Decentralized Variable Structure Control for Interconnected Systems with Mismatched Uncertainties

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Abstract-- In this paper, a problem of decentralized control is investigated for a class of nonlinear interconnected systems with mismatched uncertainties. The interconnected systems consist of matched and mismatched uncertainties. Based on variable structure control theory and by exploiting the structure of the interconnected systems, a new approach of decentralized variable structure control is presented. In contrast with other existing results, the controllers are simpler. The effectiveness of the proposed decentralized variable structure controllers is illustrated through an numerical example.

Index Terms-- nonlinear interconnected systems, uncertainty, decentralized variable structure control

I. INTRODUCTION

Large-scale systems are often characterized as being widely distributed in space or being of high order. These systems are commonly modeled as dynamic equations composed of subsystems and interconnections. When the systems were studied, decentralized control is often applied because the design of large-scale systems can be simplified and realization is easy in practical.

In practical systems, uncertainties of parameters and external disturbances exist. So, variable structure control (VSC) was proposed as early as 1950s and has attracted many researchers to pay more attentions since 1970s [1]. Robustness, insensitivity and simplicity of design, which are advantages of VSC, make sliding controllers a powerful approach [2], [3], [4].

In recent years, a great deal of attention has been devoted to the large-scale systems with uncertainties and many useful results have been obtained. In [5], base on VSC theory, stabilization of interconnected large-scale systems has been considered. The design of variable structure controller for large-scale systems has been studied in [6].The case of sliding mode control has been tackled for linear discrete-time systems with matched perturbations in [7]. In [8], decentralized control of large-scale systems with uncertain interconnections has been studies, where the interconnections were known and its approach is complicated. Decentralized VSC for interconnected systems with uncertainties has been discussed, but without uncertain interconnections in [9]. In [10], decentralized VSC of large-scale systems with uncertain interconnections have been investigated, where matched conditions were satisfied and choice of the switch function was related to the inter-connections. In the above studies of decentralized VSC for large-scale systems with uncertainties, matched conditions are required. In practice, mismatched uncertainties exist in many physical systems. In this case, however, the invariance of parameters and the robustness of sliding motion can't be ensured. In order to deal with mismatched uncertainties, backsteping approach has been applied in [11], [12].

In this paper, a class of nonlinear interconnected systems with mismatched uncertainties is considered. No structural information is required for uncertainties in subsystems and interconnections except that they are bounded by known. By exploiting the structure of the interconnected systems, a new approach of decentralized VSC is proposed. In contrast with other existing results, the controller of the associated closed-loop interconnected system is simpler, and makes its implementation much easier.

II. SYSTEM DESCRIPTION

We consider an uncertain system Σ consisting of N subsystems $\Sigma_1, \Sigma_2, \cdots, \Sigma_N$ that are interconnected. The subsystem Σ_i , $i \in \Omega = \{1,\ 2,\ \ldots,\ N\}$ is described by the equation:

$$\Sigma_i: \quad \dot{x}_i = A_i x_i + f_i(x_i) + B_i[I_i + \Delta E_i(t,v)]u_i + \sum_{j=1, j\neq i}^{N}[h_{ij}(t,x_j) + \Delta h_{ij}(t,v,x_j)] \tag{1}$$

where $u_i \in R$ is the input, and $x_i \in R^{n_i}$ is the state at time t. A_i and B_i are scale system matrix, $f_i(x_i)$ is mismatched nonlinear uncertainty of the subsystem Σ_i ; ΔE_i is matched uncertainty; v is uncertain parameter; h_{ij} and Δh_{ij} are interconnection and uncertain interconnection for the subsystem Σ_i due to the subsystem Σ_j ($j \neq i$, $i, j \in \Omega$); $I_i \in R^{n_i \times n_i}$ is identity matrix.

Note that the interconnection term and uncertain interconnection term are mismatched.

For the subsystem Σ_i , we assume with reason:

Assumption: non-negative constants ξ_i , η_i , α_i , β_i

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exist such that:

$$\|f_i(x_i)\| \le \xi_i \|x_i\|, \quad \|\Delta E_i\| \le \eta_i < 1,$$
$$\|h_{ij}\| \le \alpha_i \|x_j\|, \qquad \|\Delta h_{ij}\| \le \beta_i, \tag{2}$$

Here, $\|\bullet\|$ is Euclidian norm.

The goal is designing a decentralized variable structure controller $u_i = u_i(x_i)$ such that state x_i can be convergent to origin in arbitrary initialization state $x_i(0)$.

III. DESIGN OF DECENTRALIZED VARIABLE STRUCTURE CONTROLLER

A. Choice of the switch surface

Consider the subsystem Σ_i, we define the switch surface according to the following equation:

$$S_i(t) = G_i x_i(t) \tag{3}$$

where $G_i \in R^{1\times n_i}$ is the switch matrix. Choice of G_i should make sliding motion of the subsystem Σ_i is asymptotically stable.

After the subsystem Σ_i turn into sliding mode, there is

$$S_i(t) = \dot{S}_i(t) = 0$$

Then, equivalent control is

$$u_i^{eq} = -[G_i B_i (I_i + \Delta E_i)]^{-1} G_i [A_i x_i + f_i(x_i) + \sum_{j=1, j\ne i}^{N} (h_{ij} + \Delta h_{ij})] \tag{4}$$

Formula (4) is substituted to formula (1), and assumption is considered, we can obtain sliding equation of the subsystem Σ_i:

$$\begin{aligned} \dot{x}_i &= [I - B_i (G_i B_i)^{-1} G_i] A_i x_i \\ S_i &= G_i x_i(t) = 0 \end{aligned} \tag{5}$$

It is obvious that sliding motion of the subsystem is relational to G_i. Therefore, choice of G_i can be based on demand to convergence speed of sliding motion, and G_i is determined by formula (5).

B. 2.2 Design of decentralized variable structure controller

Lemma[13]: the system Σ_i is globally asymptotically stable if the following global reaching condition satisfy:

$$\sum_{i=1}^{n} P_i(t) S_i(t) \dot{S}_i(t) < 0 \tag{6}$$

where $P_i(t) > 0$, and satisfy $P_i(t)|S_i(t)| = w_i$, w_i is a positive constant.

It is necessary to point out, w_i is a given arbitrary positive constant. In a general way, if a proper w_i is picked, dynamic characteristic of the system can be improved, and reaching speed can be changed, too.

Theorem 1: To the uncertain system Σ_i, closed-loop system is globally asymptotically stable if control law u_i is

$$u_i = u_i^{\mathrm{L}} + u_i^{\mathrm{N}} \tag{7}$$
$$u_i^{\mathrm{L}} = -(G_i B_i)^{-1} G_i A_i x_i \tag{8}$$
$$u_i^{\mathrm{N}} = -\frac{1}{1-\eta_i} (G_i B_i)^{-1} [\sigma_i + \delta_i \|x_i\|] \cdot \mathrm{sign}(S_i) \tag{9}$$

where $\sigma_i \ge N\beta \|G_i\|$, $\delta_i \ge \|G_i\|(\xi_i + \eta_i \|A_i\|) + \dfrac{NwR}{w_i}$, and
$\beta = \max\{\beta_i, i = 1,2,\cdots,N\}$, $w = \max\{w_i\}$,
$R = \max\{\alpha_i \|G_i\|, i, = 1,2,\cdots,N\}$.

Proof: From Lemma 1, reaching condition of sliding mode can be described as:

$$\begin{aligned} V(t) &= \sum_{i=1}^{N} P_i(t) S_i(t) \dot{S}_i(t) \\ &= \sum_{i=1}^{N} P_i(t) \{G_i A_i x_i S_i(t) \\ &\quad + G_i f_i(x_i) S_i(t) \\ &\quad + G_i B_i (I + \Delta E_i) u_i S_i(t) \\ &\quad + G_i \sum_{j=1, j\ne i}^{N} h_{ij}(x_j) S_i(t) \\ &\quad + G_i \sum_{j=1, j\ne i}^{N} \Delta h_{ij}(x_j) S_i(t)\} \end{aligned}$$

Based on control law (7)~(9) and the assumption, there is

$$\begin{aligned} V(t) &= \sum_{i=1}^{N} P_i \{(G_i f_i(x_i) - \Delta E_i G_i A_i x_i) S_i \\ &\quad - \frac{1+\Delta E_i}{1-\eta_i} \left(\sigma_i + \delta_i \|x_i\|\right) |S_i| \\ &\quad + G_i \sum_{j=1, j\ne i}^{N} h_{ij}(x_j) S_i \\ &\quad + G_i \sum_{j=1, j\ne i}^{N} \Delta h_{ij}(x_j) S_i\} \\ &\le \sum_{i=1}^{N} P_i \{\left(\|G_i\| \|f_i(x_i)\| + \eta_i \|G_i\| \|A_i\| \|x_i\|\right) |S_i| \\ &\quad - \left(\sigma_i + \delta_i \|x_i\|\right) |S_i| \\ &\quad + \sum_{j=1, j\ne i}^{N} \|G_i\| \alpha_i \|x_j\| |S_i| \\ &\quad + \sum_{j=1, j\ne i}^{N} \|G_i\| \beta_i |S_i| \} \\ &\le \sum_{i=1}^{N} P_i \{\left(\|G_i\| \xi_i + \eta_i \|G_i\| \|A_i\|\right) \|x_i\| |S_i| \\ &\quad - \left(\sigma_i + \delta_i \|x_i\|\right) |S_i| \end{aligned}$$

$$+\sum_{j=1,j\neq i}^{N}\|G_i\|\alpha_i\|x_j\||S_i|$$

$$+\sum_{j=1,j\neq i}^{N}\|G_i\|\beta_i|S_i|\}$$

$$=V_1(t)+V_2(t)$$

$$V_1(t)=\sum_{i=1}^{N}P_i\{[\|G_i\|(\xi_i+\eta_i\|A_i\|)-\delta_i]\|x_i\||S_i|$$

$$+\sum_{j=1,j\neq i}^{N}\alpha_i\|G_i\|\|x_j\||S_i|\}$$

$$=\sum_{i=1}^{N}[\|G_i\|(\xi_i+\eta_i\ \|A_i\|)-\delta_i]\|x_i\|w_i$$

$$+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\alpha_i\|G_i\|\|x_j\|w_i$$

$$\leqslant -wR\sum_{i=1}^{N}\sum_{j=1}^{N}\|x_j\|+wR\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\|x_j\|$$

$$<0$$

$$V_2(t)=\sum_{i=1}^{N}P_i\left(\sum_{j=1,j\neq i}^{N}\beta_i\ \|G_i\|-\sigma\right)|S_i|$$

$$\leqslant\sum_{i=1}^{N}(N\beta\|G_i\|-\sigma)w_i$$

$$\leqslant 0$$

Then $V(t)<0$. Therefore, the uncertain interconnected system Σ_i is globally asymptotically stable. ■

IV. NUMERICAL EXAMPLE

In this section, we present an numerical example to illustrate the utility of the proposed decentralized variable structure controller.

Let us consider the two identical pendulums which are coupled by a spring and subject to two distinct inputs as shown in Fig. 1 [9].

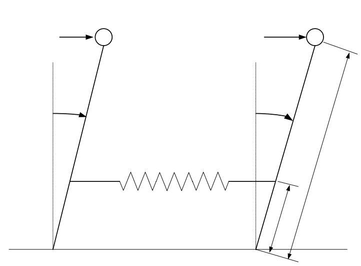

Fig.1 Two Inverted pendulums.

Choosing the state vectors $x_1=(x_{11},x_{12})^T=(\theta_1,\dot{\theta}_1)^T$ and $x_2=(x_{21},x_{22})^T=(\theta_2,\dot{\theta}_2)^T$, the model of the pendulums can be described as

$$\begin{pmatrix}\dot{x}_{11}\\ \dot{x}_{12}\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x_{11}\\x_{12}\end{pmatrix}+\begin{pmatrix}0&0\\-\dfrac{a^2(t)}{(1+\Delta m_1)l^2}&0\end{pmatrix}\begin{pmatrix}x_{11}\\x_{12}\end{pmatrix}$$
$$+\begin{pmatrix}0\\1\end{pmatrix}\left(1-\frac{\Delta m_1}{1+\Delta m_1}\right)u_1$$
$$+\begin{pmatrix}0&0\\ \dfrac{a^2(t)}{(1+\Delta m_1)l^2}&0\end{pmatrix}\begin{pmatrix}x_{21}\\x_{22}\end{pmatrix}$$

$$\begin{pmatrix}\dot{x}_{21}\\ \dot{x}_{22}\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x_{21}\\x_{22}\end{pmatrix}+\begin{pmatrix}0&0\\-\dfrac{2a^2(t)}{(1+\Delta m_2)l^2}&0\end{pmatrix}\begin{pmatrix}x_{21}\\x_{22}\end{pmatrix}$$
$$+\begin{pmatrix}0\\2\end{pmatrix}\left(1-\frac{2\Delta m_2}{1+\Delta m_2}\right)u_2$$
$$+\begin{pmatrix}0&0\\ \dfrac{2a^2(t)}{(1+\Delta m_2)l^2}&0\end{pmatrix}\begin{pmatrix}x_{11}\\x_{12}\end{pmatrix}$$

where $|\Delta m_1|<0.1$, $|\Delta m_2|<0.05$, $a(t)/l\in[0,1]$ is an uncertain value at time t.

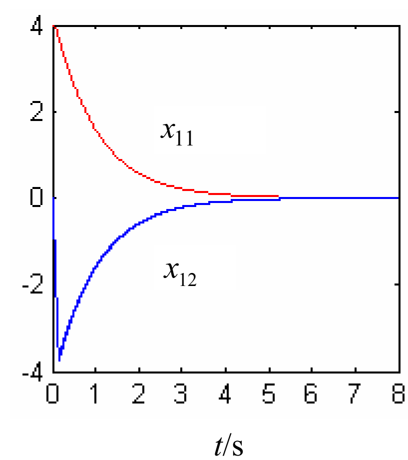

Fig.2 States response of the first subsystem

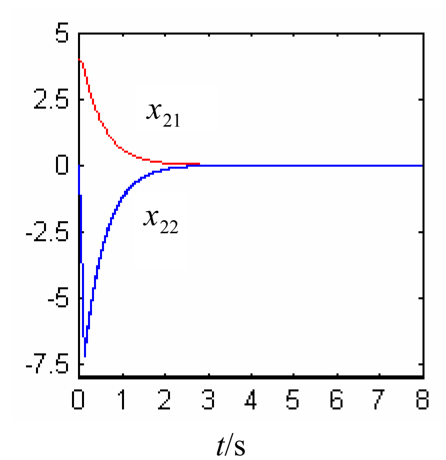

Fig.3 States response of the second subsystem

Let G_1 =[1 1], G_2 =[1 0.5]. We assume that the initial condition $x_1(0)$ =[4 0] and $x_2(0)$ =[4 0]. For the given system, we can design decentralized variable structure controller based on formula (7)~(9).

Fig.2 and Fig.3 are simulation results. Obviously the system is asymptotically stable and the sliding motion tends to the origin in finite time.

V. CONCLUSION

The principal contribution of this paper is that a new approach of decentralized variable structure control was proposed for uncertain interconnected system with parameters perturbations and relating influence. If the uncertainties are bounded and the interconnected items are unmatched, the new approach based on sliding mode variable structure control solved the control problem for a class of uncertain interconnected system. Compared with the existing results, the controller is simple. The approach can be applied for uncertain time variable system.

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