Inversion of LPV systems

Z. Szabó, J. Bokor,G. Balas

Abstract— **This paper investigates the problem dynamic system inversion for linear parameter varying (LPV) systems, where the system matrix depends affinely from the parameters. The paper presents a method for the construction of the inverse for LPV systems by using parameter varying invariant subspaces and the results of classical geometrical system theory.**

Keywords— **Inverse system, LPV systems.**

I. Problem formulation

The solution of the problem of dynamic inversion of systems gave rise to considerable attention in the control literature in the past years. Silverman *e.g.,* considered the properties and calculation of the inverse of LTI systems in his classical paper [15] guaranteeing neither minimality (or observability, detectability) nor stability properties of the resulting inverse system. The problem was also considered by Fliess [4] for nonlinear input-output systems. For certain classes of nonlinear state space representations Isidori provided algorithms and also sufficient or necessary conditions of invertibility in [6]. Dynamic inversion based controllers are popular in aerospace control, see e.g. [7], [8], [9].

Throughout this paper the problem of dynamic inversion for the class of linear parameter–varying (LPV) systems of which state matrix depends affinely on the parameter vector will be considered. This class of systems can be described as:

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t) + B(\rho)u(t) \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where C is right invertible and

$$A(\rho) = A_0 + \rho_1 A_1 + \cdots + \rho_N A_N, \tag{2}$$
$$B(\rho) = B_0 + \rho_1 B_1 + \cdots + \rho_N B_N, \tag{3}$$

and ρ_i are time varying parameters. It is assumed that each parameter ρ_i and its derivatives $\dot{\rho}_i$ ranges between known extremal values $\rho_i(t) \in [-\overline{\rho}_i, \overline{\rho}_i]$ and $\dot{\rho}_i(t) \in [-\overline{\dot{\rho}}_i, \overline{\dot{\rho}}_i]$, respectively. Let us denote this parameter set by $\mathcal{P}$.

The paper attempts to provide a better understanding of the inversion procedure for LPV systems. In the discussions the concepts of geometrical system theory is used. We derive a procedure based on the concept of invariant subspaces and on the related coordinate transforms that result in an inverse system supposed it is given in state space form and it is left invertible. A numerical example is presented which demonstrates the theoretical results and the procedure on which the inverse calculation is based.

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II. Linear Time Invariant(LTI) systems

Let us recall first the results for LTI systems. An LTI system is invertible if the corresponding input–output map is injective, i.e., whenever u_1 and u_2 are distinct input functions then the corresponding output functions y_1 and y_2 are different. An LTI system is invertible if and only if $\mathcal{R}^* = 0$, where $\mathcal{R}^*$ is the maximal controllability subspace contained in $kerC$, see [13]. If V^* denotes the maximal (A,B)-invariant subspace contained in $kerC$, then the invertibility conditions can be formulated as, [1], [17]:

$$dim\, ImB = m, \quad V^* \cap ImB = 0. \tag{4}$$

Let us observe, that if these conditions are fulfilled, one can always choose a basis of the state space as

$$\{\ \Lambda\ \ ImB\ \ V^*\ \}, \quad \Lambda \subset V^{*\perp},$$

that induces a coordinate transform of the form

$$x = T^{-1}z, \text{ where } \quad T^{-1} = \begin{bmatrix} \Lambda & ImB & V^* \end{bmatrix}.$$

Accordingly, the system will be decomposed to:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1 u \tag{5}$$
$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 \tag{6}$$
$$y = C_1 x_1. \tag{7}$$

It is clear that $ImB = ImB_1$. Follows, that applying the feedback

$$w = F_1 x_1 + F_2 x_2 + u, \tag{8}$$

such that V^* is $(A + BF, B)$ invariant, one can obtain the system:

$$\dot{x}_1 = A_{11}x_1 + B_1 u \tag{9}$$
$$y = C_1 x_1. \tag{10}$$

For the dynamical system (9) the subspace of unknown-input state unobservability by means of differentiators is exactly V_1^*, the maximal (A_{11},B)-invariant subspace contained in $kerC_1$. Moreover, the system (9) can be inverted for u belonging to the complementary subspace of $B^{-1}V_1^*$, see [1]. By the maximality of V^* follows that $V_1^* = 0$, i.e., both x_1 and u can be expressed as functions of y and its derivatives.

Follows, that (6) and (8) gives the inverse system equations, moreover, this realization is minimal.

Denoting by c_i the rows of C_1 let us consider

$$span\{c_1, \cdots, c_1 A_{11}^{\gamma_1}, \cdots, c_p, \cdots, c_p A_{11}^{\gamma_p}\} \tag{11}$$

where $c_i A_{11}^l B_1 = 0$, for $l < \gamma_i$, and γ_i are chosen such that the system to be linearly independent.

By choosing a solution of the equation $A_{12}+BF_2=0$ one can set $F_1=0$. Follows, that choosing the basis (11) for $V^{*\perp}$, one has a particulary simple form of the decomposition (6) and feedback (8) with $F_1=0$, revealing immediately the structure of the minimal inverse system.

III. Invariant subspaces

These ideas can be also extended to the LPV case. To do this we have to introduce first the parameter varying counterparts of the LTI invariant subspaces.

For the parameter varying case one can extend these notions, and introduce the *parameter varying (A,B)-invariant subspaces*, as follows, [16]:

Definition 1: Let $\mathcal{B}(\rho)$ denote Im $B(\rho)$. Then a subspace $\mathcal{V}$ is called a parameter-varying (A,B)-invariant subspace (or shortly $(\mathcal{A},\mathcal{B})$-invariant subspace) if for all $\rho\in\mathcal{P}$ one has

$$A(\rho)\mathcal{V}\subset\mathcal{V}+\mathcal{B}(\rho). \tag{12}$$

As in the classical case one has the following characterization of the parameter varying (A,B)-invariant subspaces:

Proposition 1: $\mathcal{V}$ is a parameter varying (A,B)-invariant subspace if and only if for any $\rho\in\mathcal{P}$ there exists a state feedback matrix $F(\rho)$ such that

$$(A(\rho)+B(\rho)F(\rho))\mathcal{V}\subset\mathcal{V}. \tag{13}$$

The set of all parameter varying (A,B)-invariant subspaces containing a given subspace $\mathcal{C}$, is an upper semilattice with respect to the intersection of subspaces. This semilattice admits a maximum, denoted by

$$\mathcal{V}^*_{p.v.}(\mathcal{C}):=\max\mathcal{V}(A(\rho),B(\rho),\mathcal{C}). \tag{14}$$

In what follows the subscript $p.v.$ will be dropped. This maximum which can be computed from the $(\mathcal{A},\mathcal{B})$-$\mathcal{I}$nvariant $\mathcal{S}$ubspace $\mathcal{A}$lgorithm:

$$\begin{aligned}\mathcal{ABISA}:\quad \mathcal{V}_0 &= \mathcal{K} &(15)\\ \mathcal{V}_{k+1} &= \mathcal{K}\cap\bigcap_{i=0}^{N}A_i^{-1}(\mathcal{V}_k+\mathcal{B}). &(16)\end{aligned}$$

The limit of this algorithm will be denoted by $\mathcal{V}^*$ and its calculation needs at most n steps, for details see [16].

IV. Inversion of LPV systems

Let us consider the class of linear parameter-varying (LPV) systems of m inputs and p outputs that can be described as:

$$\begin{aligned}\dot{x}(t) &= A(\rho(t))x(t)+B(\rho(t))u(t) &(17)\\ y(t) &= Cx(t) &(18)\end{aligned}$$

where

$$\begin{aligned}A(\rho(t)) &= A_0+\rho_1(t)A_1+\ldots+\rho_N(t)A_N, &(19)\\ B(\rho(t)) &= B_0+\rho_1(t)B_1+\ldots+\rho_N(t)B_N, &(20)\\ & &(21)\end{aligned}$$

and the dimension of the state space is supposed to be n.

Let us recall, first, some elementary definitions and facts from [6] and [12] stated for nonlinear input affine system Σ :

$$\begin{aligned}\dot{x} &= f(x)+\sum_{i=1}^{m}g_i(x)u_i &(22)\\ (y_j)_{j=1,p} &= (h_j(x))_{j=1,p}.\end{aligned}$$

A smooth connected submanifold M which contains the point x_0 is said to be *locally controlled invariant* at x_0 if their is a smooth feedback $u(x)$ and a neighborhood U_0 of x_0 such that the vector field $\tilde{f}(x)=f(x)+g(x)u(x)$ is tangent to M for all $x\in M\cap U_0$, i.e., M is locally invariant under $\tilde{f}$.

Let us denote by Z^* the locally maximal output zeroing submanifold. Then the invertability conditions can be stated as:

$$dim\,span\{\,g_i(x_0)\,|\,i=1,m\}=m, \tag{23}$$

and

$$dim\,span\{\,g_i(x)\,|\,i=1,m\}\cap T_xZ^*=0. \tag{24}$$

An algorithm for computing Z^*, the zero dynamics algorithm, for a general case can be found in [6] and [12]. However, in some cases Z^* can be determined relative easily relating it to the maximal controlled invariant distribution Δ^* contained in $kerdh$.

It is not hard to figure out that if some technical conditions for the parameter functions (persistency) are fulfilled, then $T_xZ^*=\mathcal{V}^*$, where $\mathcal{V}^*$ is the maximal $(\mathcal{A},\mathcal{B})$-invariant subspace contained in $kerC$.

Conditions (23) and (24) reduce then to:

$$dim\,ImB=m,\quad \mathcal{V}^*\cap ImB=0.$$

Let us observe, that if these conditions are fulfilled, one can always choose a coordinate transform of the form

$$z=Tx,\text{ where}\quad T^{-1}=\left[\begin{array}{ccc}\Lambda & ImB & \mathcal{V}^*\end{array}\right],\quad \Lambda\subset\mathcal{V}^{*\perp}.$$

Accordingly, the system will be decomposed to:

$$\begin{aligned}\dot{x}_1 &= A_{11}(\rho)x_1+A_{12}(\rho)x_2+B_1u &(25)\\ \dot{x}_2 &= A_{21}(\rho)x_1+A_{22}(\rho)x_2 &(26)\\ y &= C_1x_1. &(27)\end{aligned}$$

Follows, that applying the feedback

$$w=F_1(\rho)x_1+F_2(\rho)x_2+u, \tag{28}$$

such that $\mathcal{V}^*$ is $(\mathcal{A}+\mathcal{B}F,\mathcal{B})$ invariant, one can obtain the system:

$$\begin{aligned}\dot{x}_1 &= A_{11}(\rho)x_1+B_1u &(29)\\ y &= C_1x_1. &(30)\end{aligned}$$

If starting from the rows c_i of C_1, one can choose a linearly independent system such that the dual space of X_1 is spanned by

$$\{c_1,\cdots,S_1^{\gamma_1}(\rho),\cdots,c_p,\cdots,S_p^{\gamma_p}(\rho)\}, \tag{31}$$

where $S_i^l(\rho)B_1 = 0$, for $l < \gamma_i$, and

$$\begin{aligned} S_i^0(\rho) &= c_i, & (32)\\ S_i^{k+1}(\rho) &= \dot{S}_i^k(\rho) + S_i^k(\rho)A_{11}(\rho), & (33)\end{aligned}$$

see [19], [18], then one can define a coordinate transform $\mathcal{S}(\rho)$ that maps x_1 to $\tilde{y}$, where

$$\tilde{y} = \left[y_1, \cdots, y_1^{(\gamma_1)}, \cdots, y_p, \cdots, y_p^{(\gamma_p)}\right]^T. \tag{34}$$

Since one can chose $F_1(\rho) = 0$, the inverse system is given by:

$$\begin{aligned} \dot{\eta} &= A_{22}(\rho)\eta + A_{21}(\rho)S^{-1}(\rho)\tilde{y}, & (35)\\ u &= F_2(\rho)\eta + B_1^{-r}S^{-1}(\rho)(\dot{\tilde{y}} - \\ &- (S(\rho)A_{11}(\rho)S^{-1}(\rho) + \dot{S}(\rho)S^{-1}(\rho))\tilde{y}),\end{aligned}$$

where B_1^{-r} is the right inverse of B_1.

Remark 1: In general the structure of the matrix $S(\rho)$, i.e., the indices γ_i, can change during the time. From a practical point of view this is an unconvenience since one might prefer to work with a fix set of derivatives.

Let us denote by

$$A_{\mathbf{k},11} = \{A_{i_1,11}A_{i_2,11}\cdots A_{i_k,11} \mid i_j \in \{0, 1, \cdots, N\}\}.$$

In order to determine a good matrix $S(t)$, one can compute the sets

$$\{\ c_i, \cdots, c_i A_{\gamma_i^k,11}\ \},$$

where $c_i A_{\mathbf{l},11}B_1 = 0$ for all $l < \gamma_i^k$, and one has to determine the indices $\gamma_i = \min_k \gamma_i^k$.

If the set

$$\{c_1, \cdots, S_1^{\gamma_1}(\rho), \cdots, c_p, \cdots, S_p^{\gamma_p}(\rho)\}$$

span the dual space of X_1, then the matrix $S(\rho)$ will be a good choice in order to ensure that its structure remains unchanged, i.e., one can always use the same set of outputs and derivatives.

Remark 2: It is clear that the method presented above can also be applied for quasi LPV systems.

One can observe that to compute the matrix $S(\rho)$ one needs certain derivatives of the parameter functions $\rho_i(y)$, i.e., certain derivatives of the output y, but the order of these derivatives are bounded by $\max_i \gamma_i$.

V. Example

As an illustrative example for the (q)LPV inversion scheme let us consider the following linearized parameter varying model:

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t) + Lu(t)\\ y(t) &= Cx(t),\end{aligned}$$

where $A(\rho) = A_0 + \rho_1 A_1 + \rho_2 A_2$.

The state matrices are:

$$A_0 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & -1 & 1 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 1\\ 0 & 0\\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

The parameter varying subspace $\mathcal{V}^* = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$. Applying the transformation

$$Ti = \begin{bmatrix} 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 & 1\\ 0 & 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (, i.e.,) \quad \begin{bmatrix} 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

the system splits as

$$\begin{bmatrix} A_{11}^0 & A_{12}^0\\ A_{21}^0 & A_{22}^0 \end{bmatrix} = \left[\begin{array}{cccc|c} -1 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & -1 & 0\\ \hline 0 & 0 & 0 & 0 & -1 \end{array}\right],$$

$$\begin{bmatrix} A_{11}^1 & A_{12}^1\\ A_{21}^1 & A_{22}^1 \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0\\ \hline -1 & 0 & 0 & 0 & 0 \end{array}\right],$$

$$\begin{bmatrix} A_{11}^2 & A_{12}^2\\ A_{21}^2 & A_{22}^2 \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ \hline 0 & 0 & 1 & 0 & 0 \end{array}\right],$$

$$\begin{bmatrix} L_1\\ 0 \end{bmatrix} = \left[\begin{array}{cc} 0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\\ \hline 0 & 0 \end{array}\right], \quad \begin{bmatrix} C_1 & 0 \end{bmatrix} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0 & 0 \end{array}\right].$$

The matrix $F(\rho) = F_0 + \rho_1 F_1 + \rho_2 F_2$, is given by $F_0 = 0$, $F_2 = 0$ and $F_1 = \begin{bmatrix} 0 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

The transformation $S(\rho) = S_0 + \rho_1 S_1 + \rho_2 S_2$, where

$$S_0 = \begin{bmatrix} 1 & 0 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 1 & 1 & 0 & 0 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix},$$

maps x_1 to $\tilde{y} = \begin{bmatrix} y_1 & \dot{y}_1 & y_2 & y_3 \end{bmatrix}^T$.

One can figure out that

$$S^{-1}(\rho) = \begin{bmatrix} 1 & 0 & 0 & 0\\ -1 & 0 & 0 & 1\\ \frac{1}{\rho_1} & \frac{1}{\rho_1} & -\frac{\rho_2}{\rho_1} & 0\\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$\dot{S}(\rho)S^{-1}(\rho) = \begin{bmatrix} 0 & 0 & 0 & 0\\ \frac{\dot{\rho}_1}{\rho_1} & \frac{\dot{\rho}_1}{\rho_1} & -\frac{\dot{\rho}_1\rho_2}{\rho_1} + \dot{\rho}_2 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It follows, that

$$S(\rho)A_{11}(\rho)S^{-1}(\rho) = \begin{bmatrix} -1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{bmatrix} +$$

$$+\rho_1 \begin{bmatrix} \frac{1}{\rho_1} & \frac{1}{\rho_1} & -\frac{\rho_2}{\rho_1} & 0\\ -\frac{2}{\rho_1} & -\frac{1}{\rho_1} & \frac{\rho_2}{\rho_1} & 0\\ 1 & 0 & 0 & 0\\ \frac{1}{\rho_1} & \frac{1}{\rho_1} & -\frac{\rho_2}{\rho_1} & 0 \end{bmatrix} + \rho_2 \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 \end{bmatrix},$$

and

$$L_1^{-r}S^{-1}(\rho) = \begin{bmatrix} \frac{1}{\rho_1} & \frac{1}{\rho_1} & -\frac{\rho_2}{\rho_1} & 0\\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

For the inverse system one has

$$\dot{\eta} = -\eta + (\frac{\rho_2}{\rho_1} - \rho_1)y_1 + \frac{\rho_2}{\rho_1}\dot{y}_1 - \frac{\rho_2^2}{\rho_1}y_2,$$

and

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \eta + \begin{bmatrix} \frac{1}{\rho_1} & \frac{1}{\rho_1} & -\frac{\rho_2}{\rho_1} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} (\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} -$$

$$- \begin{bmatrix} 0 & 1 & 0 & \rho_1 \\ \frac{\dot{\rho}_1 - 2\rho_1}{\rho_1} & \frac{\dot{\rho}_1 - 2\rho_1}{\rho_1} & \frac{\dot{\rho}_2\rho_1 - \dot{\rho}_1\rho_2}{\rho_1} & 0 \\ \rho_1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ y_3 \end{bmatrix}).$$

VI. Conclusion

This paper investigated the problem of input reconstruction by means of dynamic system inversion for linear parameter varying (LPV) systems, where the system matrix depends affinely from the parameters. A procedure for the construction of the inverse, based on the geometric concept of parameter varying invariant subspaces and on the related coordinate transform was given.

Acknowledgments

This research was supported in part by NASA Langley, NASA Grant NCC-1-337 Dr. Christine M. Belcastro technical monitor and in part by the Hungarian National Science Foundation (OTKA) under Grant T 030182.

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