# Controller Tuning for Integrating Processes with Time Delay
# Part II: Robustness Analysis under Structured Parametric Uncertainty

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***Abstract*— In this paper, pseudo-derivative feedback controllers, which are designed and tuned in order to control integrator plus dead time (IPDT) processes, are analyzed in terms of robustness. The robustness analysis reported in the paper, is performed in terms of the structured parametric uncertainty description. This analysis constitutes a basis, for a comparison of the control and tuning methods presented in [1], with existing tuning methods for conventional PI/PID controllers of IPDT processes. This comparison reveals that the proposed control and tuning methods provide satisfactory robustness, and offer larger parametric stability margins than those provided by most of the existing tuning methods.**

***Index Terms*— Pseudo-derivative feedback, controller tuning, integrating processes, dead-time processes, robust stability, structured parametric uncertainty.**

## I. Introduction

In the past, integrator plus dead time (IPDT) process model has been the focus of interest by many control and process designers (see [2]-[4] and the references therein). This increased interest stems from the simplicity of this model and its ability of adequately representing process dynamics. Moreover, since, most of the controller implementations in process industry is of the PI/PID type, simple controller tuning for IPDT processes has received an adequate attention in the past (see [4]-[12] and the references therein). A common feature of the existing PI/PID tuning methods for IPDT processes is that they give large overshoot in the servo response of the closed-loop system [1]. To avoid such an overshoot and to obtain a smoother closed-loop response to set-point changes, it has been proposed to use setpoint weighting or a controller configuration with filtered derivative or finally set point filters [3], [4], [11], [12].

An alternative way to reduce the closed-loop system overshoot, is to use the pseudo-derivative feedback (PDF) configuration [13], instead of that of a PI/PID controller, in order to control IPDT processes. The PDF structure avoids abrupt responses to set point changes, because naturally ramps the controller effort, since it internalizes the pre-filter that one would apply in the PI/PID control configuration, to reduce overshoot. In the first part of this paper [1], two simple types of PDF controllers, namely the PD-0F controller and the PD-1F controller are analyzed and simple methods for tuning their settings are presented. As it has been shown by simulation in [1], the proposed control and tuning methods provide also fast regulatory control and sufficient robustness against parametric uncertainty.

The purpose of this second part of the paper is to perform a thorough analysis of PD-0F and PD-1F controllers, which are designed and tuned in order to control integrator plus dead time (IPDT) processes, in terms of robustness. The analysis of robustness of PDF controllers presented in this paper is based on the performance of the loop transfer function and on the structured parametric uncertainty description. Structured uncertainty is preferred to use in the analysis instead of the unstructured uncertainty description, due to the conservative nature of the later. Based on this analysis, a comparison of the control and tuning methods presented in [1], with existing tuning methods for conventional PI/PID controllers of IPDT processes, is accomplished. This comparison reveals that the proposed control and tuning methods provide satisfactory robustness, and offer larger parametric stability margins than those provided by most of the existing tuning methods.

## II. PD-0F and PD-1F Controller Settings for IPDT Processes

The PD-0F and the PD-1F controllers are special cases of the general PDF control structure, depicted in Fig. 1. More precisely, the PD-0F controller corresponds to the case where $K_{D,i}=0$, for i=1,… ,n-1 and $K_{D,0}\equiv K_P\neq 0$, while the PD-1F controller corresponds to the case where $K_{D,0}\equiv K_P\neq 0$,

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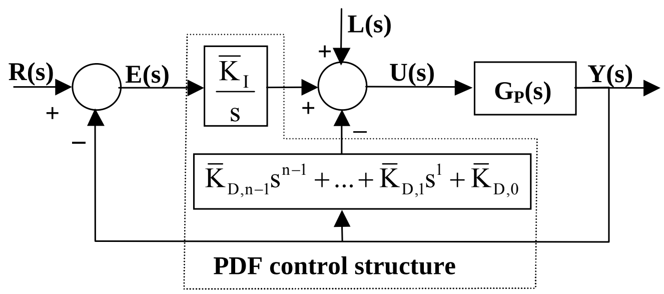


Fig. 1. The general PDF control structure.

$K_{D,1}=K_d\neq 0$ and $K_{D,i}=0$, for i=2,…,n-1. As it has been shown in [1], for IPDT process models of the form

$$G_P(s)=Kexp(-ds)/s \tag{1}$$

where $K$ and $d$ are the process gain and the time delay, respectively, the loop transfer function is given by

$$G_L(s)=\frac{K(K_P s+K_I)exp(-ds)}{s^2} \tag{2}$$

in the case of the PD-0F controller, while in the case of the PD-1F controller is given by

$$G_L(s)=\frac{K(K_d s^2+K_P s+K_I)exp(-ds)}{s^2} \tag{3}$$

In [1], based on approximations of the delay term through first order Taylor and Padé expansions and of the crossover frequency of the Nyquist plot of the loop transfer function, the following PD-0F and PD-1F controller settings have been proposed.

*PD-0F Controller Settings for IPDT models*

The PD-0F controller parameters can be chosen as

$$K_P=4\pi^2\left[\left((8-\alpha)\pi^2+\alpha^2\right)dK\right]^{-1} \tag{4a}$$

$$K_I=\alpha\left(\pi^2-\alpha\right)\left[\left((8-\alpha)\pi^2+\alpha^2\right)d^2K\right]^{-1} \tag{4b}$$

where $\alpha$ is an adjustable parameter, whose value can be specified by the designer in order to obtain a desired damping ratio of the closed-loop system or to minimize classical integral criteria (see [1], for details).

*Alternative PD-0F Controller Settings for IPDT models*

The PD-0F controller parameters can alternatively be chosen as

$$K_P=4\left[(8-\beta)dK\right]^{-1} \tag{5a}$$

$$K_I=\beta\left[(8-\beta)d^2K\right]^{-1} \tag{5b}$$

where $\beta=\alpha(\pi^2-\alpha)\pi^{-2}$ is an adjustable parameter.

*PD-1F Controller Settings for IPDT models*

The PD-1F controller parameters can be chosen as

$$K_P=16\left[(16-3\gamma)dK\right]^{-1} \tag{6a}$$

$$K_I=4\gamma\left[(16-3\gamma)d^2K\right]^{-1} \tag{6b}$$

$$K_d=(8-\gamma)\left[(16-3\gamma)K\right]^{-1} \tag{6c}$$

where $\gamma$ is an adjustable parameter, whose value can be specified in order to obtain a desired damping ratio for the closed-loop system or to minimize classical integral criteria (see [1], for details).

## III. Robustness of PD-0F and PD-1F Controllers under Parametric Uncertainty

In order to study the robustness of the proposed tuning methods, we consider the structured uncertainty description given by the following equations

$$K_{act}=K_{nom}(1+r_K)\ ,\ d_{act}=d_{nom}(1+r_d) \tag{7}$$

where $K_{nom}$ and $d_{nom}$ are the nominal values of the parameters and $r_K$ and $r_d$ are the relative uncertainties. In this case, the real plant has the following transfer function

$$G_{P,act}(s)=\frac{K_{nom}(1+r_K)e^{-d_{nom}(1+r_d)s}}{s} \tag{8}$$

Suppose, now, that a PD-0F controller is designed for the nominal plant, and that it is tuned according to relations (4a), (4b). In this case, the actual loop transfer function is given by

$$G_{L,act}(s)=\frac{K_{nom}(1+r_K)(K_P s+K_I)}{s^2}e^{-d_{nom}(1+r_d)s} \tag{9}$$

Substituting (4a) and (4b) in (9) yields

$$G_{L,act}(s)=\frac{(1+r_K)\left[\dfrac{4\pi^2}{Ad_{nom}}s+\dfrac{\alpha(\pi^2-\alpha)}{Ad_{nom}^2}\right]}{s^2}e^{-d_{nom}(1+r_d)s} \tag{10}$$

where $A=(8-\alpha)\pi^2+\alpha^2$. The parameter values for which the controlled system will be marginally stable are those for which the Nyquist plot passes through the (-1,0) point. These values are close related to the parametric stability margin discussed in [14]. Using equation (10) and setting the real part equal to -1 and the imaginary part equal to 0 gives

$$tan\left[d_{nom}(1+r_d)\omega\right]=\frac{4\pi^2 d_{nom}\omega}{\alpha(\pi^2-\alpha)} \tag{11}$$

$$1+r_K = \frac{Ad_{nom}^2\omega^2}{\alpha(\pi^2-\alpha)cos(B)+4\pi^2 d_{nom}\omega\, sin(B)} \quad (12)$$

where $B = d_{nom}(1+r_d)\omega$ , or equivalently

$$\begin{aligned} 1+r_K &= \frac{Ad_{nom}^2\omega^2}{cos(B)\left\{\alpha(\pi^2-\alpha)+4\pi^2 d_{nom}\omega\, tan(B)\right\}} \\ &= \frac{\alpha(\pi^2-\alpha)\left[(8-\alpha)\pi^2+\alpha^2\right]d_{nom}^2\omega^2}{cos\left[d_{nom}(1+r_d)\omega\right]\left\{\alpha^2(\pi^2-\alpha)^2+16\pi^4 d_{nom}^2\omega^2\right\}} \end{aligned} \quad (13)$$

Thus, for a given value of the adjustable parameter $\alpha$ and a given value of the relative uncertainty in the time delay ($r_d$), the value of the frequency $\omega$ that satisfies equation (11) is calculated first. Then, the maximum relative gain uncertainty, that can be tolerated without the closed loop system to become unstable, is calculated using equation (12). The set of equations (11) and (12) (or (13)) is a powerful robustness analysis tool since using these equation, we can calculate the exact values of the uncertainty, in the model parameters, for which the closed loop system becomes unstable.

In order to obtain an explicit relationship between the relative uncertainty of the model parameters and the adjustable parameter $\alpha$, an approximate solution for the equations (11) and (13) is presented. To this end, rewrite equation (11) as

$$d_{nom}(1+r_d)\omega = tan^{-1}\left[\frac{4\pi^2 d_{nom}\omega}{\alpha(\pi^2-\alpha)}\right] \quad (14)$$

and use the approximation $tan^{-1}(x) \approx 0.5\pi - x^{-1}$ in the right hand side of (14) to obtain

$$d_{nom}(1+r_d)\omega \approx \frac{\pi}{2} - \frac{\alpha(\pi^2-\alpha)}{4\pi^2 d_{nom}\omega}$$

The solution of the above equation is

$$\omega \approx \frac{\pi}{4d_{nom}(1+r_d)}\left[1+\sqrt{1-\frac{4\alpha(\pi^2-\alpha)(1+r_d)}{\pi^4}}\right] \quad (15)$$

Finally, substitution of (15) into (13) yields

$$1+r_K \approx \frac{\alpha(\pi^2-\alpha)\left[(8-\alpha)\pi^2+\alpha^2\right]}{16\, cos\left[\frac{\pi}{4}(1+\Gamma)\right]\left[\pi^4+\frac{(1+r_d)^2\alpha^2(\pi^2-\alpha)^2}{\pi^2(1+\Gamma)^2}\right]} \quad (16)$$

where

$$\Gamma = \sqrt{1-\frac{4\alpha(\pi^2-\alpha)(1+r_d))}{\pi^4}}$$

We now make use of the following approximation for the cosine function [15]

$$cos(x) \approx \mu_1 x^2 + \mu_2 x + 1 \quad (17a)$$

$$\mu_1 = \frac{8(1-\sqrt{2})}{\pi^2} \;,\; \mu_2 = \frac{2(2\sqrt{2}-3)}{\pi} \quad (17b)$$

and we obtain

$$1+r_K = \frac{\alpha(\pi^2-\alpha)\left[(8-\alpha)\pi^2+\alpha^2\right]}{16\left\{\mu_1\left[\frac{\pi}{4}(1+\Gamma)\right]^2+\mu_2\left[\frac{\pi}{4}(1+\Gamma)\right]+1\right\}\left[\pi^4+\frac{(1+r_d)^2\alpha^2(\pi^2-\alpha)^2}{\pi^2(1+\Gamma)^2}\right]} \quad (18)$$

I Note that the fittings for the cosine function, provided by (17a) are exact at the points $x=0, \pi/4, \pi/2$.

Equation (18) is an extremely accurate approximation and can be used even for controller synthesis, since it provides the controller parameter $\alpha$ for given values of the parameter uncertainties.

Suppose, now, that a PD-0F controller is designed for the nominal plant, and that it is tuned according to relations (5a) and (5b). Then, the actual loop transfer function is given by

$$G_{L,act}(s) = \frac{(1+r_K)\left[\frac{4}{(8-\beta)d_{nom}}s+\frac{\beta}{(8-\beta)d_{nom}^2}\right]}{s^2}e^{-d_{nom}(1+r_d)s}$$

By performing a similar analysis as before we obtain

$$tan\left[d_{nom}(1+r_d)\omega\right] = \frac{4d_{nom}\omega}{\beta} \quad (19)$$

$$1+r_K = \frac{\beta(8-\beta)d_{nom}^2\omega^2}{cos\left[d_{nom}(1+r_d)\omega\right]\left(\beta^2+16d_{nom}^2\omega^2\right)} \quad (20)$$

Solving (19), according to the procedure presented above, yields

$$\omega \approx \frac{\pi}{4d_{nom}(1+r_d)}\left[1+\sqrt{1-\frac{4\beta(1+r_d)}{\pi^2}}\right] \quad (21)$$

Substituting (21) in (20) yields

$$1+r_K \approx \frac{\beta(8-\beta)}{16\, cos\left[\frac{\pi}{4}(1+\Delta)\right]\left[1+\frac{\beta^2(1+r_d)^2}{\pi^2(1+\Delta)^2}\right]} \quad (22)$$

$$\Delta = \sqrt{1-\frac{4\beta(1+r_d)}{\pi^2}}$$

Using approximation (17) in (22), we finally obtain

$$1+r_K \approx \frac{\beta(8-\beta)}{16\left\{\mu_1\left[\frac{\pi}{4}(1+\Delta)\right]^2+\mu_2\left[\frac{\pi}{4}(1+\Delta)\right]+1\right\}\left[1+\frac{\beta^2(1+r_d)^2}{\pi^2(1+\Delta)^2}\right]} \quad (23)$$

Equation (23) provides the controller parameter $\beta$ for given values of the parameter uncertainties.

Finally, suppose that a PD-1F controller is designed for the nominal plant, and that it is tuned according to relations (6a)-(6c). In this case, it is not difficult to check that the actual loop transfer function is given by

$$G_{L,act}(s)=\frac{(1+r_K)\left[(8-\gamma)d_{nom}^2 s^2+16d_{nom}s+4\gamma\right]}{(16-3\gamma)s^2}e^{-d_{nom}(1+r_d)s}$$

By performing a similar analysis as before, we obtain

$$\tan\left[d_{nom}(1+r_d)\omega\right]=\frac{16d_{nom}\omega}{4\gamma-(8-\gamma)d_{nom}^2\omega^2} \quad (24)$$

$$1+r_K=\frac{(16-3\gamma)\left[4\gamma-(8-\gamma)d_{nom}^2\omega^2\right]\omega^2}{\cos\left[d_{nom}(1+r_d)\omega\right]\left\{\left[4\gamma-(8-\gamma)d_{nom}^2\omega^2\right]^2+256d_{nom}^2\omega^2\right\}} \quad (25)$$

An approximate solution of (24) is given by

$$\omega \approx \frac{4\pi}{d_{nom}\left[16(1+r_d)+\gamma-8\right]}\left[1+\sqrt{1-\frac{\gamma\left[16(1+r_d)+\gamma-8\right]}{4\pi^2}}\right] \quad (26)$$

and a relation analogous to relations (18) and (23), can be easily obtained by substituting (26) into (25) and then using approximation (17) in the resulting equation.

## IV. Comparison with Existing PI/PID Tuning Methods

In order to perform a comparison of our methods with known PI/PID tuning methods for IPDT processes, in terms of robustness, we next briefly review some of them, and in particular those reported in [2] (T-L method), [5] (C-method), [6] (K-L-A method), [7] (P-P method), [8], [9] (W-C method), [10] (V-method) and [12] (C-PS method).

The methods reported in [2], [5], [6] and [7] for PI controller tuning, propose controller settings of the following form

$$K_P=c_1/(dK)\ ,\ \theta=c_2 d \quad (27)$$

where

$c_1$=0.4863 , $c_2$=8.7527 (T-L method)
$c_1$=0.671 , $c_2$=3.6547 (C-method)
$c_1$=0.6042 , $c_2$=5 (K-L-A method)
$c_1$=0.5325 , $c_2$=4.16 (P-P method)

By considering the structured uncertainty description of the form (7) and by performing a similar robustness analysis for the methods reported in [2], [5], [6] and [7], we obtain

$$\tan\left[d_{nom}(1+r_d)\omega\right]=c_2 d_{nom}\omega \quad (28)$$

$$1+r_K=\frac{c_2 d_{nom}^2\omega^2}{c_1\cos\left[d_{nom}(1+r_d)\omega\right]\left\{1+c_2^2 d_{nom}^2\omega^2\right\}} \quad (29)$$

$$\omega \approx \frac{\pi}{4(1+r_d)d_{nom}}\left[1+\sqrt{1-\frac{16(1+r_d)}{\pi^2 c_2}}\right] \quad (30)$$

In the methods reported in [8], [9] and [12] control-ler settings depend on adjustable parameters.

In particular, in [12], the following rules for tuning a PI or a PID controller have been proposed:

*PI Controller Settings [12]*

$$K_P=\frac{2\alpha_1}{(1+\alpha_1)dK}\ ,\ \theta=\frac{0.5(\alpha_1+1)d}{\alpha_1-1} \quad (31)$$

*PID Controller Settings [12]*

$$K_P=\frac{4\alpha_1^2}{(1+\alpha_1)^2 dK}\ ,\ \theta=\frac{0.5(\alpha_1+1)d}{\alpha_1-1}\ ,\ \delta=\frac{0.25(\alpha_1+1)d}{\alpha_1} \quad (32)$$

By considering the structured uncertainty description of the form (7) and by performing a similar robust-ness analysis, in the case of a PI controller, we obtain

$$\tan\left[d_{nom}(1+r_d)\omega\right]=\frac{(\alpha_1+1)d_{nom}\omega}{2(\alpha_1-1)} \quad (33)$$

$$1+r_K=\frac{(\alpha_1+1)^2(\alpha_1-1)d_{nom}^2\omega^2}{\cos\left[d_{nom}(1+r_d)\omega\right]\left\{4\alpha_1(\alpha_1-1)^2+\alpha_1(\alpha_1+1)^2 d_{nom}^2\omega^2\right\}} \quad (34)$$

$$\omega \approx \frac{\pi}{4(1+r_d)d_{nom}}\left[1+\sqrt{1-\frac{32(\alpha_1-1)(1+r_d)}{\pi^2(\alpha_1+1)^2}}\right] \quad (35)$$

while in the case of a PID controller we obtain

$$\tan\left[d_{nom}(1+r_d)\omega\right]=\frac{4\alpha_1^2(\alpha_1+1)d_{nom}\omega}{8\alpha_1^2(\alpha_1-1)-\alpha_1(\alpha_1+1)^2 d_{nom}^2\omega^2} \quad (36)$$

$$1+r_K=\frac{(\alpha_1+1)^3 E d_{nom}^2\omega^2}{\cos\left[d_{nom}(1+r_d)\omega\right]\left\{E^2+16\alpha_1^4(\alpha_1+1)^2 d_{nom}^2\omega^2\right\}} \quad (37)$$

$$E=8\alpha_1^2(\alpha_1-1)-\alpha_1(\alpha_1+1)^2 d_{nom}^2\omega^2$$

$$\omega \approx \frac{\pi}{(3\alpha_1+4\alpha_1 r_d-1)d_{nom}}\left[1+\sqrt{1-\frac{8(\alpha_1-1)(3\alpha_1+4\alpha_1 r_d-1)}{\pi^2\alpha_1(\alpha_1+1)}}\right] \quad (38)$$

In [8], [9], the following rules for tuning a PID controller have been proposed:

*PID Controller Settings [8], [9]*

$$K_P = \frac{1}{f_1(\beta_1)dK} \quad , \; \theta = df_2(\beta_1) \quad , \; \delta = d / f_3(\beta_1) \tag{39}$$

where, for $\zeta_1$=1

$$f_1(\beta_1) = 0.5080\beta_1 + 0.6208 \tag{40a}$$
$$f_2(\beta_1) = 1.9885\beta_1 + 1.2235 \tag{40b}$$
$$f_3(\beta_1) = 1.0043\beta_1 + 1.8194 \tag{40c}$$

while, for $\zeta_1 = \sqrt{2}/2$

$$f_1(\beta_1) = 0.7138\beta_1 + 0.3904 \tag{41a}$$
$$f_2(\beta_1) = 1.4020\beta_1 + 1.2076 \tag{41b}$$
$$f_3(\beta_1) = 1.4167\beta_1 + 1.6999 \tag{41c}$$

Note that that in [8], [9] the design specification is to obtain a closed-loop transfer function of the form

$$G_{CL}(s) = \frac{(2\tau_e\zeta_1 + 1)ds + 1}{\tau_e^2 d^2 s^2 + 2\tau_e\zeta_1 ds + 1} exp(-ds) \tag{42}$$

By considering once again the structured uncertainty description of the form (7) and by performing a similar robustness analysis, as for the previous tuning methods, we obtain

$$tan\left[d_{nom}\left(1 + r_d\right)\omega\right] = \frac{f_2(\beta_1) f_3(\beta_1) d_{nom}\omega}{f_3(\beta_1) - f_2(\beta_1) d_{nom}^2 \omega^2} \tag{43}$$

$$1 + r_K = \frac{f_1^2(\beta_1) f_2(\beta_1) f_3(\beta_1) Z d_{nom}^2 \omega^2}{cos\left[d_{nom}(1 + r_d)\omega\right] f_1(\beta_1)\left\{Z^2 + f_2^2(\beta_1) f_3^2(\beta_1) d_{nom}^2 \omega^2\right\}} \tag{44}$$

$$Z = f_3(\beta_1) - f_2(\beta_1) d_{nom}^2 \omega^2$$

$$\omega \approx \frac{\pi f_3(\beta_1)}{4d_{nom}\left[f_3(\beta_1)(1 + r_d) - 1\right]}\left[1 + \sqrt{1 - \frac{16\left[f_3(\beta_1)(1 + r_d) - 1\right]}{\pi^2 f_2(\beta_1) f_3(\beta_1)}}\right]$$

Finally, in [10] the following PID controller settings have been reported for controlling IPDT processes

$$K_P = h_1 / (dK) \quad , \; \theta = h_2 d \quad , \; \delta = h_3 d \tag{45a}$$
$$h_1 = 1.37 \quad , \; h_2 = 1.49 \quad , \; h_3 = 0.59 \tag{45b}$$

A robustness analysis under the structured uncertainty description (7) yields

$$tan\left[d_{nom}\left(1 + r_d\right)\omega\right] = \frac{h_2 d_{nom}\omega}{1 - h_2 h_3 d_{nom}^2 \omega^2} \tag{46}$$

$$1 + r_K = \frac{h_2 d_{nom}^2 \omega^2}{h_1 \cos\left[d_{nom}(1 + r_d)\omega\right]\left\{(1 - h_2 h_3 d_{nom}^2 \omega^2)^2 + h_2^2 d_{nom}^2 \omega^2\right\}} \tag{47}$$

$$\omega \approx \frac{\pi}{4d_{nom}\left(1 + r_d - h_3\right)}\left[1 + \sqrt{1 - \frac{16\left(1 + r_d - h_3\right)}{\pi^2 h_2}}\right]$$

Before proceeding to the comparison of the above methods in terms of robustness, it should be noted that, although the sets of equations (11) and (13), (19) and (20), (24) and (25), (28) and (29), (33) and (34), (36) and (37), (43) and (44), (46) and (47) depend on $d_{nom}$, the relation between the relative uncertainties $r_K$ and $r_d$ does not. This implies a universal robustness analysis. Moreover, it is worth noticing that all these sets of equations have the trivial solution $\omega$=0 and $r_K$=-1. This solution corresponds to the well-known fact that any system with positive feedback is unstable and will not be considered in our analysis.

Consider now the IPDT process with $K$=1, $d$=1. In Fig. 2, the solutions of equations (11) and (13), (19) and (20), and (28), (29) for various values of the adjustable parameters $\alpha$, $\beta$ and $c_1$, $c_2$ are depicted. Note that as it is expected, the solutions of equations (11), (13) and (19), (20) are identical. Clearly, our method provides almost the same stability margins as the P-P method, while it offers larger parametric stability margins as compared to those offered by the C-method and the K-L-A method. The T-L method gives the largest stability margins among the methods under comparison. Moreover, in Fig. 3, the solutions of equations (11) and (13), (19) and (20), and (33), (34) for various values of the adjustable parameters $\alpha$, $\beta$ and $\alpha_1$ are depicted. Clearly, our method offers larger parametric stability margins, as compared to that offered by the method in [12]. Finally, in Fig 4, the solutions of equations (24) and (25), (36) and (37), (43) and (44), and (46), (47) are depicted for various values of the adjustable parameters $\gamma$, $\alpha_1$ and $\beta_1$. Clearly, the W-C PID controller tuning method offers the largest parametric stability margins, while, for the same set-point tracking performance (which is obtained for $\gamma$= 1.3333 and $\alpha_1$=1.4 or for $\gamma$=1.0309 and $\alpha_1$=1.3 and with the use of a set-point filter of the form $1/(\theta s+1)$ in the implementation of the PID control configuration, when the C-PS method is applied), the stability margins obtained by our method is larger than those obtained by the C-PS PID controller tuning method and comparable to those obtained by the V-method [10].

Finally, in order to compare the robustness of the proposed methods with the robustness offered, when an unstructured description of the uncertainty is used, we consider the solution of the robust performance problem with unstructured uncertainty [16]

$$\left|S(j\omega)W(j\omega)\right| + \left|T(j\omega)\,\bar{\ell}_m\right| < 1, \;\; \forall\omega \tag{48}$$

where $W(s)$ is the performance weight, $S(s)$ and $T(s)$ are the sensitivity and complementary sensitivity function, respectively, and the multiplicative uncertainty description is given by

$$G_{P,act}(j\omega) = G_{P,nom}(j\omega)\left(1 + \ell_m(j\omega)\right) \; , \; \left|\ell_m(j\omega)\right| \le \bar{\ell}_m(\omega) \tag{49}$$

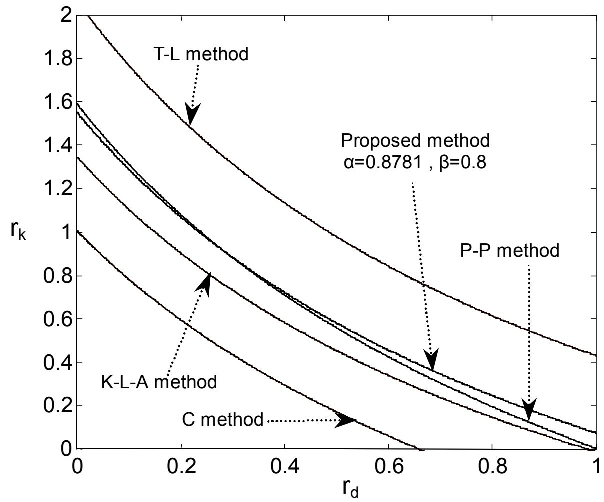


Fig. 2. Parametric stability margins for the proposed PD-0F controller tuning method and the PI control-ler tuning method reported in [2], [5], [6] and [7].

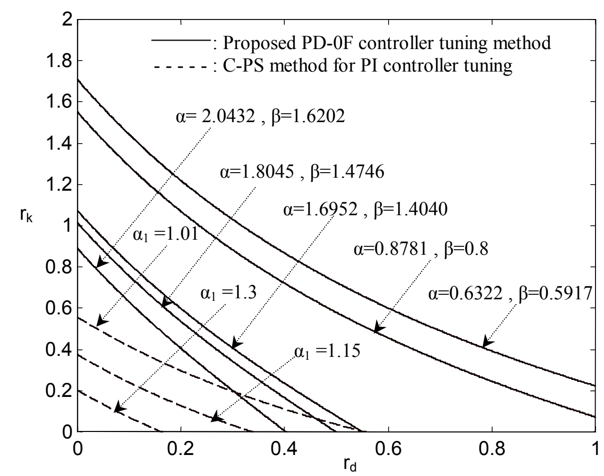


Fig. 3. Parametric stability margins for the proposed PD-0F controller tuning method and the C-PS PI controller tuning method ([12]).

Consider the IPDT process with $K$=1, $d$=1. Let the expected parametric uncertainty in the delay time be 50% and the uncertainty in static gain be 70%. A description for the unstructured uncertainty is given by [17]

$$\bar{\ell}_m(\omega) = \sqrt{r_K^2 + 2(1+r_K)\{1-\cos[(1+r_d)\omega]\}},\ \omega < \frac{\pi}{1+r_d} \quad (50)$$

$$\bar{\ell}_m(\omega) = r_K + 2,\ \omega \geq \pi/(1+r_d) \quad (51)$$

Consider also that a PD-0F controller is designed for the nominal plant according to relations (5a) and (5b). Solving the robust stability problem, with the performance weight having the simplified form [18]

$$W^{-1} = M_P = 3$$

gives no solution (i.e. there are no feasible controller settings, such that (48) to be satisfied). The fact that no feasible con-

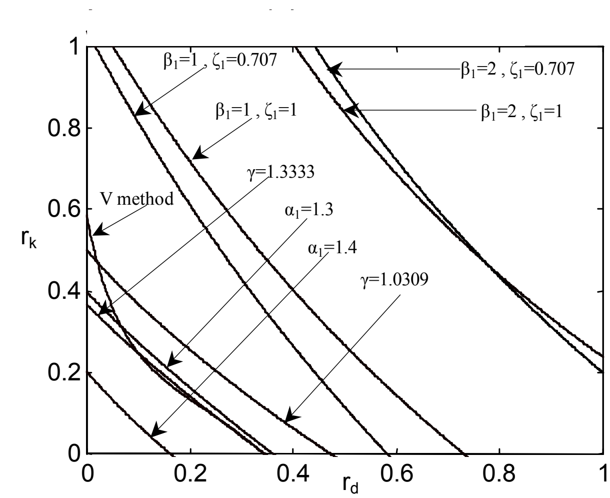


Fig. 4. Parametric stability margins for the proposed PD-1F controller tuning method and the PID controller tuning methods reported in [8], [9], [10], [12].

troller exist as a solution of the robust stability problem is due to the conservative nature of the unstructured uncertainty description. If we apply the proposed method based on the structured uncertainty description (7), we obtain $\beta$=0.6275 for $r_K$=0.7 and $r_d$=0.5, and the PD-0F controller settings solving the robust stability problem are $K_P$= 0.5426 and $K_I$=0.0851. Thus, the proposed method gives a feasible solution to the robust stability problem whereas the method based on the unstructured uncertainty description, fails to solve the problem.

## V. SIMULATION STUDIES

Since a comparison, of the proposed tuning methods for PD-0F and PD-1F controllers has already been performed in Part I of the present paper [1], our aim here is to demonstrate the effectiveness of the proposed methods for robust tuning and to provide a comparison with the $H_\infty$ tuning based on the unstructured uncertainty description. To this end, the IPDT model with nominal parameter values $K$=1 %/min and $d$=1 min is once again considered.

Suppose that our aim is to design a PD-0F controller for this process. The settings of PD-0F controller as obtained from the application of the proposed method are given by relation (20) for $\zeta_{des}$=1 ($\alpha$=0.8781 or $\beta$=0.8) and have the values $K_P$=0.7427, $K_I$= 0.1520. On the other hand for $\alpha$=1.8045 or $\beta$=1.4746 (which corresponds to the minimum of the ISENDCO_SP criterion) the respective settings are given as $K_P$=0.8195, $K_I$= 0.3091.

Consider now the case where a 20% uncertainty in the process gain and a 15% uncertainty in the process delay are to be tolerated. In order to obtain an $H_\infty$-PD-0F controller and to compare its performance with the proposed PD-0F controller tuning method, we next use the performance weight $W^{-1}$= $M_P$=3, and using equations (50) and (51), we solve the robust performance problem (48) for $\beta$=0.157. Its solution is $K_P$= 0.51, $K_I$=0.02.

In Fig. 5, the servo response of the PD-0F controller, which is tuned according to the proposed tuning method, is given, in

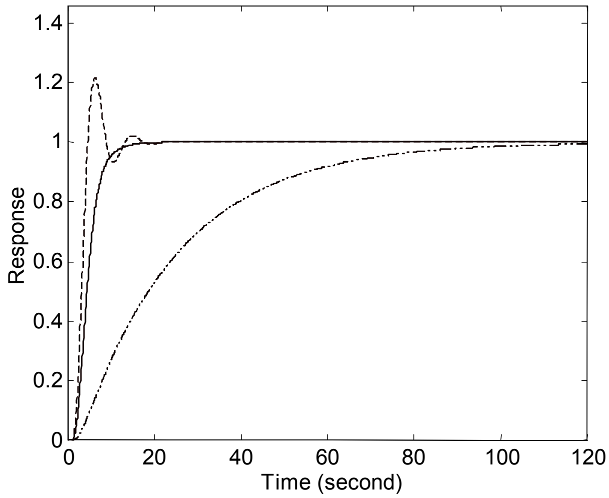


Fig. 5. Servo response for different methods for PD-0F controller tuning, in case of nominal process parameters. Solid: Proposed method for $\alpha$=0.8781 or $\beta$=0.8; dash: Proposed method for $\alpha$=1.8045 or $\beta$=1.4746; dash-dot: $H_\infty$-tuning.

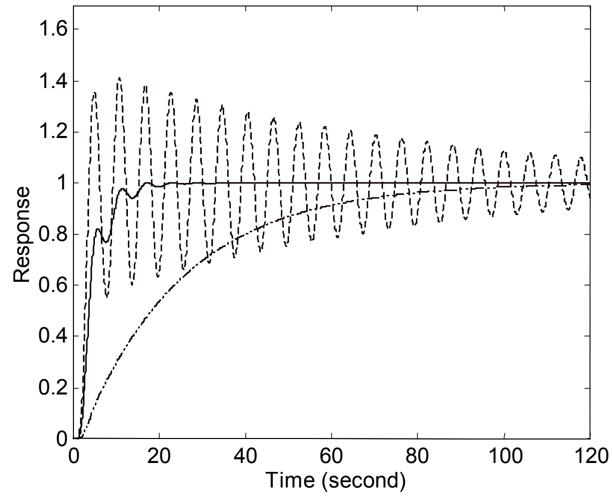


Fig. 7. Servo response for different methods for PD-0F controller tuning, in case of 20% in process gain and 15% uncertainty in process delay. Other legend as in Fig. 5.

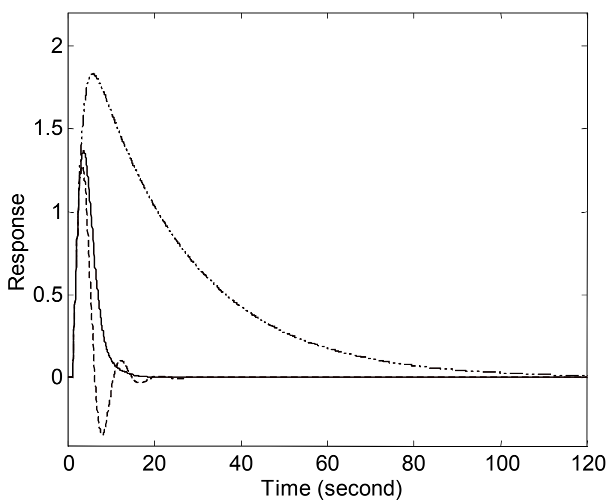


Fig. 6. Regulatory response for different methods for PD-0F controller tuning, in case of nominal process parameters. Other legend as in Fig. 5.

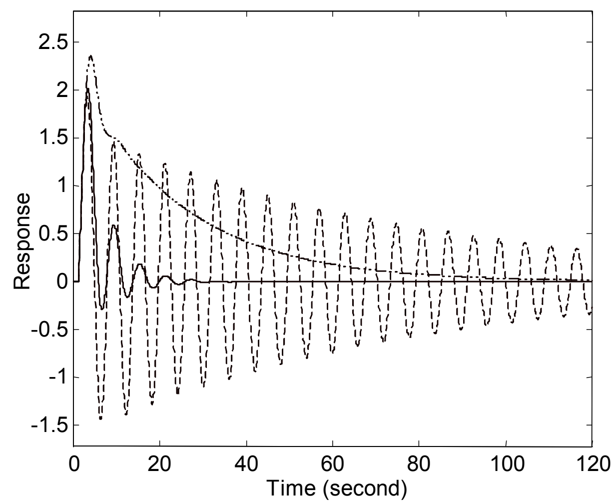


Fig. 8. Regulatory response for different methods for PD-0F controller tuning, in case of 20% in process gain and 15% uncertainty in process delay. Other legend as in Fig. 5.

the case where no uncertainty is present, together with that obtained from the application of the $H_\infty$-PD-0F controller. Fig. 6 illustrates the respective regulatory responses. From this simulation, it is clear that the $H_\infty$-PD-0F controller gives the most conservative tuning. The reader should compare the responses obtained when the $H_\infty$-PD-0F controller is applied, with those obtained in [1], in the case of a PI controller with set-point filter tuned according to the T-L method or the C-PS method with $\alpha_I$=1.1 (see Figs. 3c and 3e in [1]), in order to realize how conservative is the $H_\infty$ tuning based on the unstructured uncertainty description. Figs. 7 and 8 illustrate the servo and the regulatory response, respectively in the case of the assumed simultaneous uncertainty. It is worth noticing that, with $\beta$>0.157, the robust performance problem (48), has no solution. That is for $\beta$=0.157, the system must be conditionally stable for the assumed simultaneous parametric uncertainty. However, as it can be easily checked by performing an analysis similar to that presented in Sections 3 and 4, for $r_d$=0.15, the closed-loop system becomes conditionally stable for $r_K$=1.1735. Thus, the actual stability margin is much larger than the stability margin calculated through the solution of the robust stability problem using the unstructured uncertainty description. This once again certifies, our conclusion about the conservativeness of the unstructured uncertainty description.

## VI. Conclusions

The present paper investigates the robustness properties of PDF controllers, which are designed and tuned for controlling IPDT processes. The reported robustness analysis relies on the structured parametric uncertainty description, which is preferred to apply here, instead of unstructured uncertainty, due to the conservative nature of the later. As it has been shown in the paper, the proposed control and tuning methods provide satisfactory robustness, and offer parametric stability margins larger than those provided by most of the existing PI/PID tuning methods. Taking into account the present analysis and the results reported in the companion paper [1], among existing tuning methods, only the method reported in [8], [9] really provides better results, in terms of closed-loop performance and stability robustness.

Before closing, it is worth noticing that, in the preceding analysis of stability robustness, only variations in the parameters of the model are addressed. However, since a simple model, as the IPDT model considered in this paper, is usually an approximation of a more complex process, variations in the process structure should also be addressed. In the case where the variation in the process structure is not very important, the analysis presented in this paper is sufficient. In the case where, the variation in the process structure is indeed very important, then, the unstructured uncertainty used in the performance criterion given by equation (48), as well as the Internal Model Control approach reported in [16] (see Chapter 6 therein), can be used in order to settle this concern. However, a more thorough analysis is needed, in order to fully address this problem, which is beyond the scope of this paper

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