On-line Gradient Optimization of a Fractional Order Hold

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***Abstract*— This work deals with a problem of optimization of a fractional order hold (FROH) parameter. All approaches known from literature and related to this problem are based on location of limiting zeros of open loop discrete system composed of a hold device, continuous-time system and a sampler in series. In this work in order to optimization of a FROH parameter a quadratic performance index is defined. The performance index is minimized on-line on a gradient way using structural sensitivity analysis. Such an approach makes possible an optimization of a FROH parameter not only for linear systems but also for nonlinear systems, for different reference signals and in presence of different types of disturbance signal. Results of experiments depending on simultaneous gradient optimization of FROH parameter and parameters of discrete-time PID controller shows that in all investigated cases the FROH device is better than a zero order hold (ZOH) and a first order hold (FOH) under the same sample time.**

Index Terms— **Fractional order hold, gradient optimization, adaptive control, sampled data systems.**

I. Introduction

The most frequently used device which holds a control signal in discrete time control systems with continuous time plants is a zero order hold (ZOH). Another, less frequently used, but frequently discussed in the literature related to sampled data systems, is a first order hold (FOH). A very interesting problem, discussed in the literature, is a question about the influence of the type of the holding device and the value of the sampling period on a discrete time transfer function of the system composed of the holding device, a continuous time plant and a sampler in series. A commonly used approach [1, 6, 9, 12, 14, 17] is an analysis of zero location of such a system with special attention paid on so called limiting zeros obtained for the sampling time $T_s \to 0$. An investigation of zeros of obtained discrete time system is very important, specially when control algorithms, depending on zeros and roots cancellation, are used. In such a case it is very desirable to obtain zeros as stable as possible. It means zeros lying inside the unit circle on the z plane the closest to its center. In a work [11] a comparison of systems with ZOH and FOH devices has been made. It was shown that FOH device is not better than ZOH, as far as the location of zeros is concerned. In works [15, 16] a fractional order hold (FROH) has been proposed as an alternative to ZOH and FOH. In addition it was shown in paper [15] that it is possible to locate zeros of pulse transfer function with the FROH element for continuous time plants for which the ZOH fails to do so. Ishitobi in work [13] shown that the FROH element with the parameter $-1 < \beta < 0$ locates all zeros of the sampled system in the unit circle for larger class of continuous time systems than the ZOH device. In a paper [2] an analytical method for choosing optimal value of β, basing on the generalized root locus method, has been presented. In works [3] and [4] the FROH device has been used for control of a DC motor and a read-write head in computer hard disc. In a work [5] a multilayer perceptron has been used to approximate the function mapping the optimal value of β on the basis of parameters of continuous time plant and the sampling period.

Up to now all methods of evaluating of control systems with different holding elements were based on the analysis of zeros of open-loop system with the holding device. In this work a quite different approach to the problem of choosing of optimal value of β is proposed. It depends on a definition of a performance index which is a square of instantaneous value of the control error. This performance index is minimized on-line using gradient methods. So called structural sensitivity methods [7, 8, 10, 18] are applied in order to calculate the gradient of the performance index.

A modification of the parameter β, besides changing of the zeros location, causes an alteration of the gain. Hence an optimization of only β parameter would cause in a higher degree an adaptation to the optimal value of the gain, but not to the optimal shape of the control signal. Hence it is necessary an simultaneous optimization of at least two parameters: β and the gain of the controller. In this work a discrete time PID controller is used. Hence four parameters are optimized: parameter β and three parameters of the controller. Four sensitivity models, which work simultaneously and generate partial derivatives of the performance index, are used in order to adaptation of these algorithms.

II. The fractional order hold

The FROH element is a holding device generating on its output a continuous time signal $x(t)$ basing on a discrete time input signal $u(kT_s)$ according to the formula

$$x(t) = u(kT_s) + \beta \left[\frac{u(kT_s) - u\left((k-1)T_s\right)}{T_s} \right] (t - kT_s) \quad (1)$$
$$kT_s \le t \le (k+1)T_s$$

where T_s is the sampling period and β is a real number. The FROH can be expressed as a serial connection of an ideal pulser and a continuous transfer function

$$K_\beta(s) = \left(1 - \beta e^{-sT_s}\right) \frac{1 - e^{-sT_s}}{s} + \frac{\beta}{s^2 T_s} \left(1 - e^{-sT_s}\right)^2 \quad (2)$$

Hence, the discrete transfer function of the FROH, the continuous time system $K_P(s)$ and the sampling device connected in series is given by the formula

$$\begin{aligned} K_{\beta P}(z) &= \mathbf{Z} \left\{ \left[\left(1 - \beta e^{-sT_s}\right) \frac{1 - e^{-sT_s}}{s} + \right.\right. \\ &\left.\left. + \frac{\beta}{s^2 T_s} \left(1 - e^{-sT_s}\right)^2 \right] K_P(s) \right\} = \\ &= \frac{\beta(1 - z^{-1})}{T_s} \mathbf{Z} \left[\frac{1 - e^{-sT_s}}{s} \frac{K_P(s)}{s} \right] + \\ &+ \beta(1 - z^{-1}) \mathbf{Z} \left[\frac{1 - e^{-sT_s}}{s} K_P(s) \right] \quad (3) \end{aligned}$$

ZOH and FOH elements are special cases of the FROH element for $\beta = 0$ and $\beta = 1$ respectively.

The FROH is presented in a Fig. 1a as a block diagram. This is not straight realization of (2) but it is composed of two ZOH devices and non-stationary linear element $q(t)$ which gain varies in time, see Fig. 1b. Such realization requires less numerical effort.

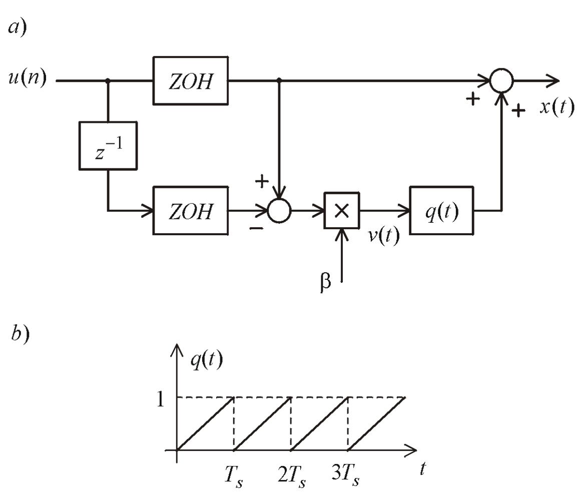

Fig. 1. The fractional order hold: a) a block diagram, b) a signal $q(t)$ acting in the block diagram

III. Problem Formulation

Let us consider a discrete-time control system with FROH device presented in Fig. 2. In this system P is a continuous-time plant and C is a discrete-time PID controller. Following

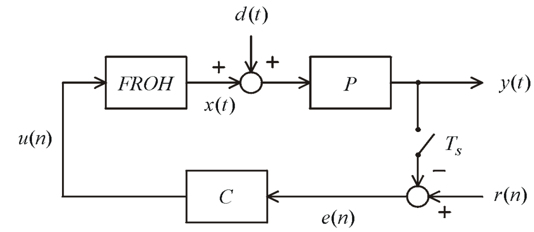

Fig. 2. A discrete-time control system with the FROH device

continuous-time signals act in the system: an output signal of the FROH device $x(t)$, a disturbance $d(t)$ and an output signal of the plant $y(t)$. Discrete signals are: a reference signal $r(n)$, an control error $e(n)$ and a control signal $u(n)$ coming from the controller. In addition we assume that a continuous reference signal $r(t)$, which after sampling gives $r(n)$, is known.

The minimized performance index is as follows

$$J(t) = \frac{1}{2} e^2(t) \quad (4)$$

In order to perform the gradient minimization of (4) one have to know following partial derivative

$$\frac{\partial J(t)}{\partial \beta} \quad (5)$$

The partial derivative (5) will be calculated using a sensitivity model.

IV. Gradient Calculation

The sensitivity model will be constructed using so called structural sensitivity methods [7, 8, 18]. The sensitivity model is created using a set of simply rules on the basis of the block diagram of the original system. In this work a structural sensitivity analysis approach proposed in [10] is utilized. This approach, fully mnemonic, requires some modifications of the original block diagram to obtain a system where the differentiated signal is an output of the system, and the parameter (signal) with respect to which the differentiation is performed is an input signal. The modified block diagram of analyzed in this paper system is presented in Fig. 3. In addition the sampler is moved to another place. In order to obtain the sensitivity model, all nonlinear elements of the system shown in Fig. 3. should be linearized. Under the assumption that the plant is linear, two elements should be linearized: the element calculated the quadratic performance index and the FROH device because it is a nonlinear element as well (see Fig. 1). As a result the sensitivity model presented in Fig. 4 is created. In this block diagram elements $\bar{P}$ and $\bar{C}$ are the sensitivity models of the plant and the controller. If they are linear, the only difference, in comparison to the original elements, are zero initial condition assumed in sensitivity models. The sensitivity model of the FROH device is shown in Fig. 5. The partial derivative (5) obtained on the output of the system from Fig. 4 is used for adaptation on-line of the parameter β.

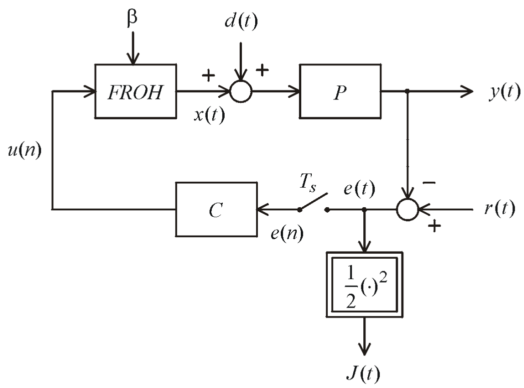

Fig. 3. A modified block diagram of the control system

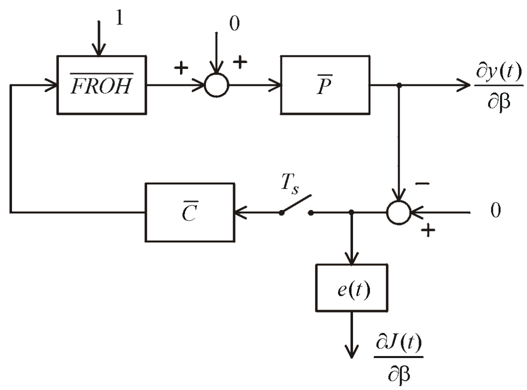

Fig. 4. A sensitivity model generating the partial derivative $\partial J(t)/\partial\beta$

The adaptation of parameter β is done using simply gradient method:

$$\beta(t) = \beta(0) - \alpha \int_0^t \left[\frac{\partial J(\tau)}{\partial \beta}\right] \mathrm{d}\tau \tag{6}$$

where α is a small positive number and $\beta(0)$ is an initial value of the parameter β.

Using similar structural approach three sensitivity models for parameters k_p, k_i, k_d of the discrete in time PID controller has been constructed. All these models simulated on-line perform the adaptation of four parameters to minimize the performance index (4). Since α is a small number, the changes of the parameters are very slow and the minimization of the performance index (4) can be treated as a minimization of its mean value.

V. A Numerical Example

The numerical simulation has been done for the continuous time plant P of the transfer function

$$K_P(s) = \frac{1}{(1+s)^2} \tag{7}$$

The desired signal $r(t)$ was a sum of a constant value 2 and a square wave with the amplitude $A_r = 1$ and the frequency $f_r = 0.02$[1/sec].

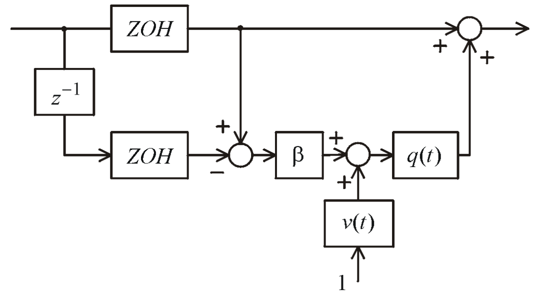

Fig. 5. A sensitivity model of the fractional order hold

A non-stationary disturbance $d(t)$ acted in the system. It was changed with about $8.33 \cdot 10^2$[sec] long periods. During even periods the disturbance was a square wave of the amplitude 0.3 and the frequency 0.07[1/sec]. In odd periods the disturbance was a random signal of the amplitude 0.05. Fig. 6 presents the changes of adapted parameters k_p, k_i, k_d and β for the $\alpha = 0.2$. The optimal value of β is about -0.5 and in analyzed case does not depend on the form of the disturbance $d(t)$. The variation of the value of β is only the result of the fact that after the change of disturbance the parameters of the controller are not adjusted. But it not the general rule. Other investigated examples shown that the optimal value of β may vary with the disturbance type, although it is much more dependent on the desired signal, the transfer function of the plant and the sampling period. Fig. 7 presents the performance index signal after passing by low-pass filter.

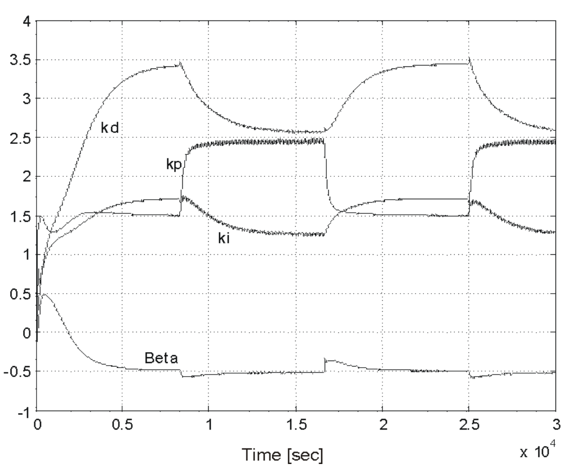

Fig. 6. Values of all four optimized parameters during their on-line opimization

Fig. 8 presents signals $r(t), y(t)$ and $x(t)$ obtained for optimal values of parameters k_p, k_i, k_d and β under square disturbance. The mean value of the performance index is about 0.098. On the subsequent figures the signals for optimal parameters k_p, k_i, k_d with ZOH device (Fig. 9) and with FOH device (Fig. 10) are presented. The mean values of the performance index reached values of about 0.106 for ZOH and

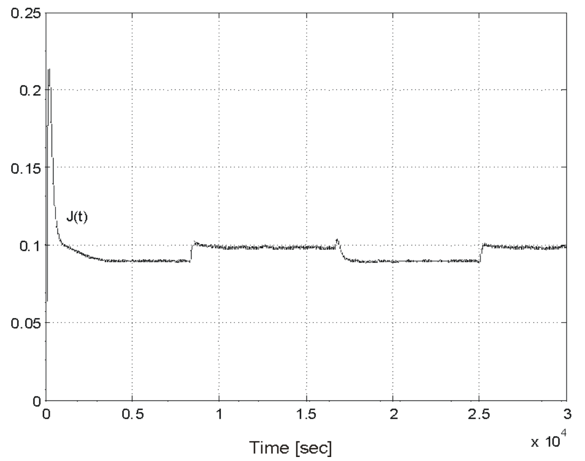

Fig. 7. The performance index

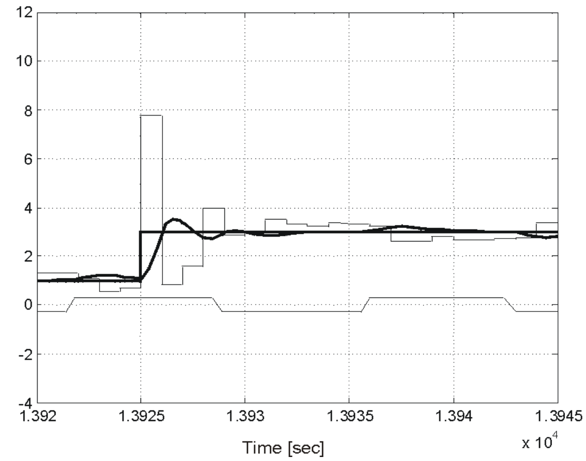

Fig. 9. Signals obtained for ZOH device and optimal values of controller parameters

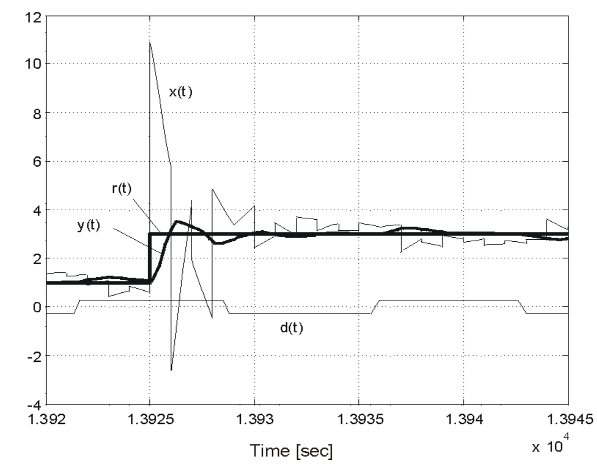

Fig. 8. Signals obtained for optimal values of β and controller parameters

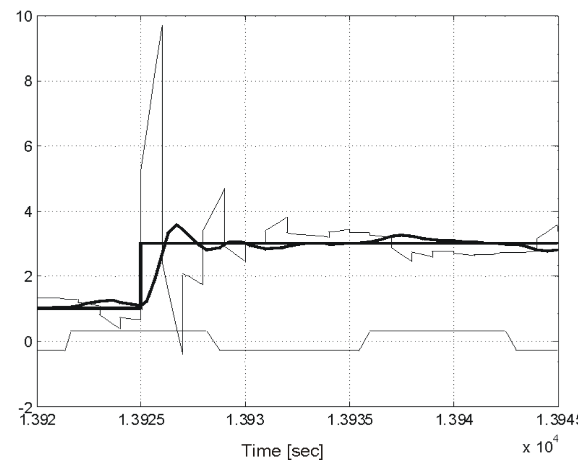

Fig. 10. Signals obtained for FOH device and optimal values of controller parameters

0.118 for FOH elements. They were greater by 8% and 20% in comparison to the optimal FROH device.

VI. Conclusion

The paper presents a new approach to the problem of optimization of a fractional order hold. All proposed in literature solutions to this problem rely on location of zeros of sampled data system with hold device. Here, a sensitivity approach to gradient minimization of a quadratic performance index is proposed. The minimization is performed on-line and not only β parameter of the FROH is adapted but also three parameters of discrete time PID controller. A numerical examples show that sampled data control system with FROH device and properly adjusted parameters is superior to systems with ZOH and FOH devices with optimal PID controller parameters.

The question, how the optimal value of β depends on parameters of the plant, disturbances, and the sampling period is an open problem and it will be possible to answer it in the future by using proposed in this paper approach.

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