Robust stabilization of discrete linear repetitive processes with application to a physical example

Wojciech Paszke, Krzysztof Gałkowski, Eric Rogers, David Owens

Abstract— **This paper considers the problem of stabilizing an uncertain discrete linear repetitive process. Such processes are a distinct class of $2D$ linear systems which have many practical applications. The main intrinsic feature which distinguishes them from other classes of $2D$ linear systems is that information propagation in one of the two independent directions only occurs over a finite duration. This, in turn, means that a distinct systems theory must be developed for them. Based on a model of an industrial process (metal rolling), robust stability and stabilization under norm-bounded uncertainty in the model are studied. In particular, stabilizing feedback control laws based on the solutions of linear matrix inequalities are developed and illustrated by a numerical example.**

Keywords— **linear repetitive processes, linear matrix inequalities, robust stabilization, metal rolling**

I. Introduction

Repetitive processes are a distinct class of 2D systems of both system theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

The analysis of linear repetitive processes has received considerable attention in the literature — see, for example, [9], [5], [8], [10]. Although these processes are well known, many control design problems for them are still (relatively) open. This stems from the fact that control of these processes using standard (or $1D$) systems theory/algorithms fails (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass to pass and along a given pass, and the pass initial conditions are reset before the start of each new pass.

Metal rolling is one of a number of physically based problems which can be modeled as a linear repetitive process [10]. In this paper, we use metal rolling as a basis to illustrate numerically algorithms for solutions we develop to currently open robust stability and stabilization problems for the underlying sub-class of so-called discrete linear repetitive processes. Based on the state-space model of such systems, robust stability and stabilization (using an appropriately specified control law) conditions are solved in terms of the feasibility of some linear matrix inequalities (LMIs). These inequalities, in turn, can be solved with well established effective numerical algorithms [2], [7]. Also, it is shown that by incorporating additional LMI constraints a solution with substantially better numerical properties can be obtained.

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Throughout this paper, the null matrix and the identity matrix with appropriate dimensions are denoted by 0 and I, respectively. Moreover, $M > 0$ denotes a matrix M which is real symmetric and positive definite. The following well known results are extensively used throughout the paper.

Lemma 1: Let Σ_1, Σ_2, Σ_3 and Δ be real matrices of appropriate dimensions.
1. [6] For any $\Delta^T\Delta \leq I$ and a scalar $\epsilon > 0$ the following holds

$$\Sigma_1\Delta\Sigma_2 + \Sigma_2^T\Delta\Sigma_1^T \leq \epsilon^{-1}\Sigma_1\Sigma_1^T + \epsilon\Sigma_2^T\Sigma_2 \tag{1}$$

2. (Schur complement) [1] For matrices Σ_1, Σ_2, and Σ_3 where $\Sigma_1 > 0$ and $\Sigma_3 = \Sigma_3^T$ then

$$\Sigma_3 + \Sigma_2^T\Sigma_1^{-1}\Sigma_2 < 0$$

if and only if

$$\begin{bmatrix} \Sigma_3 & \Sigma_2^T \\ \Sigma_2 & -\Sigma_1 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Sigma_1 & \Sigma_2 \\ \Sigma_2^T & \Sigma_3 \end{bmatrix} < 0 \tag{2}$$

II. Metal rolling as a linear repetitive process

Metal rolling is an extremely common industrial process where, in essence, deformation of the workpiece takes place between two rolls with parallel axes revolving in opposite directions. Figure 1 is a schematic diagram of the process where one approach is to pass the stock (i.e. the metal to be rolled to a pre-specified thickness (also termed the gauge or shape)) through a series of rolls for successive reductions which can be costly in terms of the equipment required. A more economic route is to use a single two high stand, where this process is often termed ‘clogging’.

In practice, a number of models of this process can be developed depending on the assumptions made on the underlying dynamics and the particular mode of operation under consideration. Here, however, we will restrict attention to a linearized model of the dynamics of the case shown schematically in Figure 2. In particular, following, for example, [3] the model considered is a so-called discrete

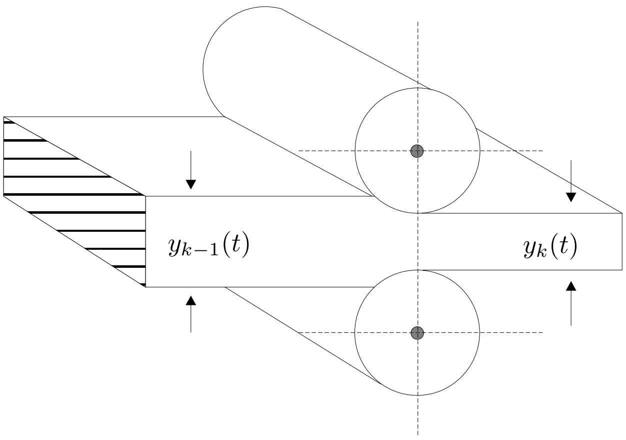

Fig. 1. Metal rolling process

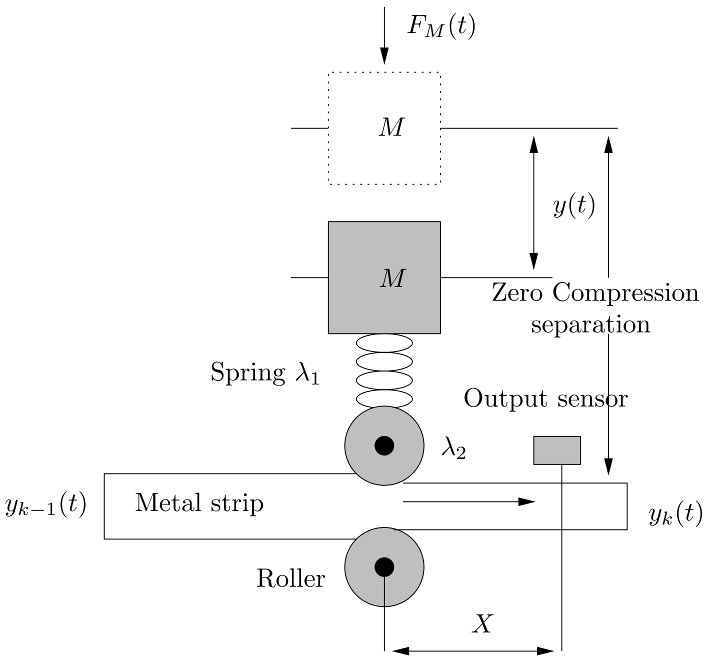

Fig. 2. Metal rolling process

linear repetitive process whose state space model is of the form

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + Bu_k(p) + B_0 y_{k-1}(p) \\ y_k(p) &= Cx_k(p) + Du_k(p) + D_0 y_{k-1}(p), \end{aligned} \tag{3}$$

Here on pass k, $x_k(p) \in \mathbb{R}^n$ the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector, $u_k(p) \in \mathbb{R}^l$ is the vector of control inputs. To detail the structure for our metal rolling example, first introduce

$$u_k(p) = F_M(p),$$

$$x_k(p) = \begin{bmatrix} y_k(p-1) & y_k(p-2) & y_{k-1}(p-1) & y_{k-1}(p-2) \end{bmatrix}^T,$$

$$A = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ B = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ B_0 = \begin{bmatrix} a_3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \tag{4}$$
$$C = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \end{bmatrix}, \ D = b, \ D_0 = a_3.$$

where

$$a_1 = \frac{2M}{\lambda T^2 + M}, \ a_2 = \frac{-M}{\lambda^2 T + M}, \ a_3 = \frac{\lambda}{\lambda T^2 + M}\left(T^2 + \frac{M}{\lambda_1}\right),$$

$$a_4 = \frac{-2\lambda M}{\lambda_1(\lambda T^2 + M)}, \ a_5 = \frac{\lambda M}{\lambda_1(\lambda T^2 + M)}, \ b = \frac{-\lambda T^2}{\lambda_2(\lambda T^2 + M)}.$$

The systems variables in above expressions are: $y_{k-1}(t)$ and $y_k(t)$, which denote thickness of the metal on the current and previous pass respectively, M is the lumped mass of the roll-gap adjusting mechanism, λ_1 is the stiffness of the adjustment mechanism spring, λ_2 is the hardness of the metal strip, $\lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ is the composite stiffness of the metal strip and the roll mechanism. Finally, $F_M(t)$ is the force developed by the motor and T is sampling period.

In the design studies given here, the data used is $\lambda_1 = 600$, $\lambda_2 = 2000$, $M = 100$ and $T = 0.1$. This yields $\lambda = 461.54$ and the following matrices in (4)

$$A = \begin{bmatrix} 1.9118 & -0.0047 & -1.4706 & 0.7353 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0.7794 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$
$$B = \begin{bmatrix} -2.2059 \cdot 10^{-5} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ D = 2.2059 \cdot 10^{-5}, \ D_0 = 0.7794,$$
$$C = \begin{bmatrix} 1.9118 & -0.0047 & -1.4706 & 0.7353 \end{bmatrix}. \tag{5}$$

To complete the process description it is necessary to specify the pass length and the initial, or boundary, conditions, i.e. the pass state initial vector sequence and the initial pass profile. Here these are taken to be of the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \ k \geq 0 \\ y_0(p) &= y(p), \ 0 \leq p \leq \alpha, \end{aligned} \tag{6}$$

where d_{k+1} is an $n \times 1$ vector with constant entries and $y(p)$ is an $m \times 1$ vector whose entries are known functions of p.

III. Stability analysis

For processes described by (3) with boundary conditions (6), several equivalent sets of necessary and sufficient conditions for stability along the pass [10] have been reported but the following set is required.

Theorem 1: [10] Discrete linear repetitive processes described by (3) and (6) are stable along the pass if, and only if, the $2D$ characteristic polynomial

$$C(z_1, z_2) := \det \begin{bmatrix} I_n - z_1 A & -z_1 B_0 \\ -z_2 C & I_m - z_2 D_0 \end{bmatrix}, \tag{7}$$

satisfies

$$C(z_1, z_2) \neq 0, \ \forall \ (z_1, z_2) \in \overline{U}^2, \tag{8}$$

where

$$\overline{U}^2 = \{(z_1, z_2) : |z_1| \leq 1, |z_2| \leq 1\}. \tag{9}$$

Note that (8) gives the necessary conditions that $r(D_0) < 1$ (asymptotic stability) and $r(A) < 1$ which should be verified before proceeding further with any stability analysis.

Next, define the following matrices from the state space model (3)

$$A_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix}. \tag{10}$$

Then we have the following sufficient condition for stability along the pass of a process described by (3) and (6) in terms of an LMI.

Theorem 2: [4] Discrete linear repetitive processes described by (3) and (6) are stable along the pass if $\exists$ matrices $P > 0$ and $Q > 0$ satisfying the following LMI

$$\begin{bmatrix} A_1^T P A_1 + Q - P & A_1^T P A_2 \\ A_2^T P A_1 & A_2^T P A_2 - Q \end{bmatrix} < 0. \tag{11}$$

As shown in the remainder of this paper, this theorem provides an efficient basis for controller design in the case when there is uncertainty in the process description (in a sense to be defined next).

IV. Model with norm-bounded uncertainty

In all practically relevant cases there will be uncertainty associated with the (linear) model used for analysis/controller design. Hence robust control is also a key aspect for these processes and is a subject for which relatively few results are currently available. In this paper we derive and illustrate on the metal rolling data controller design algorithms for the following model structure where the basic assumption is that the uncertainty can be modeled as an additive perturbation to the nominal model dynamics

$$\begin{aligned} x_k(p+1) =&(A + \Delta A)x_k(p) + (B + \Delta B)u_k(p) \\ &+ (B_0 + \Delta B_0)y_{k-1}(p) \\ y_k(p) =&(C + \Delta C)x_k(p) + (D + \Delta D)u_k(p) \\ &+ (D_0 + \Delta D_0)y_{k-1}(p). \end{aligned} \tag{12}$$

The matrices ΔA, ΔB, ΔB_0, ΔC, ΔD, ΔD_0 represent the uncertainty in the modeling. These matrices are unknown except for the facts that they have constant entries and are norm-bounded, i.e. each matrix ΔM from the set $\{\Delta A$, ΔB, ΔB_0, ΔC, ΔD, $\Delta D_0\}$ satisfies

$$\Delta M = HFE \tag{13}$$

where H and E are some known constant matrices with compatible dimensions and F is an unknown, constant matrix which satisfies

$$F^T F \leq I. \tag{14}$$

To further simplify notation, introduce the so-called augmented process and input matrices respectively for the nominal model as

$$S = \left[\begin{array}{c|c} A & B_0 \\ \hline C & D_0 \end{array}\right], \quad R = \left[\begin{array}{c} B \\ \hline D \end{array}\right]. \tag{15}$$

Then, noting the assumed uncertainty structure (13) and (14) the following uncertainty matrices can be defined

$$\Delta S = \left[\begin{array}{c|c} \Delta A & \Delta B_0 \\ \hline \Delta C & \Delta D_0 \end{array}\right], \quad \Delta R = \left[\begin{array}{c} \Delta B \\ \hline \Delta D \end{array}\right]. \tag{16}$$

Also the uncertainty can now be modeled in the form

$$\begin{aligned} \Delta A_1 &= \left[\begin{array}{c|c} \Delta A & \Delta B_0 \\ \hline 0 & 0 \end{array}\right] = \widehat{H}_1 F \widehat{E} \\ &= \left[\begin{array}{c} \widetilde{H}_1 \\ \hline 0 \end{array}\right] F \left[\begin{array}{c|c} E_1 & E_2 \end{array}\right], \end{aligned} \tag{17}$$

and

$$\begin{aligned} \Delta A_2 &= \left[\begin{array}{c|c} 0 & 0 \\ \hline \Delta C & \Delta D_0 \end{array}\right] = \widehat{H}_2 F \widehat{E}, \\ &= \left[\begin{array}{c} 0 \\ \hline H_1 \end{array}\right] F \left[\begin{array}{c|c} E_1 & E_2 \end{array}\right] \end{aligned} \tag{18}$$

where

$$\widetilde{H}_1 = \left[\begin{array}{c} H_1 \\ \hline 0 \end{array}\right]$$

$$\Delta B_1 = \left[\begin{array}{c} \Delta B \\ \hline 0 \end{array}\right] = \widehat{H}_1 F E_3 \tag{19}$$

$$\Delta B_2 = \left[\begin{array}{c} 0 \\ \hline \Delta D \end{array}\right] = \widehat{H}_2 F E_3. \tag{20}$$

Note the that the non-zero parts of $\widehat{H}_1$ and $\widehat{H}_2$ both only consist of the row vector H_1.

V. Stability and stabilization of the uncertain model

Applying the LMI sufficient condition for stability along the pass given by (11), gives the following result for any process where the uncertainty associated with the model to be used for analysis is modeled by the structure defined in the previous section.

Theorem 3: Discrete linear repetitive processes described by (3) and (6) whose defining matrices have the uncertainty structure of (13) and (14) are stable along the pass if $\exists$ $P > 0$, $Q > 0$ and a positive scalar ϵ such that

$$\begin{bmatrix} -P & PA_1 & PA_2 & H_1 \\ A_1^T & Q - P + \epsilon E_1^T E_1 & \epsilon E_1^T E_2 & 0 \\ A_2^T & -Q + \epsilon E_2^T E_1 & \epsilon E_2^T E_2 & 0 \\ H_1^T & 0 & 0 & -\epsilon I \end{bmatrix} < 0. \tag{21}$$

Proof: The LMI condition of Theorem 11 can be rewritten for the uncertain process of (12) as

$$\begin{bmatrix} \widehat{A}_1^T P \widehat{A}_1 + Q - P & \widehat{A}_1^T P \widehat{A}_2 \\ \widehat{A}_2^T P \widehat{A}_1 & \widehat{A}_2^T P \widehat{A}_2 - Q \end{bmatrix} < 0 \tag{22}$$

where

$$\begin{aligned} \widehat{A}_1 =& A_1 + \Delta A_1 \\ \widehat{A}_2 =& A_2 + \Delta A_2. \end{aligned} \tag{23}$$

Applying the Schur complement (2) formula to the LMI of (22) now yields

$$\begin{bmatrix} -P & P\widehat{A}_1 & P\widehat{A}_2 \\ \widehat{A}_1^T P & Q-P & 0 \\ \widehat{A}_2^T P & 0 & -Q \end{bmatrix} < 0. \tag{24}$$

Now apply to (24) the first result of Lemma 1, and note (17) and (18) to obtain

$$\begin{bmatrix} -P+\epsilon^{-1}HH^T & PA_1 & PA_2 \\ A_1^T & Q-P+\epsilon E_1^T E_1 & \epsilon E_1^T E_2 \\ A_2^T & -Q+\epsilon E_2^T E_1 & \epsilon E_2^T E_2 \end{bmatrix} \tag{25}$$

Finally, by applying the Schur complement (2) to this last expression we obtain the LMI condition (21). □

In this paper, a stabilizing control law over $0 \le p \le \alpha$, $k \ge 0$ of the form

$$u_k(p) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_k(p) \\ y_{k-1}(p) \end{bmatrix} = K \begin{bmatrix} x_k(p) \\ y_{k-1}(p) \end{bmatrix} \tag{26}$$

is sought, where K_1 and K_2 are appropriately dimensioned matrices to be designed. In effect, this control law is composed of the weighted sum of current pass state feedback and feedforward of the previous pass profile (see [4] for further background on this form of control action).

The existence of stabilizing K_1 and K_2 can be characterized in LMI terms as follows.

Theorem 4: Discrete linear repetitive processes described by (3) whose defining matrices have the uncertainty structure defined by (13) and (14) are stable along the pass under control law (26) if $\exists$ a scalar $\epsilon > 0$ and matrices $Y > 0$, $Z > 0$, and N such that the following LMI holds

$$\begin{bmatrix} -W+\epsilon\widehat{H}_1\widehat{H}_1^T+\epsilon\widehat{H}_2\widehat{H}_2^T & A_1W+B_1N & A_2W+B_2N & 0 & 0 \\ WA_1^T+N^TB_1^T & Z-W & 0 & W\widehat{E}^T+N^TE_3^T & 0 \\ WA_2^T+N^TB_2^T & 0 & -Z & 0 & W\widehat{E}^T+N^TE_3^T \\ 0 & \widehat{E}W+E_3N & 0 & -\epsilon I & 0 \\ 0 & 0 & \widehat{E}W+E_3N & 0 & -\epsilon I \end{bmatrix} < 0 \tag{27}$$

In this case, the matrix K is given by

$$K = NW^{-1}. \tag{28}$$

Proof: First, consider the stabilization problem in the absence of uncertainty in the model. Then on applying (26), the corresponding closed-loop system is

$$\begin{aligned} x_k(p+1) &= (A+BK_1)x_k(p) + (B_0+BK_2)y_{k-1}(p) \\ y_k(p) &= (C+DK_1)x_k(p) + (D_0+DK_2)y_{k-1}(p). \end{aligned} \tag{29}$$

Hence on using (11) it is clear that the closed loop process is stable along the pass if $\exists$ symmetric matrices $P > 0$ and $Q > 0$ satisfying the following matrix inequality

$$\begin{bmatrix} \widetilde{A}_1^T P\widetilde{A}_1 + Q - P & \widetilde{A}_1^T P\widetilde{A}_2 \\ \widetilde{A}_2^T P\widetilde{A}_1 & \widetilde{A}_2^T P\widetilde{A}_2 - Q \end{bmatrix} < 0. \tag{30}$$

where

$$\begin{aligned} \widetilde{A}_1 &= A_1 + B_1K, \\ \widetilde{A}_2 &= A_2 + B_2K. \end{aligned} \tag{31}$$

Note that the matrix inequality (30) is not an LMI because it is nonlinear with respect to its parameters. Consequently apply the Schur complement (2) to yield

$$\begin{bmatrix} -P & P(A_1+B_1K) & P(A_2+B_2K) \\ (A_1+B_1K)^TP & Q-P & 0 \\ (A_2+B_2K)^TP & 0 & -Q \end{bmatrix} < 0. \tag{32}$$

Now set $P = W^{-1}$ and pre- and post-multiply (32) by $\operatorname{diag}(W,W,W)$, to obtain

$$\begin{bmatrix} -W & (A_1+B_1K)W & (A_2+B_2K)W \\ W(A_1+B_1K)^T & -W+Z & 0 \\ W(A_2+B_2K)^T & 0 & -Z \end{bmatrix} < 0, \tag{33}$$

where $Z = WQW$.

Writing this last inequality with the norm-bounded uncertainties included in the model gives

$$\begin{bmatrix} -W & (A_1+B_1K)W & (A_2+B_2K)W \\ W(A_1+B_1K)^T & -W+Z & 0 \\ W(A_2+B_2K)^T & 0 & -Z \end{bmatrix} + \begin{bmatrix} 0 & (\Delta A_1+\Delta B_1K)W & (\Delta A_2+\Delta B_2K)W \\ W(\Delta A_1+\Delta B_1K)^T & 0 & 0 \\ W(\Delta A_2+\Delta B_2K)^T & 0 & 0 \end{bmatrix}. \tag{34}$$

and the second term in this last inequality can be written in the form

$$\begin{bmatrix} 0 & \widehat{H}_1 & \widehat{H}_2 \end{bmatrix} \begin{bmatrix} F & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & F \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \widehat{E}W+E_3N & 0 \\ 0 & 0 & \widehat{E}W+E_3N \end{bmatrix}$$

Making use of Lemma 1 we can now write

$$\begin{aligned} &\begin{bmatrix} 0 & (\Delta A_1+\Delta B_1K)W & (\Delta A_2+\Delta B_2K)W \\ W(\Delta A_1+\Delta B_1K)^T & 0 & 0 \\ W(\Delta A_2+\Delta B_2K)^T & 0 & 0 \end{bmatrix} \\ &\le \operatorname{diag}\Big(\epsilon\widehat{H}_1\widehat{H}_1^T + \epsilon\widehat{H}_2\widehat{H}_2^T,\ \epsilon^{-1}(E_1W+E_3N)^T \\ &\qquad \times(E_1W+E_3N), \epsilon^{-1}(E_2W+E_3N)^T(E_2W+E_3N)\Big). \end{aligned}$$

The result then follows by an obvious (and hence note detailed here) Schur complement (2) and congruence transform. □

A. Minimization of the condition number

A great advantage of an LMI approach is a possibility to include many different specifications in the design problem. In particular, it enables us to find the solution with many design constraints —- a key aspect which is often a non-trivial task to achieve with classical design methods.

As an example to illustrate the above observation, note that to compute the controller matrices K_1 and K_2 it is necessary to compute the inverse of the matrix W — a task where numerical problems could well arise if this matrix is badly scaled or almost singular. One option in this case is to obtain the the solution with the smallest condition number.

The problem of minimizing the condition number of the matrix W can be formulated as a Generalized Eigenvalues Problem (GEVP) [1] of the following form

$$\begin{aligned} &\text{minimize } \gamma \\ &\text{subject to } F(x) > 0, \;\; \mu > 0, \;\; \mu I < W(x) < \gamma\mu I \end{aligned} \tag{35}$$

which can be solved by, for example, the LMI solver (for example Matlab LMI Control Toolbox [2]). However, the computational complexity of this approach is 'quite high' and it turns out that the so-called EVP [1] problem is more efficient. This proceeds by re-formulating this last problem as follows.

First, rewrite the LMI constraint $F(x) > 0$ and the positive definite matrix $W(x)$ from (35) as

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i, \;\; W(x) = W_0 + \sum_{i=1}^{m} x_i W_i$$

Next, define the new variables $\nu = 1/\mu$ and $\widetilde{x} = x/\mu$, which yields the following EVP problem:

$$\begin{aligned} &\text{minimize } \gamma \\ &\text{subject to } \begin{cases} \nu F_0 + \sum_{i=1}^{m} \widetilde{x}_i F_i > 0, \\ I < \nu W_0 + \sum_{i=1}^{m} \widetilde{x}_i W_i < \gamma I \end{cases} \end{aligned} \tag{36}$$

In the reminder of this paper we apply the results obtained to the metal rolling example.

VI. Numerical Example

Consider the metal rolling process whose nominal model for controller design/evaluation is defined by (12) and (5). The matrices, which describe the uncertainty model (17) and (18) are taken to be

$$H_1 = \begin{bmatrix} 0.0170 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; H_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.0170 \end{bmatrix}, \; E_2 = 0.0480,$$

$$E_1 = \begin{bmatrix} 0.1786 & 0.0714 & 0.0979 & 0.1919 \end{bmatrix}, \;\; E_3 = 7.7584 \cdot 10^{-5}$$

Application of the controller design procedure of Theorem 4 (27), one solution of the LMI for this example is

$$W = \begin{bmatrix} 5.6684 & -2.1322 & 3.7439 & -8.6996 & 0.3329 \\ -2.1322 & 23.1669 & -0.6607 & 5.0831 & 0.2097 \\ 3.7439 & -0.6607 & 7.5433 & 0.9278 & 1.4752 \\ -8.6996 & 5.0831 & 0.9278 & 24.9010 & 0.6378 \\ 0.3329 & 0.2097 & 1.4752 & 0.6378 & 2.3759 \end{bmatrix}$$

$$Z = \begin{bmatrix} 2.3563 & -0.7834 & 1.4817 & -3.7157 & -0.2357 \\ -0.7834 & 11.9855 & -0.3760 & 1.7207 & 0.2058 \\ 1.4817 & -0.3760 & 3.1808 & 0.6209 & -0.0788 \\ -3.7157 & 1.7207 & 0.6209 & 11.0339 & 0.3483 \\ -0.2357 & 0.2058 & -0.0788 & 0.3483 & 0.8386 \end{bmatrix}$$

$$N = 1.0 \cdot 10^4 \begin{bmatrix} 0.6298 & 1.3710 & -2.6457 & 2.7844 & -0.2523 \end{bmatrix},$$
$$\epsilon = 51.1677.$$

and the corresponding controller matrix is

$$K = 1.0 \cdot 10^4 \begin{bmatrix} 7.8062 & -0.0254 & -4.7855 & 2.9978 & 0.9687 \end{bmatrix}. \tag{37}$$

and the condition number of W is

$$\text{cond}(W) = 118.7091. \tag{38}$$

Using the condition number minimization procedure of (35) now yields

$$W = 1.0 \cdot 10^{-14} \times \begin{bmatrix} 0.3036 & 0.0003 & 0.0039 & -0.0006 & -0.0006 \\ 0.0003 & 0.3463 & 0.0011 & -0.0004 & 0.0001 \\ 0.0039 & 0.0011 & 0.3225 & -0.0002 & 0.0011 \\ -0.0006 & -0.0004 & -0.0002 & 0.3494 & -0.0002 \\ -0.0006 & 0.0001 & 0.0011 & -0.0002 & 0.2991 \end{bmatrix}$$

$$Z = 1.0 \cdot 10^{-14} \times \begin{bmatrix} 0.0222 & 0.0000 & -0.0001 & -0.0012 & -0.0004 \\ 0.0000 & 0.2308 & 0.0001 & -0.0003 & -0.0000 \\ -0.0001 & 0.0001 & 0.0148 & 0.0008 & 0.0001 \\ -0.0012 & -0.0003 & 0.0008 & 0.2325 & -0.0004 \\ -0.0004 & -0.0000 & 0.0001 & -0.0004 & 0.0141 \end{bmatrix}$$

$$N = 1.0 \cdot 10^{-9} \begin{bmatrix} 0.2596 & -0.0013 & -0.2109 & 0.1159 & 0.1031 \end{bmatrix},$$
$$\epsilon = 7.6410 \cdot 10^{-12}.$$

and the controller matrix now is

$$K = 1.0 \cdot 10^4 \begin{bmatrix} 8.6516 & -0.0213 & -6.6528 & 3.3300 & 3.4919 \end{bmatrix}$$

and the condition number of W is now

$$\text{cond}(W) = 1.1693, \tag{39}$$

which much less than in the previous design.

In the case of EVP procedure (36), the following result is obtained

$$W = \begin{bmatrix} 1.0090 & -0.0007 & 0.0119 & 0.0008 & -0.0004 \\ -0.0007 & 1.0484 & 0.0078 & -0.0014 & -0.0000 \\ 0.0119 & 0.0078 & 1.0210 & -0.0045 & 0.0023 \\ 0.0008 & -0.0014 & -0.0045 & 1.0490 & 0.0001 \\ -0.0004 & -0.0000 & 0.0023 & 0.0001 & 1.0040 \end{bmatrix}$$

$$Z = \begin{bmatrix} 0.0206 & -0.0001 & -0.0048 & -0.0019 & 0.0032 \\ -0.0001 & 0.6989 & 0.0002 & -0.0009 & -0.0000 \\ -0.0048 & 0.0002 & 0.0154 & 0.0014 & -0.0029 \\ -0.0019 & -0.0009 & 0.0014 & 0.6985 & -0.0008 \\ 0.0032 & -0.0000 & -0.0029 & -0.0008 & 0.0101 \end{bmatrix}$$

$$N = 1.0 \cdot 10^4 \begin{bmatrix} 8.6695 & -0.0848 & -6.7136 & 3.5351 & 3.4979 \end{bmatrix},$$
$$\epsilon = 3.3934 \cdot 10^3.$$

with corresponding controller matrix

$$K = 1.0 \cdot 10^4 \begin{bmatrix} 8.6699 & -0.0213 & -6.6696 & 3.3341 & 3.5025 \end{bmatrix}$$

and the condition number of W is now

$$\mathrm{cond}(W) = 1.0524, \tag{40}$$

This is comparable to that obtained with the GEVP based algorithm. Detailed simulation studies (omitted here for brevity) have confirmed that better all round performance is obtained with the design where the minimization of the condition number of the matrix W is included.

In the reminder, the simulation study is presented which confirms that the obtained controller works properly for all models encountered by assumed uncertainty set. Below for the controller of (37), the resulted closed loop model matrices for two choices of the matrix F ($F = 0.97$, $F = 0.1$) together with simulations of a pass profile variable $y_k(p)$ are presented. Assume that there is no control action, $u_k(p) = 0$ and the boundary condition $y_0(p) = 1$ and $x_p(0) = 0$, $p = 0, 1, \dots, \alpha - 1$. It is seen that in both cases, the process has been stabilised.

$$A = \begin{bmatrix} 0.2926 & 0.0018 & -0.4746 & 0.1115 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0.5789 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$
$$D_0 = 0.5789,\ C = \begin{bmatrix} 0.2926 & 0.0018 & -0.4746 & 0.1115 \end{bmatrix}.$$

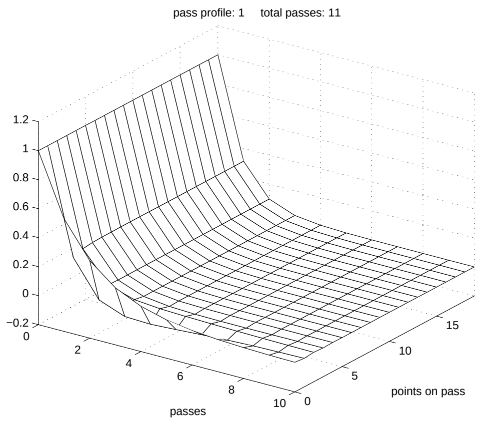

Fig. 3. Pass profile of the model ($F = 0.97$)

$$A = \begin{bmatrix} 0.2004 & 0.0010 & -0.4211 & 0.0783 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0.5671 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$
$$D_0 = 0.5671,\ C = \begin{bmatrix} 0.2004 & 0.0010 & -0.4211 & 0.0783 \end{bmatrix}.$$

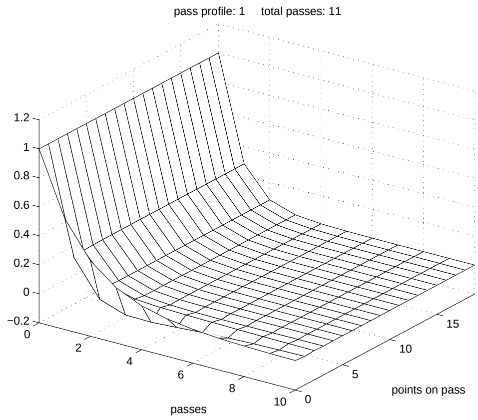

Fig. 4. Pass profile of the model ($F = 0.1$)

VII. Conclusions

In this paper, we have developed new results on the robust control of discrete linear repetitive processes and illustrated them by application to the model of a metal rolling process (one of the first physical examples to be recognized as a repetitive process). By enhancing the design computations through minimization of the condition number of a key matrix, it has also been shown that better numerical properties (relative to the basic case) can be achieved. On going work is aimed at robust controller design with performance specifications and this will be reported on in due course.

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