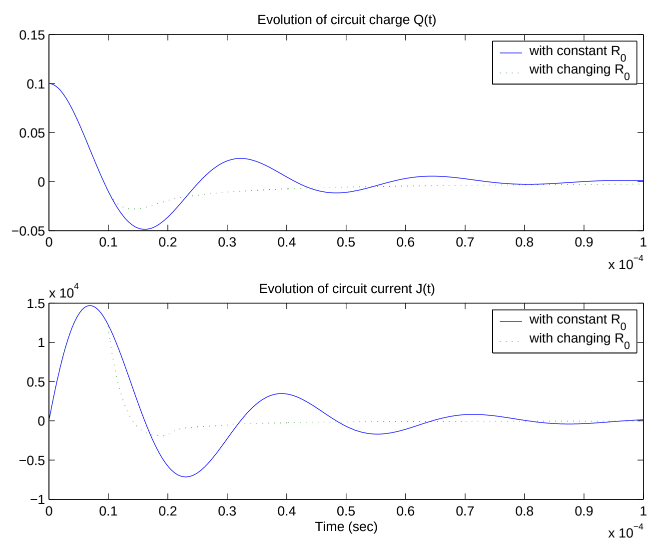

Fig. 4. Effects of variable resistance on circuit.

consider the so-called "nominal" system (11) wherein the disturbance term $d(\mathbf{x}) \equiv 0$. Using a constant value of $\beta = 10^{-4}$, we studied the effects of α on the terminal slug velocity at the terminal time $t = 10$. In Fig. 5, one may observe the terminal velocity increases proportional to $\sqrt{\alpha}$. The opposite is observed when one uses a nominal value of $\alpha = 0.1$ and varies β. Fig. 6 depicts this dependence of the terminal velocity on β. The terminal velocity is inversely proportional to β, and therefore one may consider increasing α and decreasing β as a strategy to increase the terminal velocity.

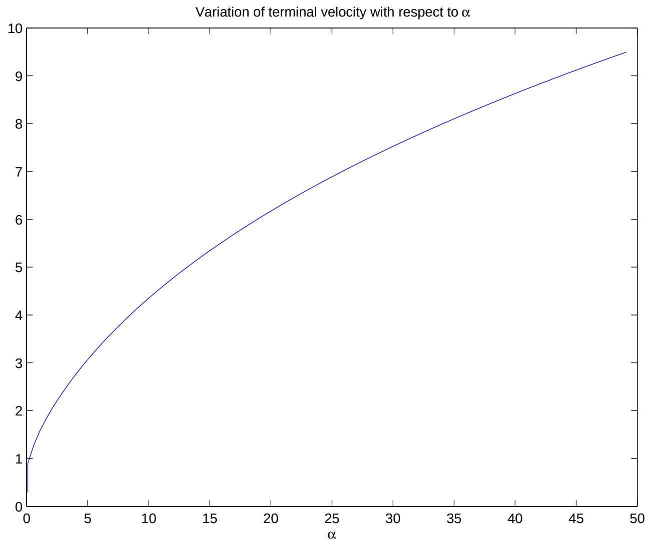

Fig. 5. Effects of α on terminal velocity.

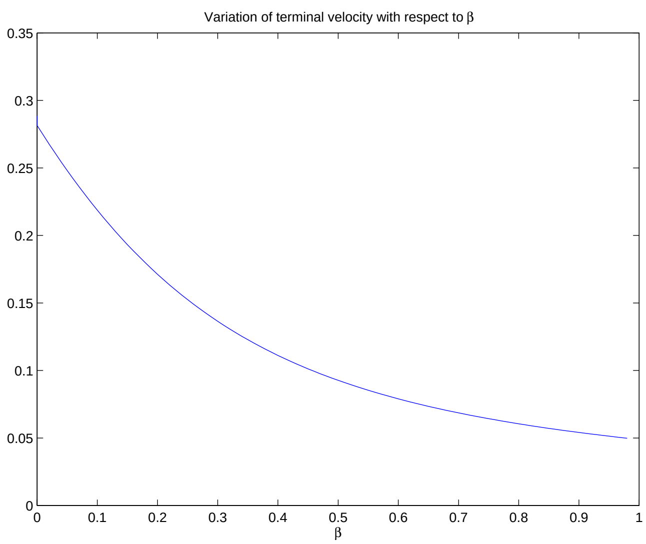

Fig. 6. Effects of β on terminal velocity.

VI. HYBRID CONTROL

Using a diffusion type for the sink term, we simulated the above system in (11) with the particle acceleration now given by

$$\dot{x}_2(t) = \alpha x_3^2(t) - d x_2(t), \quad d \geq 0. \tag{14}$$

This has the effect of reducing the terminal slug velocity as depicted in Fig. 7 for the nominal values of $\alpha = 10$, $\beta = 0.05$ and with $d = 0.1$ (dashed). The nominal system ($d = 0$, solid line) produces a terminal velocity $v_{ss} = 4.1544$ whereas the system with the disturbance ($d = 0.1$, dashed line) yields $v_{ss} = 1.8349$. The proposed hybrid controller takes the form

$$\alpha_{k+1} = \alpha_k + \Delta\alpha, \; \beta_{k+1} = \beta_k - \Delta\beta, \tag{15}$$

where k denotes the discrete time index in the (nondimensionalized) time interval of interest $[0,10]$; i.e. $\alpha_k = \alpha(k\Delta T)$ with

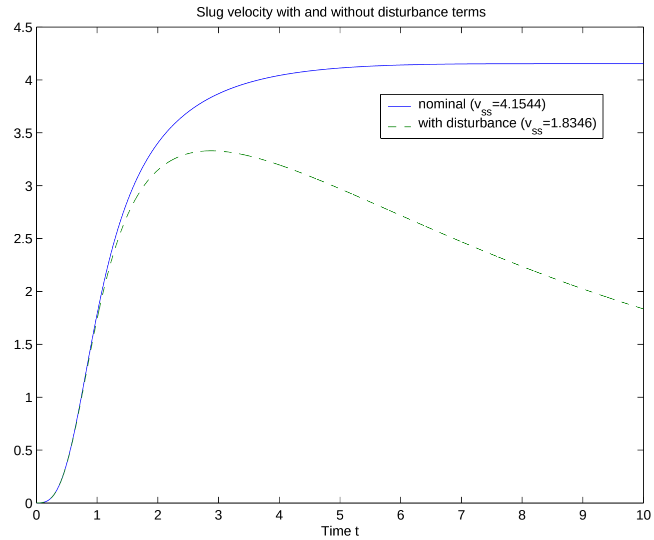

Fig. 7. Effects of disturbances on terminal velocity.

$\Delta T = T_{final}/ns$, where ns : # of switchings over a time interval $[0, T_{final}]$. Table 1 depicts the various values of the system parameters α, β for the ideal case and when there are two switchings in $[0,10]$, i.e. update the values of α, β once in the time interval with $\alpha = \alpha_1$, $\beta = \beta_1$ for $t \in [0,5)$ and $\alpha = \alpha_2$, $\beta = \beta_2$ for $t \in [5,10]$. The highest value of terminal velocity ($v_{ss} = 4.1544$) is achieved in the case of zero disturbances ($d \equiv 0$). Even in this case, one may increase the terminal velocity when α, β are changed to a value of $v_{ss} = 4.32$. By changing the values of the parameters, one may increase the terminal velocity even in the presence of disturbances. For example, when the circuit parameters are chosen equal to the initial values $\alpha = 10, \beta = 0.05$ throughout $[0,10]$ and $d = 0.1$, the corresponding terminal velocity is $v_{ss} = 1.8349$. When the circuit parameters change in the interval $[5,10]$ to $\alpha = 60, \beta = 0.02$, the corresponding terminal velocity is $v_{ss} = 1.9641$. Continuing, one may achieve higher terminal velocities with the values of α, β in $[5,10]$ chosen as $\alpha = 210, \beta = 0.04$; in this case the terminal velocity becomes $v_{ss} = 2.4567$.

While the proposed parameter switching increases the terminal velocity, it cannot reach the levels acquired by the nominal system. Increasing the switching frequency would in theory increase the terminal velocity, but at the same time would invalidate the modelling equations, since a ΔT less than 4.472 (nondimensionalized) time units falls below the pulse duration ($t = \bar{t}/\sqrt{L_0 C} = 4.472 \times 10^4 \bar{t} = 4.472 \times 10^4 \times 10^{-4} = 4.472$). This suggest further investigation on the (nonlinear) controllability issues arising by considering the intermittent parameter adaptation of the proposed hybrid controller.

The proposed open loop hybrid controller, while considering circuit and modelling restrictions, provided encouraging results that translated into increased terminal velocity, a more comprehensive model would allow one to consider entrainment effects and hence arrive at a model better describing the PPT dynamics. This along with controllability and stability issues, and a form of closed-loop hybrid controller are currently under investigation by the authors.

d	α	$\Delta\alpha$	β	$\Delta\beta$	ns	v_{ss}
0	10	0	0.05	0	0	4.1544
0	10	50	0.05	0.03	2	4.3200
0.1	10	0	0.05	0	0	1.8349
0.1	10	50	0.05	0.03	2	1.9641
0.1	10	40	0.05	0.002	2	1.9642
0.1	10	80	0.05	0.004	2	2.0906
0.1	10	100	0.05	0.01	2	2.1534
0.1	10	200	0.05	0.01	2	2.4567

TABLE I

EFFECTS OF PARAMETER SWITCHING ON TERMINAL VELOCITY.

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