Dynamic Focusing of Awareness in Fuzzy Control Systems

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***Abstract*— Adaptive fuzzy logic control systems with Gaussian membership functions are described. A stability proof is given. A systematic simulation study of 'dynamic focusing of awareness' in fuzzy logic control systems is provided. This study shows how the final steady-state values of the membership functions change in response to varying initial membership functions, changing desired trajectory, and varying system nonlinearities. It is shown that the fuzzy logic control system is focusing on a different region of the state-space depending on these varying factors. Conclusions on higher-level behavior of the fuzzy logic control system are drawn.**

***Index Terms*— adaptive systems, fuzzy logic, nonlinear control,**

I. Introduction

Adaptive fuzzy logic (FL) systems are becoming more and more popular in control systems due to the ability to select initial membership functions (MFs) based on experience and intuition, and the ability to tune the MFs to learn about the unknown dynamics of the system. By now, proofs of the stability and performance of FL systems have been provided by a variety of researchers ([1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11] and others). However, the cognitive behavior of FL controllers has yet to be investigated. Specifically, it is not known how the MFs adapt in response to changing initial MF selections, different desired trajectories, and changing system dynamics. and how FL control systems emulate the higher human functions of consciousness including focusing of awareness and the filtering out of irrelevant details and noise.

In this paper, we provide a systematic simulation study of "dynamic focusing of awareness" and cognitive features in FL control systems. It appears that some cognitive functions of human awareness are reflected in the behavior of FL control systems. We define here cognitive function of the FL system as the ability of the FL system to acquire knowledge of the controlled plant in it's entirety, or , in another words, to establish the base functions needed for system identification over the whole state space involved in the system operation, including the history and the present states (we connect this with the "awareness" of the system), and the ability of the FL system to focus on the current state trajectories (we connect this with making proper judgment of which part of the space of awareness is important at the moment). Similar proofs for multiple input multiple output (MIMO) neural network control can be found in the literature and the work on extending the ideas described here is the current research topic of the authors.

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II. Dynamic System Preliminaries

In this paper the Frobenius norm $\|A\|_F^2$ is denoted by $\|\cdot\|$, unless otherwise specified.

Let the process have dynamics of the general nonlinear form

$$\dot{x}_1 = x_2;\ \dot{x}_{n-1} = x_n;\ldots;\dot{x}_n = f(x)+u(t)+d(t) \qquad \text{(II-1)}$$
$$y = x_1,$$

with the state $x = \left[x_1\ \ x_2 \ldots . x_n\right]^T$, $u(t)$ the control input to the plant, and $y(t)$ the output of interest. Signal $d(t)$ denotes the unknown disturbance. The system nonlinearities are given by the smooth function $f(x): R^n \to R^m$ .Given a desired trajectory and its derivative values

$$x_d(t) = \begin{bmatrix} x_d & \dot{x}_d & \ldots & x_d^{(n)} \end{bmatrix}^T, \qquad \text{(II-2)}$$

define the tracking error as

$$e(t) = x_d(t) - x(t). \qquad \text{(II-3)}$$

Define the filtered tracking error $r(t) \in R^m$:

$$r(t) = [\,\Lambda^T \quad 1\,]\cdot e(t), \qquad \text{(II-4)}$$

where $\Lambda = \begin{bmatrix}\lambda_{n-2} & \lambda_{n-3} & \cdots & \lambda_1\end{bmatrix}^T$ is an appropriate chosen vector of constant values so that $\left|s^{n-1} + \lambda_{n-2}s^{n-2} + \cdots + \lambda_1\right|$ is stable. This means that $e(t) \to 0$ exponentially as $r(t) \to 0$. When $r(t)$ is small, the system performance is good. Using equations (II-1), (II-2) and (II-3) the dynamics of the performance measure signal (II-4)

$$\dot{r} = g(x, x_d) - u(t) - d(t), \qquad \text{(II-5)}$$

where $g(x, x_d)$ is a complex unknown nonlinear function of the state and desired trajectory vectors $x(t)$ and $x_d(t)$, respectively.

III. BACKGROUND ON FUZZY SYSTEMS

To fully take advantage of the learning abilities of FL systems, in this section we describe a nonlinearly-parametrized control function. In this FL control function we tune the output representative values, but also the MF centroids and spreads. Let $\phi_{A_i^l}(z_i, a_i^l, b_i^l)$ be Gaussian membership functions defined by

$$\phi_{A_i^l}(z_i, a_i^l, b_i^l) = e^{\left(-a_i^{l\,2}(z_i - b_i^l)^2\right)}. \qquad \text{(III-1)}$$

The output of the fuzzy logic system can be expressed in a vector notation as

$$y = W^T \Phi(z, a, b). \qquad \text{(III-2)}$$

Note that b_i^l are the MF centroids and a_i^l determines the MF spread or width. The adjustable parameters are W, a_i^l, b_i^l so that this FL system is nonlinear in adjustable parameters.

IV. FL LOGIC CONTROL ARCHITECTURE

There are two distinct parts of the proposed FL control architecture: a proportional-plus-derivative (PD) outer tracking loop, and a nonlinear adaptive FL loop. Without the FL adaptive loop this scheme boils down to PD control and its performance deteriorates. The FL loop is fed by plant states and desired trajectories and in essence it is used to approximate the nonlinear function $g(x, x_d)$ in (II-5). Instead of desired states x_d, the error vector can be used as an input to the FL system since $e = x_d - x$. This FL logic control architecture is shown in Fig. IV-1.

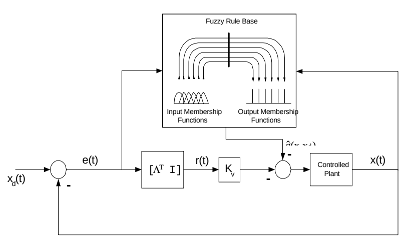

Fig. IV-1: Fuzzy control system architecture

We now derive an adaptive FL logic controller for nonlinear systems. According to the approximation properties of fuzzy logic systems, the continuous nonlinear function $g(x, x_d)$ in (II-5) can be represented as

$$g(x, x_d) = W^T \Phi(z, a, b) + \varepsilon(z), \qquad \text{(IV-1)}$$

where z is the input vector to the fuzzy system and the approximation error ε(z) is bounded on a compact set by a known constant ε_N. The ideal parameters a, b and W that approximate $g(\cdot)$ are unknown. Vector z may be selected as $\begin{bmatrix} x^T & x_d^T \end{bmatrix}^T$ or as $\begin{bmatrix} x^T & e^T \end{bmatrix}^T$.

Let the control input $u(t)$ be given by

$$u(t) = K_v r + \hat{g}(x, x_d), \qquad \text{(IV-2)}$$

where $\hat{g}(x, x_d)$ is provided by the fuzzy system and the control gain matrix is $K_v = K_v^T > 0$. Let the fuzzy functional estimate for the continuous nonlinear function $g(x, x_d)$ be:

$$\hat{g}(x, x_d) = \hat{W}^T \Phi(z, \hat{a}, \hat{b}), \qquad \text{(IV-3)}$$

where $\hat{a}, \hat{b}, \hat{W}$ are the actual current parameters of the FL system. Then, the filtered error dynamics (II-5) can be rewritten as

$$\begin{aligned} \dot{r} = {} & -K_v r + W^T \Phi(z, a, b) \\ & - \hat{W}^T \Phi(z, \hat{a}, \hat{b}) + d(t) + \varepsilon \end{aligned}. \qquad \text{(IV-4)}$$

Some required mild assumptions are now stated. These assumptions will be true in most practical situations and are standard in the existing literature.

Assumption 1: The ideal FL parameters W, a and b are bounded by known positive values so that the Frobenius norms satisfy

$$\|W\| \le W_M, \quad \|a\| \le a_M, \quad \|b\| \le b_M. \qquad \text{(IV-5)}$$

Assumption 2: The desired trajectory is bounded in the sense, for instance that

$$\left\| \begin{matrix} q_d & \dot{q}_d & \dots & \overset{n}{q}_d \end{matrix} \right\| \le Q_d. \qquad \text{(IV-6)}$$

Assumption 3: The disturbance and FL approximation error are bounded in the sense

$$\|d\| \le b_d; \quad \|\varepsilon\| \le \varepsilon_N. \qquad \text{(IV-7)}$$

Now, the following theorem can be formulated.

Theorem

Suppose that assumptions 1, 2 and 3 hold. Let the system be given by (II-1). Let FBF functions be defined as in (III-1) and fuzzy system output as in (III-2). Let control signal be defined by

$$u(t) = K_v r + \hat{g}(x, x_d),. \qquad \text{(IV-8)}$$

Let the tuning laws for the FL system be

$$\begin{aligned} \dot{\hat{W}} = {} & K_W (\hat{\Phi} - A\hat{a} - B\hat{b}) r^T \\ & - k_W K_W \hat{W} \|r\| \end{aligned}, \qquad \text{(IV-9)}$$

$$\dot{\hat{a}} = K_a A^T \hat{W} r - k_a K_a \hat{a} \|r\|, \qquad \text{(IV-10)}$$

$$\dot{\hat{b}} = K_b B^T \hat{W} r - k_b K_b \hat{b} \|r\|, \qquad \text{(IV-11)}$$

where $K_v, K_a, K_b, K_w, k_a, k_b, k_w$ are design parameters. Then the filtered error r and FL parameters W, a and b will be uniformly ultimately bounded. In addition, the filtered error can be made as small as desired by increasing gain K_v.□ The detailed proof is given in the appendix.

V. COGNITIVE BEHAVIOR OF FL CONTROL SYSTEM

As linguistic systems, FL systems have a long history of applications involving the emulation of human cognitive functions. This history has not been tied to adaptive FL control systems, where the literature has been more concerned with mathematical proofs of stability of signals in

the hardware control loops. In this section we propose to study through computer simulations the effects on the learned final membership functions of changing initial MF information stored in the FL approximator portion of the control signal. We begin by noting that the FL component (IV-3) in the control signal (IV-2) has two components. The short-term memory resides in the values of the control representative values W, and the long-term memory resides in the shape (a,b) of the MFs. Indeed, we observed, and shall discuss, the interesting behavior that the W weights tune faster and with more activity than the (a,b) weights.
As controlled plant, a Van der Pol oscillator was used. It has dynamics given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}x_2\\ -a(x_1^2-1)x_2-x_1+u\end{bmatrix}, \qquad \text{(V-1)}$$

$$y=x_1 . \qquad \text{(V-2)}$$

Simulation studies for the system with adaptive fuzzy control architecture were performed for sinusoidal reference $x_{1d}=2\cdot sin(0.5\Pi t)$. All simulations were performed for 100 s. No disturbance was introduced in the system. Cases of two different sets of initial membership functions of the fuzzy system were investigated.
The fuzzy control system parameters were: $\Lambda=5$, $K_v=diag\{3\}$, $K_w = diag\{10.15\}$, $k_w = 0.0985$, $K_a = diag\{0.01\}$, $k_a = 0.1$, $K_b = diag\{0.355\}$, $k_b = 0.0282$. The fuzzy system input vector z was defined as $z=\begin{bmatrix}x_1 & x_2 & e_1 & e_2\end{bmatrix}^T$.
Membership functions were defined as Gaussians with 3 membership functions per each of state dimensions x and 3 membership functions per each of errors, which totals to 81 membership functions with the given fuzzy system architecture.

A. Detailed Plots for Sinusoidal Reference Input

Here are provided detailed plots in order to show the performance characteristics of the proposed FL control architecture. Also, the performance of the fuzzy controller is compared with a nonadaptive PD controller. Detailed plots are given for a sinusoidal reference input $x_{1d}=2\cdot sin(0.5\Pi t)$ and with oscillator having parameter $a = 0.1$.
A phase-plane plot for the oscillator controlled by FL is given in Fig. V-3. It can be seen that the state trajectories converge to the required ellipse in state space for the given reference input.
The complete FL controller was compared to a standard PD controller in order to show the superiority of the FL controller. Parameters of the PD controller for comparison were: proportional gain K_p = 15 and derivative gain K_d=3. The error plot in Fig. V-4 confirms that the error with FL control is indeed small. The solid line in Fig. V-4 denotes error for the system controlled by the FL controller, dashed line for the system controlled by PD controller.
Spreads and centroids of MFs are shown in Fig. V-5. It can be seen that the changes are smooth and relatively slow when compared with reference input. No higher frequency components can be observed in tuning of spreads and centroids of MFs. It can be also seen that spreads and centroids do not exhibit any short-term changes, but are tuned steadily over the time. An analysis in following Sections shows that spreads and centroids are indeed tuned in such way that MFs cover the whole state space in which state trajectories lay. This behavior exhibits the long-term memory which maps the space in which the state trajectories are moving during the operation of the FL controlled system. In this way an appropriate function space is set for the linear tuning of the output layer values.
Output layer values of the FL controller are shown in Fig. V-6. In contrast to the spreads and centroids of MFs one can observe a significant higher frequency components in tuning output layer weights *W*. Output layer weights exhibit the higher frequency component with the same frequency as the desired trajectory x_d. That is, output layer weights exhibit a short-term memory adapting themselves to fast changes.

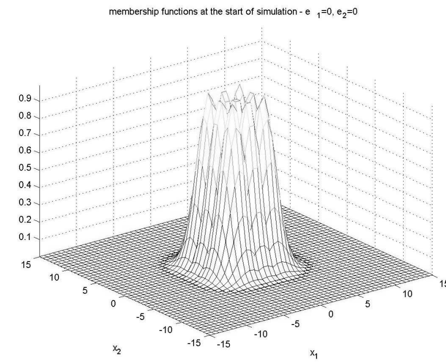

Fig. V-1: Membership functions at start of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0

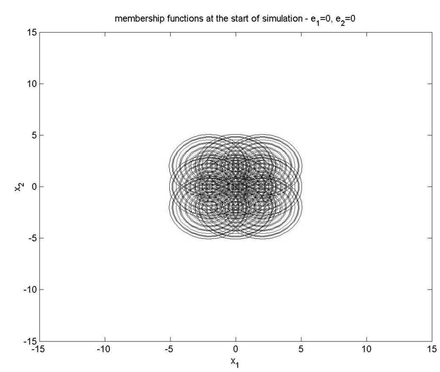

Fig. V-2: Membership functions at start of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0 – contour plot

Membership functions at the end of simulation for sinusoidal reference input in Fig. V-7 have a very different shape than the initial membership functions in Fig. V-1. Also they cover a broader area than the original MFs, but with the area covered by initial MFs almost completely included (Fig. V-8). Recall the phase-plane plot for sinusoidal input in Fig. V-3. It can be seen that ellipsoid-like contour of final MFs covers the region that comprises state trajectories of the system completely. It can be said that FL input layer membership functions dynamically focus to cover the area in which state trajectories are moving over time.

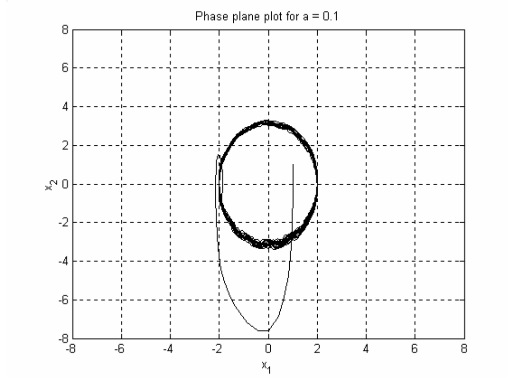

Fig. V-3: Phase-plane plot for sinusoidal input with A=2, f=0.25 Hz, a=0.1, system controlled by fuzzy controller

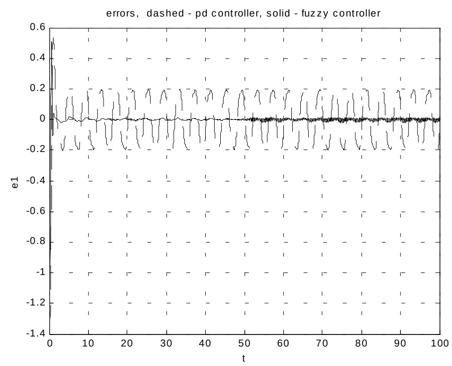

Fig. V-4: Error for sinusoidal input with A=2, f=0.25 Hz, a=0.1

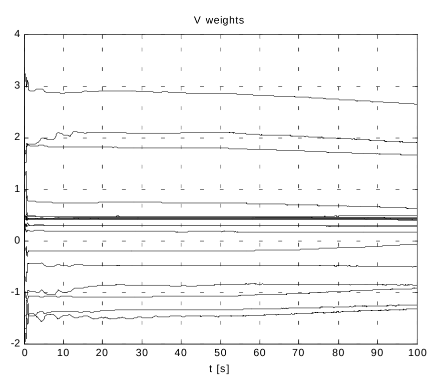

Fig. V-5: Spreads and centroids of membership functions for sinusoidal input with A=2, f=0.25 Hz, a=0.1

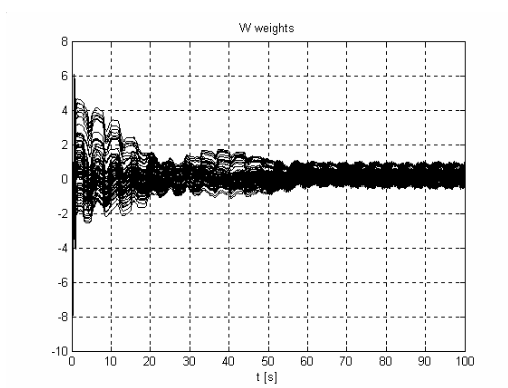

Fig. V-6: Output layer weights for sinusoidal input with A=2, f=0.25 Hz, a=0.1

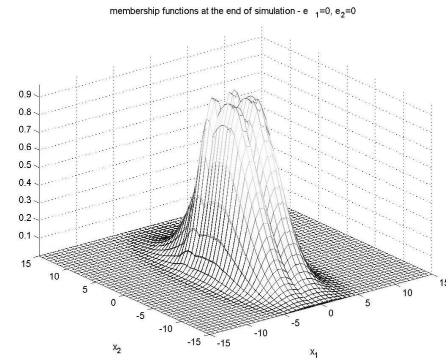

Fig. V-7: Membership functions at the end of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0

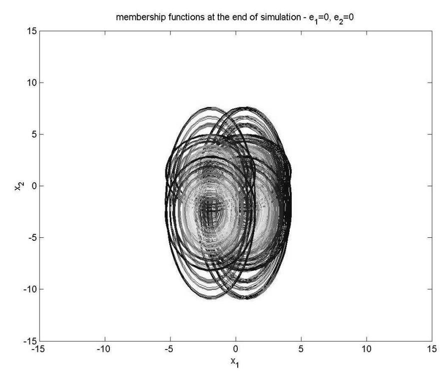

Fig. V-8: Membership functions at the end of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0 – contour plot

However, the final MFs cover much broader area than needed for final state trajectories. It seems that MFs do not focus only exactly on the area in which the final state trajectories are lying, but cover the area that includes states trajectories during the whole simulation. This reveals the presence of the cognitive focusing of awareness both of long-term memory and short-term memory.
It is important to note that the sinusoidal reference is PE for the given plant.

B. Effect of Shifted Initial Membership Functions on Final Membership Functions

In this subsection is provided a simulation study of the effect of shifted initial MFs on final values of MFs. This reveals the effect of initial incorrect information on the final information learned by the FL system. A contour plot of Initial weights for all simulations in Section B, assuming that errors are 0, is shown in Fig. V-9. Initial states x_0 are marked with "x" in Fig. V-9. It is important to note that the initial system states lay outside area covered by these initial FL MFs (i.e. value of MF's in that area is very small).Final MFs and phase-plane plots are shown in Fig. V-10 - Fig. V-12 Results show that initial MFs do have significant effect on final MFs. The FL system does focus on the final state trajectories area and does covers the whole state space, but final MFs are in general different than in Section A.

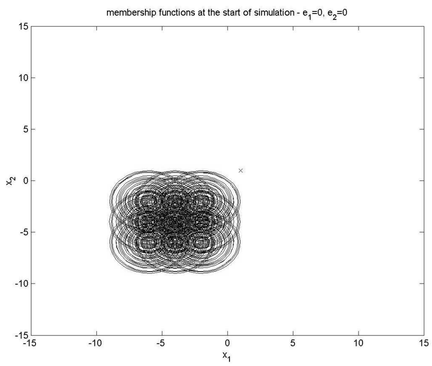

Fig. V-9: Membership functions with shifted centroids at start of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0 – contour plot

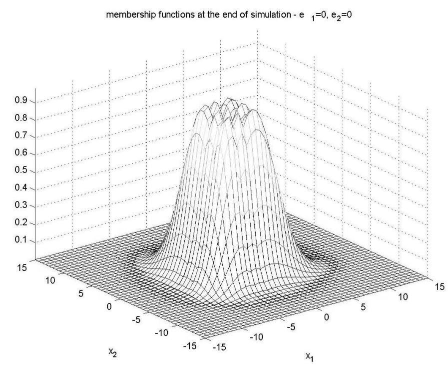

Fig. V-10: Membership functions with shifted centroids at the end of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0

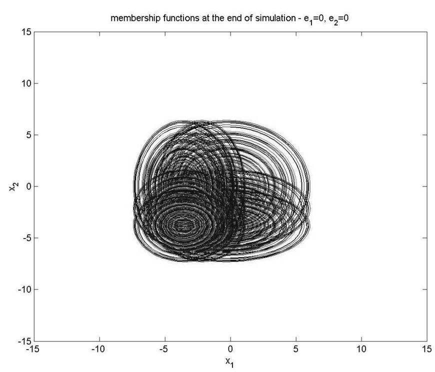

Fig. V-11: Membership functions with shifted centroids at the end of simulation for sinusoidal input with A=2, f=0.25 Hz, a=0.1 assuming error vector e=0 – contour plot

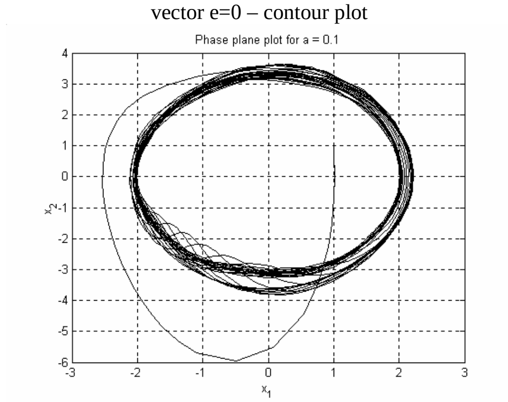

Fig. V-12: Phase-plane plot for sinusoidal input with A=2, f=0.25 Hz, a=0.1 – initial MFs with shifted centroids

Effect of initial values of MFs can be clearly seen. This result leads to a conclusion that while FL system will try to cover the space inside which state trajectories are comprised and in the same time to focus on the final state trajectories area, the final MFs will be affected by initial MF's. In another words, a different "state knowledge" will be reached depending on the initial MFs while trying to reach the same goal. Mathematically speaking, this illustrates the fact that in the case when no unique solution exists, the final state of FL logic system will depend very much on it's initial state, converging to a local minimum. It can be also noted that the area covered by MFs at the end (Fig. V-9) is shifted toward the area covered by the final MFs in the case when the initial centroids were not shifted (Fig. V-8). Comparison with phase-plane plot in Fig. V-12 shows that FL system tries again to cover the whole space in which state trajectories are comprised and to focus towards the area in which final state trajectories lay.

VI. CONCLUSION

A fuzzy logic control architecture is described in the paper. A detailed stability proof is given. The proposed FL control algorithm does not require any assumptions on the initial parameters of FL controller for the system to be stable. The performance of the FL controller is studied in detail using simulation studies. Effects of changes in initial FL parameters, plant nonlinearity, and different reference trajectories on the final FL MFs were investigated with the intention of drawing some conclusions on how the learned MFs adapt to changing environments. It was observed that input layer MFs are changing in such a way that the whole state space in which plant states are contained is covered by MFs, but also that MFs are trying to put more weight on approximating plant states at their steady trajectories after the transient is finished if the excitation is PE. Simulations were also performed for the step reference input. It was noted that in the case of step reference although the fuzzy controller kept the system stable, the learning was very slow and it included only long-term memory first layer weights. Also, the final trajectories (only a point in this case), was just barely covered by the final MFs. The conclusion was that no meaningful learning process occurred due to lack of information in step reference signal. Step signal is not PE signal for the Vad der Pol oscillator. The plots are not given due to space constraints.

VII. APPENDIX – PROOF OF THEOREM

.Using equations (II-1), (II-2) and (II-3) can be written as:

$$\dot{r} = g(x, x_d) - u(t) - d(t), \qquad \text{(VII-1)}$$

where $g(x, x_d)$ is a complex nonlinear function of the state and desired trajectory vectors x and x_d, respectively. Note that this function includes the original system unknown nonlinear function $f(x)$. The continuous nonlinear function $g(x, x_d)$ in (VII-1) can be represented by (IV-1). Let the control input $u(t)$ be given by (IV-2). Let the fuzzy functional estimate for $g(x,x_d)$ be given by (IV-3). Then, filtered error dynamics (VII-1) can be expressed as in (IV-4) and the functional estimation error is defined as $\tilde{g}(x,x_d) = g(x,x_d) - \hat{g}(x,x_d)$.

Define:

$$\begin{aligned} &\tilde{W} = W - \hat{W},\ \tilde{\Phi} = \Phi(z,\tilde{a},\tilde{b}),\\ &\hat{\Phi} = \Phi(z,\hat{a},\hat{b}),\ \tilde{a} = a - \hat{a},\ \tilde{b} = b - \hat{b}. \end{aligned} \qquad \text{(VII-2)}$$

Now, Taylor expansion of Φ can be written as

$$\Phi = \hat{\Phi} + A\tilde{a} + B\tilde{b} + H, \qquad \text{(VII-3)}$$

where A and B are suitable jacobians and H represents higher order terms.

$$A = \left\langle \frac{\partial \Phi}{\partial a} \right|_{a=\hat{a},b=\hat{b}},\quad B = \left\langle \frac{\partial \Phi}{\partial b} \right|_{a=\hat{a},b=\hat{b}}. \qquad \text{(VII-4)}$$

Introduce (IV-2) into (II-5):

$$\dot{r} = -K_v r + \tilde{W}^T\tilde{\Phi} + \hat{W}^T\tilde{\Phi} + \tilde{W}^T\hat{\Phi} + d + \varepsilon. \qquad \text{(VII-5)}$$

With Taylor expansion (VII-3) equation (VII-5) can be written as

$$\begin{aligned} \dot{r} &= -K_v r + \tilde{W}^T(A\tilde{a} + B\tilde{b} + H)\\ &+ \hat{W}^T(A\tilde{a} + B\tilde{b} + H) + \tilde{W}^T\hat{\Phi} + d + \varepsilon \end{aligned}. \qquad \text{(VII-6)}$$

Define Lyapunov candidate

$$L = \frac{1}{2}r^T r + \frac{1}{2}tr(\tilde{W}^T K_W^{-1}\tilde{W}) + \frac{1}{2}\tilde{a}^T K_a^{-1}\tilde{a} + \frac{1}{2}\tilde{b}^T K_b^{-1}\tilde{b}, \qquad \text{(VII-7)}$$

where K_W, K_a, and K_b are design matrices and $K_W = K_W^T > 0;\ K_a = K_a^T > 0;\ K_b = K_b^T > 0$.

$$\dot{L} = r^T\dot{r} + tr(-\tilde{W}^T K_W^{-1}\dot{\hat{W}}) - \tilde{a}^T K_a^{-1}\dot{\hat{a}} - \tilde{b}^T K_b^{-1}\dot{\hat{b}}. \qquad \text{(VII-8)}$$

Substituting (VII-6) into (VII-8) and introducing tuning laws (IV-9), (IV-10) and (IV-11) yields

$$\begin{aligned} \dot{L} &= -r^T K_v r + k_w\|r\|\cdot tr(\tilde{W}^T\hat{W}) + k_a\|r\|\tilde{a}^T\hat{a} + k_b\|r\|\tilde{b}^T\hat{b}\\ &+ r^T W^T H + r^T\tilde{W}^T(Aa + Bb) + r^T(d + \varepsilon). \end{aligned} \qquad \text{(VII-9)}$$

For Gaussian FBFs there exists the following bound [4]:

$$\left\| r^T W^T H + r^T\tilde{W}^T(Aa+Bb) \right\| \le \|r\|\cdot(c_0 + c_1\|\tilde{W}\| + c_2\|\tilde{a}\| + c_3\|\tilde{b}\|). \qquad \text{(VII-10)}$$

Now, with (VII-10) and with assumptions 1 and 3 holding, the following inequality can be obtained:

$$\begin{aligned} \dot{L} \le &-\|r\|\left\{K_{v\,min}\|r\| - k_w\cdot\left[\|\tilde{W}\|(W_M - \|\tilde{W}\|)\right]\right\}\\ &+ \|r\|k_a\cdot\left[\|\tilde{a}\|(a_M - \|\tilde{a}\|)\right] + \|r\|k_b\cdot\left[\|\tilde{b}\|(b_M - \|\tilde{b}\|)\right]\\ &+ \|r\|\left[D + c_1\|\tilde{W}\| + c_2\|\tilde{a}\| + c_3\|\tilde{b}\|\right],\\ &D = c_0 + b_d + \varepsilon_N. \end{aligned} \qquad \text{(VII-11)}$$

where K_{vmin} is the minimum singular value of K_v.
Let us define auxiliary variables D_W, D_a, D_b and Δ:

$$\begin{aligned} &D_W = \frac{k_W W_M + c_1}{2\sqrt{k_W}},\ D_a = \frac{k_a a_M + c_2}{2\sqrt{k_a}},\\ &D_b = \frac{k_b b_M + c_3}{2\sqrt{k_b}},\quad \Delta = D_W^2 + D_a^2 + D_b^2 + D \end{aligned} \qquad \text{(VII-12)}$$

Using (VII-12) and completing squares in (VII-11), it can be seen that $\dot{L} < 0$ as long as:

$$\|r\| > \frac{\Delta}{K_{v\,min}},\ \|\tilde{W}\| > \frac{D_W}{k_W} + \sqrt{\frac{\Delta}{k_W}}, \qquad \text{(VII-13)}$$

$$\|\tilde{a}\| > \frac{D_a}{k_a} + \sqrt{\frac{\Delta}{k_a}},\ \|\tilde{b}\| > \frac{D_b}{k_b} + \sqrt{\frac{\Delta}{k_b}} \qquad \text{(VII-14)}$$

According to a standard Lyapunov theorem extension, that proves that r, $\tilde{W}$, $\tilde{a}$ and $\tilde{b}$ are UUB. Since W, a and b are bounded, $\hat{W}$, $\hat{a}$ and $\hat{b}$ are also bounded. This fact concludes the stability proof.

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