Space Time Adaptive Processing Using State-Space Methods and Kalman Filtering

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***Abstract*— In this paper, we introduce a novel approach of estimating the waveform of planar waves impinging on array of antennas. Our approach concurrently addresses issues in Space-Time Adaptive Processing design: 1) array gain, 2) system uncertainties–include gain and phase response errors introduced by receiver channel and antenna location perturbation–and 3) mutual coupling. A pre-processor consisting of a Kalman filter and shooting point method is constructed to mitigate the degrading effects of these uncertainties in radar systems. The results of the simulations show that the pre-processor can isolate discrete interferers and jammers and can reject clutter and system uncertainties.**

***Index Terms*—Kalman filter, Space-Time Adaptive Processing**

I. Introduction

In radar systems, Space Time Adaptive Processing (STAP) is utilized to detect the presence, and to pin-point the location and the velocity of small, maneuverable target in the presence interference and noise. The interference may include natural as in ground and weather clutters, deliberate as in jammers and coincidental presence of another discrete scatterer. The STAP weights applied to the received signals are designed to preserve the target signal while rejecting the effects of the interference signals and noise. STAP is used in conjunction with spatially separated array antennas emitting coherent pulses. The spatial separations of the antennas are essential to provide the spatial processing dimension. Repetitive identical pulses provide the temporal processing dimension.

Some issues that STAP designers are concerned with in optimizing their algorithms are array gain, sensor perturbations, mutual coupling and finite sampling. Incorrect methods of dealing with these issues lead to performance below optimum level.

The environment of the desired signal is surrounded by spatially separated interferences and uncorrelated measurement receiver noise. Existing methods of dealing with discrete interferers and jammers involve placing nulls on the beamformer in the direction of the interferences. The drawbacks of nulls on the beamformers are the reduction of available degree of freedom and the decrease of the array gain [1]. In fact, some researchers [i.e.8, 9] discourage insertion of constraints, i.e. null contraints, on the spatial weights vector because it may loose the orthogonality property of the transformation matrix from space-time domain to beam-space domain.

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Errors in the modeling of the antenna and receiver channel characteristics lead to incorrect derivation of the weight vector of the STAP. Sensor modeling uncertainties due to antenna-location perturbation and gain and phase response errors are currently being compensated by adding diagonal loading on the spatial covariance matrix of the measured signals. The amount of loading added is arbitrarily assigned and is not representative of the actual system. Receiver channels, which distort the received signal as it propagates in the circuitry (modulator, low pass filter, A/D converter, matched filter), are partially compensated by attempting to equalize the channel effect. The errors in the calibration of the channel amplify the amplitude and add distortion in the phase of the received signal.

Array theory neglects the effect of mutual coupling. However, it has been shown in the literatures [2, 3], that the response of an isolated antenna difference considerably with array of antennas. The degradation of array performance with mutual coupling has been shown in [4, 5, 6].

Our approach to solve these concerns is by appending a Kalman filter utilizing a shooting point method as a pre- processor to the beamformer. Its functions are to reject the errors due to sensor perturbations, gain and phase response errors, mutual coupling, and isolated and null directional interference if it exists. It is a recursive method operating on the actual range bin under test.

The paper is organized to show how the format of the signal prior to the STAP process. Section II starts by defining the signal transmitted by the array and the reflected planar wave. Events, such as system uncertainties and mutual coupling, which distort the received signals, are discussed. Section III discusses the insertion of the pre-processor to the conventional STAP radar process. Section IV derives the Kalman filter used in the pre-processor. And Section V shows simulation results of our approach.

II. System model

1) Propagating signals: In a phase-array antenna (refer to [7, 10, 11] for a more complete derivation), a narrow-band emitted signal, $e(t)$, consists of a pulse modulated by a sinusoid with carrier frequency f_c

$$e(t) = \sqrt{2P_e}E(t)\cos(2\pi f_c t) \tag{1}$$

where P_e is the transmission power and $E(t)$ is the envelop of the pulse. All the transmitted pulses are identical and are periodically transmitted at constant Pulse Repetition Frequency (PRF) within a Coherent Processing Interval (CPI).

When the emitted signal reaches an object, a wave is reflected towards the antenna array of the form

$$r(t) = \sqrt{2P_r}E(t-\tau)\cos(2\pi f_c(t-\tau)) \tag{2}$$

where P_r is the power of the received propagating wave, and τ is the round-trip delay in terms of the distance between the array and the target and speed of light c.

Equation 2 goes through the receiver channel of the antenna to arrive with $s(t)$:

$$s(t) = \sqrt{2P_r}e^{j2\pi f_c \frac{2v_{rad}}{c}t} = \sqrt{2P_r}e^{j\omega_D t} \tag{3}$$

where ω_D is known as the Doppler frequency.

Equation 3 contains the information about the object that reflected the planar wave. The reflected power is proportional to the size of the target and the Doppler frequency describes the motion of the target with respect to the array platform. The presence of the target is declared by comparing P_r to a threshold value and the motion is determined from non-zero Doppler frequency. It is important to note that Equation 3 is the desired signal that STAP algorithms try to estimate.

2) Received signals: Each of the antenna and its receiving channel amplifies the impinging signal as a function of the arrival direction of the planar wave:

$$\mathbf{G}(\theta,\phi) = \begin{bmatrix} \gamma_1(\theta,\phi)e^{j\psi_1(\theta,\phi)} \\ \gamma_2(\theta,\phi)e^{j\psi_2(\theta,\phi)} \\ \vdots \\ \gamma_N(\theta,\phi)e^{j\psi_N(\theta,\phi)} \end{bmatrix} \tag{4}$$

where γ_i and ϕ_i are the gain and phase response, respectively, of the i^{th} antenna and its corresponding channel.

The signal impinging on each antenna in a vector form is

$$\mathbf{s}(t,\mathbf{r}) = \begin{bmatrix} s(t-\tau_0) \\ s(t-\tau_1) \\ \vdots \\ s(t-\tau_{N-1}) \end{bmatrix} \tag{5}$$

where $\mathbf{r}$ is the array location matrix, τ_i is the time delay of the propagating signal to reach the i^{th} antenna in an N element array, and $s(t)$ is the signal that would have been received at the origin (XYZ plane of the array). It can be further simplified as

$$\mathbf{s}(t,\mathbf{r}) = s(t)\mathbf{V}(\mathbf{k}) \tag{6}$$

where $\mathbf{V}(\mathbf{k})$ is known as the spatial steering vector and $\mathbf{k}$ is known as the wavenumber given by

$$\mathbf{V}(\mathbf{k}) = \begin{bmatrix} e^{-j\mathbf{k}^T\mathbf{r}_0} \\ e^{-j\mathbf{k}^T\mathbf{r}_1} \\ \vdots \\ e^{-j\mathbf{k}^T\mathbf{r}_N} \end{bmatrix}, \quad \mathbf{k} = -\frac{2\pi}{\lambda}\begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\phi \end{bmatrix} \tag{7}$$

and $\mathbf{r}_i$ is the XYZ coordinate of the i^{th} antenna.

$s(t)$ as defined in (3) incorporates the information of the reflected planar wave, $\mathbf{V}(\mathbf{k})$ contains the spatial characteristics of the antenna array and $\mathbf{k}$ contains the statistics of the source of the signal.

If the signal is sampled at the same speed as the PRF within a M-pulse CPI, then the well known space-time data, $\mathbf{y}(t)$, is acquire:

$$\mathbf{y}(t) = |s(t)|\mathbf{T}(\omega_{\mathbf{D}}) \otimes \mathbf{V}(\mathbf{k}) \tag{8}$$

where $\otimes$ represents Kronecker product and the temporal steering vector, $T(\omega_D)$, is given by

$$T(\bar{\omega}) = \begin{bmatrix} 1 & e^{j2\pi\bar{\omega}} & e^{j4\pi\bar{\omega}} & \cdots & e^{j(M-1)\pi\bar{\omega}} \end{bmatrix}^T \tag{9}$$

3) System uncertainties: Combining Equations (3), (4), and (7), the spatial sample of a single reflected signal from a target object to the antenna can be summarized by:

$$\mathbf{x}(t) = \gamma(\theta,\phi)e^{j\psi(\theta,\phi)}e^{-j\mathbf{r}^T\mathbf{k}}s(t). \tag{10}$$

Equation (10) is the ideal signal that would be received if the antenna characteristic and response are accurately modeled. Unfortunately, circumstances, including sensor perturbation, gain and phase errors and mutual coupling, arise that will cause it to deviates from this ideal case.

a) Sensor perturbation: Sensor perturbation exists when the antenna is located in a place away from the designed value. Sensor perturbation may occur when the platform of the antenna is temporarily tilted in an angle during operation. This is common when the array platform is placed on an airplane, which encounters constant turbulence.

The mathematical equations of the sensor perturbation are

$$(r_x)_i = (r_x)_i^n + (\triangle r_x)_i \tag{11}$$
$$(r_y)_i = (r_y)_i^n + (\triangle r_y)_i \tag{12}$$
$$(r_z)_i = (r_z)_i^n + (\triangle r_z)_i \tag{13}$$

where subscript i represents i^{th} antenna in the array, superscript n represents nominal value, and $(\triangle r_x)$, $(\triangle r_y)$, and $(\triangle r_z)$ are the position perturbation in the X,Y,Z plane respectively.

The extent of the errors due to $(\triangle r_x)$, $(\triangle r_y)$, and $(\triangle r_z)$ can be shown to be:

$$e^{-j[(r_x)_i^n,(r_y)_i^n,(r_z)_i^n]k}j\frac{2\pi}{\lambda}(\triangle r_x)_i\sin\theta\cos\phi \tag{14}$$
$$e^{-j[(r_x)_i^n,(r_y)_i^n,(r_z)_i^n]k}j\frac{2\pi}{\lambda}(\triangle r_y)_i\sin\theta\sin\phi \tag{15}$$
$$e^{-j[(r_x)_i^n,(r_y)_i^n,(r_z)_i^n]k}j\frac{2\pi}{\lambda}(\triangle r_z)_n\cos\phi) \tag{16}$$

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b) Gain and phase errors: Gain and phase response occurs when the antenna and/or its corresponding receiver channel drifts performance over time and/or the responses has been inaccurately measured in the lab because the process of generating angles of the directional waves is an estimate. The gain and phase errors can be modeled as:

$$\gamma(\theta,\phi)_i = \gamma(\theta,\phi)_n^i(1+\triangle\gamma(\theta,\phi)_i) \tag{17}$$
$$\psi(\theta,\phi)_i = \psi(\theta,\phi)_i^n + \triangle\psi(\theta,\phi)_i \tag{18}$$

where $\triangle\gamma(\theta,\phi)$ and $\triangle\psi(\theta,\phi)$ are the gain and phase errors, respectively. Similarly, it can be shown that faults contributed by these errors are:

$$\gamma(\theta,\phi)_n^n e^{\psi(\theta,\phi)_n^n}\triangle\psi_n \quad (19)$$
$$\gamma(\theta,\phi)_n^n e^{\psi(\theta,\phi)_n^n} j\triangle\gamma_n \quad (20)$$

c) Mutual coupling: Mutual coupling in the receiver mode occurs when portion of the planar wave received by neighboring antennas is retransmitted and is received by the antenna as part of the original planar wave. The coupling between antennas is proportional to the impedance, Z, parameters of the array and is determined by the voltage-current relationship

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix}^T = Z \begin{bmatrix} i_1 & i_2 & \cdots & i_N \end{bmatrix}^T \quad (21)$$

where v_j is the voltage at j^{th} antenna port, i_j is the current flowing in the antenna's circuitry, and Z is given by

$$\begin{bmatrix} z_L+z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & z_L+z_{22} & \cdots & z_{2N} \\ \vdots & \cdots & \ddots & \vdots \\ z_{N1} & \vdots & \cdots & z_L+z_{NN} \end{bmatrix} \quad (22)$$

where z_L is the load impedance, z_{kk} is called self impedance and $z_{jk}, j \neq k$ is mutual impedance measured with the following equations:

$$z_{jj} = \frac{v_j}{i_j}\Big|_{v_k=0, j\neq k}, \quad z_{jk} = \frac{v_j}{i_k}\Big|_{I_i=0} \quad (23)$$

Normalizing **Z** with z_L:

$$\begin{bmatrix} 1+\frac{z_{11}}{z_L} & \frac{z_{12}}{z_L} & \cdots & \frac{z_{1N}}{z_L} \\ \frac{z_{21}}{z_L} & 1+\frac{z_{22}}{z_L} & \cdots & \frac{z_{2N}}{z_L} \\ \vdots & \cdots & \ddots & \vdots \\ \frac{z_{N1}}{z_L} & \vdots & \cdots & 1+\frac{z_{NN}}{z_L} \end{bmatrix} \quad (24)$$

where z_{jk}/z_L is the coupling contribution of the k^{th} antenna to j^{th} antenna. Notice that the mutual coupling matrix depends only on the configuration and response of the antenna and is independent on the received signal.

III. Pre-processor

As shown in Figure 1, the pre-processor is a Kalman filter utilizing a shooting point method. The inputs to the Kalman filter are spatial samples for a fixed time of the received signal. The Kalman filter propagates through antenna measurement (the first N elements of the space-time data which has the same format as Equation (8)) to estimate corrupted signals due model uncertainties, mutual coupling and noise. The shooting point method is used to propagate backward and estimate the initial state of the Kalman filter for the new set of spatial samples in the next sampling time (the next noisy N elements of the space-time data)). The initial state of the Kalman filter is a vector of estimate of all discrete scatterer including the desired target signal.

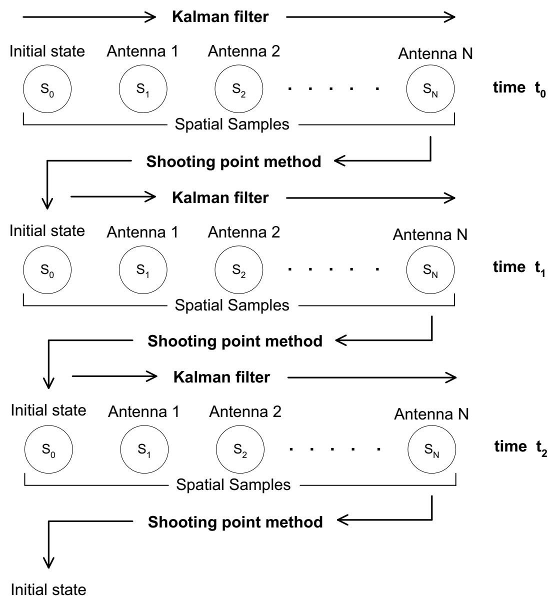

Fig. 1. Pre-processor block diagram

The estimate state converges to a solution independent of the initial conditions within several forward (Kalman Filter)-backward (Shooting Point Method) iterations. The states of the shooting point methods are signals of both target and interference, which the direction of the source is known.

The signals passed from the pre-processor to the beamformer consist of only the estimated target signal or the estimated target signal corrupted only by other discrete interferer not accounted for in the pre-processor. Directional discrete interferences that are estimated by the pre-processor are not passed to the beamformer. In addition, the effects of thermal noise, sensor perturbation, and coupling has been suppress before signals are passed.

IV. State Estimator

The Kalman filter is constructed from the signal models (similar derivation has been done in [12, 13]) derived in Section II condensed in a matrix form as:

$$X = Z(G+\triangle G)\odot(V+\triangle V)S \quad (25)$$

where $\odot$ denotes Hadamard product, Z is given by Equation (24), G by Equation (4), $\triangle G$ by Equations (19), and (20), V by Equation (7), $\triangle V$ by Equations (14), (15), and (16) and S by Equation (3). It is assumed that $\triangle r_x$, $\triangle r_y$, $\triangle r_z$, $\triangle\gamma(\theta,\phi)$, and $\psi(\theta,\phi)$ to have Gaussian distribution.

The Kalman filter is used primarily to filter out the contribution of $\triangle G$, $\triangle V$ and Z of the received signal. It uses apriori knowledge of the distribution of $\triangle G$ and $\triangle V$ to determine the error signals and uses Z to decouples the measured signals. The shooting point method uses the states of Kalman filter to estimate the actual waveforms of the reflected signals.

The system and measurement models for the Kalman filter assuming the major source of mutual coupling for the antennas are its two immediate neighbors:

$$x_{k+1}(t) = \Phi_k x_k(t) + w_k(t) \quad k \in [0, N-1] \quad (26)$$

$$z_k(t) = H_k x_k(t) + v_k(t) \quad k \in [1, N] \tag{27}$$

$$x_0\ (\bar{x}_0, P_{x_0}), \qquad w_k\ (0, Q_k), \qquad v_k\ (0, R_k) \tag{28}$$

where

- $x_k \; \epsilon \; R^{3M\times 1}$ is the state vector containing three s(t) received by three different antennas and M is the number of target and interference signals. The second M elements of x_k is the s(t) by the k^{th} antenna and the first and third M elements of x_k are the s(t)s of the immediate neighbors of the k^{th} antenna.
- $\Phi_k \; \epsilon \; R^{3M\times 3M}$ is the state transition matrix defined by:

$$\Phi(k) = diag\left[v_{k+1} \odot v_k^*\right] \tag{29}$$

$$\Phi_k = \begin{bmatrix} \Phi(k-1) & 0 & 0 \\ 0 & \Phi(k) & 0 \\ 0 & 0 & \Phi(k+1) \end{bmatrix} \tag{30}$$

  where v_k^T is the k^{th} row of the vector manifold matrix in Equation (7) and v_0 is column vector of 1s.
- w_k is an additive white processing noise consisting of

$$w(k) = \triangle\psi_k + j\triangle\gamma_k + j(\triangle r_x)_k \frac{2\pi}{\lambda}\sin\theta\cos\phi \tag{31}$$
$$+j(\triangle r_y)_k \frac{2\pi}{\lambda}\sin\theta\sin\phi + j(\triangle r_z)_k \frac{2\pi}{\lambda}\cos\phi$$

  and w_k is defined by

$$w_k = \begin{bmatrix} w(k-1) \\ w(k) \\ w(k+1) \end{bmatrix} \tag{32}$$

- $z_k \; \epsilon \; R^{1\times 1}$ represents the sample measured by the k^{th} antenna
- $H_k \; \epsilon \; R^{1\times 3M}$ is the measurement matrix defined by

$$\begin{bmatrix} \beta_k G_{k-1} & G_k & \alpha_k G_{k+1} \end{bmatrix} \tag{33}$$

  where G_k is the k^{th} row of the gain and phase response matrix in Equation (4); and β_k and α_k are the strength of the neighboring coupling and can be obtained from the elements of the impedance matrix in Equation (22).
- v_k is an additive white measurement noise describing thermal and mechanical noise present on the antenna.

The processing noise sequence $w_k(t)$ and measurement noise sequence $v_k(t)$ are independent of each other and have Gaussian distribution with zero means and covariances:

$$E\left[w_i(t)w_j^T(t)\right] = Q \quad \delta_{ij} \tag{34}$$

$$E\left[v_i(t)v_j^T(t)\right] = R \quad \delta_{ij} \tag{35}$$

The spatial update propagation of the filter states and covariance matrix is defined by the following equations:

$$\hat{x}_k^-(t) = \Phi_k \hat{x}_{k-1}^+(t) + B_k u_k(t) \tag{36}$$

$$P_k^-(t) = \Phi_k P_{k-1}^+(t)\Phi_k^T + G_k Q_k G_k^T \tag{37}$$

where the $\hat{x}_k^-(t)$ and $P_k^-(t)$ are the state estimate and covariance, respectively, of the k^{th} array element before the measurement is available.

Similarly, the measurement update propagation of the filter states and covariance matrix is defined by the following equations:

$$K_k(t) = P_k^-(t)(H_k(t)P_k^-(t)H_k^T(t) + R_k(t))^{-1} \tag{38}$$

$$P_k^+(t) = (I - K_k(t)H_k(t))P_k^-(t) \tag{39}$$

$$\hat{x}_k^+(t) = \hat{x}_k^-(t) + K_k(t)(z_k(t) - H_k(t)\hat{x}_k^-(t)) \tag{40}$$

where the $K_k(t)$ is the Kalman filter gain and $\hat{x}_k^+(t)$ and $P_k^+(t)$ are the state estimate and covariance, respectively, of the k^{th} array element after the measurement is available.

V. Representative examples

Consider a 54 element planar circular array. The antenna-locations are uniformly placed on the XY plane in a radius of 24 feet. Only the half 27 antennas facing the target object are active in transmitting signals at 300 MHz and receiving propagating waves.

In all of the simulations, we created one target object at broadside ($\theta = 0^0, \phi = 0^0$) and four directional interference sources at $[(-45^0, 0^0), (-50^0, 0^0), (30^0, 0^0), (35^0, 0^0)]$. The target signal and each of the interference signals are uncorrelated. Each point source is generating as continuous sinusoidal signal with varying amplitude and the frequencies are normalized to $\sqrt{2}$ of the carrier frequency. White noise due to thermal noise and vibrations of the sensors are added.

Three types of simulation scenario were constructed. The first simulation scenario, Section V-A, describes the circular array functioning in nominal mode. The second scenario, Section V-B, describes a situation when the responses and configuration of the array has been perturbed. And the third scenario, Section V-C, depicts the array with system uncertainties and the interfering signals are not completely modeled.

A. Without system uncertainties and complete knowledge of the interferences

In this simulation, it is assumed that the antenna configuration is perfectly modeled. All antenna locations are exactly known with no sensor perturbation. Planar waves pass through the antenna sensors with no amplification and no phase distortion. Each of the antenna elements is isolated and no coupling between elements occurs.

The direction of the target and the interference signals are completely known. All the source signals is generated with equipower at 0 dB. Background noise power of -30dB has been formed. Figure 2 shows the target (left graph) and interference (right graph) signals waveforms. All the estimated signals closely follow the desired signals. Only the target signal is passed through to the beamformer. If the beamformer is Minimum Variance Distortionless Response MVDR, then the output of the beamformer is the actual reflected target signal. The interference signals are properly detected and rejected by the pre-processor. Thus, no nulling operations are needed in the beamformer for the interference signals.

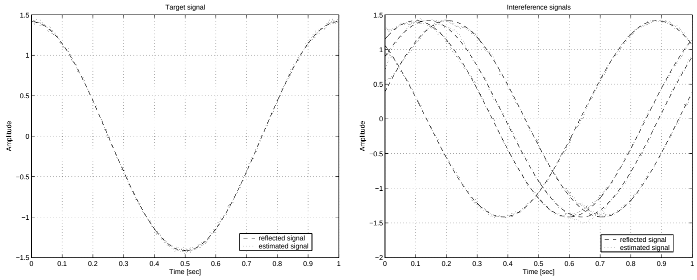

Fig. 2. Signal waveforms in a precisely modelled array and the Direction of Arrival (DOA) of the interference signals are known.

B. With system uncertainties and complete knowledge of the interferences

In this simulation, it is assumed that the array has antenna-location perturbation and response mismodelling. The sensors are displaced in the X, Y, Z plane with a variance of 10^{-3} m, 10^{-3} m, and 10^{-3} m respectively. The antenna gain and phase response has been measured incorrectly with a variance of 10^{-3} times the unity gain and 5^0, respectively. Each antenna element is experiencing coupling from its immediate neighbors with intensity of 0.1 times the received power of its immediate neighbors.

The direction of the target and the interference signals are completely known. All the source signals are generated with equipower at 0 dB. Background noise power of -30dB has been formed.

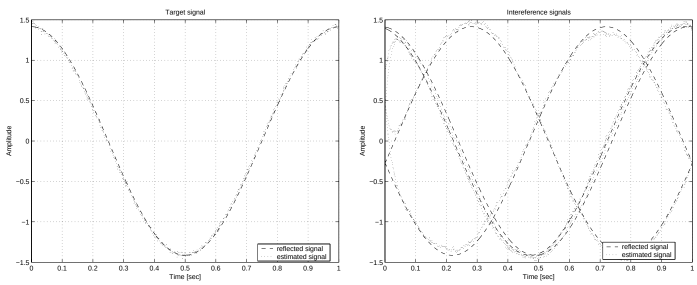

Fig. 3. Signal waveforms in a mismodeled array and the DOAs of the interference signals are known.

Figure 3 is target (left) and interference (right) signal waveforms, respectively. The graphs in the figures exhibit similar behaviors Figure 2 in Section V-B.

C. With system uncertainties and partial knowledge of the interferences

Similar case as in V-B but two additional interference signals at $[(\theta = -15, \phi = 0), (\theta = 25, \phi = 0)]$ are introduced and are intentionally not modeled in the pre-processor. The power of the target and the four original interference signals remain at 0 dB and the two new interference signals are set at -20 dB. Background noise power of -30dB has been formed. Figure 4 is the target (left) and interference (right) signal waveforms. The estimated signal waveforms deviate from the reflected signal waveforms due to the mismodeling of the Kalman filter (refer to [14]). However, it has been compensated for by the beamformer as shown in the left graph of Figure 4 when the estimated signal is process by an MVDR beamformer.

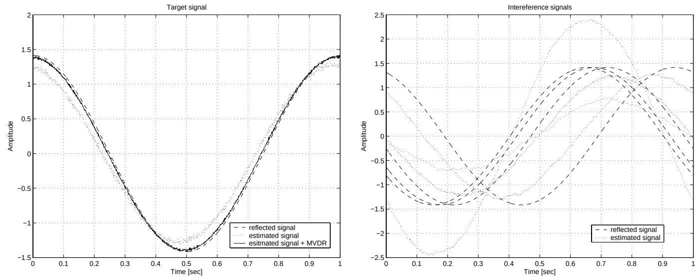

Fig. 4. Inaccurately modeled array and partial knowledge of the interference signals.

VI. Conclusion

The simulation shows that beamformers are not required to set nulls on interference signals with known direction of arrivals if the pre-processor is used. The waveforms of each propagating directional wave are accurately estimated and isolated by the pre-processor.

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