Aliasing Probability Calculations in Testing Sequential Circuits

E. Athanasopoulou and C. N. Hadjicostis

***Abstract*— This paper focuses on testing sequential circuits using a simple form of signature analysis as a compaction technique. More specifically, the paper describes a systematic methodology for calculating the probability of aliasing when a randomly generated test input vector sequence is applied to a given finite state machine (FSM) and the final FSM output is used to verify the functionality of the FSM. We also explore how the aliasing probability is affected when the output mapping (from the set of states to the set of outputs) of the FSM under test changes.**

***Index Terms*— Aliasing probability, response compaction, signature analyzer, finite state machines, Markov models.**

I. INTRODUCTION

Compaction techniques are employed at the testing stage of a circuit to decrease the number of bits in the original circuit response and hence reduce the test application time and the memory requirements on the testing circuitry [1], [2]. Figure 1 shows the basic structure used in test compaction. The circuit under test (CUT) is driven by a known sequence of test input vectors $\mathbf{i}[0], \mathbf{i}[1], ..., \mathbf{i}[L]$. The possibly erroneous output vector sequence of the circuit $\mathbf{y}_f[0], \mathbf{y}_f[1], ..., \mathbf{y}_f[L]$ is fed into a compactor, i.e., a finite state machine (FSM) whose final output is the *signature* of the CUT. Once a particular test vector sequence has been randomly generated, the error-free response $\mathbf{y}[0], \mathbf{y}[1], ..., \mathbf{y}[L]$ of the CUT can be pre-computed and its signature can be compared to the signature obtained by applying the same test vector sequence to the CUT; a disagreement between the error-free and the obtained signature indicates the existence of defects in the CUT.

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The information loss due to the compaction of a circuit response can lead to situations in which the signature of a faulty circuit matches the error-free signature. This unwanted scenario allows a faulty circuit to pass the testing process and is called *aliasing*. Given a randomly generated test vector sequence, the probability of aliasing essentially determines how effective a particular compaction technique is.

For combinational circuits, the probability of aliasing under various compaction methodologies (such as signature analysis, parity checking or transition count) has been calculated (see, for example, [3]–[9] and [1], [2] for an overview). However, the aliasing probability when compaction techniques are used to test *sequential* circuits has not explicitly been computed analytically yet.

In this paper we focus on analyzing a simple compaction method for testing a sequential circuit. More specifically, we use the final output vector $\mathbf{y}_f[L]$ as the signature of the compaction method. In this simplified scenario, *aliasing* occurs when the final output of a faulty CUT agrees with the final output of its fault-free response, i.e., when $\mathbf{y}[L] = \mathbf{y}_f[L]$. Similar concepts appear in [10], [11] which address the error latency of a fault when testing a sequential circuit. More specifically, the error latency model (ELM) depends on the *product state table* of the fault-free FSM and the faulty FSM. The error latency of a fault is defined in [11] as the number of input vectors that need to be applied to the CUT while the fault is active before the first incorrect output vector due to that fault is observed. The product machine (which essentially keeps track of the fault-free state and the faulty state) produces an output of "1" when the first discrepancy between the fault-free and the faulty FSM is observed.

The paper is organized as follows. In Section II we introduce the necessary notation for our development and in Section III we describe our fault model. In Section IV we develop a methodology to calculate analytically the probability of aliasing when an FSM is tested and its final state is used as the signature. In Section V we extend our methodology for calculating the aliasing probability for the more general case when the final output of the FSM under test is used as the signature.

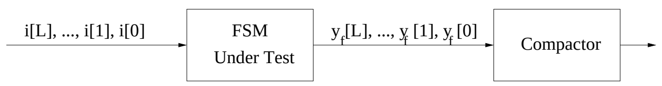

Fig. 1. Testing of a sequential circuit using a compactor.

Finally, conclusions and future directions are discussed in Section VI.

II. NOTATION AND PRELIMINARIES

Let us consider a synchronous FSM described by a set of states $\mathbf{Q} = \{q_i\}_{1 \le i \le N}$, a set of inputs $\mathbf{X} = \{x_k\}_{1 \le k \le M_I}$, and a set of outputs $\mathbf{Y} = \{y_l\}_{1 \le l \le M_O}$. The next state $q[t+1]$ of the FSM is specified by its state $q[t]$ and its input $x[t]$ at time step t via the *next-state function* $q[t+1] = \delta(q[t], x[t])$. To make the connection with the Markov chains more transparent, we will denote the FSM state at time step t by an N-dimensional binary indicator vector $\mathbf{q}[t]$ which has exactly one nonzero entry with value equal to "1." This single nonzero entry denotes the state of the system (i.e., if the i^{th} entry of $\mathbf{q}[t]$ equals "1," then the FSM is in state q_i at time step t). If input x_k is applied at time step t, then the state evolution of the system can be captured by an equation of the form

$$\mathbf{q}[t+1] = \mathbf{A}_k \mathbf{q}[t],$$

where $\mathbf{A}_k$ is the $N \times N$ state transition matrix associated with input $x[t] = x_k$. Specifically, $\mathbf{A}_k$ is such that each of its columns has exactly one nonzero entry with value "1" (i.e., matrix $\mathbf{A}_k$ has a total of N nonzero entries, all with value "1"). A nonzero entry at the (j^{th}, i^{th}) position of $\mathbf{A}_k$ denotes a transition from state q_i to state q_j under input x_k. (Clearly, the constraint that each row of $\mathbf{A}_k$ has exactly one nonzero entry simply reflects the requirement that there can only be one transition out of a particular state under a particular input.)

The output $\mathbf{y}[t]$ of the FSM at a given time step t is generally a function of its present state $q[t]$ and its input $x[t]$, i.e., it is captured by an *output function* $y[t] = \lambda(q[t], x[t])$. Here, we focus on the special case when this output function is restricted to be a mapping of the set of states to the set of outputs, i.e., $y[t] = \lambda(q[t])$. This restricted model describes Moore machines while the more general model describes Mealy machines. Our results can be easily extended to cover Mealy machines as well.

We assume that the input sequence applied to a given FSM is white, i.e., that the inputs are statistically independent from one time step to another and that their probability distribution is fixed so that, at any given time step t, input $x[t] = x_k$ takes place with probability p_k (where $\sum_{k=1}^{M_I} p_k = 1$). Since the FSM makes a transition to the next state depending on both the present input and the present state, the FSM behaves as a homogeneous Markov chain, i.e., a Markov chain in which the transition probabilities are not a function of time [12]. This Markov chain can be obtained from the given FSM by assigning to each transition a probability that depends on the probabilities of the primary inputs that cause it (the only distinction is that the Markov chain has no primary inputs, hence the transition depends probabilistically only on the present state of the chain).

If we denote the state transition probabilities by $a_{ji} = P\{(\mathbf{q}[t+1] = q_j) | (\mathbf{q}[t] = q_i)\}$, the state transition matrix of the Markov chain is given by $\mathbf{A} = (a_{ji})_{1 \le i,j \le N}$ and captures how state probabilities evolve in time via the evolution equation

$$\boldsymbol{\pi}[t+1] = \mathbf{A}\boldsymbol{\pi}[t].$$

Here, $\boldsymbol{\pi}[t]$ is an N-dimensional vector, whose i^{th} entry denotes the probability that the Markov chain is in state q_i at time step t. The N-dimensional probability vector $\boldsymbol{\pi}[t]$ has elements that are nonnegative and sum to 1. Clearly, the state transition matrix $\mathbf{A}$ of the Markov chain can be written as

$$\mathbf{A} = \sum_{k=1}^{M_I} p_k \mathbf{A}_k,$$

where p_k and $\mathbf{A}_k$ are the probability and state transition matrix associated with input x_k.

The *stationary* probability vector of the Markov chain $\mathbf{v} = (v_i)_{1 \le i \le N}$ represents the frequencies with which states are visited in the long run. For a connected FSM, the corresponding Markov chain is irreducible and has a unique stationary distribution vector that satisfies

$$\mathbf{A}\mathbf{v} = \mathbf{v}.$$

This vector can be extracted either from the eigenvalues of the transition matrix $\mathbf{A}$ or from simulation of a particular FSM for typical input sequences.

In our analysis we will need to consider two FSMs that operate in parallel, as well as the Markov chain that describes their behavior. To capture this concisely, we will make use of the Kronecker product notation [13]. The Kronecker product of an $N_1 \times M_1$ matrix $\mathbf{A}$ with an $N_2 \times M_2$ matrix $\mathbf{B}$ is denoted by $\mathbf{A} \otimes \mathbf{B}$ and is defined as the partitioned matrix

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} \alpha_{11}\mathbf{B} & \alpha_{12}\mathbf{B} & \dots & \alpha_{1M_1}\mathbf{B} \\ \alpha_{21}\mathbf{B} & \alpha_{22}\mathbf{B} & \dots & \alpha_{2M_1}\mathbf{B} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N_1 1}\mathbf{B} & \alpha_{N_1 2}\mathbf{B} & \dots & \alpha_{N_1 M_1}\mathbf{B} \end{bmatrix},$$

where α_{ij} is the entry at the i^{th} row, j^{th} column position of matrix $\mathbf{A}$. Note that $\mathbf{A} \otimes \mathbf{B}$ is of dimension $N_1N_2 \times M_1M_2$.

III. FAULT MODEL

In our discussion we consider faults that affect the transition behavior of the sequential circuit under test. More specifically, we consider permanent transition faults that cause the FSM to take, from a given state and under a certain input, a transition to an incorrect state. Such faults can be caused by various factors, such as impurities and defects in materials, equipment malfunctions, or human errors [1].

Let the CUT be an FSM with N states, M_I inputs, and M_O outputs. The sequence of applied test inputs vectors $\mathbf{i}[0], \mathbf{i}[1], ..., \mathbf{i}[L]$ in Figure 1 is generated randomly so that the test input vector at any given time step is chosen independently from other steps. More specifically, we assume that each input x_k is chosen with probability p_k at any given time step.

Suppose that a fault in the hardware implementation of the FSM causes a fault in the state transition mechanism under input x_m. More specifically, while a fault-free FSM would take a transition from state q_i to state q_j under input x_m, this faulty FSM takes a transition from state q_i to state $q_{j'}$. In terms of the transition matrix model, the matrix $\mathbf{A}_m$ that corresponds to input x_m becomes corrupted, i.e., instead of a "1" at the (j^{th}, i^{th}) position, there is a "1" at the ($(j')^{th}, i^{th}$) position. In effect, the state transition matrix $\mathbf{A}'_m$ for the faulty FSM is given by

$$\mathbf{A}'_m = \mathbf{A}_m + \mathbf{E}_m,$$

where $\mathbf{E}_m$ is the fault matrix with two nonzero entries: a "-1" at the (j^{th}, i^{th}) position and a "$+1$" at the ($(j')^{th}, i^{th}$) position.

IV. CALCULATION OF THE ALIASING PROBABILITY

Recall that, in order to keep things simple, we treat the final output of the FSM as its signature. More specifically, we compare the final output of the faulty FSM against the output of the fault-free FSM (under the same input vector sequence). Therefore, by examining the probabilistic relationship between the signature of the faulty FSM under test and the signature of the fault-free FSM, we can compute the probability of aliasing. In this section, we initially study aliasing when the output function λ implies an one-to-one correspondence between the output of the FSM and its internal state (i.e., we study the case when the final FSM state serves as the signature).

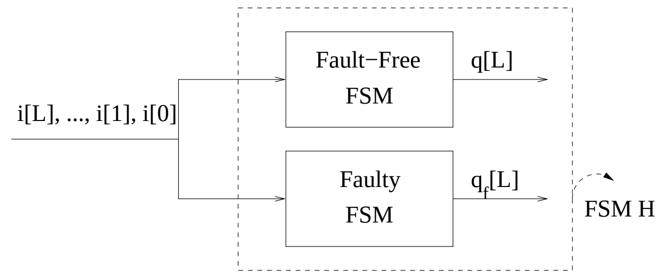

Fig. 2. Simultaneous modeling of the fault-free and the faulty operation of the FSM.

This description is summarized in Figure 2, where we simultaneously apply the same test input vector sequence to the fault-free and the faulty FSMs. The probability of aliasing AP in this case is given by

$$\begin{aligned} AP &= \Pr\{\mathbf{y}[L] = \mathbf{y}_f[L]\} = \Pr\{\mathbf{q}[L] = \mathbf{q}_f[L]\} \\ &= \sum_{i=1}^{N} \Pr\{\mathbf{q}[L] = \mathbf{q}_f[L] = q_i\}. \end{aligned}$$

The dotted system H in Figure 2 is an FSM with M_I inputs and N^2 states and can be described in terms of pairs of the form $(q_i, q_{i'})$, where q_i captures the state of the fault-free FSM and $q_{i'}$ denotes the state of the faulty FSM. The state of the FSM H at time step t can be represented by a binary column vector $\mathbf{q}_h[t]$ with N^2 entries and exactly one nonzero entry with value "1," which denotes the state of the system. More specifically, we arrange the states of H in the order $(q_1, q_1), (q_1, q_2), ..., (q_1, q_N), (q_2, q_1), ..., (q_2, q_N), ..., (q_N, q_1), ..., (q_N, q_N)$. Note that, if FSM H is in state $(q_i, q_{i'})$ at time step t, then the $((i-1)N + i')^{th}$ entry of vector $\mathbf{q}_h[t]$ is equal to "1," while every other entry is equal to "0." Incorporating this notation for the state vector $\mathbf{q}_h[t]$ of FSM H, we notice that it is simply the Kronecker product

$$\mathbf{q}_h[t] = \mathbf{q}[t] \otimes \mathbf{q}_f[t],$$

where $\mathbf{q}[t]$ is the binary indicator vector for the state of the fault-free FSM and $\mathbf{q}_f[t]$ is the vector for the state of the faulty FSM in Figure 2.

As stated previously, the state transition matrix $\mathbf{A}$ of the fault-free FSM and the state transition matrix $\mathbf{A}'$ of the faulty FSM can be written as

$$\mathbf{A} = \sum_{k=1}^{M_I} p_k \mathbf{A}_k, \quad \text{and} \quad \mathbf{A}' = \sum_{k=1}^{M_I} p_k \mathbf{A}'_k.$$

We would also like to express the state transition matrix $\mathbf{A}_h$ of the FSM H in similar manner, i.e.,

$$\mathbf{A}_h = \sum_{k=1}^{M_I} p_k \mathbf{A}_{h_k}.$$

Using a well-known property of the Kronecker product [13], it follows that

$$\mathbf{A}_k\mathbf{q}[t] \otimes \mathbf{A}'_k\mathbf{q}_f[t] = (\mathbf{A}_k \otimes \mathbf{A}'_k)(\mathbf{q}[t] \otimes \mathbf{q}_f[t]).$$

Hence,

$$\begin{aligned}\mathbf{A}_h &= \sum_{k=1}^{M_I} p_k \mathbf{A}_{h_k} = \sum_{k=1}^{M_I} p_k(\mathbf{A}_k \otimes \mathbf{A}'_k) = \\ &= p_m(\mathbf{A}_m \otimes \mathbf{E}_m) + \sum_{k=1}^{M_I} p_k(\mathbf{A}_k \otimes \mathbf{A}_k),\end{aligned}$$

where $\mathbf{E}_m$ is the fault matrix that corresponds to the state transition fault under input x_m.

Let $\mathbf{v}_h$ be a vector that satisfies

$$\mathbf{v}_h = \mathbf{A}_h\mathbf{v}_h.$$

We can immediately distinguish between three cases:

1) If the matrix $\mathbf{A}_h$ has a single eigenvalue at $\lambda = 1$, then the corresponding eigenvector $\mathbf{v}_h$ denotes the unique stationary distribution of the FSM. In this case, the Markov chain corresponding to the FSM H is irreducible and FSM H is connected, i.e., all states are reachable from each other through a finite sequence of inputs.
2) If there are multiple solutions to equation $\mathbf{v}_h = \mathbf{A}_h\mathbf{v}_h$, then the stationary distribution is still well-defined if we know the initial state of the FSM. In this case, the Markov chain is reducible and the FSM H is not connected.
3) If the matrix $\mathbf{A}_h$ has D eigenvalues of unit magnitude given by $\lambda = e^{\frac{j2\pi d}{D}}$, $d \,\epsilon\, \{0, 1, 2, ..., (D-1)\}$, then the eigenvector that corresponds to $\lambda_0 = 1$ denotes the unique stationary distribution of the FSM. The stationary distribution at time t, however, depends on the initial probability distribution of the Markov chain (given by $\mathbf{v}_h[0]$) and the value of $(t \bmod D)$ [12].

Due to space limitations we assume that $\lambda = 1$ is the only eigenvalue of unit magnitude so that $\mathbf{v}_h$ is unique. Our discussion can easily be extended to the more complicated cases listed above.

For a large number of steps L the probability of aliasing can be calculated as the probability that the FSM H ends up in a state of the form $(q_i, q_i), 1 \leq i \leq N$. Hence, the aliasing probability AP is given by the sum of the entries of the stationary vector $\mathbf{v}_h$ that correspond to this type of states. This discussion leads to the following proposition.

Proposition 1: Let S be an FSM under test with N states and assume that the randomly generated test input vector sequence is long enough and that each test input vector is chosen independently between different time steps. If we treat the final *state* of S as its *signature*, then the probability of aliasing AP is given by

$$AP = \sum_{i=1}^{N} \mathbf{v}_h((i-1)N + i),$$

where $\mathbf{v}_h$ is the stationary distribution of H.

V. THE GENERAL CASE AND AN EXAMPLE

In the previous section we calculated the probability of aliasing under the assumption that the outputs of the FSM are in one-to-one correspondence with its states. However, due to the specific structure of FSMs, the same output can be produced by multiple states. Since there is no general way to determine the final state of the FSM by observing its final output, the probability of aliasing in these cases will increase. In terms of Figure 2, the outputs of the fault-free and the faulty FSM denoted by $\mathbf{y}[L]$ and $\mathbf{y}_f[L]$ respectively, are given by

$$\mathbf{y}[L] = \lambda(\mathbf{q}[L]) \quad \text{and} \quad \mathbf{y}_f[L] = \lambda(\mathbf{q}_f[L]).$$

For large L, the probability of aliasing is the probability that FSM H ends up in a state of the form $(q_i, q_{i'})$, $1 \leq i, i' \leq N$, for which $\lambda(q_i) = \lambda(q_{i'})$. The following proposition is a generalization of Proposition 1.

Proposition 2: Let S be an FSM under test with N states and assume that the randomly generated test input vector sequence is long enough and that each test input vector is chosen independently between different time steps. If we treat the final *output* of S as its *signature*, then the probability of aliasing AP is given by

$$AP = \sum_{i=1}^{N} \sum_{(i' \text{ such that } \lambda(q_{i'}) = \lambda(q_i))} \mathbf{v}_h((i-1)N + i'),$$

where $\mathbf{v}_h$ is the stationary distribution of H (which is assumed to be unique).

Intuitively, we can think of the entries of the vector $\mathbf{v}_h$ as the entries of an $N \times N$ table, where the $(i^{th}, (i')^{th})$ entry of such table represents the probability that FSM H ends up in state $(q_i, q_{i'})$. (This is shown in the table below for $N = 4$ states.)

$\mathbf{q}[L]$ / $\mathbf{q}_f[L]$	q_1	q_2	q_3	q_4
q_1	$\mathbf{v}_{11}$	$\mathbf{v}_{12}$	$\mathbf{v}_{13}$	$\mathbf{v}_{14}$
q_2	$\mathbf{v}_{21}$	$\mathbf{v}_{22}$	$\mathbf{v}_{23}$	$\mathbf{v}_{24}$
q_3	$\mathbf{v}_{31}$	$\mathbf{v}_{32}$	$\mathbf{v}_{33}$	$\mathbf{v}_{34}$
q_4	$\mathbf{v}_{41}$	$\mathbf{v}_{42}$	$\mathbf{v}_{43}$	$\mathbf{v}_{44}$

Clearly, the smallest possible probability of aliasing is the sum of the diagonal entries of the table and denotes the probability that both the fault-free and the faulty FSM end up in the same state. This minimum value for the aliasing probability is achieved if the mapping λ is invertible. However, if the mapping results in the same output for the states q_i and $q_{i'}$, then the AP will increase and will be given by the sum of the diagonal entries, plus the entry $\mathbf{v}_h((i-1)N+i')$, plus its symmetric entry $\mathbf{v}_h((i'-1)N+i)$.

Next, we calculate the aliasing probability for a particular FSM S with $N=7$ states and $M_I=2$ inputs that occur with equal probability ($x_1=x_2=0.5$). The state transition matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ corresponding to inputs x_1 and x_2 are chosen to be

$$\mathbf{A}_1 = \begin{bmatrix} 0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1\\ 0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0 \end{bmatrix}.$$

We consider different number of outputs M_O and three different output functions λ_1, λ_2, and λ_3 as follows:

q_i	$\lambda_1(q_i)$
q_1	y_1
q_2	y_2
q_3	y_3
q_4	y_4
q_5	y_5
q_6	y_6
q_7	y_7

q_i	$\lambda_2(q_i)$
q_1	y_1
q_2	y_1
q_3	y_2
q_4	y_2
q_5	y_3
q_6	y_3
q_7	y_4

q_i	$\lambda_3(q_i)$
q_1	y_1
q_2	y_2
q_3	y_1
q_4	y_2
q_5	y_1
q_6	y_2
q_7	y_1

We compute the AP for each of the $N(N-1)=42$ possible single transition faults that can occur under input x_1 and we choose the worst case scenario, i.e., the maximum AP value. We do this for each of the three different mappings λ. The results are shown in the table below.

λ_1	$MaxAP_1=0.1429$
λ_2	$MaxAP_2=0.2857$
λ_3	$MaxAP_3=0.5714$

As the table confirms, if we use the mapping λ_1 to map the set of states to the set outputs, we get the smallest possible aliasing probability. The reason is that there is one-to-one correspondence between the states and the outputs of the FSM. We reach the same conclusion if we use any invertible mapping. If we use the mapping λ_2 some pairs of states are mapped to the same output, hence the aliasing probability increases. Lastly, if we use the mapping λ_3 four states are mapped to output y_1 and three states are mapped to output y_2 and the aliasing probability becomes very large.

VI. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper we have discussed an analytical methodology to calculate the probability of aliasing when testing a sequential circuit. The compaction technique uses the final output of the FSM under test as a signature. Our analysis evaluates how aliasing probability changes for different mappings from the set of states to the set of outputs of the FSM.

Clearly, the next step is to apply more complex compaction techniques to test sequential circuits. For example, the compactor can be an FSM with known initial state that receives as input the output of the FSM under test. The output of the compactor after a large number of states L can then serve as the signature of the system.

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