Extending classical pipeline models with Neural Nets

Gerhard Geiger, Drago Matko, Thomas Werner

Abstract— **The paper deals with the comparison of different mathematical models with regard to their usage in the model-based leak-monitoring scheme. The pipeline is represented as a two - port system. Four models are compared: a nonlinear distributed parameters model, a linearised model whose transcendent transfer function is obtained by a Laplace transformation and corresponding initial and boundary conditions, a simplified lumped parameter model and an extended neural net based model. All four models are tested on a real pipeline data with an artificially generated leak.**

Keywords— **Environmental and Safety Systems, Fault and Uncertainty Modelling in Dynamical systems, Process Supervision, Neural nets**

I. Introduction

Many of fluids transported by pipelines are in some sense dangerous. It is therefore often necessary to install leak-monitoring systems, especially due to legal regulations. Such systems can be treated as Technical Fault Diagnosis Systems [5]. Different methods are available, e.g. volume balance method, pressure wave method and gradient intersection method [3]. This paper is concerned with a model-based approach – with the use of a mathematical model description of a pipeline in the form of a pipeline observer. In the case of a leak, the leak flow and leak position can be calculated [4].

The aim of this paper is to compare different mathematical models with regard to their usage in the model-based leak-monitoring scheme. The basic model is a nonlinear distributed parameter model obtained by applying the principle of mass conservation and Newton's second law of motion [11]. Next two models are obtained by linearization and Laplace transformation leading to Multi-Input Multi-Output (MIMO) models [6], [7]. Last model is a neural net model which is is an addendum to nonlinear distributed parameter model and is obtained by the generalization of one of the linear models.

II. Observer Based Leak Monitoring

Observer-based leak monitoring requires a pipeline model in the form of a pipeline observer to compute the pipeline states assuming no leak. Further discussion will be focused on a discrete-time data processing scheme. The difference between the measured and estimated flow at inlet and outlet

$$x(k) = v(0,k) - \hat{v}(0,k) \quad (1)$$
$$y(k) = v(L,k) - \hat{v}(L,k) \quad (2)$$

where $v(0,k)$ and $\hat{v}(0,k)$ are the measured flow and the flow of the observer at the inlet of the pipeline, while $v(L,k)$ and $\hat{v}(L,k))$ are the measured flow and the flow of the observer at the outlet of the pipeline, will be referred to as residuals. The leak flow and position can be estimated by

$$v_{leak} = x(k) - y(k) \quad (3)$$
$$x_{leak}(k) = \frac{-y(k)}{x(k)-y(k)} \times L_p \quad (4)$$

where L_p denotes the length of the pipeline [4]. In order to eliminate flow measurement noise, a filter will be applied to the residuals leading to

$$\hat{v}_{leak} = x_f(k) - y_f(k) \quad (5)$$
$$\hat{x}_{leak}(k) = \frac{-y_f(k)}{x_f(k)-y_f(k)} \times L_p \quad (6)$$

with

$$x_f(k) = \frac{1}{N} \times \sum_{i=1}^{N} x(k-i+1) \quad (7)$$

and

$$y_f(k) = \frac{1}{N} \times \sum_{i=1}^{N} y(k-i+1) \quad (8)$$

In the above equations N is the depth of a FIFO-buffer and should be chosen so as to be appropriate for the noise statistics. In Figure 1 all basic parts of the described approach to the leak detection and classification can be seen.

III. Models for the observer design

Observer-based leak detection and localization schemes require a pipeline model to compute the states of a pipeline without a leak [2], [10]. The basic mathematical model of a pipeline is a nonlinear distributed parameter model. It describes the one-dimensional compressible fluid flow through the pipeline and is represented by a set of nonlinear partial differential equations [12]. No general closed-form solution of this equations are known yet. Numerical approaches like the Method of Characteristics must be used instead [1]. If the pipeline is operating in the vicinity of a working point, a linear model can be exploited. The transfer function of such model is obtained by the Laplace transformation of the linearized equations and corresponding initial and

Gerhard Geiger and Thomas Werner are with the Fachhochschule Gelsenkirchen, Neidenburger Str. 10, 45877 Gelsenkirchen, Germany. E-mail: gerhard.geiger@fh-gelsenkirchen.de; thomas.werner@fh-gelsenkirchen.de.

Drago Matko is with the Faculty of Electrical Engineering University of Ljubljana, Tržaška 25, 1000 Ljubljana Slovenia. E-mail: drago-matko@fe.uni-lj.si

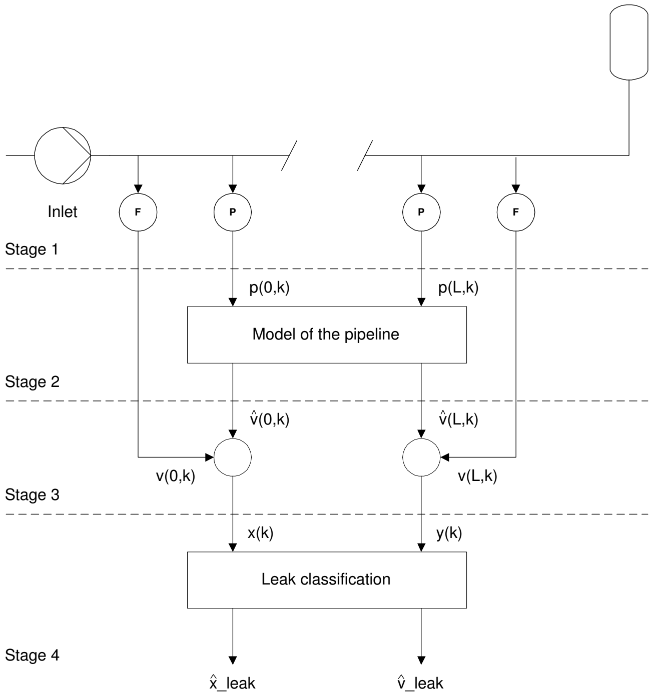

Fig. 1. Observer based leak monitoring

boundary conditions. The resulting transfer function is transcendent. Simple models of the pipeline in the form of a lumped parameter system can be obtained by a Taylor series expansion of transcendent transfer functions. The resulting algorithms are less time-consuming and hence better suited for critical real time applications. With neural net models the gray box approach is used: They are only an addendum to nonlinear distributed parameter models; their structure is obtained from linear models, as are the initial values of the weights. After teaching a nonlinear model is obtained. All four mathematical models of the pipeline will be given next. For simplicity reasons outputs of the models (which are actually outputs of observers) will be denoted by p and v rather than $\hat{p}$ and $\hat{v}$ respectively.

A. Non – linear pipeline model with distributed parameters.

The non-linear pipeline model with distributed parameters is obtained by application of the physical principles of mass conservation and Newton's second law. Applying these equations leads under the assumptions that the fluid is compressible, viscous, isentropic, homogenous and one-dimensional to the following coupled non-linear set of partial differential equations [1], [13]:

$$\frac{1}{a^2\bar{\rho}}\frac{\partial p}{\partial t} = -\frac{\partial v}{\partial x} \tag{9}$$

$$\bar{\rho}\frac{\partial v}{\partial t} + \bar{\rho} g \sin\alpha + \frac{\lambda(v)\bar{\rho}}{2D}v|v| = -\frac{\partial p}{\partial x} \tag{10}$$

where p is the pressure, v is the flow velocity, a the velocity of sound, $\bar{\rho}$ the constant density of the homogenous fluid, α the pipeline inclination, λ the dimensionless friction coefficient and D the diameter of the pipeline. The continuity and momentum equations (9) and (10) form a pair of quasilinear hyperbolic partial differential equations in term of two dependent variables, mass flow velocity $v(x,t)$ and pressure $p(x,t)$, and two independent variables, distance along the pipeline x and time t. A general solution is not available; however, a transformation into four ordinary differential equations grouped to two pairs of equations by the characteristics method is possible [14]. This method was realized in a special program PIPESIM which was used for the simulation of the nonlinear distributed parameters model.

B. Linear pipeline model – distributed parameters.

Nonlinear Eqns.(9, 10) are linearised and written in a form using notations common in the analysis of electrical transmission lines. Also, the gravity effect can be included into the working point so $\alpha = 0$ is supposed. The corresponding system of linear partial differential equations is

$$L\frac{\partial v}{\partial t} + Rv = -\frac{\partial p}{\partial x} \tag{11}$$

$$C\frac{\partial p}{\partial t} = -\frac{\partial v}{\partial x} \tag{12}$$

where $L = \bar{\rho}$, $R = \frac{\bar{\rho}\lambda|\bar{v}|}{D}$ ($\bar{v}$ is the flow velocity at the working point) and $C = \frac{1}{a^2\bar{\rho}}$ are the inertance (inductivity), resistance and capacitance per unit length, respectively. Introducing the *characteristic impedance* $Z_K = \sqrt{\frac{Ls+R}{Cs}}$ and $n = \sqrt{(Ls+R)\cdot Cs}$ the linearised model of the pipeline can be written in one of the following four forms which differ from each other with respect to the model inputs (independent quantities) and outputs (dependent quantities)

- *Hybrid representation: Inputs V_0,P_L, outputs V_L,P_0:*

$$\begin{bmatrix} P_0 \\ V_L \end{bmatrix} = \begin{bmatrix} \frac{1}{\cosh(nL_p)} & Z_K\tanh(nL_p) \\ -\frac{1}{Z_K}\tanh(nL_p) & \frac{1}{\cosh(nL_p)} \end{bmatrix} \begin{bmatrix} P_L \\ V_0 \end{bmatrix} \tag{13}$$

- *Hybrid representation: Inputs V_L,P_0, outputs V_0,P_L:*

$$\begin{bmatrix} P_L \\ V_0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\cosh(nL_p)} & -Z_K\tanh(nL_p) \\ \frac{1}{Z_K}\tanh(nL_p) & \frac{1}{\cosh(nL_p)} \end{bmatrix} \begin{bmatrix} P_0 \\ V_L \end{bmatrix} \tag{14}$$

- *Impedance representation: Inputs V_0,V_L, outputs P_0,P_L:*

$$\begin{bmatrix} P_0 \\ P_L \end{bmatrix} = \begin{bmatrix} Z_K\coth(nL_p) & Z_K\frac{1}{\sinh(nL_p)} \\ -Z_K\frac{1}{\sinh(nL_p)} & -Z_K\coth(nL_p) \end{bmatrix} \begin{bmatrix} V_0 \\ V_L \end{bmatrix} \tag{15}$$

- *Admittance representation: Inputs P_0,P_L, outputs V_0,V_L:*

$$\begin{bmatrix} V_0 \\ V_L \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_K}\coth(nL_p) & -\frac{1}{Z_K}\frac{1}{\sinh(nL_p)} \\ \frac{1}{Z_K}\frac{1}{\sinh(nL_p)} & -\frac{1}{Z_K}\coth(nL_p) \end{bmatrix} \begin{bmatrix} P_0 \\ P_L \end{bmatrix} \tag{16}$$

Linear models with distributed parameters were simulated by the convolution of the input signals and impulse responses of the corresponding transcendent transfer functions. The impulse responses were obtained by the inverse Fourier transformation of frequency responses. They were calculated in the discrete form and numeric problems were encountered resulting in small oscillations on both sides of impulses which represent the real impulse response. After applying the Hamming window to the frequency response the oscillations were nearly reduced, but the impulse response become "blurred". The reason for this is that the multiplication of the frequency response by the Hamming function $\frac{1}{2}*(1+\cos\frac{2*\pi*m}{N}) = \frac{1}{2}+\frac{1}{4}*e^{\jmath\frac{2*\pi*m}{N}})+\frac{1}{4}*e^{-\jmath\frac{2*\pi*m}{N}})$, where m is the frequency index and N is the number of points, results as the sum of three functions in the time domain. According to the theorems of the Fourier transforms, the first function is the half of the real inverse Fourier transform, the second and the third ones are one fourth of the real inverse Fourier transforms shifted for one sample to the right and left respectively. Since the sampling time was small this distortion has practically no effect on the calculated impulse responses for the function which has no immediate response $(\frac{1}{Z_K}\frac{1}{\sinh(nL_p)})$. However with functions with immediate response $(\frac{1}{Z_K}\coth(nL_p)$ the one sample shift to the left represents the shift to the right for $N-1$ samples, i. e. to the end of the periodic signal. This effect results in an incorrect impulse response. It was eliminated by setting the first value of the impulse response to the sum of the first and last value and by setting the last value of the impulse response to zero.

C. Linear pipeline model – lumped parameters.

Each of the transcendent transfer functions of the impedance and admittance forms can be presented as a second order transfer function by expanding the transcendent transfer function into a Taylor series. Only admittance representations, used in the observer based leak monitoring will be given here and are as follows:

1. **Admittance representation:** $\frac{1}{Z_K}\coth(nL_p)$

• Input admittance at downstrem reservoir: $\frac{V_0}{P_0}\Big|_{P_L=0}$

• Negative output admittance at upstream reservoir: $-\frac{V_L}{P_L}\Big|_{P_0=0}$

$$\frac{1}{Z_K}\coth(nL_p) = \frac{\sqrt{Cs}\coth(\sqrt{(Ls+R)Cs}L_p)}{\sqrt{Ls+R}} \approx$$
$$\approx \frac{(\frac{1}{2}{L_p}^2LC+\frac{1}{24}{L_p}^4R^2C^2)s^2+\frac{1}{2}{L_p}^2RCs+1}{\frac{1}{3}{L_P}^3RLCs^2+(L_pL+\frac{1}{6}{L_p}^3R^2C)s+L_pR} \tag{17}$$

This transfer function describes the change of the flow velocity at one end of the pipeline if the pressure is changing at the same end while the pressure at the other end remains constant.

2. **Admittance representation:** $\frac{1}{Z_K\sinh(nL_p)}$

Negative reverse transadmittance at upstream reservoir: $-\frac{V_0}{P_L}\Big|_{P_0=0}$

Transadmittance at downstream reservoir: $\frac{V_L}{P_0}\Big|_{P_L=0}$

$$\frac{1}{Z_K\sinh(nL_p)} = \frac{\sqrt{\frac{Cs}{Ls+R}}}{\sinh(\sqrt{(Ls+R)Cs}L_p)} \approx$$
$$\approx \frac{1}{\frac{1}{3}{L_P}^3RLCs^2+(L_pL+\frac{1}{6}{L_p}^3R^2C)s+L_pR} \tag{18}$$

This transfer function describes the change of the flow velocity at one end of the pipeline if the pressure is changing at the other end while the pressure at the same end remains constant.

Both transfer functions (17) and (18) have a static gain $\frac{1}{L_pR}$ which corresponds to the static change of the flow due to changing pressure.

D. Yet another linear model of the pipeline

In this Section a linear model will be derived which enables a simple design of Neural net models. The flow at the outlet of the pipeline can be expressed from Eq. (16) as follows:

$$\begin{aligned} V_L &= \frac{2}{e^{nL_p}-e^{-nL_p}}\frac{P_0}{Z_K} - \frac{e^{nL_p}+e^{-nL_p}}{e^{nL_p}-e^{-nL_p}}\frac{P_L}{Z_K} \\ &= \frac{2e^{-nL_p}}{1-e^{-2nL_p}}\frac{P_0}{Z_K} - \frac{1+e^{-2nL_p}}{1-e^{-2nL_p}}\frac{P_L}{Z_K} \end{aligned} \tag{19}$$

or equivalently

$$V_L - e^{-2nL_p}V_L = 2e^{-nL_p}\frac{P_0}{Z_K} - \frac{P_L}{Z_K} - e^{-2nL_p}\frac{P_L}{Z_K} \tag{20}$$

The same flow can be expressed also from Eq. (13):

$$\begin{aligned} V_L &= \frac{2}{e^{nL_p}+e^{-nL_p}}V_0 - \frac{e^{nL_p}-e^{-nL_p}}{e^{nL_p}+e^{-nL_p}}\frac{P_L}{Z_K} \\ &= \frac{2e^{-nL_p}}{1+e^{-2nL_p}}V_0 - \frac{1-e^{-2nL_p}}{1+e^{-2nL_p}}\frac{P_L}{Z_K} \end{aligned} \tag{21}$$

or equivalently

$$V_L + e^{-2nL_p}V_L = 2e^{-nL_p}V_0 - \frac{P_L}{Z_K} + e^{-2nL_p}\frac{P_L}{Z_K} \tag{22}$$

Adding Eqns. (20) and (22) and dividing by 2 we get the equation which describes the outlet flow in dependence of the inlet flow and both (inlet and outlet) pressures:

$$V_L = e^{-nL_p}V_0 + e^{-nL_p}\frac{P_0}{Z_K} - \frac{P_L}{Z_K} \tag{23}$$

In the same way the inlet flow can be rewritten from Eq. (16):

$$\begin{aligned} V_0 &= -\frac{2}{e^{nL_p}-e^{-nL_p}}\frac{P_L}{Z_K} + \frac{e^{nL_p}+e^{-nL_p}}{e^{nL_p}-e^{-nL_p}}\frac{P_0}{Z_K} \\ &= -\frac{2e^{-nL_p}}{1-e^{-2nL_p}}\frac{P_L}{Z_K} + \frac{1+e^{-2nL_p}}{1-e^{-2nL_p}}\frac{P_0}{Z_K} \end{aligned} \tag{24}$$

or equivalently

$$V_0 - e^{-2nL_p} V_0 = -2e^{-nL_p} \frac{P_L}{Z_K} + \frac{P_0}{Z_K} + e^{-2nL_p} \frac{P_0}{Z_K} \quad (25)$$

and also from Eq. (14):

$$\begin{aligned} V_0 &= \frac{2}{e^{nL_p} + e^{-nL_p}} V_L + \frac{e^{nL_p} - e^{-nL_p}}{e^{nL_p} + e^{-nL_p}} \frac{P_0}{Z_K} \\ &= \frac{2e^{-nL_p}}{1 + e^{-2nL_p}} V_L + \frac{1 - e^{-2nL_p}}{1 + e^{-2nL_p}} \frac{P_0}{Z_K} \end{aligned} \quad (26)$$

or equivalently

$$V_0 + e^{-2nL_p} V_0 = 2e^{-nL_p} V_L + \frac{P_0}{Z_K} - e^{-2nL_p} \frac{P_0}{Z_K} \quad (27)$$

Adding Eqns. (25) and (27) and dividing by 2 the equation which describes the inlet flow in dependence of the outlet flow and both (inlet and outlet) pressures is obtained:

$$V_0 = e^{-nL_p} V_L - e^{-nL_p} \frac{P_L}{Z_K} + \frac{P_0}{Z_K} \quad (28)$$

Equations (23) and (28) indicate that the flow on one side of the pipeline depends on the pressure on the same side (corresponding transfer function is $1/Z_K$) and on the pressure and flow on the other side of the pipeline (corresponding transfer functions are e^{-nL_p}/Z_K and e^{-nL_p} respectively). This fact will be used when designing the neural net model of the pipeline.

E. Neural Net model of the pipeline

The models derived in sections III-B, III-C and III-D are linear models. The pipeline is, however a nonlinear process. Neural nets are capable to model nonlinear phenomena, so it seems to be reasonable to apply them. However neural nets are essentially lumped parameter systems. Lumped parameter models in Section III-C were obtained from distributed parameter models in Section III-B by expanding the transcendent transfer function in Taylor series. Such models can be easily realized by a neural net - actually there is only one linear neuron and some delays at the input and feedback. So one way to construct a neural net model is to extend a linear lumped parameter model, i.e. a linear neuron, by one or several layers. A drawback of this model is that it does not involved the time delays, which are inherently involved in a distributed parameter model. An other way to design a neural net model of the pipeline is to use models of Section III-D An attempt to approximate the transcendent transfer functions $1/Z_K = \sqrt{Cs}/\sqrt{Ls+R}$ and $e^{-\sqrt{(Ls+R)\cdot CsL_p}}$ by rational ones was made in [8]. Following the same paradigm a neural net could be designed; but since the square root has no Taylor series expansion around zero (in the mentioned paper the expansion was done by Padé approximation and is not valid for $s = 0$), the long term stability of such models is doubtful. Approach used in this paper is similar to the previous mentioned one, however the neural net is used for correction only. Actual pipeline neural net model consist of two parts. Basic part is the nonlinear distributed parameter model (the so-called PIPSIM model). Parallel to it there is a neural net correction model, with outputs $\tilde{v}(0, k)$ and $\tilde{v}(N, k)$ which are corrections of the inlet and outlet velocities respectively. The inputs to the neural net are determined by equations (23) and (28) which indicate that the flow at inlet depend on the pressure at inlet (transfer function $1/Z_K$), on the pressure at the outlet (transfer function e^{-nL_p}/Z_K) and on the flow at the outlet (transfer function e^{-nL_p} respectively). Since the transfer function e^{-nL_p} involves a time delay (time needed by waves to travel along the pipeline is N time steps, where N is the number of segments in the PIPESIM model), N-steps delayed signals of $p(N)$ and $v(N)$ are used as input to the neural net. Additional n input delays are used to enable the neural net to correct dynamic phenomena which are not involved in the PIPESIM model. So the inlet and outlet velocity corrections can be written as

$$\begin{aligned} \tilde{v}(0, k) = f(&p(0, k), \ldots, p(0, k-n), p(N, k-N), \ldots, \\ &p(N, k-N-n), v(N, k-N), \ldots, v(N, k-N-n)) \end{aligned} \quad (29)$$

$$\begin{aligned} \tilde{v}(N, k) = f(&p(0, k-N), \ldots, p(0, k-N-n), p(N, k), \\ &\ldots, p(N, k-n), v(0, k-N), \ldots, v(N, k-N-n)) \end{aligned} \quad (30)$$

Equations 29 and 30 were realized by two neural nets. A three layers (3+8+1 neuron) neural net, depicted in Fig. 2 has proven itself as suitable trade-off between complexity and time needed to be taught. A tapped delay line with delays of 1 to 11 seconds was used at the input which has made the neural net dynamic.

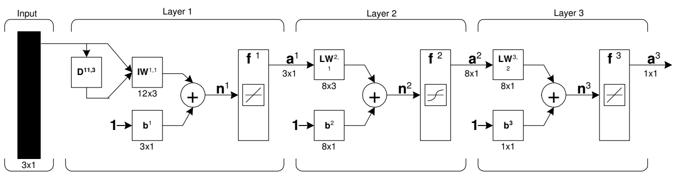

Fig. 2. Neural net

IV. Application to a real pipeline

The models were tested using data obtained by a leak experiment on a real pipeline with the following data: Length of the pipeline $L_p = 9854\ m$, diameter $D = 0.2065\ m$, relative roughness $k_c = 0.0602\ mm$, inclination $\alpha = -0.1948\ ^o$ and the fluid data: Density $\rho = 680\ kg/m^3$, kinematic viscosity $v = 7.0 \times 10^{-7}\ m^2/s$ and velocity of sound $a = 951\ m/s$. The stationary fluid velocity prior to the leak occurrence was 2.45 m/s. A 5%-leak rate (0.1225 m/s corresponding to 14.77 m^3/h) was generated at $t = 473\ s$ at 56.4 % of the pipeline length where the outrunning fluid was filled into a tank lorry.

The parametrization of programs was done by an other no leak experiment with similar (2.46 m/s) stationary fluid

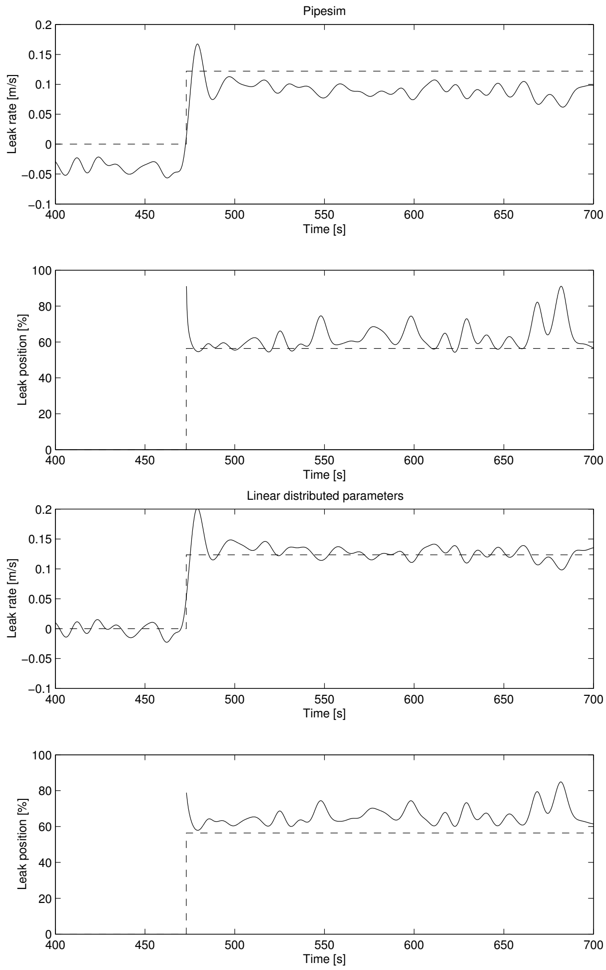

Fig. 3. Estimated leak rate (upper) and estimated leak location (lower) for the PIPESIM (left) and the Linear distributed parameter model (right) models respectively

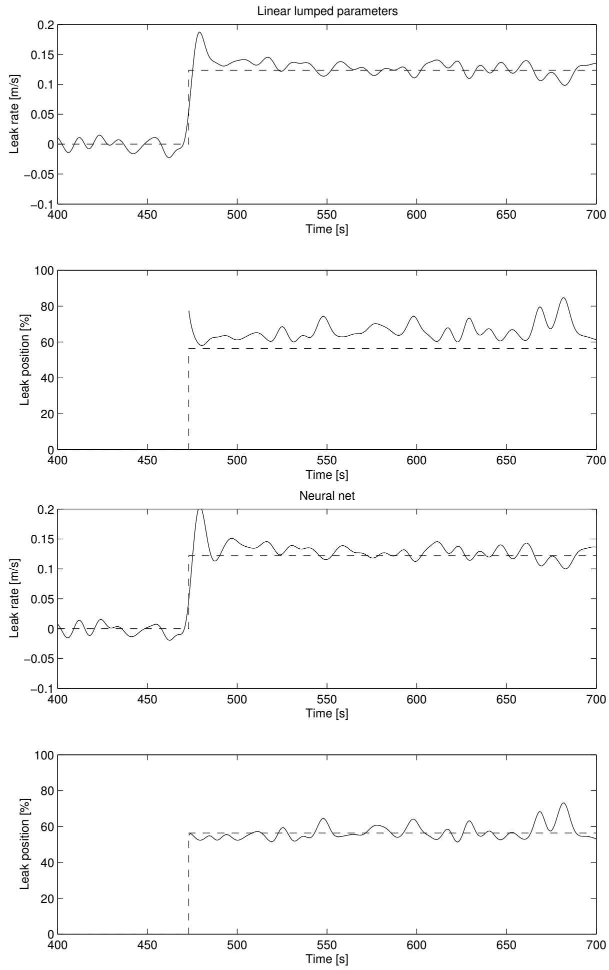

Fig. 4. Estimated leak rate (upper) and estimated leak location (lower) for the Linear lumped parameter (left) and the Neural net models (right) respectively

velocity. By this experiment a shunt valve at the beginning of the pipeline was closed leading to a quick drop in pressure and causing fluid transients. For the PIPESIM program only the relative roughness was calculated from the stationary working point. For both linear models additionally the stationary working points (at the inlet and outlet respectively) of the mentioned experiment were used. Neural net was taught by the residuals (without leak) of the PIPESIM results.

In Figures 3 and 4 the estimated leak rate and leak location are depicted for the PIPESIM, the Linear distributed parameter, the Linear lumped parameter and the Neural net model, respectively. Standard deviations (with respect to the real mean values) and deviations of the mean values of the rate and position errors for all models in percent are depicted in Tables I and II respectively.

The PIPESIM program results are biased in both, the estimation of leak rate and position. This is due to the bias in the fluid velocity sensors and due to the non-modelled dynamics. All other methods exhibit a small bias (approximately 3%) for the leak rate. This is probably due to approximate evaluation of the real leak rate, which was done manually. Linear models use the working point of a similar experiment, so the offsets in leak rate (which are actually the differences of residuals) were eliminated. However bias in the estimate of the leak position remained due to non-modelled effects. Some of these effects were incorporated in the neural net model, which has the smallest bias of the estimated position.

TABLE I

Standard deviations of the rate and position errors for all models in percent

%	PIPESIM	Lin. distr.	Lin. lump.	NNet
Rate	28.4249	8.7816	8.6543	9.3034
Position	9.4026	10.8520	10.8318	4.2284

TABLE II

Deviation of the mean values of rate and position errors for all models in percent

%	PIPESIM	Lin. distr.	Lin. lump.	NNet
Rate	27.1487	3.0323	3.2328	3.8471
Position	6.3811	9.6956	9.6815	0.5350

V. Conclusion

Four models of the pipeline: the nonlinear distributed parameters model, the linear distributed parameters model, the linear lumped parameters model and the Neural net model were used for leak detection and localization. The nonlinear distributed parameters model was simulated using a special program PIPESIM. In its original form it serves biased (in leak rate and position) results due to offsets in sensors and some non-modelled effects. Linear models eliminate the sensor offsets by definition, however they provide useful results only for very small changes of the signals around a working point. Besides they can not cope with comprehensive nonlinear effects of the pipeline, so the leak position estimate is biased. Best results were obtained with an neural net model as an addendum to the PIPESIM model. The neural net eliminates the sensor bias and incorporates some nonlinear effects which are not modelled by the PIPESIM program.

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